# A 5D Chern-Simons theory on warped space-time and detecting its topological anomalous couplings at the LHC

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based on arXiv:1003.5725

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# Outline

# Motivation

- Anomalous physics are observed. Recent data from PAMELA, ATIC, and PPB-BETS, have motivated considering the possibility that there is a light gauge boson that couples to the SM charged particles with a much suppressed coupling.
- "An anomalous positron abundance in cosmic rays with energies 1.5.100 GeV"(PAMELA, NATUA,458,607), "An Excess Of Cosmic Ray Electrons At Energies Of 300.800 Gev(ATIC, NATUA,456,362)", "High-energy electron observations by PPB-BETS flight in Antarctica(PPB-BETS, arXiv:0809.4472)","Measurement of the Cosmic Ray e+ plus e- spectrum from 20 GeV to 1 TeV with the Fermi Large Area Telescope"(The Fermi LAT, arXiv:0905.0025)
- The coupling of a GeV scale vector boson (U boson) to the SM charged particles with a natural size of strength $\leq 10^{-3} - 10^{-4}$, assuming originates from loops of heavy particles and comparing to that of the photon, have been probed in various low energy experiments (such as Babar, Belle, BEPC, HyperCP, CLEO and KLOE) "Searching for the light dark gauge boson in GeV-scale experiments"(Matt Reece, 0904.1743). Assuming an effective interaction with tensor structure $\sim \epsilon^{\mu\nu\rho\sigma}$ couplings.

# Motivation

- Last Wednesday, Jure Zupan(CERN), "Global interpretation of direct dark matter searches ", on DAMA DATA, Leptophilic DM signal, the mediating DM does not have to be light, spin dependent operators, with two Majarana fermions, a Dimension 6 operator,

$$\mathcal{L}_{int} = \frac{C_T}{\Lambda^2}[\bar{\psi}\sigma_{\mu\nu}\psi][\bar{q}\sigma^{\mu\nu}q]$$

  Not clear how to obtain it from a realistic theory.
- One possibility: A heavy neutral gauge boson that couples to the SM neutral gauge bosons with an effective loop level suppressed couplings to provide more high energy electrons and photons than the SM.
- Topological quantum field theory(TQFT) with a tensor structure $\sim \epsilon^{\mu\nu\rho\sigma}$ might be one of possible candidates to explain these phenomenology.

# History

- Fundamental laws are expressed in TQFT, which are independent of geometry. The candidate has been formed, Chern Simons theory in dealing chiral gauge theory(B.Zumino etc. 1984).
- In the early time before the popularity of gauge-gravity duality, it has been realized that a 2+1 dimensional gravity is exactly solvable at classical and quantum level with vanishing beta function, thus (2+1)D general relativity is a renormalizable quantum gravity. This is because the (2+1)D Einstein Hilbert action is closely related to (2+1)D quantum Yang-Mills theory with a purely Chern-Simons 3-form term, which turns out to be an exactly solvable quantum filed theory and is intimately connected with Jones polynomial of knot theory in 3 D(E.Witten 1988). Thus in (2+1)D, gravity is related to non-Abelian gauge theory with a pure CS term.

$$S_{CS3} = c \int dx^3\, Tr[AdA + \frac{2}{3}A^3]$$

# Topological invariance

- The physical meaning of topological invariance is general covariance(GC) for a physicist. Any quantity computed in a GC quantum field theory(QFT) without of a choice of metric will be a topological invariant. Conversely, a QFT in which all the observables are topological invariants can naturally be seen as a generally covariant QFT.
- Fundamental law of physics, the Gauge and Gravity theory is formed in terms of the geometric curvature of space and time. General relativity gives a prototype for construct a QFT without a priori choice of metric- one introduce a metric and then integrate over all metrics. Thus one tend to think of a GC theory, by definition, is a theory in which the metric is a dynamical variable. There are highly non-trivial QFT in which the GC is achieved not by integrating over metrics but because one begin with a gauge invariant Lagrangian that are metric independent.

