REENTRANT CAVITIES HAVE HIGHEST GRADIENT AND MINIMAL LOSSES SIMULTANEOUSLY

Valery Shemelin
Laboratory for Elementary-Particle Physics, Cornell University, Ithaca, NY 14853

INTRODUCTION

As it was shown in tutorials at the SRF 2005 workshop [1], the consecutive usage of optimization algorithm for minimal losses leads to a reentrant shape of the cavity cell. The reentrant shape was also obtained as the optimal shape for maximal accelerating gradient if we believe that maximal gradient is limited by magnetic field and we minimize this field for a given overvoltage on the iris [2]. Now we will show that optimization for H_{pk}/E_{acc} leads to nearly the same geometry as optimization for $G \cdot R/Q$, at least with difference in these parameters less than 0.2 %. Here, H_{pk}/E_{acc} denotes ratio of peak magnetic field on the cell surface to accelerating gradient in this cell. This ratio is defined by geometry, and the lower is it the higher gradient can be achieved. $G \cdot R/Q$ is the product of the geometry factor and the geometric shunt impedance. This value is a measure of losses in the cavity. For a given surface resistance, losses are inversely proportional to it.

THE GEOMETRY FOR OPTIMIZATION

For this optimization, as we did earlier [2], we employ the construction of the cell profile line as two conjugated elliptic arcs (Fig. 1). The radius of the iris aperture is chosen by some additional consideration and is not the task of this optimization, the length L of the half-cell is taken as a quarter of the wave-length, and boundary conditions correspond to the π-mode. The value of the equatorial radius *Req* is used for tuning to the working frequency. Sure, a more intricate profile line can give a better eventual result, and we used earlier a description of the profile with 6 circular arcs [3]. However, an improvement of H_{pk}/E_{acc} was not more than 1 % in the case of 6 circle arcs in comparison to 2 elliptic arcs. Usage of a straight segment between 2 arcs, as it is done for the TESLA profile [4], also can be replaced by 2 elliptic arcs without a loss of parameters. Adoption of an elliptic arc for the equatorial area is crucial. The problem of cavity electric strength made to take the iris edge in a shape of ellipse far ago. We apply an ellipse to the inductive part of the cell because now we have a problem of magnetic strength.

And the last but not least argument in favor of 2 elliptic arcs is that in this case we have only 3 independent parameters for optimization: 3 half-axes (*A*, *B*, and *a*), the fourth one (*b*) is defined by geometrical restrictions. The value of the equatorial radius *Req* is used for tuning to the working frequency.

Calculations were done with TunedCell code that is a wrapper code for SLANS and was developed specially for fast optimization [5]. The SLANS code is known as a code with high accuracy [6] that is necessary for this optimization.

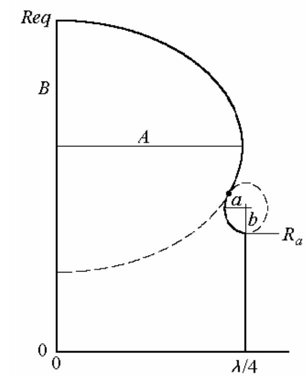

Fig. 1: Geometry for calculation.

RESULTS OF OPTIMIZATION

Optimization of the cell shape consisted in search of minimal value of H_{pk}/E_{acc} or maximal value of $G \cdot R/Q$ for a cell with a given value of the surface overvoltage E_{pk}/E_{acc}, where E_{pk} is peak electric field on the surface. Calculations are done for the cell frequency of 1.3 GHz but the results are suitable for any frequency because the issue of optimization is the shape, not dimensions.

As can be seen from the Table, optimizations for maximal gradient and for minimal losses give practically same results. We examined 3 cases: different apertures (iris radius R_a) with same E_{pk}/E_{acc} (see “a” and “b” in the Table), and different values of E_{pk}/E_{acc} for the same aperture (cases “a” and “c”).

