Three Dimension Emittance Calculation Code - 3DE*

Z. Greenwald †, Cornell University, Ithaca, NY

Abstract

It is expected to get emittance growth in an asymmetiric structure due to different kicks and non linearities seen by the head and the tail of the bunch. The code 3DE was developed in order to calculate the transverse emittance of a three dimensional structure where the electro-magnetic field in the structure is given (calculated, for example, by MAFIA[1] or MWS[1]). The code handles particles with velocities ranging from classic to relativistic. Space charge effect are not included in this code.

1 INTRODUCTION

The procedure used in the 3DE code to calculate emittance is described in the following sections. The emittance calculation is done by tracking the particles along the structure. It is calculated using Equation 1.

$$\epsilon_{RMS} = \sqrt{\sigma_y^2 \cdot \sigma_{y'}^2 - \sigma_{y,y'}^2} \qquad (1)$$

Where y and y' are the vertical displacement and angle, and σ_y and $\sigma_{y'}$ are the rms beam sizes. Various examples of particle tracking, focusing due to the RF field in the structure and emittance growth calculation are brought to demonstrate the 3DE code capabilities.

2 3DE ALGORITHM

The block diagram in Figure 1 outlines the steps used in the 3DE code to calculate particle trajectories, focusing due to RF field and emittance.

3 PRE RUN

The pre run block includes the steps shown in Figure 2.

3.1 Input Electromagnetic Field Simulation

The electromagnetic field in the structure, for the mode of interest, should be given in a three dimensional grid spaced by $\Delta_{x,y,z}$ of about one tenth of the smallest structure detail. The transverse dimension of the field grid should be at least 5 times the bunch rms sizes, σ_x, σ_y, in order to accurately simulate the particles in the tail of the bunch. The input electromagnetic field calculated by the RF codes are standing wave, therefore, it is necessary to add the time variation and phase, φ, in the form $e^{j\omega t+\varphi}$. ω is the resonance frequency of the structure mode of interest, and the phase φ is the phase between the RF field and the particle that corresponds to maximum acceleration. In

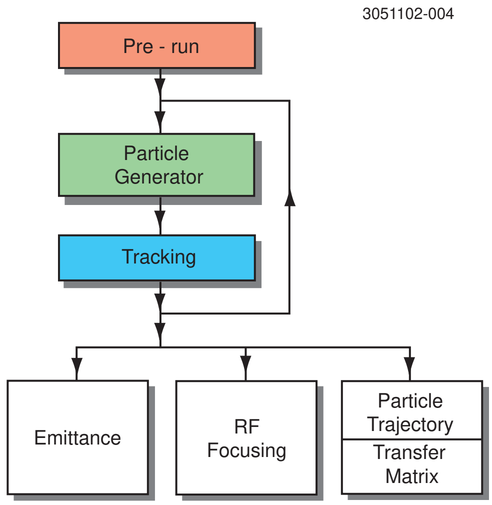

Figure 1: 3DE outline

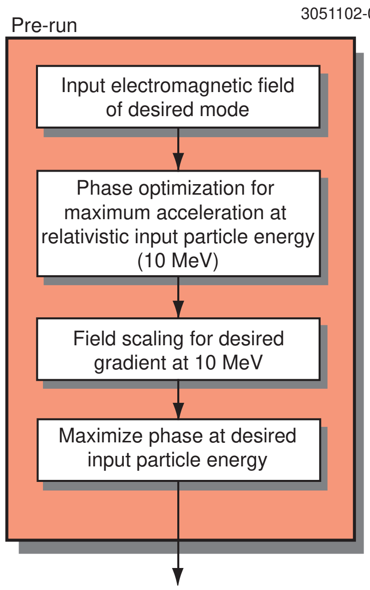

Figure 2: Pre Run outline

cases where the RF field is a traveling wave, like around an RF coupler, the simulated field should be constructed by combining the fields calculated for both boundary conditions of electric and magnetic walls [2]. Examples of the accelerating field E_z. The transverse fields E_y and B_x calculated using MWS [3] for boundary conditions for electric and magnetic walls are seen in Figures 3-5. The combined accelerating and transverse fields seen by a particle entering the structure at the center of the beam pipe are seen in figures 6,7.

