COHERENT SYNCHRO-BETATRON BEAM-BEAM MODES: EXPERIMENT AND SIMULATION

I.N. Nesterenko, E.A. Perevedentsev, A.A. Valishev*,
Budker Institute of Nuclear Physics, 630090, Novosibirsk, Russia

Abstract

Analytic calculation and numerical simulations revealed a multi-line structure in the spectrum of coherent dipole oscillations in the colliding beam system due to coupled synchrobetatron beam-beam modes. The model employed in the analysis assumes linearization of the beam-beam kick and takes account of the finite length of the colliding bunches. In the present paper we discuss the behaviour of the synchrobetatron beam-beam modes, obtained both analytically and numerically, and compare it to the experimental results from the VEPP-2M collider. Based on the presented models a particular case of the betatron tune close to the half-integer resonance is considered.

1 INTRODUCTION

At the majority of modern circular colliders two modes are observed in the spectrum of dipole oscillations of colliding bunches. Nature of these modes is quite well explained using the linearized interaction model, i.e. when the transverse kick exerted by the beams on each other is proportional to the distance between their centroids [1]. The mode with tune equal to the betatron oscillation tune ν_β corresponds to the inphase oscillations of the bunches relative to the Interaction Point (IP). This mode is usually referred to as the σ mode. Asymmetric mode is called the π mode and has the tune shifted by $\Delta\nu$ which is proportionate to the beam-beam parameter ξ, in the limit of small bunch intensities [2, 3]. Since generally coupling of the transverse degrees of freedom in a storage ring is small and colliding bunches are flat at the IP it is relevant to treat the horizontal and vertical motion separately for small betatron oscillation amplitudes. Further in this work we shall discuss vertical oscillations whereas as a rule vertical ξ is greater than horizontal.

With increase of the circulating bunch intensity collective effects arise due to mutual influence of the head and the tail particles interacting via so-called wake fields [4]. This effect under certain conditions can lead to instability of the transverse oscillations. Dipole betatron and synchrotron oscillations of the beam with the account of collective interaction can be described as a superposition of states oscillating with their eigenfrequencies. These states are referred to as the synchrobetatron modes.

Numerical studies of the effect of residual beam-beam interaction on the transverse mode coupling caused by the transverse impedance in LEP have shown the reduction of the instability threshold [5]. In the simulation the bunch length σ_s was neglected in calculating the beam-beam kick, i.e. the bunch was considered much shorter then the betatron function β^* at the IP.

Colliding bunches of finite length can themselves be a medium for passing the interaction from the head to the tail as they form a two-stream system. Namely, the change in transverse momentum of the head particles after collision results in the change of their coordinate over the interaction length, before collision with the tail ones. This corresponds to non-negligible disruption parameter. In that case synchrobetatron modes must appear in the spectrum of dipole oscillations of colliding bunches. The bunches bending during their collision may be considerable if the bunch length is comparable with β^*, and this case is the subject of our study. A theoretical model describing the system was presented in [6]. It was shown that no synchrobetatron mode coupling instability arises in the system with pure beam-beam interaction while the combined action of transverse impedance and beam-beam force may lead to growth of the oscillation amplitude. Numerical simulation of the beam-beam interaction with the account of the beam shape modification over the bunch length together with the conventional transverse impedance was done in [7].

The present work gives the results of experimental investigation of the frequency spectrum of colliding bunches carried out at the VEPP-2M collider in Novosibirsk. We shall briefly regard the beam-beam synchrobetatron mode calculation methods (Section 2), describe the experimental observation system (Section 3) and present the experimental results in comparison with the calculations (Section 4).

2 ANALYTICAL AND NUMERICAL MODELS

Theoretical study of the beam-beam system was done using two techniques – with matrix calculation and numerical tracking. Since the increments of the collective instabilities resulting from the mode coupling are usually much greater than the radiation damping time we omit the radiative effects. We shall also limit our consideration with the case of two bunches of equal intensity circulating in one ring, i.e. having equal betatron and synchrotron oscillation tunes. This restriction is not fundamental and it is imposed only for agreement with the experimental conditions (Section 3).

