Cosmic Strings and Their Possible Detection in the Near Future

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Abstract

Highly energetic topological defects such as cosmic strings are becoming increasingly more important in the fields of high-energy physics and cosmology, especially considering recent developments in superstring theory. In this paper, I will discuss the formation, dynamics, and intrinsic properties of cosmic strings, whose networks may be one of the sources of large scale structure in our universe. Following this will be a discussion on the detectability of cosmic strings, including descriptions of current and future detectors on and in orbit around Earth, and unsolved mysteries concerning the detectable characteristics of cosmic strings.

1 Spontaneous Symmetry Breaking and the Origin of Cosmic Strings

Cosmic strings are a result of spontaneous symmetry breaking models, such as the Higgs mechanism, developed in the late 1960's and early 1970's. Grand unification of the fundamental forces necessitates spontaneous symmetry breaking, so the physicists of this era developed a plausible mechanism of symmetry breaking that is widely accepted today. The concept is simple: a particular symmetry is spontaneously broken when a field Lagrangian which exhibits that symmetry yields a solution which does not have the same symmetry. I can illustrate this with a very simple example. Consider the Lagrangian

$$\mathcal{L} = (\partial_\mu \bar{\phi})(\partial^\mu \phi) - V(\phi) \qquad (Eq.1)$$

with potential

$$V(\phi) = \frac{1}{4}\lambda(\bar{\phi}\phi - \eta^2)^2 \qquad (Eq.2)$$

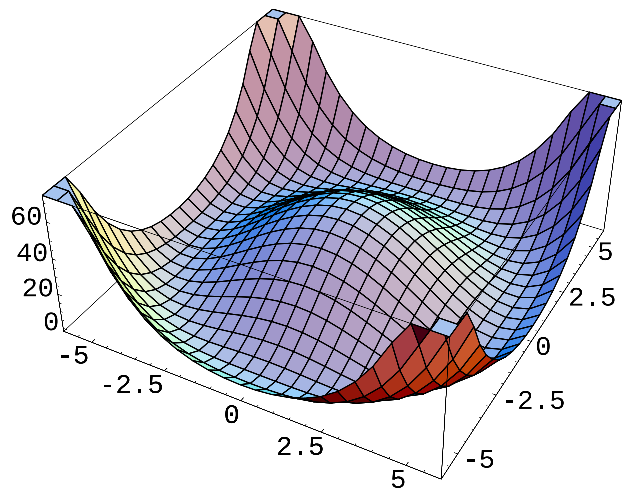

Figure 1: Graph of the potential in (2) with $\lambda = \frac{1}{4}$ and $\eta = 5$. The axes of this graph are the real and imaginary parts of the field ϕ, and the z-axis is $V(\phi)$. This potential's *vacuum manifold* is $|\phi| = 5$, and there is a local maximum at the origin of the complex plane: $V = \frac{25^2}{16}$

which describes the complex field ϕ. [The potential in (Eq. 2) is shown in Figure 1.] It is evident that this Lagrangian possesses global symmetry (invariance to the transformation $\phi \rightarrow \phi e^{i\alpha}$), although its ground state solutions (in polar coordinates, $\phi = \eta e^{i\theta}$) don't; therefore, this system is a global symmetry breaking mechanism[5]. The system described by Lagrangian (Eq. 1) is too simplistic to be physically realistic; nevertheless, it yields string solutions which would be didactically beneficial to analyze.

Cosmic strings are *topological defects* of codimension 1. They occur in spontaneous symmetry breaking models due to the continuity of fields condition. Symmetry breaking usually begins by a bubble nucleation process – the field at random points in space falls from the unstable local maximum point of the potential to random points on the *vacuum manifold*, the set of points which compose the minimum of the potential[4]. Due to continuity, the fields in the neighborhoods of the points where the field first falls must have similar values, forming "bubbles" of the new phase (see Figure 2). As these bubbles of different phases grow larger, they begin to intersect, but the field is forced to remain continuous. Topological defects occur when the field is unable to take a value on the vacuum manifold in order to remain continuous. The field must take a value on the local maximum of the potential. These places are highly energetic topological defects. In a universe where the field has two coordinates, as they do in realistic spontaneous symmetry breaking models