The cause of same result for different goals of optimization is in minimization of magnetic field in both cases. When we search geometry for highest gradient, we strive to minimize the maximal magnetic field on the cavity surface. When we are looking for geometry with minimal losses, we minimize the mean value of H^2 over the surface. Any deviation from minimally possible magnetic field will be punished with a squared value of the field. For a given acceleration in the cell (E_{acc}), the magnetic flux is nearly the same for close geometries, and optimization leads to a minimally possible value of field on the surface in both cases. On the other hand this field is achieved on a larger surface area. This means that the maximum of the magnetic field becomes very flat. The three optimized shapes and the shape of the TESLA cell are shown in Fig. 2. Distribution of the surface fields along the cavity profile line is shown in Fig. 3 for those three optimizations and, for comparison, for the TESLA cell as well. Graphically shapes as well as fields optimized for the two goals, coincide.

Optimization leads to a longer flat part of the curves for magnetic field than the TESLA cell has. The curves for electric field are smoother for the optimized shapes.

Maximal differences in the values of H_{pk}/E_{acc} and $G \cdot R/Q$ are obtained for the second geometry (see the Table), when *Rbp* = 30 mm and $E_{pk}/E_{acc} = 2.0$. In this worst case we have difference in 0.19 %. for normalized magnetic field and 0.08 % for losses. Those deviations hardly can be noticed in practical measurement and can be treated as negligibly small. Maximal difference in geometrical dimensions, 0.2 mm, is also only slightly higher than possible accuracy of fabrication.

Table. Comparison of results for two goals of optimization.

<table>
<tr><td colspan="2">a) R_a = 35 mm,
$E_{pk}/E_{acc} = 2.4$</td><td>Optimization for max gradient</td><td>Optimization for min losses</td><td colspan="2">Difference in parameters</td></tr>
<tr><td rowspan="2">Figures of merit</td><td>H_{pk}/E_{acc}, Oe/(MV/m)</td><td>37.787</td><td>37.804</td><td>$\delta(H_{pk}/E_{acc})$, %</td><td>0.04</td></tr>
<tr><td>$G \cdot R/Q$, Ohm^2</td><td>33811</td><td>33818</td><td>$\delta(G \cdot R/Q)$, %</td><td>0.02</td></tr>
<tr><td rowspan="4">Geometrical dimensions, mm</td><td>A</td><td>51.53</td><td>51.40</td><td>ΔA</td><td>-0.13</td></tr>
<tr><td>B</td><td>36.25</td><td>36.37</td><td>ΔB</td><td>0.12</td></tr>
<tr><td>a</td><td>9.19</td><td>9.32</td><td>Δa</td><td>0.13</td></tr>
<tr><td>*Req*</td><td>98.713</td><td>98.731</td><td>Δ *Req*</td><td>0.018</td></tr>
<tr><td colspan="2">b) R_a = 30 mm,
$E_{pk}/E_{acc} = 2.4$</td><td>Optimization for max gradient</td><td>Optimization for min losses</td><td colspan="2">Difference in parameters</td></tr>
<tr><td rowspan="2">Figures of merit</td><td>H_{pk}/E_{acc}, Oe/(MV/m)</td><td>35.003</td><td>35.070</td><td>$\delta(H_{pk}/E_{acc})$, %</td><td>0.19</td></tr>
<tr><td>$G \cdot R/Q$, Ohm^2</td><td>38656</td><td>38686</td><td>$\delta(G \cdot R/Q)$, %</td><td>0.08</td></tr>
<tr><td rowspan="4">Geometrical dimensions, mm</td><td>A</td><td>54.00</td><td>53.90</td><td>ΔA</td><td>-0.10</td></tr>
<tr><td>B</td><td>38.96</td><td>38.85</td><td>ΔB</td><td>-0.11</td></tr>
<tr><td>a</td><td>7.60</td><td>7.80</td><td>Δa</td><td>0.20</td></tr>
<tr><td>*Req*</td><td>97.380</td><td>97.312</td><td>Δ *Req*</td><td>0.068</td></tr>
<tr><td colspan="2">c) R_a = 35 mm,
$E_{pk}/E_{acc} = 2.0$</td><td>Optimization for max gradient</td><td>Optimization for in losses</td><td colspan="2">Difference in parameters</td></tr>
<tr><td rowspan="2">Figures of merit</td><td>H_{pk}/E_{acc}, Oe/(MV/m)</td><td>39.900</td><td>39.917</td><td>$\delta(H_{pk}/E_{acc})$, %</td><td>0.01</td></tr>
<tr><td>$G \cdot R/Q$, Ohm^2</td><td>31839</td><td>31843</td><td>$\delta(G \cdot R/Q)$, %</td><td>0.04</td></tr>
<tr><td rowspan="4">Geometrical dimensions, mm</td><td>A</td><td>45.36</td><td>45.35</td><td>ΔA</td><td>-0.01</td></tr>
<tr><td>B</td><td>36.20</td><td>36.12</td><td>ΔB</td><td>-0.08</td></tr>
<tr><td>a</td><td>12.81</td><td>12.86</td><td>Δa</td><td>0.05</td></tr>
<tr><td>*Req*</td><td>100.685</td><td>100.644</td><td>Δ *Req*</td><td>-0.041</td></tr>
</table>

