Emittance Growth Study Using 3DE Code for the ERL Injector Cavities with Various Coupler Configurations *

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Abstract

We compute emittance growth in electron bunches in high gradient accelerating cavities due to field nonlinearities and phase dependence associated with the finite bunch length. The determination of the emittance is based on tracking of individual particles through tabulated, 3-dimensional, electromagnetic fields. No symmetry is assumed. The fields for the structure, including input coupler, are calculated using MAFIA or MWS[1]. We explore particle trajectories RF focusing and emittance growth of injector cavities with various coupler configurations studied as options for Cornell Energy-Recovery-Linac proposal [2].

1 INTRODUCTION

A study of beam emittance growth in ERL injector cavities [2] was done using the new code 3DE[3]. The study was done for four different coupler configurations over the particle energy range of $0.5\ MeV - 10\ MeV$. All four structures have two identical RF cells. The input and output beam pipe dimension of the first structure is slightly different than the pipe dimensions of the other three structures. The arrangements of the couplers in each structure is listed below:

- **Structure I** : Two cells cavity - NO couplers
- **Structure II** : Two cells + ONE vertical coupler at the bottom of the output pipe located at $z = 0.281\ m$ from the input, see Figure 1.
- **Structure III** :Two cells + TWO vertical symmetric couplers at the bottom and top of the output pipe located at $z = 0.281\ m$ from the input, See Figure 2.
- **Structure IV** :Two cells + TWO vertical asymmetric couplers, the same as structure III, accept there is a $1\ mm$ vertical shift of the bottom coupler relative to the top one.

The top coupler in structures III & IV was added in order to compensate the effect of the transverse kick of the bottom coupler on the emittance. In structure IV a $1\ mm$ shift was added to simulate practical imperfection in manufacturing.

2 THREE DIMENSIONAL ELECTROMAGNETIC FIELD

The three dimensional Electromagnetic field for all the structures were calculated using the code MWS[1]. The

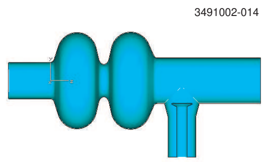

Figure 1: A two cells with one vertical coupler structure.

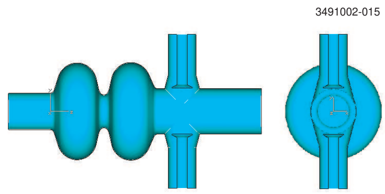

Figure 2: A two cells TWO symmetric vertical couplers structure.

fields were calculated for both electric and magnetic wall boundary conditions and were combined, in order to simulate propogating waves, using the following expressions [4]:

$$E = (-E_e + j \cdot k_n \cdot Em)\, \epsilon^{-j \cdot \omega \cdot t + \varphi} \qquad (1)$$

$$H = (H_e - j \cdot k_n \cdot Hm)\, \epsilon^{-j \cdot \omega \cdot t + \varphi} \qquad (2)$$

$E_e,\ H_e$, are fields with the electric wall boundary condition

$E_m,\ H_m$, are fields with the magnetic wall boundary condition

k_n is a normalization coefficient. The value of k_n is determined by the characteristic impedance of the coaxial line of the RF couplers. The sign of k_n is chosen to satisfies a condition for a propagating wave into the structure.

3 TRACKING

The 3DE code calculates the new kick on the particle due to the RF field at each coordinate and calls $BMAD's$ tracking routine [5], which uses Runge - Kutta algorithm, to calculate the particle new location. Using the new location

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3DE code calculates the new angles and particle energy. A comparison of the trajectories of three particles with vertical displacement of $y = -2,\ 0,\ 2\ mm$ and horizontal displacement of $x = -0,\ 0,\ 0\ mm$ entering structure II (two cells with ONE vertical coupler) and structure IV (two cells with TWO, 1 mm shifted, vertical couplers) is seen in Figure 3. The particles traveling in the ONE coupler structure are being kicked in the $-y$ direction after passing the location of the coupler at $z = 0.281\ m$, due to the transverse kick.

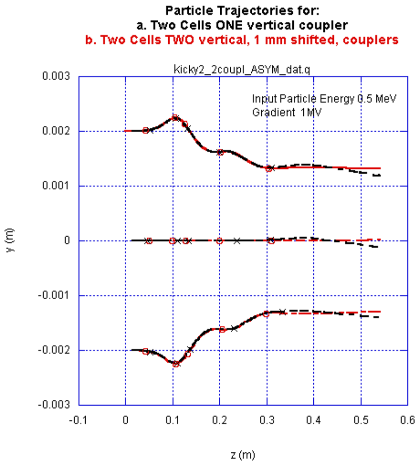

Figure 3: Particles trajectories in structures II & IV (one coupler & two asymmetric couplers)

4 RF FOCUSING

The RF cavities are acting as focusing lens, as can be seen in Figure 3. Knowing the particle trajectory and angle at each point, we can calculate the strength of the focusing lens by finding the slope of the curve $y'_{out}\ vs.\ y_{in}$. A comparison of the RF focusing versus Input-Particle-Energy for all four structures can be seen in Figure 4. The RF phase was optimized to get maximum acceleration at each input energy. The RF focusing of structures II, III, IV (two cells WITH couplers) is essentially identical and about 90% of the focusing with no couplers. When the gradient in the structure was raised from $1\ MV/l$ to $3\ MV/l$ ($l = 0.54\ m,\ total\ length\ of\ the\ structure$), the RF focusing strength was increased as seen in Figure 5 for the structure of two cells and ONE coupler.