In analytical calculation we use the linearized beam-beam force model and hence the beam-beam interaction can be described in terms of matrices transforming the be-

* e-mail: valishev@inp.nsk.su

tatron coordinates and momenta. Between collisions particles perform free betatron and synchrotron oscillations with tunes ν_β and ν_s respectively. To treat the free synchrobetatron oscillations we make use of the so-called circulant matrix [8]. In this representation the bunch consists of N elements each having equal synchrotron amplitude ("hollow beam"). Synchrotron phases (and longitudinal positions with respect to the synchronous particle) are fixed and are i/N, $i = 1 \ldots N$. Each mesh element is characterized by the dipole moment of the particles sitting inside (further referred to as the coordinate) and by the respective momentum. The matrix

$$M_{sb} = C \otimes B$$

(here $\otimes$ denotes the outer product, B is the 2×2 betatron oscillation matrix and C is the $N \times N$ circulant [6, 8]). M_{sb} transforms the $2N$-vector of the mesh coordinates and momenta over the arc. Longitudinal positions of the elements are not permuted, rather the dipole moments interchange. At the same time the eigenvectors and the eigenvalues of M_{sb} precisely correspond to the first $\pm m$ ($m = (N-1)/2$ with N odd) synchrobetatron modes of exact solution. In fact, the circulant matrix is a representation of differentiation of a function specified in N equidistant interpolation points.

The synchrobetatron matrix for the system of two non-interacting bunches is given by

$$M_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes M_{sb} \; .$$

Since the longitudinal positions of the mesh elements are not changed by the synchrobetatron motion it is very convenient to code the beam-beam interaction. It is expressed with the system of thin lens and drift matrices which represent relative longitudinal positions of the elements. Collision of the particles No. i in one bunch and No. j in the other changes their momenta according to the formula

$$\Delta p_{i,j} = \pm \frac{2\pi\xi}{\beta}(x_j - x_i) \; ,$$

where β is the betatron amplitude function, and we assume the particles to be rigid Gaussian discs in the transverse direction [9]. Multiplication of the consecutive kick matrices followed by free drifts gives the complete $4N \times 4N$ beam-beam matrix.

The one-turn matrix is the product of the arc matrix M_2 and the beam-beam matrix. Its eigenvalues and eigenvectors completely characterize the synchrobetatron modes of the beam-beam system. Since the symbolic solution for large N is quite complicated it is convenient to find the eigensystem numerically, using a computer algebra system.

Using the circulant matrix approach it is possible to create a model of the beam with arbitrary distribution in the synchrotron phase space. For this purpose the synchrotron oscillation plane is divided into K rings, each consisting of any given N_r, $r = 1 \ldots K$ mesh elements. The population of the rings is chosen to produce the desired phase space distribution. The system state is then characterized by the $(2 \cdot \sum_r N_r)$-vector of coordinates and momenta. The synchrobetatron matrix in that case consists of K blocks,

$$M_{sb} = \begin{pmatrix} C_1 \otimes B & 0 & \ldots & 0 \\ 0 & C_2 \otimes B & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \ldots & C_K \otimes B \end{pmatrix} .$$

Here C_r are the $N_r \times N_r$ circulant matrices of K rings, and B is the 2×2 betatron matrix. Each ring may have its own synchrotron tune, thus introducing the nonlinearity of the synchrotron motion.

In numerical tracking the bunches consist of a number of particles each characterized by the betatron coordinate x, the respective momentum p, the longitudinal offset with respect to the synchronous particle s, and by the energy deviation δ. The particles are seeded with initial s and δ values to have Gaussian distribution in the synchrotron plane. Transverse offsets and momenta in one bunch are zero while the other has some initial dipole moment.

The synchrotron and betatron variables are transformed independently in the arc. Before the interaction the particles are sorted by their longitudinal coordinate s and the beam-beam kicks are given in the correct order with drifts between the collisions. The stored turn-by-turn dipole moments of the beams are Fourier analysed to give the synchrobetatron beam-beam mode spectra.

Mode naming convention uses the σ and π indices to mark the beam-beam symmetry of colliding bunches with respect to the IP, and numerical indices m labeling the synchrotron wavenumber (Fig. 1).

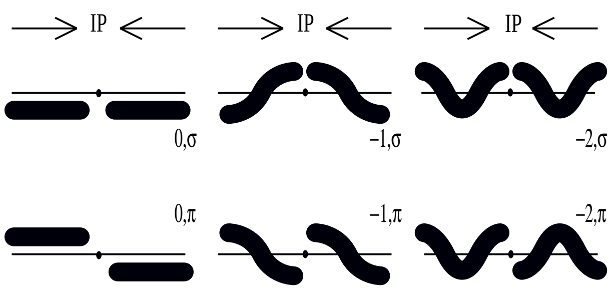

Figure 1: Notation of the synchrobetatron modes of colliding bunches.