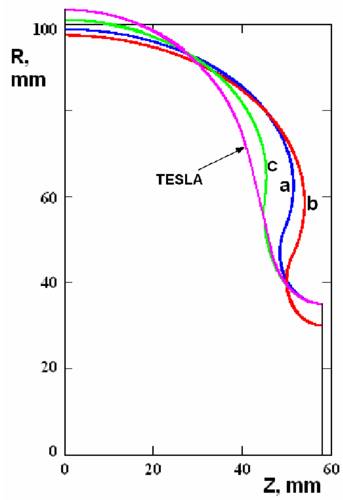

Fig. 2. Three optimized shapes and the shape of the TESLA cell.

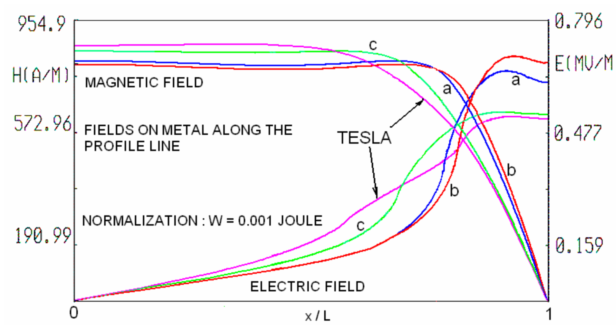

Fig. 3. Fields along the profile line for optimized shapes and for the TESLA regular cell. Curves a, b, and c correspond to 3 cases presented in the Table and in Fig. 2. The coordinate x along the profile line is normalized to the length L of this line (L is different for each shape).

CONCLUSION

It is shown that optimizations for maximal gradient and for minimal losses lead practically to the same cell shapes if these optimizations involve a possibility of reentrant shapes. Possibly, close results could be obtained also for non-reentrant shapes because in the optimization we strive to minimize maximal magnetic field on the surface or the mean value of squared magnetic field. Close goals lead to close results. Small difference between results of these optimizations is nevertheless distinctive and repeatable because the used code has high enough accuracy. A more intricate profile line than two elliptic arcs can improve the results of optimization but they cannot change too much.

REFERENCES

1. S. Belomestnykh, V. Shemelin. High-β cavity design. 12th Int. Workshop on RF superconductivity. Ithaca, NY, USA, July 10 – 15, 2005.

2. V. Shemelin, H. Padamsee, R. L. Geng. Optimal Cells for TESLA Accelerating Structure. Nucl. Instr. and Meth. A496 (2003) 1-7.

3. V. Shemelin, H. Padamsee. The Optimal Shape of Cells of a Superconducting Accelerating Section. Cornell University LNS Report SRF 0201128-01; 2002; TESLA Report 2002-1.

4. B. Aune et al. Superconducting TESLA cavities. Phys. Rev. ST – Accel. Beams **3**, 092001 (2000).

5. Dmitry Myakishev. TunedCell. Cornell University LEPP Report SRF/D 051007-01.

6. S. Belomestnykh. Spherical cavity: *Analitycal formulas. Comparison of computer codes.* Cornell University LNS Report SRF 941208-13. 1994.