

# Calculation of the B-cell cavity external $Q$ with MAFIA\* and Microwave Studio\*

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## 1. Introduction

The Cornell B-cell cavity (see, for example, [1]) should have  $Q_{ext} \approx 2 \cdot 10^5$ , to operate with a beam current of the order of 1 A. The necessary coupling of the cavity with a waveguide and the shape of the coupling slot were found experimentally and compared with the results from a simple theoretical model in the process of the cavity R&D [2, 3].

The measured values of  $Q_{ext}$  for 7 cavities with a smooth waveguide have the mean value of  $1.84 \cdot 10^5$  in the range from  $1.75 \cdot 10^5$  to  $1.99 \cdot 10^5$  for different cavities [4].

The actual design of the waveguide connected to the cavity contains a step. The value of  $Q_{ext}$  with the step measured for 4 cavities is  $2.58 \cdot 10^5$  with scatter from  $2.51 \cdot 10^5$  to  $2.67 \cdot 10^5$ .

Now we take advantage of the new possibility to compute the value of  $Q_{ext}$  using a special command file with the MAFIA code [5] and with the Microwave Studio code (MWS)[6].

## 2. Method

A method to compute the external quality factor was proposed by M. Prome [7] and improved by P. Balleyguier [8]. The essence of the method is as follows (more detailed description is in the cited work [8]). By definition

$$Q_{ext} = \omega W / P,$$

where  $\omega$  is the resonant frequency,  $W$  is the stored energy,  $P$  is the power driven from the cavity into a coupled transmission line. This power can be computed either from electric or magnetic field amplitude by integrating over the line cross-section:

$$P = \frac{1}{2\mathbf{h}} \iint |E|^2 ds = \frac{\mathbf{h}}{2} \iint |H|^2 ds.$$

If the mode is TEM and the dielectric is vacuum, the mode impedance  $\mathbf{h} = \sqrt{\mathbf{m}_0/\mathbf{e}_0}$ . The stored energy in the cavity is the integral over the cavity volume:

$$W = \frac{1}{2} \iiint \mathbf{e}_0 |E|^2 dv = \frac{1}{2} \iiint \mathbf{m}_0 |H|^2 dv.$$

So the external  $Q$  can be expressed as

$$Q_{ext} = \frac{\omega \mathbf{e}_0 \mathbf{h} \iint |E|^2 dv}{\iint |E|^2 ds} \quad (1)$$

or

$$Q_{ext} = \frac{\omega \mathbf{m}_0 \iint |H|^2 dv}{\mathbf{h} \iint |H|^2 ds}. \quad (2)$$

For the case of a line with the TEM mode both equations are written in [8] as

$$Q_{ext} = \frac{\omega \iint |F|^2 dv}{c \iint |F|^2 ds},$$

$F$  being either electric ( $E$ ) or magnetic ( $H$ ) field. Here we will keep both expressions with the aim to include in the analysis the case of a hollow-pipe waveguide used for feeding the RF cavity in our set-up.

As it is proposed in [8], inverting the sign of time gives second solution of Maxwell's equations: the same cavity is slowly gaining energy from the incoming traveling wave. One can get a standing wave by adding or subtracting two traveling wave solutions. Let us add them first. Inside the line two traveling waves drive the same power  $P$  in opposite directions, and they interfere into a standing wave. Inside the cavity two fields have phase difference  $\mathbf{j}$ , so the amplitude of the resulting field is  $|1 + e^{i\mathbf{j}}|$  times the one of the original field. Using the same formal expression as in equation (1), we can define the quantity  $Q_E$  as:

$$Q_E = \frac{\omega \mathbf{e}_0 \mathbf{h} \iint |E_1|^2 dv}{\iint |E_1|^2 ds} = \frac{|1 + e^{i\mathbf{j}}|^2}{4} Q_{ext}.$$

The integral in the denominator is taken over the reference plane where the standing wave field amplitude is twice the one of the traveling waves.