Simulation results are presented in Fig. 2 where the dependence of the calculated spectrum on the beam-beam parameter ξ is shown. No transverse mode coupling instability occurs since the system is closed and the interaction is symmetrical. Oscillations in such systems are stable unless one of the mode tunes reaches zero or 0.5 values. It is also seen that the result given by the hollow beam model agrees quite well with the numerical tracking of Gaussian distribution, and no additional modes occur in this more complete system. The reason is that the so-called radial modes in the synchrobetatron mode spectrum show up if the wake variation over the bunch length is sufficient. For the beam-beam interaction the measure of this effect is the disruption parameter $4\pi\xi\sigma_s/\beta^*$ which in our case is always much less than 1.

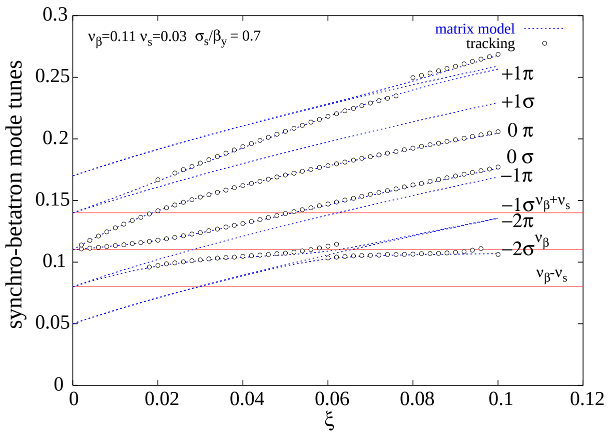

Figure 2: Synchrobetatron mode tunes vs. the beam-beam parameter ξ. Comparison of the circulant matrix hollow beam model (lines) with tracking of the Gaussian distribution (circles; number of particles in tracking was 1000). $\nu_\beta = 0.11$, $\nu_s = 0.03$, and the bunch length is $0.7\ \beta^*$.

In Figs. 3,4 results of matrix calculation are given with distribution in the synchrotron plane formed by two rings each consisting of 5 elements. Fig. 3 shows the frequency spectrum dependence on ξ while in Fig. 4 relative projections of the initial condition vector on the eigenvectors are plotted. The initial state corresponds to one bunch having zero betatron coordinates and momenta and the other bunch shifted as a whole.

In addition to the modes already seen in Fig. 2 the spectrum contains a number of radial modes (for instance two lines between 0π and 0σ). But their amplitudes in comparison with the modes 0π, 0σ, $+1\pi$, -1σ, $+2\pi$ and -2σ are negligible. The dipole moment passes from the 0 modes to the ±1 modes as they couple when ξ increases, then it turns to ±2 modes at higher ξ, and so forth.

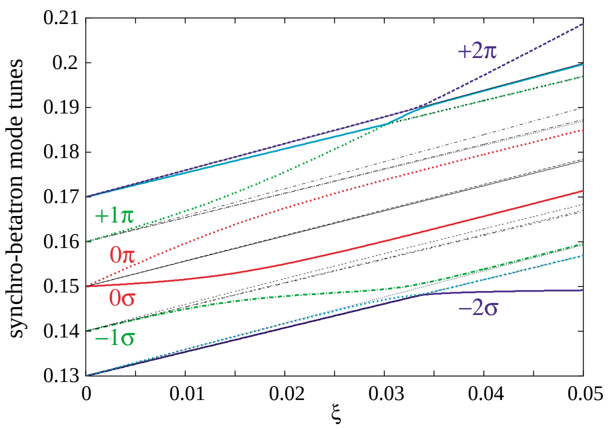

Figure 3: Synchrobetatron mode tunes vs. ξ. Two-ring hollow beam model. $\nu_\beta = 0.15$, $\nu_s = 0.01$, $\sigma_s/\beta^* = 1$.