# 5D Chern-Simons theory

$$S_{CS5} = c \int dx^5 Tr[AdAdA + \frac{3}{2}A^3 dA + \frac{3}{5}A^5]$$

where we have used differential forms, the Yang-Mills gauge fields $A_\mu^a$ is a matrix of 1-forms, and the field strength is a matrix of 2-forms

$$A = -iA_\mu^a T^a dx^\mu \quad F = dA + A^2 = -i\frac{1}{2}F_{\mu\nu}^a T^a dx^\mu \wedge dx^\nu$$

- Anomaly across the 5th dimension: CS5 term can be introduced to cancel the chiral anomaly of fermions localized on the branes, which can be seen clearly by making summation over all heavy KK fermions from $1 \to \infty$. The gauge variant CS5 term, namely CS5 anomaly cancel the anomaly from fermions localizing on the branes.
- Topological anomalous couplings between vector gauge bosons - A quantum weak-isospin incompressible quantum liquid in 4-dimension.

# Gauge invariance and Chirality

- Gauge invariance are well tested at tree level, no vertex like $Z_0 Z_0 \gamma$ appear in the pure Yang-Mills
- When one are dealing with 4D chiral gauge theory, anomaly from fermion measure through Chiral transformation in the Path integral appear, and the 4D chiral anomaly just comes from the fermion zero modes $\phi_n \gamma^5 \phi_n, n \in$ zero modes.
- The anomaly break Gauge invariance. (Ward-Takahashi Identity are broken at loop level)Vector and axial vector currents cannot be conserved at the same time.
- When dealing with lattice gauge theory, double fermion zero modes appear, one of them is un-physical in continuous limit. Chiral gauge theory in 5D is special, proper chiral boundary conditions needed to be imposed to eliminate one of the fermion zero mode, in order to be able to obtain a realistic 5D chiral gauge theory at loop level.

# Gauge invariance and Chirality

- In lower energy effective chiral gauge theory(non-linear $\sigma$ model), one has only meson field (Wilson link $U = e^{i\pi^a \vec{T}^a / f}$) satisfying gauge invariance and Lorentz invariance. Traditional approach to gauge a global theory does not work by simply replacing $\partial_\mu$ with $D_\mu$. Gauge variant but conserved Norther currents appear, which is physical and explains the $\pi^0 \to \gamma\gamma$. Meanwhile a hidden gauge variant term with a tensor structure $\sim \epsilon^{\mu\nu\rho\sigma}$ is discovered, namely Wess-Zumino Witten term(E.Witten, 1983).
- In the last ten decade, deconstruction version of chiral gauge theory, find that for every two nearest site one introduce a WZW term, under gauge transformation, the variant term is pushed towards endpoints of moose chain. In the continuous limit, the summation of all WZW term becomes CS5 term in 5D, and gauge variant term, namely CS anomaly appear on the fixed points, the UV and IR branes.

# Dimensional Deconstruction

- At high energy scale, i.e., $\Lambda_{\rm Pl}$, gauge groups are localized in theory space - a 4D renormalizable and asymptotically free thus Ultraviolet(UV) Complete theory.
- At some lower energy scale ($\Lambda_s \sim 4\pi f_s \sim \frac{1}{a}$), some gauge groups become strongly coupled $g_s \gg 1$, Dirac mass fermion condensate to generate vacuum expectation values(Vevs). A Strongly coupled gauge theory with a 4D UV cutoff scale $\frac{1}{a} = \frac{1}{gf_s}$ - A non-renormalizable non-linear $\sigma$ model with n-sites - latticed gauge theory along 5-th dimension with Wilson link variables.
- At the cutoff scale $\frac{1}{a}$, the Kaluza-Klein(KK) modes are truncated, in the continuous limit $N \to \infty, a \to 0$ - dynamically generate a 5D gauge theory with continuous space dimension, with gauge couplings $g_5^2 \sim ag^2 = g^2 R/N = g_4^2 R$.
- In the large N limit, a weakly coupled 4D gauge theory with coupling $g_4 \equiv g/N \ll 1$, perturbative calculable, KK mode is finite. At some more lower energy scale $\frac{1}{R} = \frac{1}{Na}$, 1st KK gauge bosons appear.

