Hadronic B Decays to Charm Mesons (Review)

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Abstract

We review the phenomenology of hadronic B meson decays to final states containing charm mesons charmonium mesons. After discussing test of factorization, we extract the coefficients $a_{1,2}$ of the factorized effective Hamiltonian for each of the decay groups $D\pi$, ψK, and DD_s separately. We also comment on the possible charm deficit in B meson decays.

1. Introduction

Non-leptonic decays are notoriously resistant to rigorous understanding since hadrons are surrounded by a cloud of light quark and gluon which can have infinite degrees of freedom (like the chaotic motions of fluid) and the products of the decaying quark can interact strongly with this cloud. In recent years, however, we have seen many new results coming out of CLEO, ARGUS, CDF, and LEP experiments, and also new theoretical fronts have been pushed forward based on Heavy Quark Effective Theory (HQET) and QCD sum rules. In between, models based on factorization have provided convenient framework to guide the analyses[1, 2]. Here we will focus on reviewing the phenomenological aspects while the theoretical part is left to another talk[3].

2. Test of Factorization

The idea of factorization in non-leptonic decays goes back to early 60's when Schwinger estimated the $\Delta I = 3/2$ part of Kaon decay from the semileptonic rate[4], and was thought to be good only for order of magnitude estimations. For certain heavy hadron decays, however, there are reasons to believe that the factorization can work well. We take $\bar{B}^0 \to D^{*+}\pi^-$ as an example which is caused by the 4-fermion operator

$$(\bar{d}u)_\mu (\bar{c}b)^\mu$$

where $(\bar{q}_1 q_2)_\mu \equiv \bar{q}_1^a \gamma_\mu (1-\gamma_5) q_2^a$ is color singlet $V-A$ current ($a = 1,2,3$: color index). The factorization hypothesis consists in assuming that the current $(\bar{d}u)$ creates π^-, the current $(\bar{c}b)$ generates the transition $B \to D^*$, and that they occur independently; or, that the amplitude can be 'factorized':

$$\langle D^{*+}\pi^- |(\bar{d}u)_\mu (\bar{c}b)^\mu |\bar{B}^0\rangle = \langle \pi^- |(\bar{d}u)_\mu |0\rangle \langle D^{*+}|(\bar{c}b)^\mu |\bar{B}^0\rangle$$

where (with f_π being the pion decay constant)

$$\langle \pi^- |(\bar{u}d)_\mu |0\rangle = -if_\pi q_\mu \quad (q : \pi \text{ 4-momentum}).$$

If the $d\bar{u}$ pair interacts with the color field of B meson as it passes through it, or if the π and D^* re-scatter by final-state interaction, then the two processes are not independent, and factorization would not work. However, the $d\bar{u}$ pair is colorless as it is generated by W and moving fast through the color field tightly together; thus, we expect that it will escape the color field without much interaction. Also, by the time the π is formed ($\sim$1fm/c in its rest frame, $\sim$10fm in the B rest frame), it is many times the meson radius away from D^*; thus, there will be little final-state interaction. We can hope that factorization works in this case[6].

The matrix element $\langle D^{*+}|(\bar{c}b)^\mu|\bar{B}^0\rangle$ is the same as that appearing in the corresponding semileptonic decay

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$\bar{B}^0 \to D^{*+}l^-\nu$ (at the same $q^2 = m_\pi^2$), and thus one expects that there is a simple relation between the two decay rates. In fact, we have[5, 7]

$$\frac{\mathcal{B}(\bar{B}^0 \to D^{*+}X^-)}{\frac{d\mathcal{B}}{dq^2}(\bar{B}^0 \to D^{*+}l^-\nu)\Big|_{q^2=m_X^2}} = 6\pi^2 f_X^2 |V_{ud}|^2 \qquad (1)$$

which holds for $X^- = \pi^-, \rho^-, a_1^- \ldots$ as long as factorization works and $q^2 = m_X^2$ is small. The LHS of (1) - denoted as R(measured) - can be measured and then compared to the RHS which is the expectation from factorization - denoted as R(fact.).