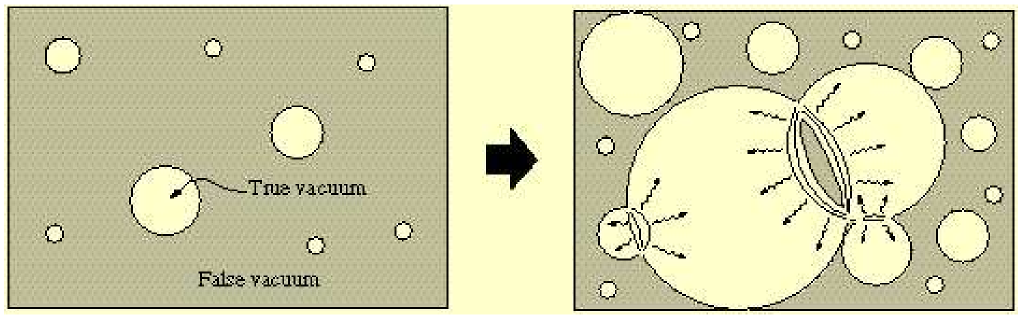

Figure 2: Depicts a symmetry breaking phase transition in the early universe. Bubbles of “new phase” form as the field falls to a particular point on the vacuum manifold. It is highly unlikely that the field will “pick” the same value in any two places. Due to the requirement that fields must be continuous, the space inside the bubbles will have similar values for the field. When bubbles of new phase meet, the field values must adjust in order to be continuous, but sometimes this is only possible if the field takes a value on the local maximum of the potential, $\phi = 0$. Graphic from [3]

like the Higgs mechanism, the defects are linear – *cosmic strings*[4][5][3].

The system described by Lagrangian (Eq. 1) is physically unrealistic, merely a simple example which demonstrates symmetry breaking and cosmic string formation clearly; but the abelian-Higgs model of electroweak symmetry breaking, which is very similiar in form to the global symmetry breaking model, is physically realistic, and has string solutions. The existence of cosmic strings in our universe as we know it is unavoidable[1]. Due to the fact that strings have very high energy (since they are stable fields on an unstable local maximum of the potential) they bend spacetime considerably and have the ability to influence the structure of the universe on large scales. They have not yet been observed, but if they could be, they would be a strong confirmation of symmetry breaking processes and the unification of the forces of the standard model. In the following sections, we will discuss how these objects may be observed.

2 Cosmic String Dynamics

In order to understand the observational properties of cosmic strings, a basic understanding of the dynamics of strings is necessary. The most basic fact about string dynamics is that the string’s action S is related to the shape of the string’s worldsheet, since the worldsheet is the history of the string’s movement through space and the action is the mathematical expression that

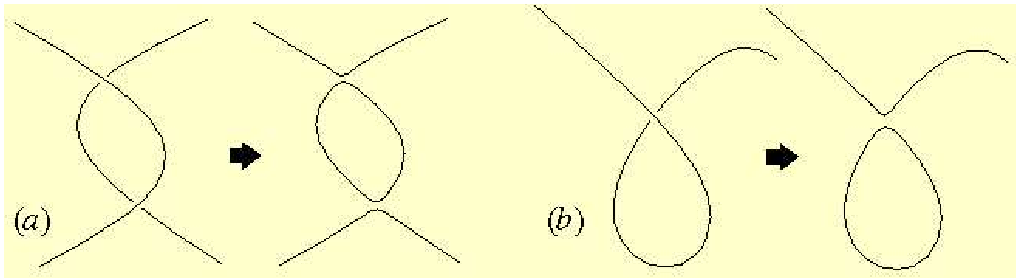

Figure 3: Shows the dynamic process of string intercommuting and loop production. A byproduct of this process is cusp and kink formation on the strings. Graphic from [3]

determines its movement. In order to do cosmic string dynamics, one must first define a metric and a coordinate system on the worldsheet of the string, then the action can be derived with knowledge about the symmetry breaking process which created the string. This is a bit out of the scope of this paper (and my understanding), although, the action which describes the motion of a string in the abelian-Higgs model is

$$S = -\mu \int \sqrt{-\gamma} d^2\zeta \qquad (Eq.3)$$

where μ is the string tension, $\gamma = det(\gamma_{ab})$, γ_{ab} is the metric tensor on the worldsheet, and (ζ_1, ζ_2) are the cooridnates on the worldsheet. Equation (Eq. 3) is called the Nambu action[1]. With the Nambu action, one can determine all sorts of interesting things, like how strings bend spacetime, interact with other strings and matter, and decay with time through emission of (mostly) gravitational radiation[5].