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† Email: zipi@cesr10.lns.cornell.edu

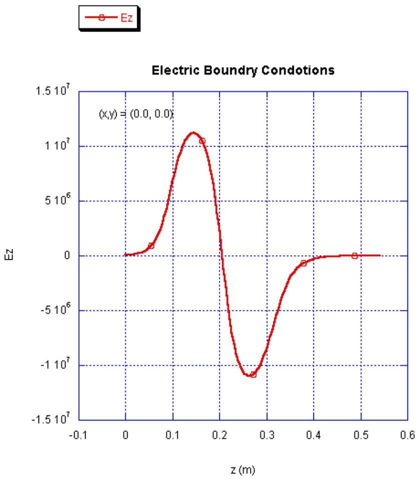

Figure 3: E_z Electric boundary condition

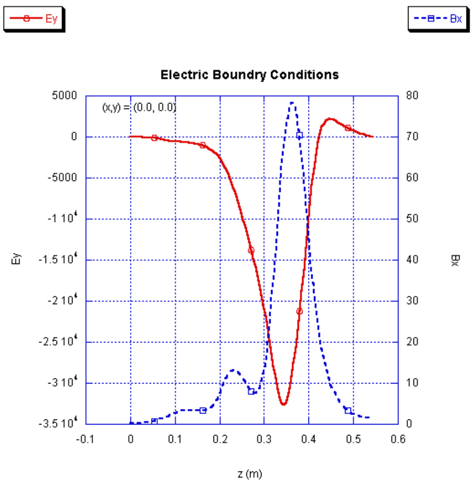

Figure 5: Transverse fields, E_y, B_x, for electric boundary conditions in a two cell one vertical coupler structure

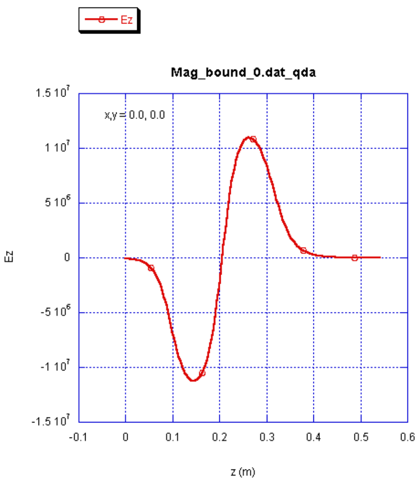

Figure 4: E_z Magnetic boundary condition

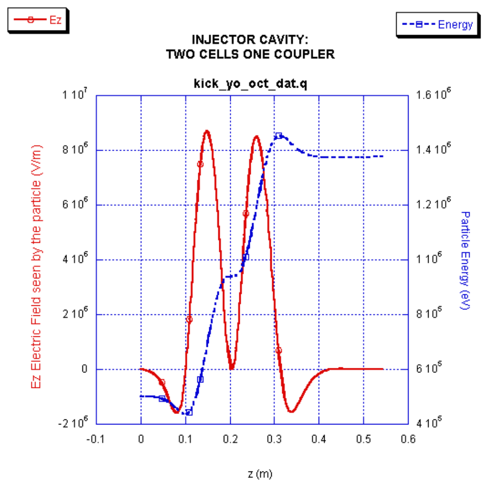

Figure 6: E_z seen by the particle

3.2 Phase, φ, Determination

In order to determine the phase for maximum acceleration, we set a particle at the input of the structure at the center of the beam pipe and calculate the integral of the field (acceleration voltage) along the particle trajectory as it travels through the structure. We change the phase untill we obtain maximum acceleration.. This procedure is done at input particle energy $10\ MeV$.

3.3 Field Gradient Scaling

The next step is to scale the input field to obtain the designed gradient of the RF structure. Holding the input energy at $10\ MeV$, and knowing the maximum value for accelerating field obtained from the raw input field in the former step, we can now calculate the scaling factor to obtain the desire gradient.

3.4 Optimizing the Phase, φ, for Lower Input Energy

The cavity dimensions were designed to work for relativistic particles. Therefore, the previous two steps wwere done at an input energy of $10\ MeV$. When the actual particle energy is much lower and its velocity is smaller than the speed of light, the particle acceleration will not be optimized. In order to maximize the acceleration volatage, we need to repeat the step of optimizing the phase,φ.