5 EMITTANCE

The emittance obtained for structure I with two cells NO coupler for a uniform square distribution bunch, at input particle energy of 0.5 MeV gradient of $1\ MV/l$ with $\sigma y_{in} = 1\ mm,\ \sigma y'_{in} = 0$ is $2.833410^{-9}\ m-rad$. Note that the input emittance is nominally zero where the emittance ϵ is:

$$\epsilon_{RMS} = \sqrt{\sigma_y^2 \cdot \sigma_{y'}^2 - \sigma_{y,y'}^2} \qquad (3)$$

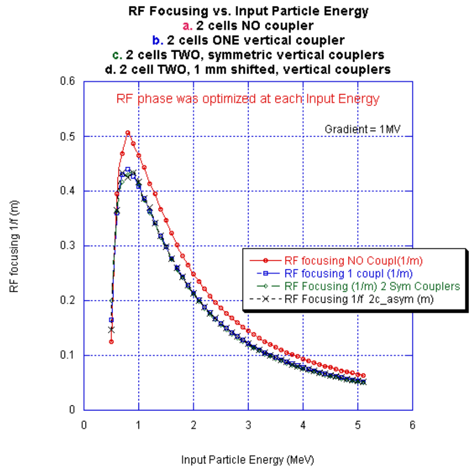

Figure 4: RF focusing versus Input-Particle-Energy for all four structures

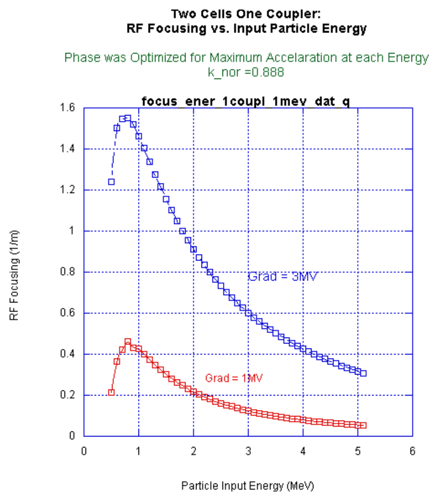

Figure 5: RF focusing versus Input-Particle-Energy for structure II (two cells with ONE coupler) with Gradients: $1\ MV/l,\ 3\ MV/l$

y and y' are the vertical displacement and angle, and σ_y and $\sigma_{y'}$ are the rms beam sizes. The corresponding phase space distribution is seen in Figure 6. The emittance calculated for a bunch with Gaussian distribution with input energy of $0.5\ MeV$ and Gradient of $1\ MV/l$, are summarized in Table I. The emittance phase space for bunch with Gaussian distribution for structures II (ONE coupler) & IV (TWO asymmetric couplers) are shown in figure 7 where the distribution of the vertical displacement y_{out} of the particles at the output of structures II & IV are shown in Figure 8, and the distribution of the transverse angle, y'_{out}, at the output are seen in Figure 9.

Calculation of emittance versus energy for II (ONE coupler) & IV (TWO asymmetric couplers) is shown in Figure 10. At low energy where the emittance is dominated by the RF focusing there is not much difference between one

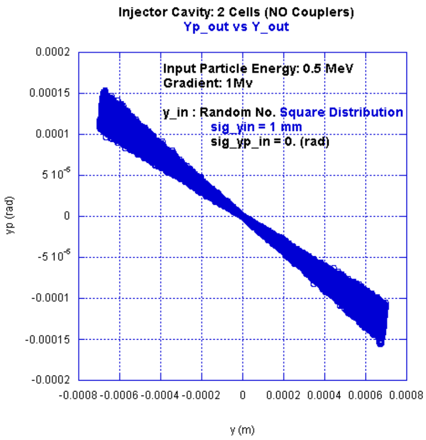

Figure 6: Phase space emittance of a 1 mm transversely squared bunch

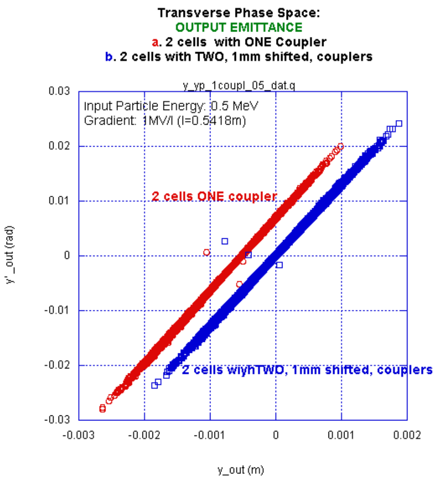

Figure 7: Phase space emittance of structures II (ONE coupler) & IV (TWO asymmetric couplers) at the output

coupler and two couplers. At high energy, where the non uniformity of transverse kick due to the coupler is the dominant contribution, we can see the effect of the cancellation in structure IV due to the second opposite coupler.

