The optimal shape of cells of a superconducting accelerating section

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Abstract

The shape of TESLA accelerating structure can be improved to decrease maximal surface magnetic field by sacrificing and increasing maximal surface electric field. This sacrifice may be necessary because the magnetic field is a hard limit but electric field emission can be decreased by processing. For the case of superconducting cavities RF magnetic strength should be of a more concern.

1. Introduction

In deciding on a cell shape of a SC accelerating section, it is necessary to ensure both electric and magnetic strength. As an indicator of a correct choice of the shape one can use the ratios of the maximal electric and magnetic field strength on the cell surface to the acceleration rate achievable in the given cell:

$$\frac{E_{\max}}{E_{acc}} = \frac{E_{\max}}{\Delta W/L} = \frac{E_{\max}}{2\Delta W/\lambda}, \quad \frac{H_{\max}}{E_{acc}} = \frac{H_{\max}}{2\Delta W/\lambda}.$$

Here ΔW is the energy gain (in volts) obtained at the cell length L equal to half wavelength. We assume that the operating mode of oscillations is the π-mode. For the TESLA accelerating cavity, as reported in [1], these values are:

$$E_{\max}/E_{acc} = 2.0, \; H_{\max}/E_{acc} = 42.6 \text{ Oe/(MV/m)}.$$

Earlier data reported in [2] for the same values were **2 and 42,** and we use here these last values as reference because
1) it is convenient to use “round” numbers;
2) the values of $E_{\max}/E_{acc}$ and $H_{\max}/E_{acc}$, according to our calculations for geometry given in [2], are slightly different from values cited in [1] and [2], and different for regular cells and end cells (moreover, two end cells have different shapes).

So, for demonstration, we will compare values of calculated fields with values from [2] and introduce for this purpose the normalized maximal electric and magnetic fields:

$$e = \frac{E_{\max}}{2E_{acc}}, \; h = \frac{H_{\max}}{42E_{acc}},$$

so that for the regular TESLA cells [2]

$$e = 1, \qquad h = 1.$$

The choice of the TESLA structure for comparison is made because it is supposed that this structure will be produced in multiple quantities, so the correct optimization can give an economic effect.

For TESLA we believe it is more important to reduce the maximal magnetic field $H_{\max}$ on the surface, even if we sacrifice and increase $E_{\max}$. This is because the critical magnetic field is a hard limit to quench of superconductivity and $E_{\max}$ is a soft limit: field emission can always be decreased by better cleanliness and by high power processing.

2. The code and geometry for calculations

We used for optimization the SLANS code [3]. This code has better accuracy [4] in comparison with earlier URMEL code used for calculation of TESLA cavities. As can be seen from Fig. 4.1 in [2] the precision of calculation of $E_{\max}/E_{acc}$ was about 1-2 %. According to [4], with SLANS we can expect accuracy between 0.01 and 0.1 %.

The SLANS code calculates the frequency, acceleration rate and values of maximal fields. Besides, the results of calculations give possibility to find the coupling coefficient and distribution of fields along the cell's profile line.

The profile line of the original TESLA cell is constructed as two arcs: elliptic and circular, and a segment of a conjugated straight line between them [1, 2] (the dashed line in Fig. 1). It is felt that more intricate line could give better values of *e* and *h.*

One calculation was performed with arcs of conjugated circles only (Fig. 2). It can be shown that any ellipse can be approximated with a good precision by 2 – 3 arcs of circles. One has greater number of free parameters in this case: an arc of ellipse is described by two half-axes and a parametric angle, two arcs of circles – by two radii and two angles. As for a stepwise change of curvature using the circles, it could be noted that it does not lead to sharp changes of field along the profile line.

The advantage of description with the help of circles is the simplicity of equations and a straightforward transition, if necessary, to a more detailed description of the shape – with a larger number of circular arcs.

One of the difficulties arising by repeating many times calculations of variants is the problem of geometry data input. For this purpose a MathCAD program was written where the values of angles and radii were used as initial data. The output of the program is an input geometry file for the SLANS code. The MathCAD program gives also the graphical display of the cell convenient for checking out (Fig. 1, 2).

We used 6 arcs (Fig. 2) to describe a half-cell (the cell is symmetric), thus we had 12 parameters (angle extents and radii of arcs) and 3 additional conditions: the length of a half-cell was equal to $\lambda/4$, the equatorial radius of a cell served to adjust the frequency, the sum of angles should be equal for the concave and convex parts of the cell. So we had 9 independent variables.