4 PARTICLE GENERATOR

Each single particle is characterized by six parameters in the vector $\mathbf{start} = (x, x', y, y', z, \delta E)$. x, y, z, are the particle coordinates, x', y' are the transverse angles within the bunch, δE is the particle initial energy. Particles can be arranged in various combinations as shown in Figure 8, by setting initial values in the vector $\mathbf{start}$. For example,

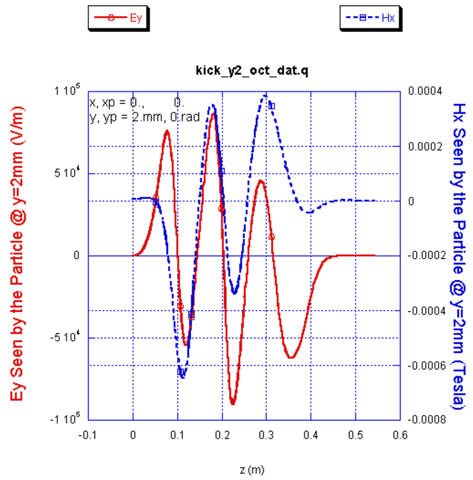

Figure 7: Total transverse fields, E_y, B_x, seen by the bunch in a two cells with ONE coupler structure.

to create a bunch with N particles and Gaussian distribution, we call the vector **start** N times and assign a random number with Gaussian distribution to $x,\ y,\ z$, and $x',\ y'$.

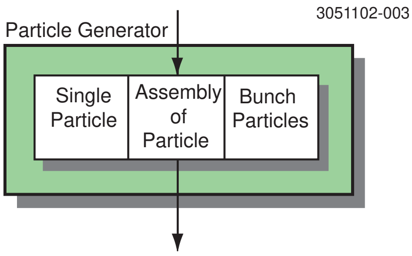

Figure 8: Particle - Generator block diagram

5 TRACKING

The block diagram describing the tracking is shown in Figure 9. The particle trajectory is calculated in small steps. At each step 3DE calculates the particle velocity, energy, and kick experienced by the particle, using the input electro magnetic fields. If the particle lands at a coordinate between the input fields grid points, then the field is calculated by a three dimension interpolator. Next, 3DE calls the BMAD tracking routine [4], which uses Runge-Kutta algorithm, to find the new coordinates the particle was kicked to. Figure 11 shows the trajectory of particles in both x and y planes for two cells with one vertical coupler, as seen in Figure10. The coupler is located at $z = 0.28\ m$ from the input causing a downward kick in the vertical trajectory.

6 RF FOCUSING

Particles in the RF structure are being focused as seen in Figure 11. In order to calculate the focusing strength of the RF structure, we launch a beam with parallel particles,

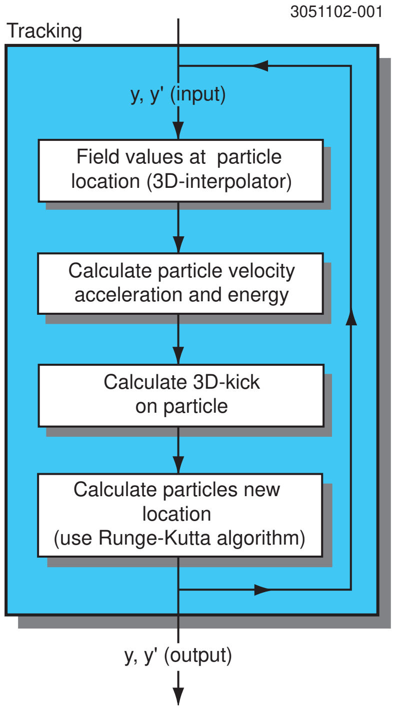

Figure 9: Tracking block diagram

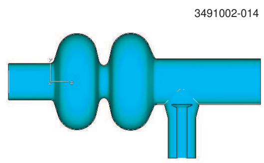

Figure 10: A two cells one vertical coupler structure.