6 CONCLUSIONS

Calculation of the emittance (not including space charge effect) shows that the highest emittance obtained in these structures will be $4.74 \cdot 10^{-8}\ m - rad$ for the first cavity after the gun (input particle energy of $0.5\ MeV$). At this energy the different arrangements of couplers makes very little effect. On the other hand, when the particle input energy increases the non uniformity of the transverse kick is dominant. At input energy of $2\ MeV$ the output emittance of structure IV (TWO asymmetirical couplers) is $\frac{1}{16}$ of the emittance of structure II (ONE coupler). See Fgure 10. Having two couplers also keeps the bunch along the center of the beam pipe, where in the case of one coupler the bunch is displaced by $0.5\ mm$ in the $-y$ direction of the center, as seen in Figure 7.

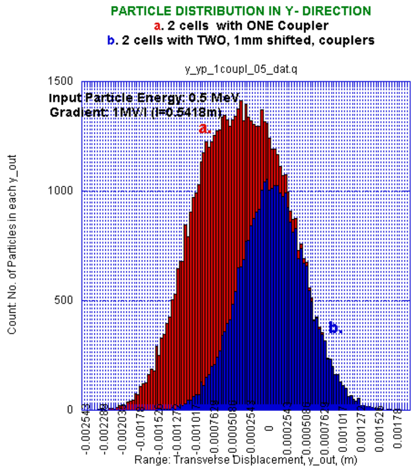

Figure 8: Particle distribution in y - direction at the output of structures II (ONE coupler) & IV (TWO asymmetric couplers)

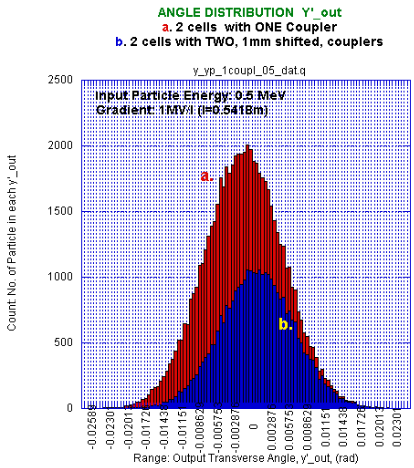

Figure 9: Distribution of the angle, y', at the output of structures II (ONE coupler) & IV (TWO asymmetric couplers)

7 ACKNOWLEDGEMENTS

I would like to thank D. Rubin for consultation and support. Many thanks to D. Sagan for his help with the BMAD tracking routine and to S. Peck for being very helpful with programing.

8 REFERENCES

[1] CST Microwave Stodio, User Guide, CST GmbH, Budinger Str. D-64289 Darmstadt, Germany

[2] V. Shemlin, SRF notes 021028-08

Table 1: Emittance calculated by 3DE for a Gaussian bunch at $0.5\ MeV$ with gradient of $1\ MV/l$. (l=0.5418m)

Structure	**ERL Injector Cavities Study**	
	Emittance m-rad	Notes
II	$4.397 \cdot 10^{-8}$	$Two\ cells\ ONE\ coupler$
$II\ off\ center$	5.35410^{-8}	$bunch\ is\ 1\ mm\ displaced\ in\ y$
III	$4.6916 \cdot 10^{-8}$	$Two\ cells -$ $TWO\ symmetric\ couplers$
IV	$4.74 \cdot 10{-}8$	$Two\ cells -$ $TWO\ asymmetric\ couplers$
$IV\ off\ center$	$4.34 \cdot 10^{-8}$	$bunch\ is\ 1\ mm\ displaced\ in\ y$

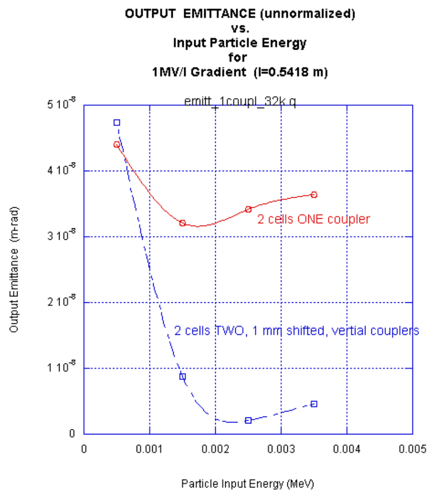

Figure 10: Emittance vs. Particles-Input-Energy of structures II (ONE coupler) & IV (TWO asymmetric couplers)

[3] Z. Greenwald, ”Three Dimension Emittance Calculation-3DE ” SRF 021203-10

[4] V. Shemelin, I. Bazarov, SRF minutes October 2002

[5] D. Sagan, Tracking subroutine ODIENT.F90