MEASUREMENT OF HADRONIC SPECTRAL MOMENTS IN τ DECAYS AND A DETERMINATION OF α_s

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ABSTRACT

We present measurements of spectral moments extracted from the invariant mass distributions of the final states of hadronic τ decays recorded in the CLEO detector. From the fit of theoretical predictions to the measurements of spectral moments and the total hadronic decay width of τ, we determine the strong coupling constant and a set of non-perturbative QCD parameters. The strong coupling constant is measured to be $\alpha_s(M_\tau)$=0.309$\pm$0.024 which, extrapolated to the Z^0 mass, yields $\alpha_s(M_{Z^0}) = 0.114\pm0.003$; where the error includes contributions from experimental and theoretical uncertainties.

1. Introduction

The τ, which is the only known lepton heavy enough to decay into hadrons, presents a clean environment to study the hadronic weak currents. The total τ hadronic width and the spectral moments (defined in next section) have been calculated[1,2] using analyticity and the Operator Product Expansion (OPE). This is one of very few processes calculated to $\mathcal{O}(\alpha_s^3)$. Hence, the missing higher order corrections are expected to be small. Thus, this process has been advertized as the best method to determine the most precise value of α_s. The ALEPH collaboration[3] has used the above procedure to determine the value of α_s from τ hadronic decays and has demonstrated that the experimental uncertainties are dominant over the theoretical uncertainties in this procedure. Thus, it becomes of great interest to use the large data sample and the excellent π^0 reconstruction capabilities of CLEO[4] in order to improve the experimental precision on this measurement.

2. Experimental and Theoretical Definitions of Spectral Moments

Analogous to the hadronic to leptonic decay width ratio in e^+e^- collisions, one may define the ratio of total hadronic decay width of τ as:

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$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau hadrons)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = \frac{1 - B_e - B_\mu}{B_l} = \frac{1}{B_l} - f_e - f_\mu \tag{1}$$

where B_e and B_μ are the branching fractions of τ decays to e and μ respectively, and f_e=1 and f_μ=0.9726 are the phase space factors. Theoretically, R_τ may be written as

$$R_\tau = \frac{1}{\Gamma_l} \int_0^{s_0} ds \frac{d\Gamma_h}{ds} \tag{2}$$

where s is the invariant mass-squared of the hadronic final state and $s_0 = m_\tau^2$. Within the existing theoretical skills, it is not possible to calculate $d\Gamma_h/ds$ directly. However, it is possible to calculate the weighted integrals of $d\Gamma_h/ds$, R_τ^{kl}, written as

$$R_\tau^{kl} = \frac{1}{\Gamma_l} \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^l \frac{d\Gamma_h}{ds}. \tag{3}$$

One should note that R_τ is the special case of R_τ^{kl} when k=0 and l=0. The normalized spectral moments, D_τ^{kl}, are defined as: $D_\tau^{kl} = \frac{R_\tau^{kl}}{R_\tau^{00}}$. The R_τ^{kl} have been predicted[5] using OPE, and can be written in the form

$$R_\tau^{kl}(u,d) = |V_{ud}|^2 \, S_{EW} r_{kl} \left[1 + \Delta_{m_q} + \Delta_p + \Delta_{np}\right] \tag{4}$$

where V_{ud}=0.9753 $\pm$ 0.0004[6] is a CKM matrix element, $S_{EW} = 1.0194$ an electroweak correction,[8] r_{kl} the terms predicted at parton level, Δ_{m_q} the quark mass corrections, Δ_p the perturbative terms in powers of α_s, and Δ_{np} are the non-perturbative corrections.

The measured values of R_τ and the spectral moments, D_τ^{kl}, are used to perform a global fit in order to determine the value of $\alpha_s(M_\tau)$ and three non-perturbative parameters labelled as $< \frac{\alpha_s}{\pi} GG >$, $\mathcal{O}(6)$ and $\mathcal{O}(8)$ following the notation of reference.[5]

3. Measurement of R_τ and Spectral Moments

The value of R_τ is determined using equation 1 where B_l is determined by the following three methods: 1) $B_l = B_e/f_e = 0.1801 \pm 0.0018$, 2) $B_l = (\frac{\tau_\tau}{\tau_\mu})(\frac{m_\tau}{m_\mu})^5 = 0.1817 \pm 0.0019$, 3) $B_l = \frac{B_\mu}{f_\mu} = 0.1816 \pm 0.0024$; where the values for leptonic branching ratios, τ lifetime, and τ mass have all been taken from reference.[6] These three measurements yield the weighted average of $B_l = 0.1810 \pm 0.0012$ which yields $R_\tau = 3.552 \pm 0.035$.