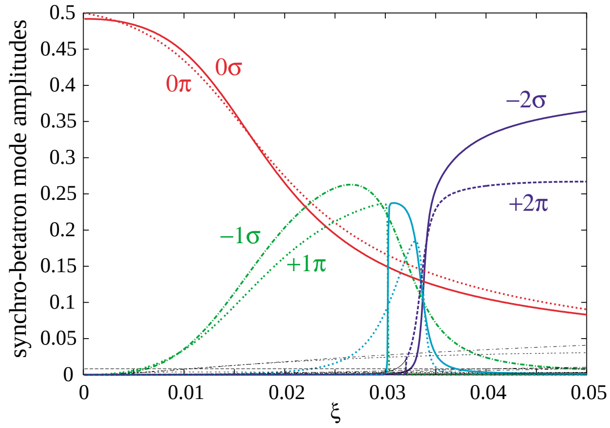

Figure 4: Synchrobetatron mode amplitudes vs. ξ. Parameters are the same as in Fig. 3.

The system may become unstable if the betatron tune is close to a half-integer and the synchrotron tune is comparable with the detuning $\nu_\beta - n/2$. Then the coherent synchrobetatron resonances arise if the mode tunes reach $n/2$ or the aliased modes couple with "normal" modes. As an example, in Fig. 5 the mode tunes are plotted vs. ξ for $\nu_\beta < \nu_s/2$, Fig. 6 shows the corresponding mode increments. The tunes of the aliased modes with $m = -1, -2$ decrease as ξ grows, and first the modes 0π and -1π couple, then follow the pairs $+1\pi$, -2π; 0σ, -1σ; $+1\sigma$, -2σ and so on. At $\xi \simeq 0.025$ the -1π mode tune reaches zero and becomes unstable. The effect is an analog of the sum resonance. Similar results have been obtained by Ohmi and Chao [10] using numerical study of the two particle model.

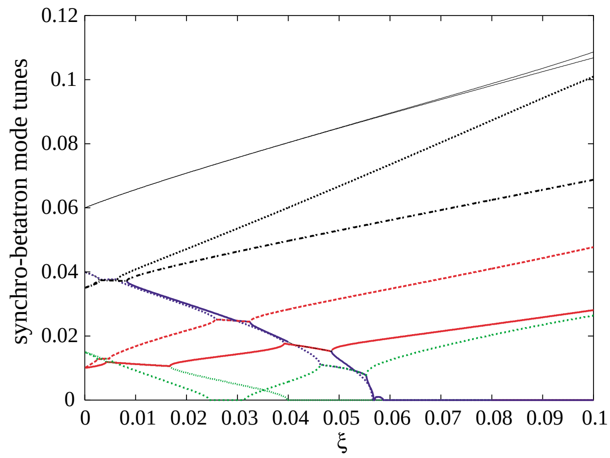

Figure 5: Synchrobetatron mode tunes vs. ξ. Hollow beam model. $\nu_\beta = 0.01$, $\nu_s = 0.025$ and $\sigma_s/\beta^* = 1$.

3 EXPERIMENTAL TECHNIQUE

Experimental observation of the frequency spectrum of colliding bunches has been done at the e^+e^- collider VEPP-2M. The storage ring was exploited in particle physics ex-

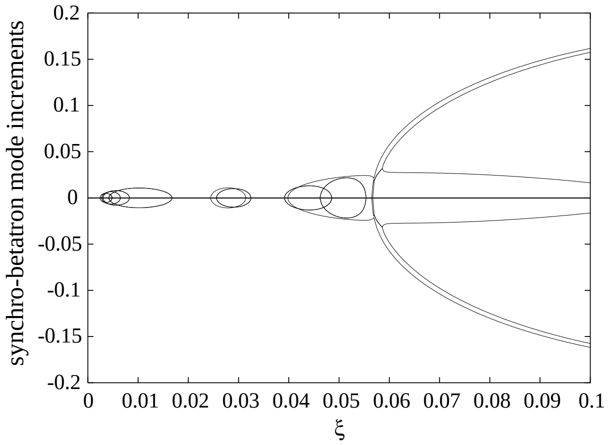

Figure 6: Synchrobetatron mode increments per turn vs. ξ. Parameters are the same as in Fig. 5.