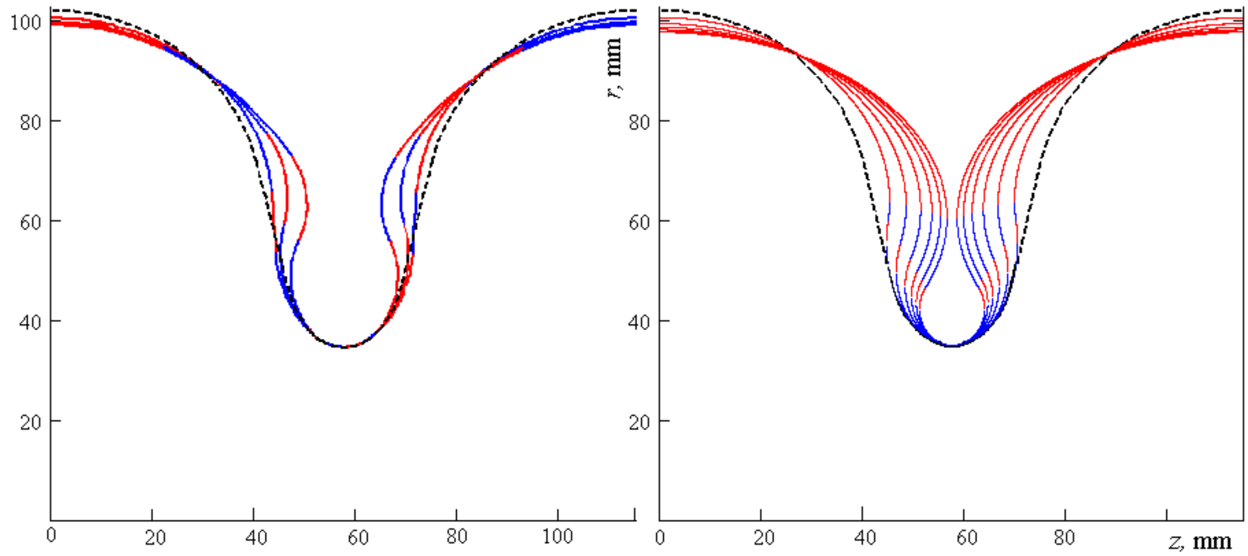

Fig. 1. Optimization of the TESLA regular cell shape. Dashed line – the present shape. On the left – the cell is described by 6 circular arcs, δe = -5, 0, +10 %. On the right – optimization with 2 elliptic arcs, δe = 0. 10, … , 50 %.

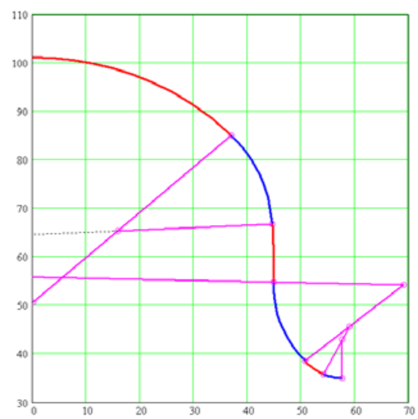

Fig. 2. A half-cell described by 6 conjugated arcs of circles.

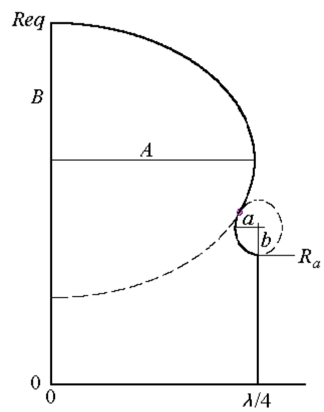

Fig. 3. Conjugated elliptic arcs.

Another approach was done with use of two elliptic arcs. This type of cavity has several features important for superconducting Nb cavities [5].

The problem of a cavity electric strength made to take the iris edge in a shape of ellipse already far ago [6]. Let us apply an ellipse to the inductive part of the cell because now we have a problem of *magnetic* strength.

After some optimization of the original geometry, the length of the straight interval conjugated to both ellipses appeared to be zero. So, we can describe the shape of the regular cell as two conjugated elliptic arcs (Fig.3). Now we have only 3 independent variables for searching the optimum: 2 half-axes of one ellipse (*A, B*) and one half-axis (*a*) of another ellipse because the conditions for the frequency and the length of the cell define other dimensions: another half-axis (*b*) and the radius of the cell equator *Req*. The aperture was taken as in the TESLA cell, R_a = 35 mm.