# Dimensional Deconstruction

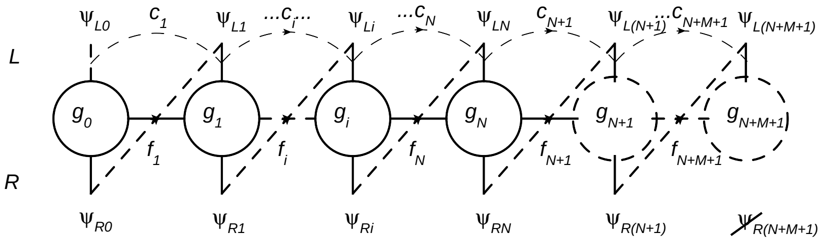



Figure: A general 4D linear moose Model :$SU(2)^{N+1} \times U(1)^{M+1}$. The circle label $N+1$ $SU(2)$ and the following $M+1$ $U(1)$ gauge groups, The Wilson link field $U_i$ belong to $(n, \bar{n})$ representation under $SU(2)_{i-1} \times SU(2)_i$. The dotted line between two different sites represents Yukawa couplings for chiral fermions, the solid line on the same site represents Dirac fermion mass term on that site.The 0-th site and $N+M+1$-th site corresponding to UV and IR brane in the continuous 5D theory. The absence of $\psi_{R(N+M+1)}$ represent the Diriclet boundary boundary for the right handed fermions on the IR branes, which leads to a left-handed fermion zero mode. The dashed curved line across two sites nearby is the Wess-Zumino Witten term added to make the chiral gauge theory gauge invariant when fermion anomaly appear at one loop level. $c_i, i = 0, \ldots N+M+1$ are coefficients in front of the WZW terms. The summation of the full WZW terms localized on each sites give a global topological invariant 5D Chern-Simons term(CS5) in the continuous limit. The gauge variant CS5 anomaly on the endpoint exactly cancel the gauge variant consistent fermions anomaly on the boundary.

# 5D Chern-Simons theory on arbitrary space-time

$$\mathcal{S}_{CS5} = \int dx^4 dz \tfrac{c}{4} \epsilon^{MNPQR} \mathrm{Tr}[A_M F_{NP} F_{QR} + i A_M A_N A_P F_{QR} \\ -\tfrac{2}{5} A_M A_N A_P A_Q A_R]$$

$$\epsilon^{M_1 \ldots M_d} = \sqrt{|g|} \epsilon^{\mu_1 \ldots \mu_d} e_{\mu_1}^{M_1} \ldots e_{\mu_d}^{M_d}$$

where $e_\mu^M$ is the 5D vierbein, satisfying $e_\mu^M \eta^{\mu\nu} e_\nu^N = g^{MN}$
i.e. for AdS metric

$$g_{MN} = (\frac{R}{z})^2 \begin{pmatrix} \eta^{\mu\nu} & \\ & -1 \end{pmatrix} \quad e_M^\mu = (\frac{R}{z}) \delta_M^\mu$$

$$g_{MN} = \begin{pmatrix} e^{-2ky} \eta^{\mu\nu} & \\ & -1 \end{pmatrix} \quad e_M^\mu = e^{-ky} \delta_M^\mu \quad e_5^\mu = \delta_5^\mu$$

# Topological invariance of 5D Chern-Simons theory on warped space-time

- We show explicitly the CS5 is independent of $AdS_5$ metric, the warp version is the same as the flat (space) version, thus it is metric independenct.
- We also demonstrate explicitly the CS5 is independent of $AdS_5$ ordinary coordinate $y$ and also conformal coordinate $z$, thus is coordinate independent.
- The CS5 is topological invariant and it looks the same in different manifold of space-time, just like Maxwell equation keep the same formalism in different reference framework. On the other hand, because CS5 has the same formalism on warped space-time as that on flat space-time, it is natural for us to choose a model on warped space-time without lose generality.

# The Randall-Sundrum Model

The AdS metric in conformal coordinate

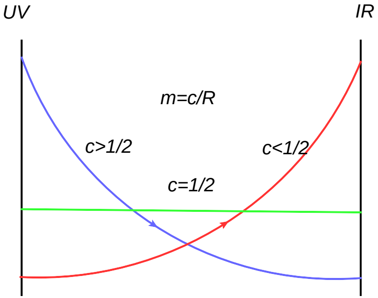
$$ds^2 = (\frac{R}{z})^2(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

- Higgs VEV on IR brane is warped down from UV scale($\sim M_{\rm Pl}$) to IR electroweak scale($\sim 246\ \rm GeV$) $\Longrightarrow$ Warping Solves big Hierarchy Problem
- Gauge fields & fermions put in bulk, avoid low IR cutoff scale.
- Extended bulk gauge symmetry to protect custodial symmetry of SM($\rho \sim 1$).
- TeV Scale KK modes detectable at LHC.
- Flavor sector, fermion localization $\Longrightarrow$ explain SM quark mass and mixing hierarchies. u,d,s $\sim 10^{-3} - 10^{-2}\ \rm GeV$, c,b $\sim 10^0$ $\rm GeV$, t $\sim 10^2\ \ \rm GeV$. CKM matrix $\lambda \sim 0.2$.
- Perturbative approachable up to $\Lambda_{pert} \sim 10\ \rm TeV$, unitarity ($\Lambda_u$)is almost saturate before theory strongly coupled.