The invariant mass distributions of the hadronic final states were computed from the selected events for the following decay channels: $\tau^- \to \pi^-\pi^0\nu_\tau$, $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$, $\tau^- \to \pi^-\pi^0\pi^0\pi^0\nu_\tau$, $\tau^- \to \pi^-\pi^0\pi^0\pi^0\pi^0\nu_\tau$, $\tau^- \to \pi^+\pi^-\pi^-\nu_\tau$, $\tau^- \to \pi^+\pi^-\pi^-\pi^0\nu_\tau$, and $\tau^- \to \pi^+\pi^-\pi^-\pi^0\pi^0\nu_\tau$. We tagged the τ events with the decay channel $\tau^- \to \mu^-\bar{\nu_\mu}\nu_\tau$ and studied the hadronic decay of the other τ (the inclusion of charge conjugate modes is implied throughout the paper). The event selection and cuts are the same as those described in reference.[7] The number of events which passed our selection criteria are listed in Table 1 for each channel. From these distributions, the background from the following sources was subtracted: 1) Cross channel contamination from other τ decay channels including Cabibbo suppressed channels, f_τ; 2) Fake π^0's constructed from unrelated photons, f_{π^0}; and 3) Contamination from hadronic events, f_h. To estimate the hadronic background, 582,500 multihadron events, generated by JETSET MC[9] and smeared through the detector effects, were put through our analysis chain. In any decay channel, not more than two out of 582,500 events could sneak through our analysis procedure. Background from other τ decay channels including the Cabibbo suppressed channels, f_τ, was determined from KORALB MC.[10] These background fractions are also listed in Table 1.

Mode	Events	Background(%)		
		f_τ	f_{π^0}	f_h
$\pi\pi^0\nu_\tau$	38388	4.71±0.10	2.07±0.07	0.0
$\pi 2\pi^0\nu_\tau$	7892	4.99±0.24	2.59±0.17	0.0
$\pi 3\pi^0\nu_\tau$	623	8.49±1.00	7.31±0.93	0.0
$\pi 4\pi^0\nu_\tau$	32	13.51 ± 6.04	5.40±3.82	0.0
$3\pi\nu_\tau$	22810	11.20±0.22	—	0.0
$3\pi\pi^0\nu_\tau$	4906	4.52±0.26	6.95±0.32	0.0
$3\pi 2\pi^0\nu_\tau$	285	20±4	7.5±1.7	0.0

Table 1: Number of selected events and the background fractions for various decay channels. For details see text.

To correct the measured mass distributions for the bin migration due to the finite resolution of the detector, a probability matrix with elements $P_{n\rightarrow m}$ was constructed using KORALB MC for each decay mode; where $P_{n\rightarrow m}$ is the probability that an event with a given decay mode generated in bin n will appear in bin m of the mass-squared distribution. The mass-squared distributions corrected by using these matrices were then corrected for the selection efficiency. The corrected distributions, normalized to unit area, weighted by the branching ratios taken from ref.,[6] were combined into a single distribution presented in Fig. 1. The π^- mass from $\pi^-\nu_\tau$ decay channel enters this distribution as a delta function (not shown in the figure). It is this distribution from which the spectral moments are computed using equation 3. The differential spectral moments are presented in Fig. 2. Their integral values are listed in Table 2 where the experimental errors are obtained by adding in quadrature the statistical errors and the systematic errors. The last column in Table 2 contains the values of the spectral moments computed from the raw data before making any background subtractions or detector corrections.

Moment	Value	$\sigma(stat)$	$\sigma(sys)$	$\sigma(exp)$	$\sigma(\%)$	Value (raw)
$D_\tau^{1,0}$	0.7287	0.0021	0.0052	0.0056	0.77	0.7265±0.0021
$D_\tau^{1,1}$	0.1553	0.0007	0.0007	0.0010	0.64	0.1542±0.0007
$D_\tau^{1,2}$	0.0559	0.0003	0.0007	0.0008	1.43	0.0554±0.0003
$D_\tau^{1,3}$	0.0250	0.0002	0.0005	0.0005	2.00	0.0246±0.0002

Table 2: The measured spectral moments from the corrected mass distributions.

The theoretical errors on the moments computed using ref.[2] are listed in Table 3. These error estimates are very similar to those in ref.[3] estimated by the similar method.