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Now, following [8], let us subtract the same solutions. Instead of the electric antinode we have now a magnetic one at the same reference plane (electric wall). The result is noted with the subscript  $H$ :

$$Q_H = \frac{\omega \mathbf{m}_0 \iiint |H_2|^2 dv}{\mathbf{h} \iiint |H_2|^2 ds} = \frac{|1 - e^{ij}|^2}{4} Q_{ext}.$$

It is easy to see that

$$Q_{ext} = Q_E + Q_H. \quad (3)$$

Now we will generalize the solution from [8] to the hollow-pipe waveguides. Partly this result was obtained in the earlier paper of P. Balleyguier [9] but for the length of the line equal to a quarter wavelength (integer number of half wavelength can be added) in a waveguide. In [8] the result is presented for arbitrary length of a line but for TEM waves only.

Let us designate the ratios of the integrals for fields as

$$R_E = \frac{\iiint |E|^2 dv}{\iint |E|^2 ds} \text{ and } R_H = \frac{\iiint |H|^2 dv}{\iint |H|^2 ds}.$$

From (1), (2), and (3) we can obtain expressions for lines with TEM, TE, and TM waves.

$$\text{TEM: } Q_{ext} = \frac{2\mathbf{p}}{I} (R_E + R_H), \quad (4)$$

$$\text{TE: } Q_{ext} = \frac{2\mathbf{p}}{I} \left( \frac{\Lambda}{I} R_E + \frac{I}{\Lambda} R_H \right), \quad (5)$$

$$\text{TM: } Q_{ext} = \frac{2\mathbf{p}}{I} \left( \frac{I}{\Lambda} R_E + \frac{\Lambda}{I} R_H \right). \quad (6)$$

For a coaxial feeding line, one can find the electric and magnetic components of the  $Q_{ext}$ :

$$Q_E = \frac{2\omega UR}{V^2}, \quad Q_H = \frac{2\omega U}{RI^2}, \quad (7)$$

where  $U$  is the full energy in the cavity,  $R$  is the impedance of the line,

$$V = E \cdot r \cdot \ln(r_o/r_i) \text{ and } I = 2\pi r \cdot H$$

are the voltage and the current in the line,  $E$  and  $H$  here are amplitudes of the electric and magnetic fields at some radius  $r$  of the coaxial at the antinode of the corresponding field,  $r_o$  and  $r_i$  are the outer and inner radii of this line.

For the often-used TE01-mode in a rectangular waveguide let us also write both components of  $Q_{ext}$  in the explicit form:

$$Q_E = \frac{8\mathbf{p}\Lambda U}{I^2 \mathbf{e}_0 E_m^2 ab}, \quad Q_H = \frac{8\mathbf{p}U}{\Lambda \mathbf{m}_0 H_e^2 ab} \quad (8)$$

Here  $E_m$  and  $H_e$  are amplitudes of the corresponding fields at the end of the waveguide in its middle plane with the magnetic and electric wall respectively,  $a$  and  $b$  are the dimensions of the waveguide cross-section,  $I$  and  $\Lambda$  are the wavelengths in a free space and in a waveguide respectively,

$$\Lambda = \frac{I}{\sqrt{1 - (I/I_c)^2}},$$

$I_c = 2a$  is the cut-off frequency of the waveguide.

Numerical integration of  $R_E$  and  $R_H$  to calculate (4) was used in the MAFIA command file supplied by LANL [10]. We replaced this expression by (5) in the MAFIA command file for TE01 wave of the B-cell cavity waveguide.

In the calculation with MWS we used the analytical expression (8) for the same waveguide.

For the waveguides with TM waves one would need to use expression (6).