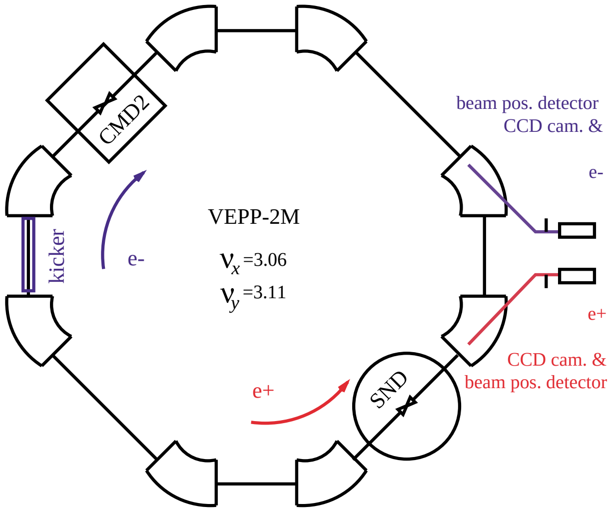

Figure 7: Layout of the experiment at VEPP-2M.

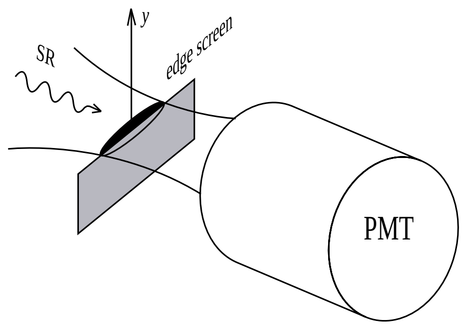

Figure 8: Scheme of the edge beam position detector.

periments in the energy range of 200 - $690\ MeV$ per beam since 1972 until the end of 2000. The beam-beam spectra observations were carried out in a number of energy points between 400 and $450\ MeV$. The collider was operated with one electron and one positron bunch of equal intensities colliding in two IP's occupied with two particle detectors SND and CMD2 which took data in parallel. The optical scheme had a 4-fold symmetry with the minimum beta values of $\beta_y = 6\ cm$ and $\beta_x = 40\ cm$ in the IP's, in the accelerating RF cavity and in the superconducting wiggler. The bunch length was about $4\ cm$ which is comparable with β_y. Maximum attainable vertical beam-beam parameter ξ_y with the wiggler off was 0.03 while the synchrotron tune was rather small and could be tuned in the range from 0.006 to 0.009. The decoherence time of small dipole betatron oscillations was in the range $4-8\cdot 10^3$ turns that with the revolution frequency of $16.7\ MHz$ allowed to have a sufficient accuracy of the turn-by-turn signal spectrum. Layout of the main diagnostics elements is shown in Fig. 7.

Vertical coherent oscillations of the electron bunch were excited with a short kicker pulse ($\sim\ 30\ ns$ which is less than one turn). Since the kicker plate terminated into matched load, motion of the positron bunch remained unaffected by the kick and its oscillations evolved only due to coupling with the electron bunch via the beam-beam force. The kick generator pulse had an adjustable magnitude and the minimum amplitude of the excited oscillations was equal to $0.2\ \sigma$, σ being the Gaussian vertical beam size.

Oscillations of the bunches were observed using the beam synchrotron radiation from the dipoles. The optical image of the beam was focused into the movable screen plane (Fig. 8). The screen was cutting off a portion of the light in the beam image plane. For a fixed edge position, a displacement of the beam centroid resulted in modulation of the light flux.

The light which passed through the optical system then fell on the PMT (Fig. 9). The PMT signal, with modulation proportional to the beam displacement, was fed to the fast ADC input. In our system we used the CAMAC-standard 8-bit ADC with the 8k read buffer and minimum conversion time of 10 ns. The PMT bandwidth was taken to observe separate turns of the bunch in the storage ring. The ADC clock rate was exactly equal to the beam revolution frequency and the phase was locked to the RF phase of the bunch. Timing of the ADC start with the high voltage beam excitation pulse was performed using the multichannel time interval generator (TIG in Fig. 9): the TIG trigger signals were passed to both the ADC and the HV generator. The similar observation channel was implemented for the positron beam. For synchronization of the electron and positron channels the clock pulse splitter was used with the delay correction in the positron channel tuned by means of the additional cable length.