The result is shown in Table 1. The branching ratios of the hadronic decays are measurements by CLEO[8] where D^{*+} is detected by the decay chain $D^{*+} \to D^0\pi^+$,$D^0 \to K^-\pi^+$, $K^-\pi^+\pi^0$,$K^-\pi^+\pi^-\pi^+$, and a_1^- is detected in the mode $a_1^- \to \rho^0\pi^-$. The differential semileptonic rate is obtained by fitting q^2 dependence of various models[9] to the combination of CLEO and ARGUS measurements[10]. By comparing R(measured) and R(fact.), one can see that factorization seems to be working for $D^{*+}\pi^-$ and $D^{*+}\rho^-$, within the error of about 25%, but not so well for $D^{*+}a_1^-$. It could be that a_1 (1230 MeV) is already too heavy and moving too slowly for factorization to hold, but the discrepancy is only 2σ which could well be due to fluctuation.

3. Effective Factorized Hamiltonian

By scaling down the operator $(\bar{d}u)_\mu(\bar{c}b)^\mu$ (or similarly for $(\bar{s}c)_\mu(\bar{c}b)^\mu$) from the W mass scale to b mass scale one obtains the QCD corrected effective Hamiltonian

$$\mathbf{H}_{\text{eff}} = c_1\mathbf{O}_1 + c_2\mathbf{O}_2$$
$$\mathbf{O}_1 \equiv (\bar{d}u)_\mu(\bar{c}b)^\mu, \quad \mathbf{O}_2 \equiv (\bar{c}u)_\mu(\bar{d}b)^\mu$$

and $c_{1,2}$ are the Wilson coefficients given by

$$\begin{aligned} &c_+ = c_1 + c_2, \quad c_- = c_1 - c_2 \\ &c_- = c_+^{-2} = [\alpha_s(\mu)/\alpha_s(M_W)]^{12/(33-2n_f)} \\ &\to c_1(m_b) = 1.12, \quad c_2(m_b) = -0.26. \end{aligned} \qquad (2)$$

Without the QCD correction ($\mu = M_W$) we have $c_+ = c_- = 1$ or $c_1 = 1, c_2 = 0$; it does not, however, mean that $\mathbf{O}_2$ isn't there without the QCD correction since $\mathbf{O}_{1,2}$ are not orthogonal. In fact, Fierz identities give

$$\mathbf{O}_1 = \frac{1}{3}\mathbf{O}_2 + \frac{1}{2}[\bar{c}u]^i_\mu[\bar{d}b]^\mu_i, \quad \mathbf{O}_2 = \frac{1}{3}\mathbf{O}_1 + \frac{1}{2}[\bar{d}u]^i_\mu[\bar{c}b]^\mu_i$$

where $[\bar{q}_1 q_2]^i_\mu \equiv (\bar{q}_1^a \lambda^i_{ab}\gamma_\mu(1-\gamma_5)q_2^b)$ is color-octet V-A current (λ^i: Gell-Mann matrices), and $\mathbf{H}_{\text{eff}}$ can be written in two ways in terms of *orthogonal basis*:

$$\begin{aligned} \mathbf{H}_{\text{eff}} &= \frac{c_1}{2}[\bar{c}u]^i_\mu[\bar{d}b]^\mu_i + (\frac{c_1}{3} + c_2)\mathbf{O}_2 \\ &= (c_1 + \frac{c_2}{3})\mathbf{O}_1 + \frac{c_2}{2}[\bar{d}u]^i_\mu[\bar{c}b]^\mu_i. \end{aligned} \qquad (3)$$

Note that $c_1 + c_2/3 = 1$ to the first order due to the relation $c_+^2 c_1 = 1$ (eq. 2); thus, one can say that the QCD correction keeps the coefficient of $\mathbf{O}_1$ intact and simply adds the octet-octet operator.