3. Optimization

The process of optimization consists in searching a cell shape with a minimal value of the maximal normalized surface field in this cell. As far as the optimized parameters are $E_{\max}/E_{acc}$ and $H_{\max}/E_{acc}$ (or *e* and *h*), the result of the optimization should be a function $h(e)$, such one that for any given normalized electric field *e* there should exist the only value of normalized magnetic field *h*. If such a function is found, the curve representing it (Fig. 4) satisfies the symmetric condition also: for any given normalized magnetic field *h* there exists the only normalized electric field *e*. It means that $h(e)$ is a monotonous function and from physical reasons this function should be decreasing.

Now the problem of maximal electrical strength is a special case of our problem: maximal electric strength is achieved with the shape corresponding to the leftmost point on the curve $h(e)$. If the problem of maximal magnetic strength exists, when the electric field can have any value, the rightmost point on this curve would correspond to this case. However, we do not analyze here the behavior of the function $h(e)$ at the extreme points.

The results obtained by optimization can be used at any different operation frequency because the values of *e* and *h* depend on the shape only, not on the dimensions of the resonant cavity. On the other hand, the value of the critical field *H* depends on the frequency [1] and the choice of the working point on the curve of Fig. 4 can be different for different frequencies.

For different ratios of the aperture to the wavelength the optimization curves will be different: the larger values of *h* for a given *e* on these curves will correspond to larger values of the ratio of the aperture radius to the wavelength: R_a/λ. Note for comparison that the cylindrical resonator with the length of $\lambda/2$ and with zero radius of the aperture has the following normalized values: $e_0 = 0.7855$, $h_0 = 0.7260$. It would be useful to have a map for different values of this ratio. The lower long-dashed curve on Fig. 4 is shown qualitatively.

The coupling coefficient *k* can be a significant factor of the choice of the cell shape. On the plot *h* vs. *e,* besides the set of curves with $R_a/\lambda = \text{const}$, one can construct a set $k = \text{const}$.

The data presented on the Fig. 4 correspond to the regular cell of the structure. The end cells have somewhat different dimensions and their optimization should be discussed separately.

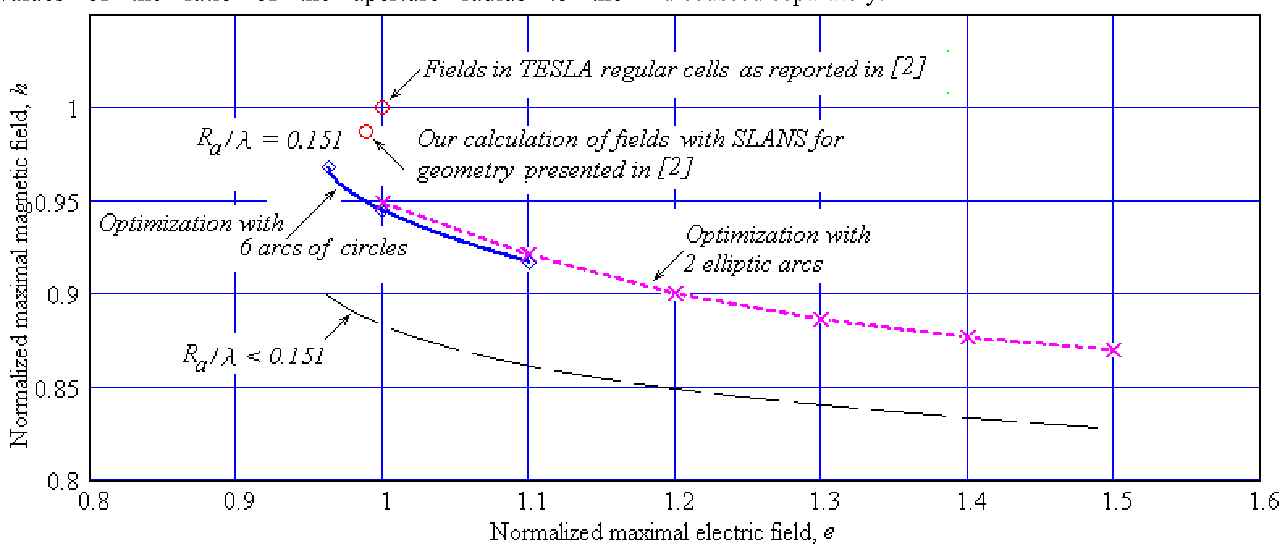

Fig. 4. Optimization curve for TESL-like geometry.
The lower long-dashed curve is shown qualitatively for a less value of R_a/λ.