In addition to the beam centroid position the vertical beam size was measured at the same points using CCD cameras. This gave a possibility to evaluate the optics deformation due to the focussing by the beam-beam force. Measuring the beam size at two orbit points allows to calculate the change of the dynamic β-function at the IP (β^*). Measured dependence of β^* vs. ξ agrees perfectly with the optics code calculation. In the calculation the colliding

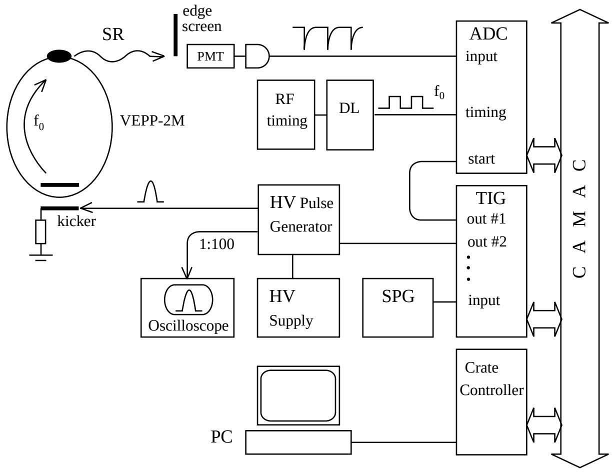

Figure 9: Block diagram of the experimental setup.

beam was represented with a lens located at the IP and focussing in both horizontal and vertical directions. The same calculation yielded the dependence of the vertical beam emittance vs. ξ which together with measured β^* gives the vertical beam size (Fig. 10). It is seen that the beam size did not change significantly at the used bunch intensities. This means that ξ was linear in beam current.

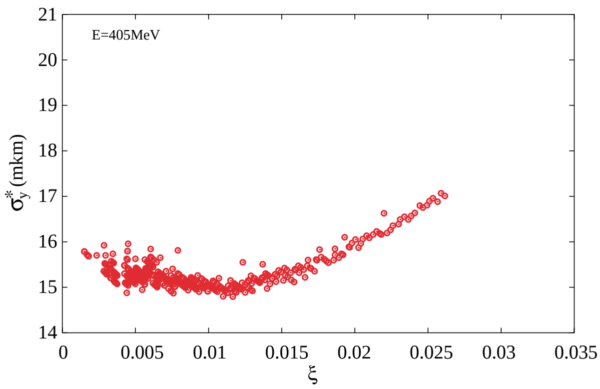

Figure 10: Vertical beam size at IP vs. ξ.

Moreover, the on-line luminosity data from two particle detectors revealed the same conclusion, since the linearity of the luminosity square root vs. the beam current was quite good (Fig. 11). This information also gave the fitting coefficient between ξ and the measured bunch current.

To study the coherent tune shifts due to beam-beam interaction, an information about the single bunch phenomena is also important. The experimental setup described in this section suits well for this purpose. We studied the single bunch coherent oscillation spectra. The betatron tune was detected very well in the spectrum even without external excitation of the beam motion. Beam current scan has shown that the single bunch coherent tune shift ~ 0.005 is negligible in comparison with the expected beam-beam effect (Fig. 12, note that maximum current was $100\ mA$, while for the beam-beam operation it was $\sim 25\ mA$), and hence the machine transverse impedance is very small.

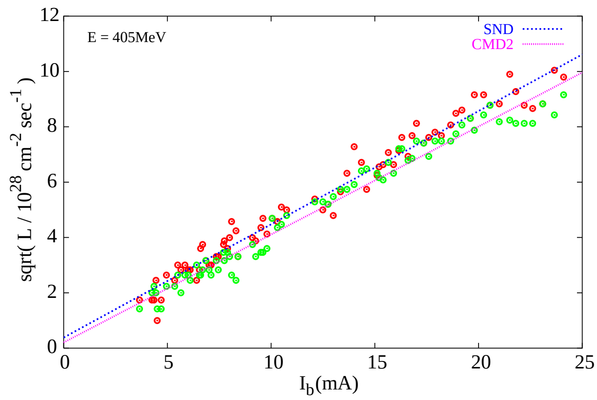

Figure 11: Square root of the luminosity vs. the colliding beam current. Data was collected at two particle detectors, CMD2 (green circles) and SND (red circles).

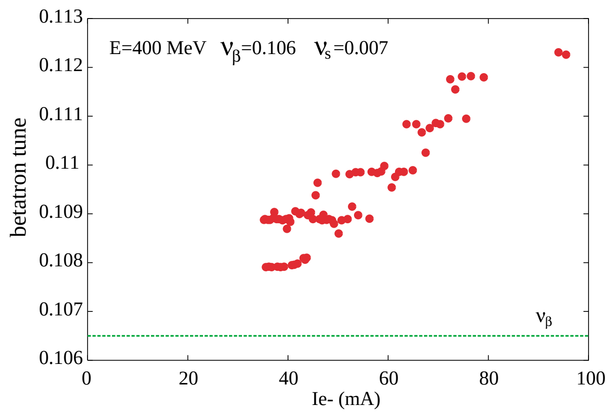

Figure 12: Betatron tune vs. electron beam current. Single bunch operation mode.