Now assume that
1) there is no contribution from the terms that put a color-octet pair into a meson, and that
2) matrix elements of $\mathbf{O}_{1,2}$ are dominated by the factorizable part $\mathbf{O}^{\text{fac}}_{1,2}$ which are defined by

$$\begin{aligned} \langle D^+\pi^-|\mathbf{O}_1^{\text{fac}}|\bar{B}^0\rangle &\stackrel{\text{def}}{\equiv} \langle\pi^-|(\bar{d}u)_\mu|0\rangle\langle D^+|(\bar{c}b)^\mu|\bar{B}^0\rangle \\ \langle D^0\pi^0|\mathbf{O}_2^{\text{fac}}|\bar{B}^0\rangle &\stackrel{\text{def}}{\equiv} \langle D^0|(\bar{c}u)_\mu|0\rangle\langle\pi^0|(\bar{d}b)^\mu|\bar{B}^0\rangle \end{aligned} \quad (4)$$

etc. Then, one can introduce a 'factorized effective Hamiltonian' $\mathbf{H}_{\text{fac}}$, which is one-level higher than $\mathbf{H}_{\text{eff}}$, by inserting the $\mathbf{H}_{\text{eff}}$ in two ways corresponding to the two ways of writing $\mathbf{H}_{\text{eff}}$ (eq.3):

$$\mathbf{H}_{\text{fac}} \equiv a_1\mathbf{O}_1^{\text{fac}} + a_2\mathbf{O}_2^{\text{fac}}$$
$$\text{with} \quad \begin{cases} a_1 = c_1 + c_2/3 + \text{nf}_1 \\ a_2 = c_1/3 + c_1 + \text{nf}_2 \end{cases}$$

where $\text{nf}_{1,2}$ (try to) account for the non-factorizable effects: namely, the matrix elements of the octet-octet operators where a octet $\bar{q}q$ pair is put into a meson, and the non-factorizable parts of $\mathbf{O}_{1,2}$. The definition of $\mathbf{O}^{\text{fac}}_{1,2}$ (eq. 4) itself depends on the types of final state mesons, and so does the octet-octet contribution. Some effects cannot even be absorbed into $a_{1,2}$. For example, CLEO measures $\mathcal{B}(B \to \chi_{c2}X) = 0.25 \pm 0.10 \pm 0.03\%$ which is not expected in naive factorization picture[13]. It seems reasonable, however, to use a same set of $a_{1,2}$ for a group of related decay modes; namely, one set for $D\pi$, $D^*\pi$, $D\rho$, and $D^*\rho$ modes, another for DD_s, D^*D_s, DD_s^*, and $D^*D_s^*$ modes etc.

4. Extraction of $a_{1,2}$ for Each Decay Group

One can distinguish 3 classes of decay modes:

Class 1: Only $\mathbf{O}_1^{\text{fac}}$ contributes and the amplitude is proportional to a_1; e.g. $\bar{B}^0 \to D^+\pi^-$ where the hadron current $B \to D$ emits π and thus the amplitude contains the pion decay constant f_π.

Class 2: Only $\mathbf{O}_2^{\text{fac}}$ contributes and the amplitude is proportional to a_2, sometimes called 'color-suppressed' decays.; e.g. $\bar{B}^0 \to D^0\pi^0$ where the hadron current $B \to \pi$ emits D thus the amplitude contains f_D.

Class 3: Both $\mathbf{O}_1^{\text{fac}}$ and $\mathbf{O}_2^{\text{fac}}$ contribute and the amplitude contains a_1 and a_2; e.g. $B^- \to D^0\pi^-$ whose amplitudes contains both f_π and f_D. This interference occurs since there are two identical quarks ($\bar{d}$) in the final state.

If both daughters are charged then it is class 1, if both are neutral then it is class 2, and if one is charged and the other is neutral then it depends on the quark

X	$\mathcal{B}(D^{*+}X^-)$ (%)$^{(a)}$	$d\mathcal{B}/dq^{2(b)}$ (%/GeV^2)	R(measured) (GeV^2)	f_X (GeV)	R(fact.) (GeV^2)
π	0.26±0.03±0.04±0.01	0.22±0.2	1.1±0.1±0.2	0.132	1.0
ρ	0.74±0.10±0.14±0.03	0.25±0.2	3.0±0.4±0.6	0.215	2.7
a_1	1.26±0.20±0.22±0.04$^{(c)}$	0.32±0.2	4.0±0.6±0.5	0.205	2.5

(a) CLEO measurements. The first error is statistical, second systematic, and the third is due to uncertainties in D^0 branching fractions.
(b) From CLEO and ARGUS measurements of $B \to D^* l^- \nu$ fitted to various models.
(c) It is assumed that $D^{*+}a_1^-$ dominates $D^{*+}\pi^+\pi^-\pi^-$ in the region $1.0 < M_{3\pi} < 1.6$ GeV.