# Chiral fermions in 5D

Fermion Zero modes of warped model:

Bulk mass determines exponential localization of zero modes, can generate hierarchies.




- fermion wave function overlaps exponentially on the IR via Dirac mass term. Choose different bulk mass for different fermions, effective 4D Yukawa couplings are hierarchical.

# Anomaly of 5D Fermions

KK fermions wave functions: i.e., with bulk fermion mass $c = 0$(special case) under proper boundary conditions:

$$g_k^L(z) = \sqrt{\tfrac{2}{R'-R}}(\tfrac{z}{R})^2 \cos[m_k(z-R)] \quad m_k = \tfrac{n\pi}{R'-R}, n = 0, 1 \ldots \infty$$

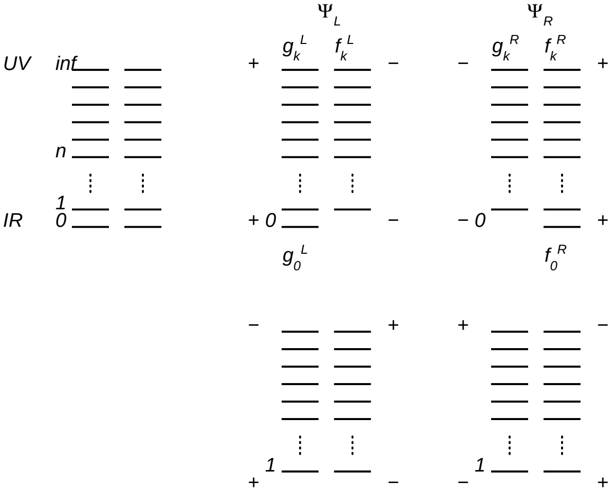
$$f_k^L(z) = \sqrt{\tfrac{2}{R'-R}}(\tfrac{z}{R})^2 \sin[m_k(z-R)] \quad m_k = \tfrac{n\pi}{R'-R}, n = 1 \ldots \infty$$

$$g_k^L(z) = \sqrt{\tfrac{2}{R'-R}}(\tfrac{z}{R})^2 \cos[m_k(R'-z)] \quad m_k = \tfrac{(n+\frac{1}{2})\pi}{R'-R}, n = 0, 1 \ldots \infty$$

$$f_k^L(z) = \sqrt{\tfrac{2}{R'-R}}(\tfrac{z}{R})^2 \sin[m_k(R'-z)] \quad m_k = \tfrac{(n+\frac{1}{2})\pi}{R'-R}, n = 0, 1 \ldots \infty$$

# Anomaly of 5D Fermions

## KK fermion Mass spectrum



# Anomaly of 5D Fermions

Sum over all the KK fermions from 0 to $\infty$: anomaly of 5D fermions $\eta^{MN}\partial_M J_N$. Norther current $J_N \equiv \sqrt{g}\bar{\Psi}\Gamma_N\Psi$, isolate the 4D chiral anomaly from the chiral zero mode, the massive KK fermion contributes the anomaly

$$\frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}(\mp)\frac{1}{2}[\delta(z-R^{'})-\delta(z-R)]$$

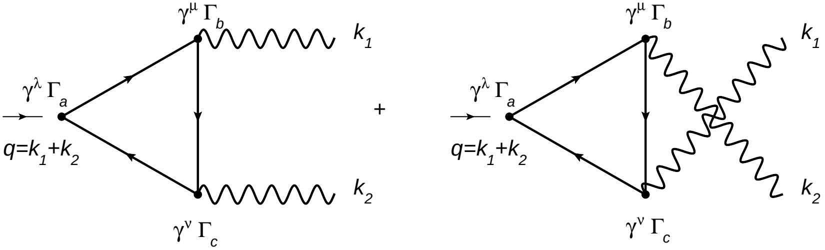



Figure: Triangle diagrams contributing to the chiral anomaly in chiral gauge theory

# Anomaly cancellation

The chiral anomaly of fermion localized on the branes can be canceled by the gauge variant CS anomaly of CS5 term.