The center of mass positions of the colliding bunches were sampled turn-by-turn over 8192 turns. The informative part of these sample was typically 4000 turns due to decoherence with the machine and beam-beam nonlinearity. The Fourier transform of the collected data gave the coherent mode spectrum, where the proposed synchrobetatron modes of the beam-beam system were experimentally detected, and their spectrum was measured as a function of the beam-beam parameter at different synchrotron tunes. To increase accuracy of the mode tune determination the interpolated Fast Fourier Transform with the Hanning data windowing [11] was used.

4 RESULTS

The complete results of the coherent beam-beam mode spectra calculation with the account of the finite bunch length are presented in [6, 7]. Since VEPP-2M had a negligible transverse impedance, we compare these experimental data with the simulation results for the case where the collective interaction is completely due to beam-beam. Figs. 13-16 show the dependence of the measured and calculated synchrobetatron mode tunes on the beam current for equal electron and positron bunch intensities at different beam energies and with various synchrotron tune values.

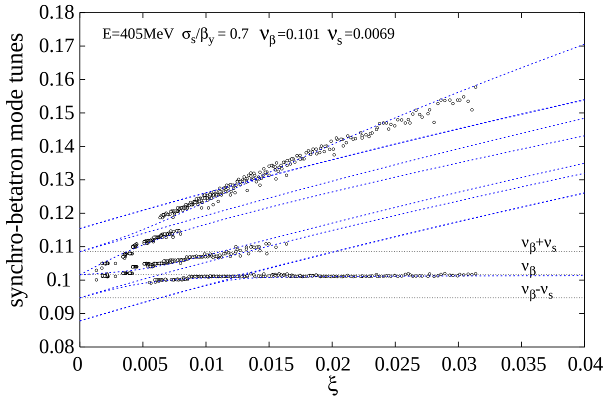

Figure 13: Synchrobetatron beam-beam mode tunes vs. ξ. Comparison of measured (circles) and calculated (lines) data. $\nu_\beta = 0.101$, $\nu_s = 0.0069$, $\beta^* = 6\ cm$, $\sigma_s = 0.7\ \beta^*$, and beam energy $E = 405\ MeV$.

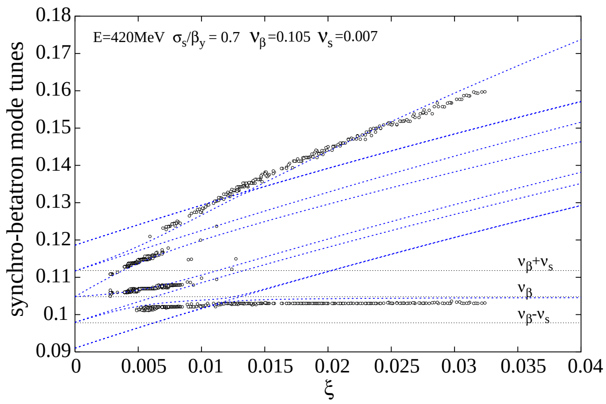

Figure 14: Synchrobetatron beam-beam mode tunes vs. ξ. Comparison of measured (circles) and calculated (lines) data. $\nu_\beta = 0.105$, $\nu_s = 0.007$, $\beta^* = 6\ cm$, $\sigma_s = 0.7\ \beta^*$, and beam energy $E = 420\ MeV$.

In perfect agreement with the theoretical model, the measurement has shown that besides the leading σ and π modes a number of synchrobetatron modes coupled via the beam-beam force exist in the dipole mode spectrum. The experimental lines seen in the figures were derived in a single measurement when at a certain bunch intensity up to four modes appear simultaneously. Since the initial state when the positron bunch does not oscillate and the electron bunch is shifted as a whole is similar to that used in calculation, the mode behaviour is well described with Fig. 4. The modes show up and disappear with the beam