Table 1. Test of factorization. Branching fraction of $B \to D^*X$ is compared to the corresponding semileptonic decay evaluated at $q^2 = m_X^2$.

contents of final state. Note that when D is emitted, it is not moving fast, thus it may interact with the color field of B meson before escaping; thus, factorization may not work well for class 2 and 3 decays of $D\pi$ decay mode group. Similarly, factorization may not hold for $D^{(*)}D_s^{(*)}$ or $\Psi^{(')}K^{(*)}$ decays where the emitted meson is $D_s^{(*)}$ or $\Psi^{(')}$, respectively.

In order to extract $a_{1,2}$ we need to express each decay rate in terms of these parameters as done in Ref.[1]. Here we will use a updated version given in Ref.[2]. For example, we have

$$A(\bar{B}^0 \to D^+\pi^-) = -i\frac{G_F}{\sqrt{2}}V_{ud}^*V_{cb}a_1 f_\pi q_\mu \langle D^+|(\bar{c}b)^\mu|\bar{B}^0\rangle$$

where the form factor in the $B \to D$ transition is obtained from semileptonic decays by fitting to HQET-inspired forms. Together with the phase space factor $p/8\pi M^2$, $\tau_B = 1.63$ ps[12], and $|Vcb| = 0.039$[14], we obtain $\mathcal{B}(D^+\pi^-) = 0.264a_1^2$. For class-2,3 decays, one needs $B \to$ light(π, ρ, K) form factors which are calculated by a relativistic harmonic oscillator model since experimental data are not available.

In Table 2, we list measurements by CLEO and the model expectations. B mass distribution for the sum of the 8 $D^{(*)}D_s^{(*)}$ modes is shown in Fig. 1. Separate fits to the decay groups $D^{(*)}\pi^{(*)}(\pi^* \equiv \rho)$, $D^{(*)}D_s^{(*)}$, and $\Psi K^{(*)}$ gives

$$\begin{aligned} a_1 &= 1.15 \pm .04 \pm .05 \pm .09 & (D^{(*)}\pi^{(*)} \text{ class } 1) \ (a) \\ a_2/a_1 &= 0.23 \pm .04 \pm .04 \pm .10 & (D^{(*)}\pi^{(*)} \text{ class } 1{,}3) \ (b) \\ a_1 &= 1.05 \pm .05 \pm .06 \pm .09 & (D^{(*)}D_s^{(*)} \text{ class } 1) \ (c) \\ |a_2| &= 0.26 \pm .01 \pm .01 \pm .02 & (\Psi K^{(*)} \text{ class } 2) \ (d) \end{aligned}$$

where the first error is statistical, second systematic, and the third is due to the uncertainties in the yield and lifetime ratios of charged vs neutral B mesons. The fits (a) and (b) are for $b \to c\bar{u}d$ transition and there exist class-3 decays; thus, it is possible to measure the relative sign of a_1 and a_2 through interference. The fits (c) and (d) are for $b \to c\bar{c}s$, and there is no corresponding class-3 decays preventing us from determining the sign of a_2/a_1. The interference effect is conveniently measured by taking the ratio of a class-3 decay rate to the corresponding class-1 decay rate, e.g. $R = \Gamma(B^- \to D^0\pi^-)/\Gamma(\bar{B}^0 \to D^+\pi^-)$, which becomes a function of a_1/a_2 as seen in Table 2. Note that $R > 1$ corresponds to $a_2/a_1 > 0$. The sign of a_2/a_1 is in a stark contrast to the case of $D \to K\pi$ decays where we had (updated with new measurements)