### 3. Geometry

The model used for calculations is presented in Fig. 1. The flutes on the left side of the cavity and the bends of the rectangular waveguide were omitted for simplicity. The step in the feeding waveguide must be taken into account during analysis because it has big influence on the value of the  $Q_{ext}$ . The step is symmetric, the dimensions of the waveguide change from 433×102 mm to 457×140 mm. If we discard the integer number of half-wavelength between the coupler and the step, the nominal position of the step is at 444 mm from the coupler.

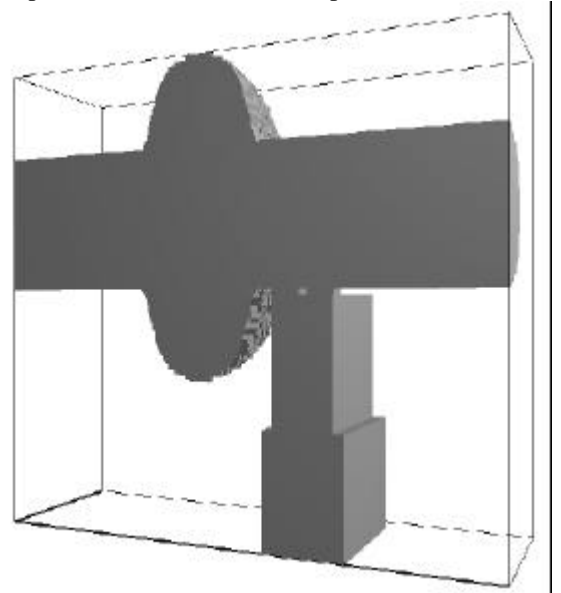


Fig. 1. The model computed with MAFIA.

The shape and dimensions of the coupling slot is the decisive factor for the value of  $Q_{ext}$ . The main features of the slot are shown in Fig. 2.

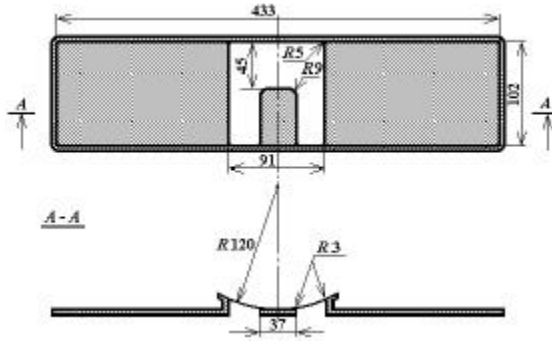


Fig. 2. Shape of the coupling slot.

## 4. Calculations

### 4.1 MAFIA

The calculations showed strong dependence of  $Q_{ext}$  on the number of the mesh cells. This number should be not less than about  $5 \cdot 10^5$  to have the variation of the value of the  $Q_{ext}$  no more than 10÷20 %. Further increase of the number of nodes leads to the unacceptable increase of the computing time. Redistribution of the mesh density is the main factor having influence on the reproducibility of the results. This could be associated with details of the tongue-like coupler: its dimensions are much smaller than dimensions of the whole assembly. On the other hand, if the mesh is kept constant everywhere but in the changing part of the assembly, the change of the  $Q_E$  and  $Q_H$  is very smooth.

Let us plot dependences of  $Q_E$ ,  $Q_H$ , and  $Q_{ext}$  in (3) on the length of the waveguide with no step (Fig. 3). We can see a sinusoidal change of  $Q_E$  and  $Q_H$  with  $Q_{ext}$  being constant. This constancy confirms validity of the obtained formulas:  $Q_{ext}$  should not depend on the length of waveguide.

As can be seen from the Fig. 3, in the case of a smooth waveguide, the calculated with MAFIA value of  $Q_{ext}$  runs close to  $2.4 \cdot 10^5$ .

For calculations of the geometry with the step the distance between the step and the end of the waveguide was 200 mm for all points in Fig. 4. As can be seen from this figure, the nominal calculated value of  $Q_{ext}$  is  $3.5 \cdot 10^5$ .