The errors listed in the first row of Table 3 contain contributions from uncertainties in non-perturbative corrections, electroweak and mass corrections, missing higher order perturbative corrections, and the renormalization scale. The second row contains contributions from all these sources except the non-perturbative corrections.

Error type	Error				
	R_τ	D_τ^{10}	D_τ^{11}	D_τ^{12}	D_τ^{13}
Σ_{th}	0.043	0.0133	0.0134	0.0032	0.0019
σ_{th}	0.033	0.0035	0.0028	0.0004	0.0002

Table 3: The theoretical errors on spectral moments computed from Reference.[2]

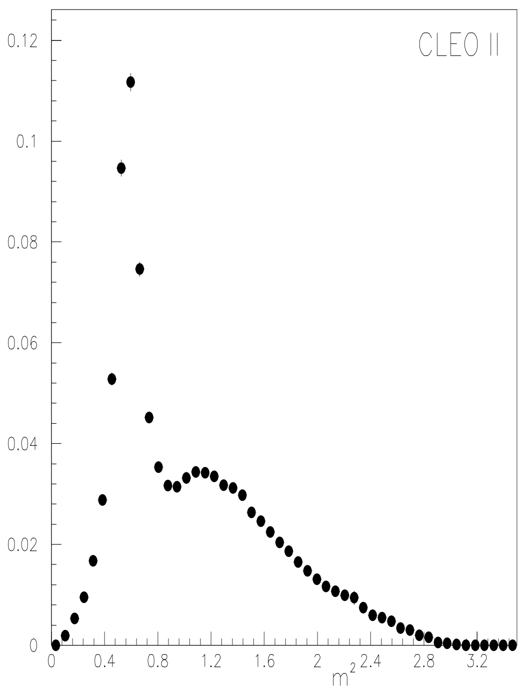

Fig. 1 Combined distribution of invariant mass squared.

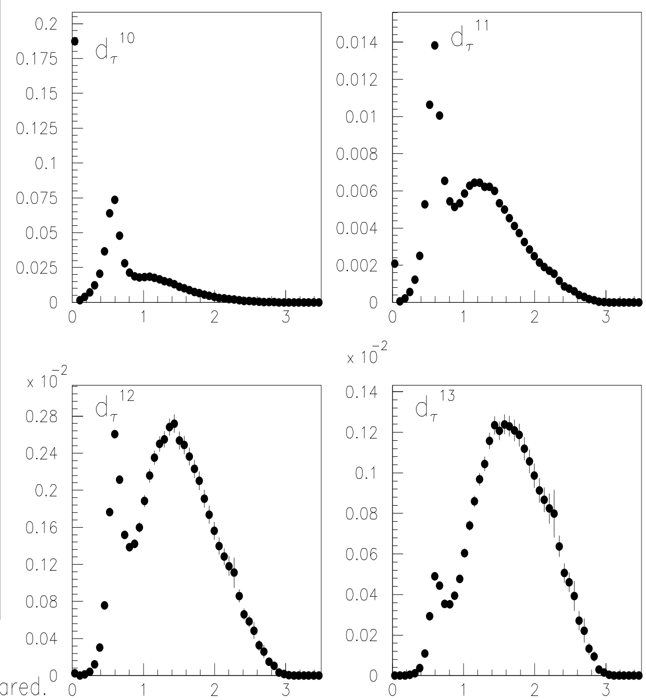

Fig. 2 Measured differential spectral moments.

4. Determination of α_s

The fits of the theoretical predictions to the experimental measurements were made for various combinations of the spectral moments and the results are listed in Table 4. In these fits, the correlations among different moments were taken into account. The simultaneous global fit to the measured values of R_τ and $D_\tau^{1,l}$ is a four-parameter fit where the values of α_s and non-perturbative parameters $< \frac{\alpha_s}{\pi} GG >$, $\mathcal{O}(6)$ and $\mathcal{O}(8)$ are determined together.This yields

$$\alpha_s(M_\tau) = 0.309 \pm 0.018 \pm 0.016, \tag{5}$$

where the first error is experimental and the second theoretical. These results are listed in the first row of Table 4. The other rows of Table 4 contain the values of $\alpha_s(M_\tau)$

determined when the non-perturbative parameters (NPP) are kept fixed at various sets of values. Row 2 is obtained when only the measured value of R_τ is compared with the theory for fixed values of NPP taken from reference.[2] Row 3 uses the measured values of $D_\tau^{1,l}$ from Table 2 and the NPP values from reference.[2] Row 4 uses the same set of values for $D_\tau^{1,l}$ but the NPP values from the global fit in row 1. The last row is the same fit but with modified theoretical errors on moments according to the errors on NPP determined from the global fit. The results for $\alpha_s(M_\tau)$ from R_τ alone and $D_\tau^{1,l}$ alone are consistent with each other. This checks the consistency of the method.