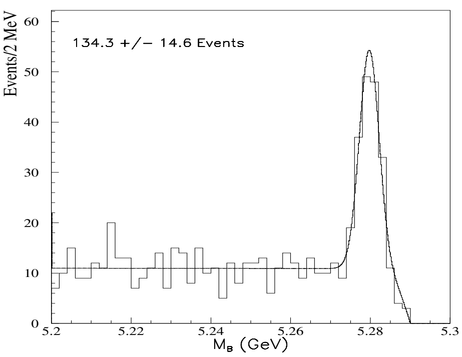

Figure 1. The beam-constrained mass for the sum of 8 $D^{(*)0,+}D_s^{(*)+}$ modes based on 1.56fb^{-1} of data collected by CLEO.

$$\begin{aligned} c_1(m_c) &= 1.26 & c_2(m_c) &= -0.51 \\ a_1 &= 1.15 \pm 0.03 & a_2 &= -0.51 \pm 0.03 \end{aligned}$$

which indicated that $c_i = a_i$ and prompted the so-called 'rule of discarding $1/N_c(N_c = 3)$ terms' in $1/N_c$ expansions[15]. For class-1 decays, it was shown by QCD sum rules that the contribution from the octet-octet term may cancel the 1/3 term in (2)[16]. For class 2 and 3 decays, however, nothing firm can be said at this point. The value of $c_{1,2}$ themselves may have significantly different values for $b \to c\bar{c}s$ vs $b \to c\bar{u}d$[17]. The positive sign of a_2/a_1, however, may have an implication on the phenomenological explanation[18] of $B_{\rm sl}$ (B semileptonic branching ratio)[14] which seems to be smaller than inclusive calculations[19].

$D^{(*)}\pi^{(*)}$ Class 1 Branching Fractions(%)			
mode	CLEO	Model	$a_1 = 1.15$
$D^+\pi^-$	0.29±.04±.03±.05	$0.264a_1^2$	0.35
$D^+\rho^-$	0.81±.11±.12±.13	$0.621a_1^2$	0.82
$D^{*+}\pi^-$	0.26±.03±.04±.01	$0.254a_1^2$	0.34
$D^{*+}\rho^-$	0.74±.10±.14±.03	$0.702a_1^2$	0.93
$D^{(*)}\pi^{(*)}$ Class 2 Branching Fractions(%)			
mode	CLEO 90% U.L.	Model	$a_2 = 0.26$
$D^0\pi^0$	<0.048	$0.201a_2^2$	0.014
$D^0\rho^0$	<0.055	$0.136a_2^2$	0.009
$D^{*0}\pi^0$	<0.097	$0.213a_2^2$	0.014
$D^{*0}\rho^0$	<0.117	$0.223a_2^2$	0.015
$D^{(*)}\pi^{(*)}$ group $R = \Gamma(\text{Class3})/\Gamma(\text{Class1})$			
mode	CLEO	Model	$a_2/a_1 = 0.23$
$D\pi$	1.89±.26±.32	$(1+1.23a_2/a_1)^2$	1.65
$D\rho$	1.67±.27±.30	$(1+0.66a_2/a_1)^2$	1.33
$D^*\pi$	2.00±.37±.28	$(1+1.29a_2/a_1)^2$	1.68
$D^*\rho$	2.27±.41±.41	$(1+0.75a_2/a_1)^2$	1.37
$D^{(*)}D_s^{(*)}$ (Class 1) Branching Fractions(%)			
mode	CLEO	Model	$a_1 = 1.05$
$D^+D_s^-$	0.75±.25±.20±.18	$1.213a_1^2$	1.33
$D^+D_s^{*-}$	0.90±.34±.20±.22	$0.859a_1^2$	0.94
$D^{*+}D_s^-$	0.99±.28±.20±.24	$0.824a_1^2$	0.90
$D^{*+}D_s^{*-}$	1.69±.50±.36±.41	$2.203a_1^2$	2.42
$D^0D_s^-$	1.55±.28±.30±.38	$1.215a_1^2$	1.33
$D^0D_s^{*-}$	0.83±.27±.20±.20	$0.862a_1^2$	0.95
$D^{*0}D_s^-$	1.00±.47±.22±.24	$0.828a_1^2$	0.91
$D^{*0}D_s^{*-}$	2.12±.75±.52±.52	$2.206a_1^2$	2.42
$\Psi K^{(*)}$ (Class 2) Branching Fractions(%)			
mode	CLEO	Model	$a_2 = 0.26$
ΨK^-	0.110±.015±.009	$1.819a_2^2$	0.123
ΨK^{*-}	0.178±.051±.023	$2.932a_2^2$	0.198
ΨK^0	0.075±.024±.008	$1,817a_2^2$	0.123
$\Psi \bar{K}^{*0}$	0.169±.031±.018	$2.927a_2^2$	0.198