4. Results

Our calculations showed that for the TESLA regular cell geometry [2] maximal fields differ slightly from the values of fields presented also there: the electric field is 1.2 % below and magnetic field is 1.3 % below for the regular cells (see Fig. 4). The coupling coefficient *k* is obtained the same: 1.87 %.

The results of calculations with 6 circular arcs are only slightly better than obtained with simpler geometry with two elliptic arcs. It looks like the elliptic shape is adequate to the task. The shapes obtained for both approaches are presented in Fig. 1.

For better comparison the results are summarized in the Table 1. Also the values of the coupling coefficient and iris thickness are presented there. This thickness becomes small at the biggest values of *e* that can be technologically inconvenient.

The re-entrant shape of optimized cells could be another technological shortcoming. In this case it is necessary to overcome troubles connected with chemical treatment and rinsing of the finished cavities.

δe, %	δh, %	*k*, %	*d*, mm
0	-5.07 (-5.55)	1.90	24.80
+ 10	-7.92 (-8.34)	2.10	18.30
+20	-10.00	2.38	12.52
+30	-11.36	2.64	8.14
+40	-12.30	2.88	4.74
+50	-12.99	3.06	2.18

Table 1. δe, δh – change of normalized electric and magnetic fields by optimization with two elliptic arcs (in parentheses – with 6 circular arcs), *k* – coupling coefficient, *d* – minimal distance between the walls of cells.

Fields on the metal surface along the profile line of the original TESLA cell and of its optimized version with $\delta e = 0$ are shown in Fig. 5. The profile of magnetic field has a flat top that extends further along the cavity surface for the optimized shape. Because of approximately the same magnetic flux, the maximal value of the magnetic field becomes lower. One can see that the electric field for the optimized cell has a smoother change, and the steeper change of it on the right side of the curve is connected with longer constancy of the magnetic field.

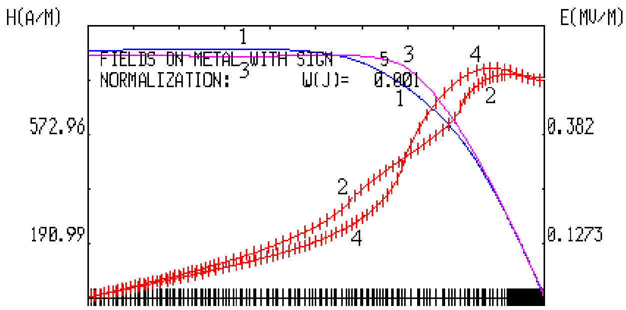

Fig. 5. Fields on the surface of the TESLA cell. The abscissa begins at the cell equator and ends at the iris center.
1, 2 – magnetic and electric fields of the original cell;
3, 4 – magnetic and electric fields of the optimized cell with the value of e = 1.

5. Conclusion

The presented results can be used for an increase of accelerating rate of the TESLA structure where the hard limit for this increase is the surface magnetic field.

One can, for example, sacrifice 20 % of electric field to gain 10 % in magnetic field and still have big enough thickness of the iris walls about half inch.

The change of the shape leads to some technological complications. It is also necessary to prove that the new shape will be free of multipacting.

The map of normalized electric and magnetic fields for optimal shapes of accelerating cells can be used both for normal and superconducting cases and can be extended for different ratios of aperture to wavelength and for broader range of fields.

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6. References

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7. Appendix

δe, %	*Req*, mm	*A*, mm	*B*, mm	*a*, mm	*b*, mm	ρ, Ohm	*G*, Ohm
0	100.71	45.25	36.75	12.99	20.45	58.70	271
+ 10	99.47	48.50	36.34	11.08	14.82	59.87	276
+20	98.72	51.39	36.35	9.34	11.82	60.29	280
+30	98.27	53.58	36.58	8.08	9.84	60.39	284
+40	97.97	55.28	36.84	7.14	8.43	60.34	286
+50	97.76	56.56	37.12	6.50	7.34	60.30	288

Table 2. Dimensions of the two-elliptic-arcs version of the optimized cells (see also Table 1 and Fig. 3) and their additional figures of merit: ρ - effective impedance defined from $P = V^2/2\rho Q_0$, and G - the geometry constant defined as $G = Q_0 \cdot R_s$, where V is the accelerating voltage for the cell, P is the power dissipated in the cell walls, Q_0 is the cell quality factor, R_s is the surface resistance.