The relative increase of  $Q_{ext}$  due to the step is practically equal in calculations and measurements. The difference between measurements and calculations is about 30 % for  $Q_{ext}$  with as well as without the step, that can be explained both by the errors of computing and discrepancy between the model and actual dimensions.

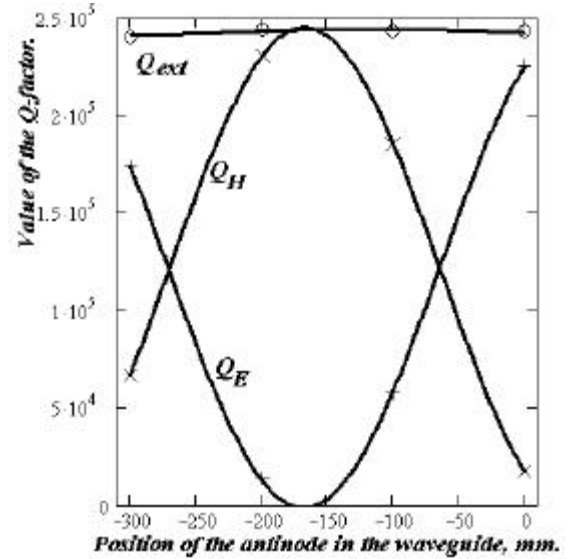


Fig. 3. Dependence of the  $Q_{ext}$  on the length of the smooth waveguide (MAFIA)\*.

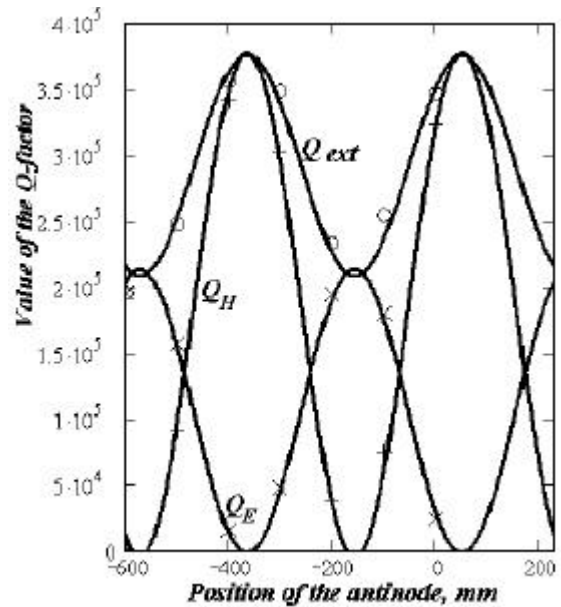


Fig. 4. Dependence of the  $Q_{ext}$  on the position of the step (MAFIA).

\* The origin of the abscissa in Figures 3, 4, 7 and 9 corresponds to the nominal position of the step: 444 mm from the coupler, but the step has zero height in the case of Fig. 3. Negative values of this coordinate correspond to moving off the axis of the cavity. Points are computed values, solid curves are sinusoidal regressions.

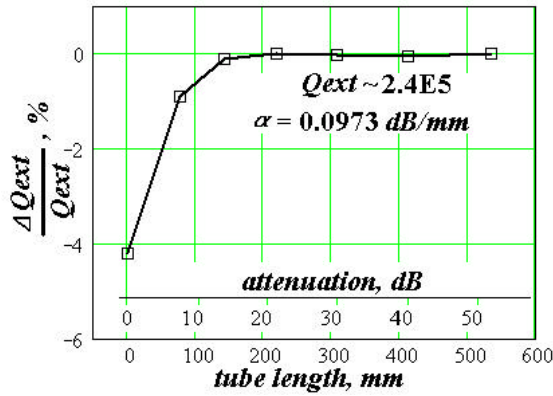


Fig. 5. Deviation of the  $Q_{ext}$  from its final value versus the length of the attached shortened beampipe.