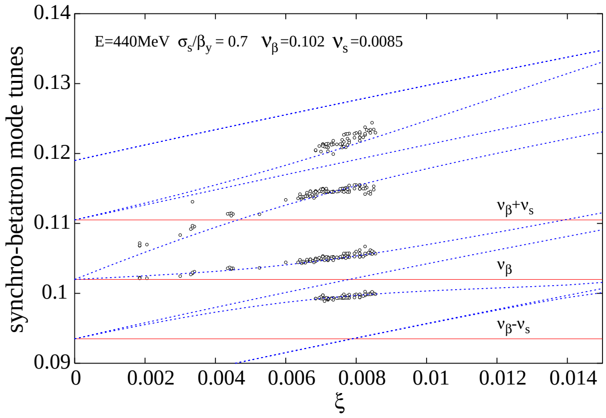

Figure 15: Synchrobetatron beam-beam mode tunes vs. ξ. Comparison of measured (circles) and calculated (lines) data. $\nu_\beta = 0.102$, $\nu_s = 0.0085$, $\beta^* = 6\ cm$, $\sigma_s = 0.7\ \beta^*$, and beam energy $E = 440\ MeV$.

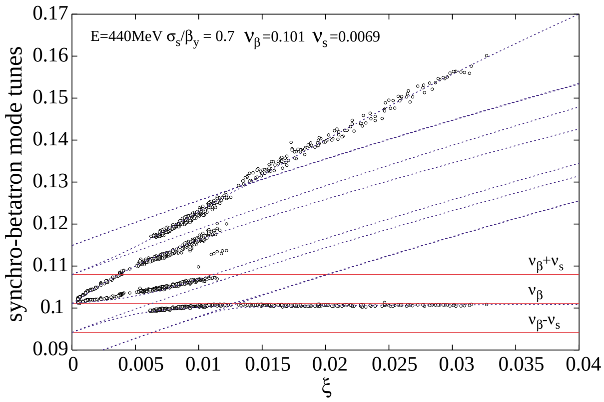

Figure 16: Synchrobetatron beam-beam mode tunes vs. ξ. Comparison of measured (circles) and calculated (lines) data. $\nu_\beta = 0.101$, $\nu_s = 0.0069$, $\beta^* = 6\ cm$, $\sigma_s = 0.7\ \beta^*$, and beam energy $E = 440\ MeV$.

current variation due to ξ-dependence of the beam-beam mode eigen-states. For the ξ value less than the synchrotron tune ν_s, the state excited with the kick mostly consists of only two beam-beam modes, σ and π, with the synchrotron wavenumber $m = 0$. In the range $\nu_s < \xi < 2\nu_s$ the initial condition is a combination of four eigenmodes: -1σ, 0σ, 0π, $+1\pi$. With larger ξ the dipole moment passes on to -2σ, $+2\pi$ and later to -3σ, $+3\pi$ σ modes. Because of small coupling of modes with large synchrotron wavenumber these transitions do not show an apparent tune split.

The experimental and calculated data fitting was done using a *single parameter*, namely the horizontal axis scaling. The parameter which describes the ratio between the coherent beam-beam kick and ξ can not be obtained within the linear theory and is the subject for a separate study [3, 12].

5 CONCLUSION

Recent analytical and numerical calculations [6, 7] predicted that the spectrum of coherent oscillations of colliding bunches contains synchrobetatron modes. The experimental setup for optical detection of the vertical coherent oscillations described in this paper allowed to discover these modes experimentally at the VEPP-2M collider. The measured spectra dependence on the beam current is in excellent agreement with analytical and numerical models.

The above presented experimental evidence of the synchrobetatron beam-beam modes adds confidence to other conclusions of their theory. One of them is that for the negligible transverse impedance the mode tunes do not intersect and the colliding beam system remains stable unless some of the mode tunes reach a half-integer resonance. The measure of the mode coupling is the ratio between the bunch length and the betatron function at the interaction point.

In the special case of betatron tune in the vicinity of a half-integer the existence of unstable synchrobetatron coupling resonance regions is predicted. However the higher order modes in electron machines are usually suppressed due to fluctuations of the synchrotron radiation, and their unstable ξ ranges are rather narrow, but the low-order resonances should be avoided since they can result in undesirable beam size growth.

On the other hand, calculations involving the machine impedance predict a coherent beam-beam *instability* without a threshold. Some, though not all, of the synchrobetatron modes can be damped by optimizing the betatron tune chromaticity. Since the theoretical models [6, 7] used the linearized beam-beam interaction, their prediction of instability is not as conclusive as the above prediction of stability. In a realistic nonlinear beam-beam system one can expect saturation of such an instability at amplitudes of the order of the vertical beam size. However, this mechanism can cause a vertical emittance blowup detrimental to the high performance of the flat-beam colliders.

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