Table 2. Values of a_1 and a_2 are fitted separately for $D^{(*)}\pi^{(*)}$, $D^{(*)}D_s^{(*)}$, and $\Psi K^{(*)}$ decay mode groups. Class 3 $D^{(*)}\pi^{(*)}$ decays are not used in the fit. The data is from Ref.[8, 11] where the first error is statistical and third systematic. When there is a third error, it reflects uncertainties in the branching ratios of charmed mesons (in particular, $\mathcal{B}(D_s^+ \to \phi\pi^+) = 3.7 \pm 0.9\%$). The model is an 'improved' BSW model of Ref.[2] where $f_D = f_{D^*} = 220$ MeV, $f_{D_s} = f_{D_s^*} = 280$ MeV, $f_\rho = 205$ MeV, $f_\Psi = 384$ MeV, and $(\tau_B/1.63\text{ps})^{1/2}|V_{cb}| = 0.039$ were used.

If we assume a_1 is the same for the $D^{(*)}D_s^{(*)}$ and $D^{(*)}\pi^{(*)}$ groups, one can obtain the D_s decay constant:

$$f_{D_s} \overset{\text{assume}}{=} f_{D_s^*} = 285 \pm 11 \pm 29 \pm 25\text{MeV}$$

where the last error reflects the model dependence of form factors, and the rate for the sum of $D^{(*)}D_s^{(*)}$ modes was extracted from the inclusive D_s spectrum.

5. Charm Deficit

Table 3 shows the B meson inclusive branching ratios as measured by CLEO. The total number of charm per B decay n_c is then 1.07 ± 0.08. For a 'reasonable set of values' $m_b = \mu(\text{renormalization scale}) = 4.8$ GeV, $m_c = 1.5$ GeV, and $\Lambda_{\overline{MS}} = 180$ MeV, the expected n_c is 1.15[20]. This is higher than the measurement, but only by 1σ. However, one possible explanation of the small $B_{\rm sl}$ is that $b \to c\bar{c}s$ mode is enhanced, either by a smaller effective m_c, or by non-perturbative effects[21, 19, 20]. Such scenario is becoming more and more unnatural, because 1) the good agreement between theory and experiment for the $D^{(*)}D_s^{(*)}$ decays as shown above, and 2) in baryonic decays data indicate $(b \to c\bar{c}s)/(b \to c\bar{u}d) = 0.20 \pm 0.13 \pm 0.04$[22]. Nonetheless, if the $b \to c\bar{c}s$ enhancement is the cause of the small $B_{\rm sl}$, then one would expect n_c in the rage 1.2 to 1.3 which makes the charm deficit significant again.

mode	CLEO Branching Ratio(%)	charm weight
D^+X	24.3±3.2±1.9±2.6[23]	1.0
D^0X	59.6±3.1±3.6±2.9[23]	1.0
D_sX	11.81±0.43±0.94±2.87*	1.0
Λ_c	6.4±0.8±0.8[24]	1.0
ΨX	1.13±0.04±0.06*	4.0
total # charm (n_c) = 1.07±0.08		

Table 3. Inclusive branching ratio measurements. New results for this conference are marked by '*'. A ΨX mode counts as 4 charms since a Ψ has two charms and we assume that there are as many charmonium states that do not decay through Ψ.

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