It is highly likely that in phase transitions which form strings, systems of many cosmic strings were formed, not just one or two which stretch across the universe, since it is highly likely that no two bubbles of new phase fell to the same point on the vacuum manifold. Therefore, one of the most important aspects of string dynamics is how strings interact with other strings. For example, what happens when two strings cross each other (called *string intercommuting*) and how does that effect their decay processes? Actually, this particular question is very important and may be the key to finding strong evidence for the existence of cosmic strings. When a string crosses another string or crosses itself, it pinches itself off to form a closed loop (loop formation) and two long kinked strings (see Figure 3)[3]. These kinks are sources for highly non-Gaussian gravitational waves that dectectors in existence today or in the very near future may have the ability to see[2]. In a paper written in 2001, Thibault Damour and Alexander Vilenkin derive a formula for the approximate amplitude of these gravitational wave bursts from cusps in terms of frequency and string tension (a measure of the string's energy), which I quote here:

$$A_{burst}(f,\mu) = 50^{\frac{2}{3}}(G\mu)^{\frac{5}{3}}(ft_0)^{\frac{-1}{3}}g(y(f)) \qquad (Eq.4)$$

with

$$g(y) = y^{\frac{-1}{3}}(1+y)^{\frac{-13}{33}}(1+yz_{eq}^{\frac{-11}{6}})^{\frac{3}{11}} \qquad (Eq.5)$$

and

$$y(f) = 10^{-2}50^{\frac{8}{3}}Rt_0(G\mu)^{\frac{8}{3}}(ft_0)^{\frac{2}{3}}. \qquad (Eq.6)$$

In the above formulas, R is the number of bursts per cusp per year, t_0 is the current age of the universe, and z_{eq} is the redshift of the epoch when there were an equal amounts of matter and radiation in the universe. Vilenkin and Damour derive this formula using the Nambu action (Eq. 3) and perturbed general relativity, plus some fundamental cosmological concepts, and they derive a similar formula for gravitational wave bursts from kinks (see [2] for detailed derivation). I include this result here because this formula may be potentially useful in determining whether a source of a particular wave burst is a cusp or kink of a cosmic string, and I spent a considerable amount of time this summer studying the physics of its derivation.

3 The Search for Cosmic Strings

Confirmation of the existence of cosmic strings may come in the very near future due to two spectacular new gravitational wave detectors: the Laser Interferometer Space Antenna (LISA) and the Laser Interferometer Gravitational Wave Observatory (LIGO). These machines may be able to detect gravitational wave bursts from the cusps and kinks of cosmic strings descibed in the previous section. Although LISA and LIGO are practically the same interferometers, with LISA in following the Earth in its orbit around the sun and LIGO on the ground, they will compliment each other if they are both commissioned. LISA's inferometer "arms" are intended to be 5 million kilometers, while LIGO's are only 4 kilometers. This means that the range of frequencies they are capable of detecting is quite different. LISA operates in the frequency band from 10^{-5} Hz to 1 Hz, with the optimal frequency 10^{-3}[6]. On the other hand, LIGO can detect waves between frequencies from 20 Hz to 1000 Hz, with an optimal range between 100 Hz and 200 Hz[7]. Guided by the calculation of Vilenkin and Damour, physicists will be able to distinguish the highly non-Gaussian bursts from the well-distributed gravitational waves from other sources, such as collapsing stars, and orbiting binaries. Also,