$y'_{in} = 0$, and calculate the angle, y'_{out}, these particles will have when coming out of the structure. Figure 12 shows the RF focusing in both the vertical and horizontal directions of a two cells cavity with one coupler. The slope of the curve $x'_{out}\ vs.\ x_{in}$, is the focusing length $\frac{1}{f}\ (m)$. The $y'_{out}\ vs.\ y_{in}$ line is offset from the center due to the kick from the coupler located in the vertical direction.

Figure 13 shows focusing curve for three particles in the vertical direction: A particle at the center of the bunch which is synchronized with the field, a second particle at $2 \cdot \sigma_l$ in the head of the bunch, and a third particle at $-2 \cdot \sigma_l$ in the tail of the bunch. It can be seen that the three particles have different focusing due to different RF phase seen by the particles. This difference causes to an emittance growth. Figure 14 shows the dependence of the RF focusing on the input particle energy. The RF phase was optimized to achieve maximum acceleration at each input energy. Also shown in Figure 14 is the variation of the ver-

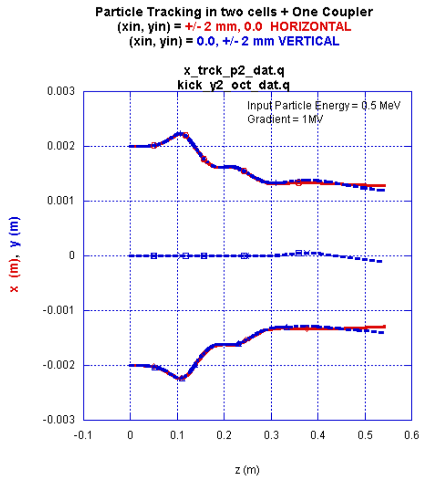

Figure 11: Particle transverse trajectories in a two cells one vertical coupler structure.

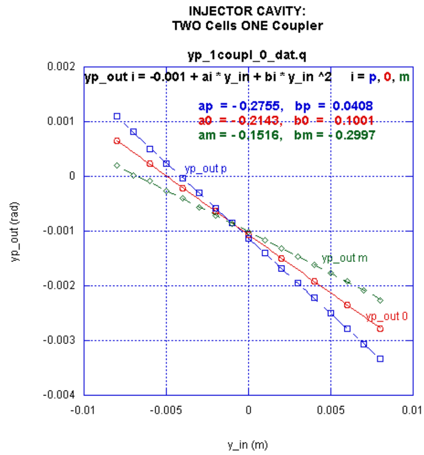

Figure 13: RF Focusing of three particles

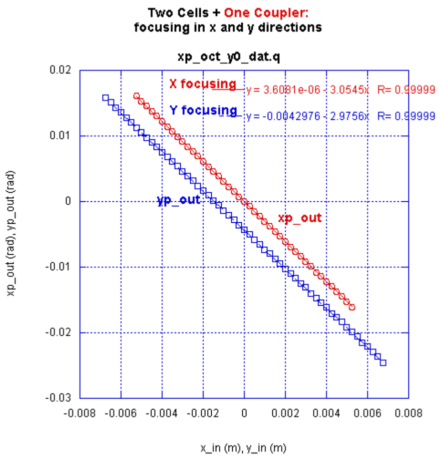

Figure 12: RF Focusing in the transverse dimension for two cells and one vertical coupler structure

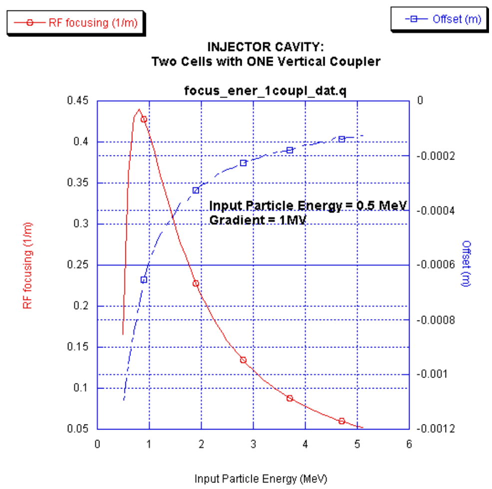

Figure 14: RF Focusing vs. input particle energy

tical offset of the bunch center with energy.