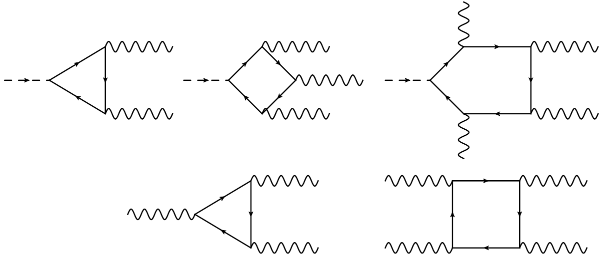


Figure: (a)The loop diagrams generate anomaly from fermions living in the boundary branes. The diagrams without pions are diagrams in the Unitary gauge. The square and pentagon diagrams also contributing to the chiral anomaly in the non-Abelian theories. (b)There are new effective vertexes from Chern-Simons terms after SSB(on mass eigenstate). They represent the new interactions between different KK modes. Note: In the above diagrams, we have neglected the crossing diagrams due to identical gauge bosons symmetry.

# CS5 interactions

Generally speaking, CS5 interactions depends on how matter are embed in the model and also how gauge symmetry is broken. Therefore, the detecting of CS5 interactions can be an approach to explore the UV completion.

- One one hand, the CS5 term coefficients will depend on the quantum numbers of fermion representation, for Abelian axial current, the anomaly is proportional to the Dynkin index while for non-Abelian case, it is proportional to the anomaly coefficients(gauge anomaly). Unsafe semi-simple Lie algebra with non-zero anomaly coefficients: $SU(N), N \geq 3$, $SO(N), N = 2, 3, 4, 6$ and $Sp(2)$.
- On the other hand, the strength of topological anomalous couplings are determined by the the BC's imposed and how electroweak symmetry is broken.

# A minimal model of Higgsless RS

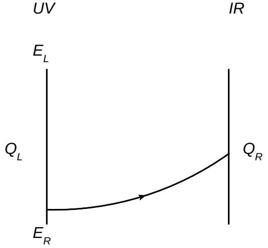
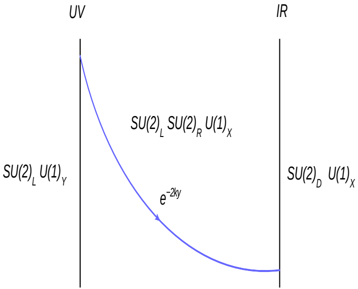
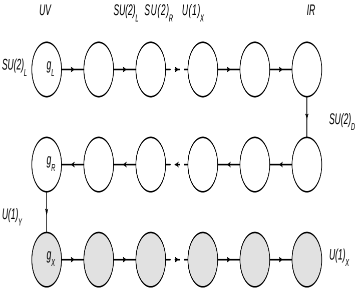



Figure: 3rd generation bulk fermions in the representations: $Q_L \in (\mathbf{3},\mathbf{2},\mathbf{1})_{1/6}$ $\quad Q_R \in (\mathbf{3},\mathbf{1},\mathbf{2})_{1/6}$ $\quad E_L \in (\mathbf{1},\mathbf{2},\mathbf{1})_{-1/2}$ $\quad E_R \in (\mathbf{1},\mathbf{1},\mathbf{2})_{-1/2}$

- A minimal model: we do not embed 3rd generation quarks into a representation that inherit a new custodial symmetry to protect the $Zb_L\bar{b}_L$ couplings.
- Fermion zero mode contributes chiral anomaly, easy to check $SU(3)^2U(1)_X$, $SU(2)^2_{L,R}U(1)_X$, $U(1)_X$ and $U(1)^3_X$ can not be exactly canceled unless CS5 term appear, the whole 5D theory is anomaly free.