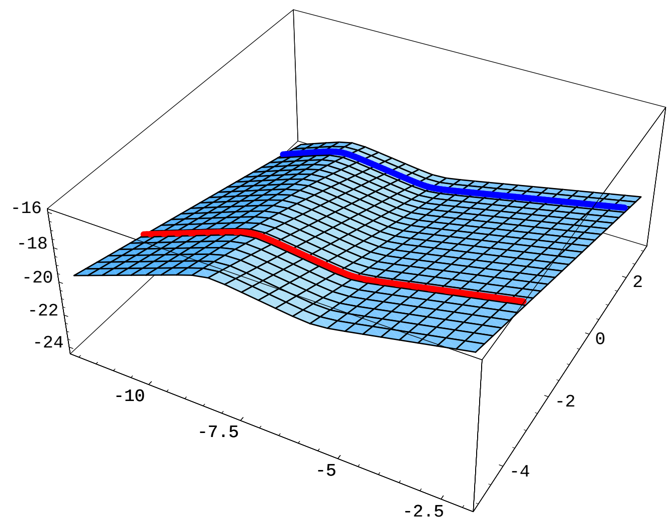

Figure 4: Depicts the log_{10} of the gravitational wave burst amplitude from cusps in cosmic strings. The x-axis is $log_{10}(G\mu)$ and the y-axis is $log_{10}(f)$. The red line denotes LISA's optimal detection frequency and the blue line denotes LIGO's. In Eqs 4,5,6, I use R = 1 burst/year, $t_0 = 10^{17.5}$s, and $z_{eq} = 10^{3.9}$. These are the same numbers Vilenkin and Damour use in [2]

using the formula for approximate gravitational wave amplitude, physicists may be able to distinguish these bursts from non-Gaussian other sources by their amplitudes. There is some question as to whether or not the sensitivity of these detectors will be strong enough to see the waves emitted by cosmic strings. One of the major results to come out of my summer is the graph in Figure 4. I digested Vilenkin and Damour's paper and graphed their results in a 3D plot, marking LISA's optimal detection frequency in red and LIGO's optimal detection frequency in blue.

There is some speculation as to why the amplitude is not a strictly increasing function of string tension. As one can see, there is a dip in the amplitude between $log(G\mu) = -7.5$ and $log(G\mu) = -5$. Next semester, I intend to continue studying the derivation of this formula in order to try to discover the reason for this dip.

4 Conclusions

As we are on the brink of possible cosmic string detection, this is a very exciting and very nerve-racking time for physics. Not finding cosmic strings with new technology like LIGO and LISA means one of three things: 1) our theory of symmetry breaking is incorrect, or 2) the detection capabilities of the technology are not strong enough, or 3) the cosmic string networks have radiated away and they no longer exist. Calculations and simulations carried out by Shellard and Allen have estimated that there are about 10 long strings stretched across the observable universe and 1000 smaller loops[3], so hopefully, option 3) is not a possibility. If 3) were true, the experimental confirmation of symmetry breaking is many years away, since accelerators are nowhere near powerful enough to produce the phase transitions that probably occured in the early universe. If 2) is the problem, it will be solved in time. If 1) is the problem, well, we're back to the drawing board.

It's appropriate to mention here that there are alternate methods of cosmic string detection that do not require gravitational wave detectors: namely, gravitational lensing and analysis of the cosmic microwave background radiation (CMBR) data provided by COBE. Since strings are highly energetic and bend spcetime, they may be positioned in a way that they act as a lens for light sources beyond them. A lucky astronomer might be able to find this kind of visual confirmation of their existence. Also, if physicists can show that the anisotropies in the CMBR data were induced by cosmic string networks, this would be very strong evidence. This strategy is already being employed by Shellard and Allen (see [3]).

Recent developments in superstring theory indicate that cosmic strings are also produced in such processes as brane inflation and collision. There is presently much theoretical effort to deduce the differences between strings produced in abelian-Higgs symmetry breaking processes in the standard model and cosmic strings produced in superstring theory. One of the major differences is that strings in a $3 + 1$ dimensional universe, as in the standard model, must intercommute when they intersect; whereas strings living in a higher dimensional space need not intercommute when they intersect. In other words, the probability of strings intersecting and intercommuting may not be 1, if our universe has more than $3 + 1$ dimensions. Thus, observation of cosmic strings might give us a better idea about the dimensionality of our universe.

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