7 EMITTANCE

The calculation of the emittance, equation 1, requires the values of y, y' at the end of the structure. This is done by tracking all the particles in the bunch through the structure, taking into a count the three dimensional kick experienced by each particle. Using these values the averages y_{ave}, y'_{ave}, the values of the rms beam width, σ_y, σ'_y, and the correlation, $\sigma_{yy'}$, are calculated at the output to give the output emittance. Figure 16,17 show the distribution of the vertical displacement and angle, y_{out}, y'_{out}, at the output of a structure with two cells and two vertical couplers that are $1\ mm$ shifted as seen in Figure 15. The vertical input coordinate is generated by a random number generator with a Gaussian distributions. Figure 18 shows the phase space $y'_{out}\ vs.\ y_{out}$ For for the same structure.

8 CONCLUSIONS

3DE code is a very useful tool in studying the dynamics of particles/bunches in three dimensional accelerator structures. A demonstration of particle tracking and focusing calculation as function of particle energy for the ERL cavities have been brought. The emittance growth due to different kicks to particles in the head and tail of the bunch and also due to nonlinearities is calculated. Since the code tracks all the particles in the bunch, as opposed to an analytical approximation which considers only the effect of the kick on the particle along the center line [5][6], the calculated emittance includes also focusing effect which can be substantial at low energies. For a detailed study of the emittance of the injector cavities for the ERL see [7]

9 ACKNOWLEDGEMENTS

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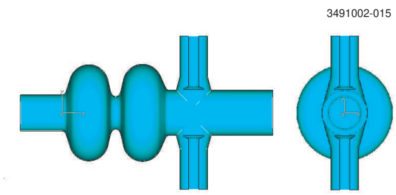

Figure 15: A structure with two cells two vertical couplers shifted by 1 mm

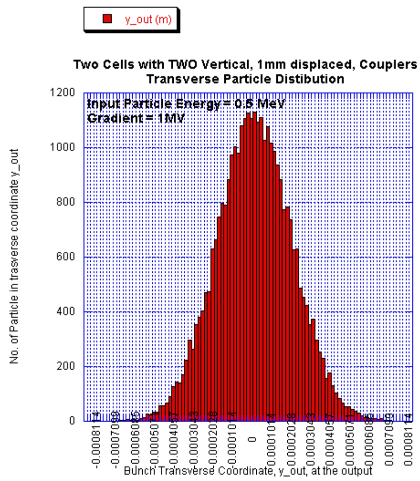

Figure 16: y_{out} vertical particle distribution

tracking routine and to S. Peck for being very helpful with programing.

10 REFERENCES

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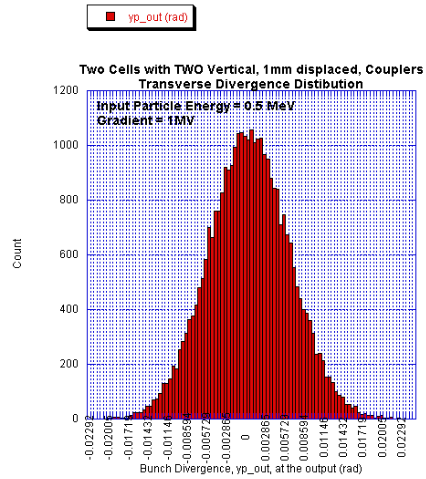

Figure 17: y'_{out}, particle's angle at the output

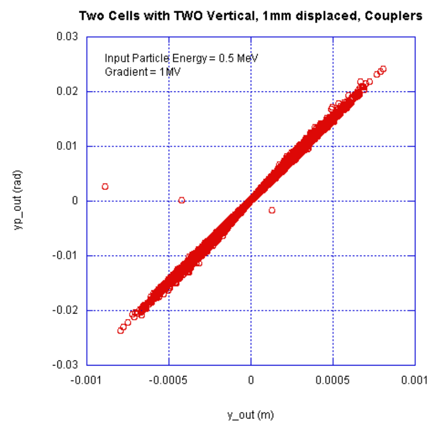

Figure 18: Emittance phase space of two cells two, 1 mm displaced, vertical couplers.