The lengths of the cavity beampipes (diameter 240 mm) should be long enough not to disturb the value of the  $Q_{ext}$ , and short enough not to use extra meshpoints. Deviation of the  $Q_{ext}$  from its final value versus the length of the right-hand-side shortened beampipe is shown in Fig.5. The length of the beampipe is measured from the right plane of the feeding waveguide. Measures were taken not to change the mesh in the cavity and the waveguide while changing the beampipe length.  $Q_{ext}$  varies by less than 0.1% when the attenuation is more than 13.8 dB. Results reported in Fig. 3 and 4 are calculated for the beampipe length equal to 240 mm (two radii).

#### 4.2 Microwave Studio

The same geometry calculated with the Microwave Studio [6] gives closer to measurements results. We believe it is due to a perfect boundary approximation (PBA) used in MWS that helps to better describe the complexity of the coupling slot. The value of coupling has high sensitivity to variations of the slot dimensions. As it is declared in [6], the PBA method increases the accuracy of the simulation by an order of magnitude in comparison to conventional simulators. A convenient dialogue mode for creating and viewing geometric structures helps to avoid errors.

A blend edge operation was used for rounding the edges in construction of the coupler tongue. This operation is a good simulation of the actual machine-tool operation. The closed-up view of the coupler tongue is shown in Fig. 6.

The results obtained with MWS for conditions similar to ones of the Fig. 4 are shown in Fig. 7. The point correspondent to the nominal value of the step position is  $Q_{ext} = 2.53 \cdot 10^5$ . The value of the quality factor without the step is  $Q_{ext} = 2.00 \cdot 10^5$ .

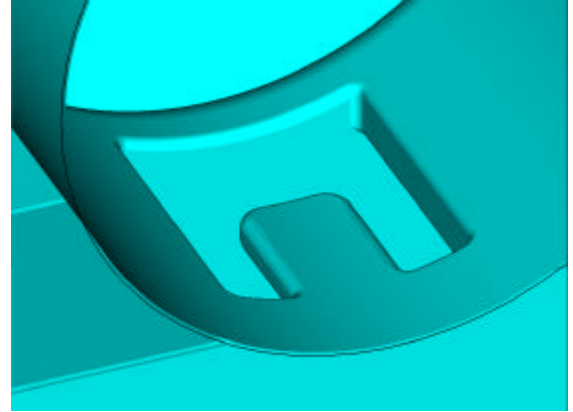


Fig. 6. The part of the model with the coupler slot (MWS).

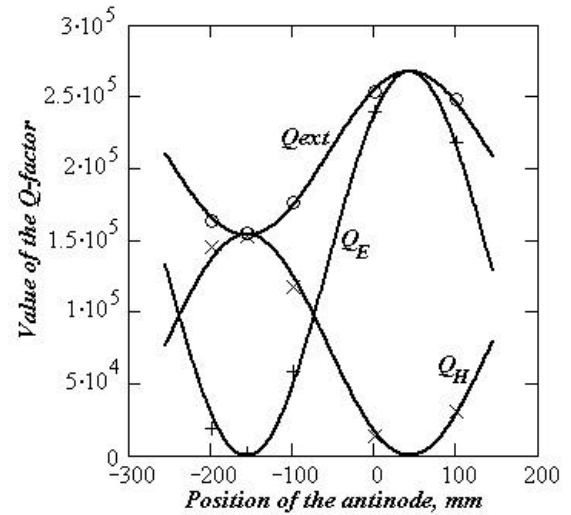


Fig. 7. Dependence of the  $Q_{ext}$  on the length of the waveguide in the case of a step (MWS).