Observable	$\alpha_s(M_\tau)$	$<\frac{\alpha_s}{\pi}GG>$	O(6)	O(8)	χ^2
$R_\tau, D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.309±0.024	0.024±0.010	-0.0036±0.0010	0.0035±0.0008	2.1/1
R_τ	0.327±0.025	0.025	0.0022	0	—
$D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.327±0.037	0.025	0.0022	0	10.81/3
$D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.377±0.046	0.024	-0.0036	0.0035	0.31/3
$D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.349±0.030	0.024	-0.0036	0.0035	1.98/3

Table 4: Values of $\alpha_s(M_\tau)$ determined from the fits of the theory to the measured spectral moments. In case of non-perturbative parameters, no error means the parameter was kept fixed in the fit.

In the global fit, as the NPP are determined from the fit, the uncertainties on their values do not contribute to the theoretical errors on the spectral moments. In the fits of $D_\tau^{1,l}$ alone, the uncertainties on the values of NPP become the dominant source of theoretical errors (Table 3). This results in the larger errors on the values of $\alpha_s(M_\tau)$ listed in rows 3 and 4. For row 5, we recompute the theoretical errors on the moments going into the fit according to the errors on the NPP parameters determined from the global fit.

5. Systematic Errors

In this analysis, systematic errors may arise from the uncertainties in the mass scale, the background evaluation, the π^0 reconstruction, the unfolding procedure, and from the limited MC statistics used to correct the data. The error due to the mass scale uncertainty of 0.25 % was estimated by multiplying the mass-squared scale (X-axis) of Fig 1. by a factor of ± 1.005 and taking the resulting shifts in the moments as the errors due to the mass scale uncertainty. The errors due to the uncertainties in the branching ratios were estimated in a rather conservative way: Spectral moments were calculated by using all the branching ratios simultaneously 1)increased by their error, and 2) decreased by their errors. The larger resulting shift in the central value of a given moment is quoted as a symmetric error on the moment due to the uncertainties in the branching fractions. The errors due to the limited MC statistics used to correct the data are also included in the systematic errors. For example, the systematic error of $\pm$ 0.0052 on $D_\tau^{1,0}$ contains contributions of ± 0.0009 from mass scale uncertainty, ± 0.0025 from MC statistics, and $\pm$ 0.0044 from branching ratio uncertainties. The errors due to other sources listed above are largely included in the errors on the branching fractions, to the extent that these sources do not affect the mass shape. However, we made various qualitative checks on the effect of those uncertainties on our final results and the resulting deviations in the final results were found of insignificant size. More accurate and detailed studies of the systematics are in progress.

6. Conclusions

Using the hadronic spectral moments in τ decays measured by CLEO and the total hadronic decay width of τ R_τ, we have determined $\alpha_s(M_\tau)$=0.309±0.018± 0.016, where the first error is experimental and the second theoretical. Our result agrees with the ALEPH measurement[3] of $\alpha_s(M_\tau)$ = 0.330± 0.046. The agreement between the values of α_s obtained from R_τ alone and from spectral moments D_τ^{1l} points towards the applicability of perturbative QCD calculations in this low energy regime.[5] The values of the non-perturbative parameters obtained from the fits of the theory to the data agree well with those previously obtained from the fits of largely non τ data.[2] Using the formulae of ref.,[12] the value of $\alpha_s(M_\tau)$=0.309±0.018±0.016, extrapolated to 10.53 GeV, yields: $\alpha_s(10.53) = 0.167 \pm 0.005 \pm 0.004$ which is in good agreement with $\alpha_s(10.53) = 0.164 \pm 0.015$ determined from jet production rates at CLEO.[13] Extrapolated to M_Z, our result yields: $\alpha_s(M_{Z^0}) = 0.114 \pm 0.002 \pm 0.002$. Adding an error of ± 0.001 from the uncertainties in the extrapolation procedure arising from the crossings of c and b thresholds, this result becomes $\alpha_s(M_{Z^0}) = 0.114 \pm 0.003$, which is consistent within errors with $\alpha_s(M_{Z^0})$=0.120±0.006 determined from LEP data[14] on jet rates and event shapes.

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