# Deconstruct RS LR Higgsless model



# A minimal linear moose model

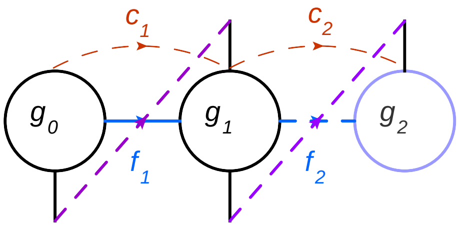



Figure: 3-Site linear moose Higgsless model :$SU(2)_1 \times SU(2)_2 \times U(1)$, $f_1, f_2$ are pions decay constants corresponding to the relevant Wilson line.$c_1, c_2$ are Chern-Simons coefficients cross over the bulk. The CS term in the bulk just cancel the chiral anomaly from the triangle diagrams of the fermions on the brane, so that the whole theory is anomaly free. The Chern-Simons terms together with brane localized terms yield the Wess-Zumino-Witten term and a residual bulk interaction amongst KK-modes, namely the anomalous couplings of KK modes.

# Decay width through topological interactions of CS5

$$\Gamma(A^n \to Z^0 Z^0) = \frac{\kappa_{n11}^2}{24\pi} \frac{m_n^3}{m_Z^2} \sqrt{1-4x}\,(1-4x)^2\,,$$

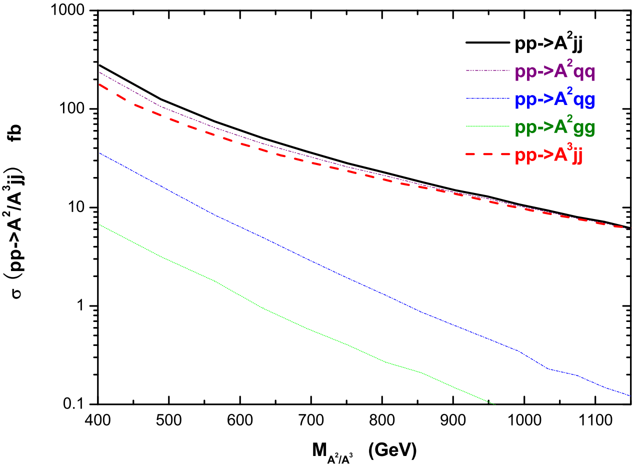
$$\begin{aligned}\Gamma(A^n \to Z^0 \gamma) = \frac{1}{24\pi} \frac{m_n^3}{m_Z^2} (1-x) \Big[\kappa_2^2 + (\kappa_1^2 - 10\kappa_1\kappa_2 - \kappa_2^2)x \\ + (10\kappa_1^2 + 8\kappa_1\kappa_2 - \kappa_2^2)x^2 + (\kappa_1+\kappa_2)^2 x^3\Big]\,.\end{aligned}$$

where $x = m_Z^2/m_n^2 \ll 1$,

- Estimate the decay width, i.e., RS LR Higgsless model with $R^{-1} = 10^8\,\mathrm{GeV}$, $R^{'-1} = 283\,\mathrm{GeV}$, $\Gamma(Z^{'} \to Z_0\gamma) \approx 0.06$ $\mathrm{GeV}$, $\Gamma(Z^{'} \to Z_0 Z_0) \approx 0.14$ $\mathrm{GeV}$.
- Comparing pion decay: $\Gamma_{exp}(\pi^0 \to 2\gamma) = (7.8 \pm 0.5) \times 10^{-6}$ $\mathrm{MeV}$, $B(\pi^0 \to \gamma\gamma) = 99\%$
- $\Gamma_{theory} = \frac{\alpha^2 N_c^2}{64\pi^3}\frac{m_\pi^3}{f_\pi^2} \approx 7.6 \times 10^{-6}$ $\mathrm{MeV}$, if $N_c = 3$, $m_\pi = 135$ $\mathrm{MeV}$, $f_\pi = 93$ $\mathrm{MeV}$, and $\alpha = 1/137$.

# Signatures at the LHC

The Production Crosssection of neutral gauge bosons at the LHC:
$Z' \to Z_0 Z_0$ and $Z' \to Z_0 \gamma$



# Summary

- CS5 is metric independent and coordinate independent, thus topological invariant.
- Based on a realistic RS LR Higgsless model, satisfying EWPT, we study the topological interactions of CS5 term, which predicts new anomalous KK couplings beyond the SM.
- We study the CS5 phenomenology, including decay width of heavy neutral gauge bosons through topological anomalous couplings and their collider signature at the LHC.
- The approach to detect these topological anomalous couplings is model independent, thus can be applied to all models that predict extra heavy neutral gauge bosons.
- We are looking forward that the near future experiments can reveal this mystery of the nature at a higher energy scale.

*Thank you!*