#### 4.3 Equivalent circuit model

Another approach to evaluate  $Q_{ext}$  change due to the waveguide step employs equivalent circuits and a matrix calculation technique (see, for example, [11, 12]). The geometry from Fig. 1 is divided into components that form an equivalent circuit as shown in Fig. 8. The cavity (transformed to the waveguide) is modeled as a parallel RLC circuit with  $R$  equal to a characteristic impedance of the waveguide, and

$$wL = 1/wC = R/Q_{ext} .$$

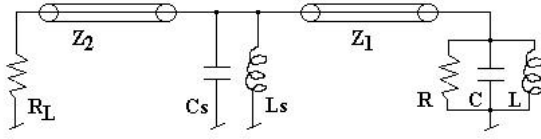


Fig. 8. The equivalent circuit.

The cavity rectangular waveguide ( $a_1 \times b_1 = 433 \times 102$  mm) and the window rectangular waveguide ( $a_2 \times b_2 = 457 \times 140$  mm) are modeled as transmission lines with characteristic impedances calculated from

$$Z_{1,2} = 120\pi \frac{\Lambda_{1,2}}{I} \frac{2b_{1,2}}{a_{1,2}}.$$

The step between two waveguides is modeled according to [12] as a parallel LC circuit. And, finally, the window waveguide is terminated by a matched load  $R_L = Z_2$ .

An ABCD matrix represents each component of the equivalent circuit. After multiplying all matrices one can use final matrix to find new  $Q_{ext}$ . This model has a free parameter, namely, initial  $Q_{ext}$  of the cavity. In this calculation we used the design value of cavity  $Q_{ext} = 2.0 \cdot 10^5$ .

Fig. 9 shows dependence of the  $Q_{ext}$  on the position of the waveguide step similar to those obtained with MAFIA (Fig. 4) and MWS (Fig. 7).

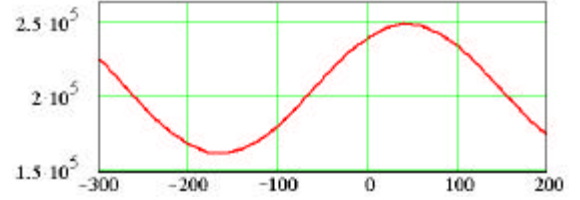


Fig. 9. Dependence of the  $Q_{ext}$  on the position of the waveguide step (in mm) calculated using equivalent circuit model for initial cavity  $Q_{ext} = 2.0 \cdot 10^5$ .

## 5. Summary

The method of  $Q_{ext}$  calculation is carried over to waveguide with TE or TM waves with an arbitrary position of the port.

All the results calculated and measured are summarized in the Table.

Table. Calculated and measured values of  $Q_{ext}$ .

	MAFIA	MWS	HFSS [13]	Equivalent circuit model	Measured
$Q_{ext}/10^5$ w/o step	2.4	2.00	2.6	(2.00)	1.84 (1.75...1.99)
$Q_{ext}/10^5$ with the step	3.5	2.53	-	2.4	2.58 (2.51...2.67)

The value of  $Q_{ext}$  for the B-cell cavity was calculated using MAFIA with a special command file and MWS. The obtained precision of 30% with MAFIA is not very high because of the complexity of the geometry and time of computing needed. Nevertheless, the validity of the method was shown when measures were taken to keep the computation mesh fixed everywhere except the changing part of the geometry. Also, the relative increase of the  $Q_{ext}$  due to the waveguide step is in good agreement with measured values.

The results obtained with MWS fit the measurements with precision of 2÷10 %. Together with better interface it makes Microwave Studio a better operative tool for more complicated geometries.

We were reported [13] about HFSS calculation results for determining the  $Q_{ext}$  of the Cornell SRF module obtained by Chaoen Wang (Taiwan, SRRC). The value of  $Q_{ext}$  obtained for the coupler with the tongue length of 57 mm was about  $2.6 \cdot 10^5$ .

Finally, equivalent circuit model approach though as not as universal tool as MAFIA or MWS, provides good results and helped us to understand the effect of the waveguide step on the  $Q_{ext}$  at the early stages [14].

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## 6. References

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