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Time Domain Cavity Simulation and Analysis*

Tom Hays

Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853

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1 Introduction

Past notes in the SRF group[1, 2] on transient cavity behavior have left out the option for field dependent Q. By constructing a differential equation in time for the cavity fields a more complete picture is obtained. One can analyze transient cavity behavior with forward power, cavity Q, and even the coupler Q_{ext} varying as functions of time or field. Furthermore, the differential equation can be rearranged to extract the cavity's Q at any instant in time from the stored energy as a function of time.

2 Differential Equation of the Cavity State

Consider a cavity driven on resonance with one coupler. By conservation of energy we can write

$$P_f = P_{diss} + P_r + \frac{dU}{dt}. \tag{1}$$

where

P_f = forward power (toward the input coupler)
P_{diss} = cavity dissipated power
P_r = reverse power
U = stored energy (inside the cavity)
t = time.

The only tricky part about this expression is the reverse power which satisfies

$$P_r = \left(\sqrt{P_f} - \sqrt{P_e}\right)^2 \tag{2}$$

where

$$P_e = \omega U / Q_{ext}. \tag{3}$$

Q_{ext} is the "external" Q of the coupler.

Equation (2) indicates that the net reverse wave results from a superposition of a wave reflected off the input coupler and a wave being emitted from the cavity. This can be seen from a model such as that in figure 1. When $Z_1 \ll Z_2$ and the length of region 2 is an integer number of half wavelengths, the conditions of having a resonant cavity are satisfied [1]. A wave incident upon the 1-2 boundary from the left will be almost entirely reflected with no change of phase (open boundary condition with a large discontinuity in impedance). The wave that does get into the cavity and gets back out has experienced a net phase shift of 180° due to the hard reflection off of the right hand side cavity wall. The phase change just due to the round trip travel is zero because the cavity is on resonance – the round trip is an integer number of wavelengths. The amplitude of this emitted wave (proportional to $\sqrt{P_e}$) depends upon how much energy is currently stored in the cavity.

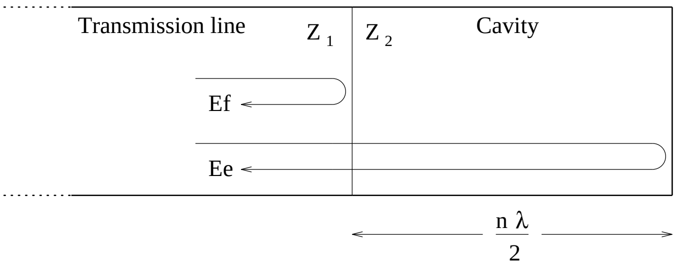

Figure 1: A transmission line model of a cavity and coupler system. $Z_1 \ll Z_2$. The electric fields of the leftward traveling waves are seen to have the opposite phase.

The above model assumes that the forward wave is at the resonant frequency of the cavity. Since having a single frequency requires no amplitude variation with time, to analyze time dependent behavior, we must make a quasistatic approximation for the forward wave. That is, its amplitude modulation must introduces a bandwidth much smaller than the width of the loaded cavity resonance. If more generality is needed, one should use a frequency domain technique.[2]

Substituting (2) into (1) and using

$$P_{diss} = \frac{\omega U}{Q_0} \tag{4}$$

for the cavity losses and

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \tag{5}$$

for the "loaded Q" as well as (3) we arrive at a differential

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[1] Choosing $Z_1 \gg Z_2$ and an appropriate cavity size would work equally well.

equation for stored energy:

$$\frac{dU}{dt} = 2\sqrt{\frac{P_f \omega U}{Q_{ext}}} - \frac{\omega U}{Q_L}. \tag{6}$$

If we find a constant κ_e such that the peak surface electric field [2] in the cavity satisfies

$$E = \kappa_e \sqrt{U} \tag{7}$$

then (6) can be written as

$$\frac{dE}{dt} = \frac{1}{2\tau_L}\left(\kappa_e \sqrt{\frac{4{\tau_L}^2 \omega P_f}{Q_{ext}}} - E\right) \tag{8}$$

where $\tau_L \equiv Q_L/\omega$.

This can be expressed in the more intuitive form of

$$\frac{dE}{dt} = \frac{1}{2\tau_L}\left(\kappa_e \sqrt{U_0} - E\right) = \frac{1}{2\tau_L}\left(E_0 - E\right) \tag{9}$$

where

$$U_0 \equiv \frac{4{\tau_L}^2 \omega P_f}{Q_{ext}} \tag{10}$$

is the steady state stored energy and E_0 is the corresponding peak surface electric field.

Equation (9) shows that the cavity has a natural time constant τ_L for response and that the field changes at a rate proportional to the displacement from its "equilibrium" value. E_0 is the equilibrium value in the sense that if all the other variables such as P_f, Q_{ext}, and Q_0 remain constant and the cavity is allowed to reach a steady state, that is the value it reaches. In other words, it is an instantaneous target field.

If the cavity-coupler system had more than one coupler, (1) would have an additional term with the form of (3) for the emitted power of each coupler. The only effect this has on the subsequent equations is to require that the definition of "loaded Q" in (5) have an additional $1/Q_{ext,k}$ term for each of the k new couplers.

From (9) and (2) the time dependent cavity behavior can be determined analytically or numerically.

The above gives E(t) from Q_0 (and other variables) but to go the other way, one has only to solve for Q_0 in (9) to get

$$\frac{1}{Q_0} = \frac{2\left(\kappa_e \sqrt{\frac{P_f \omega}{Q_{ext}}} - \frac{dE}{dt}\right)}{\omega E} - \frac{1}{Q_{ext}}. \tag{11}$$

This form may be useful for extracting the $Q(E)$ or the $Q(t)$ behavior of a cavity during pulsed operation. Again, if there are k additional couplers, they would show up as further $1/Q_{ext,k}$ terms subtracted from the right side of (11).

[2]The peak electric field is used so that $\sqrt{U}$ doesn't clutter the subsequent equations.

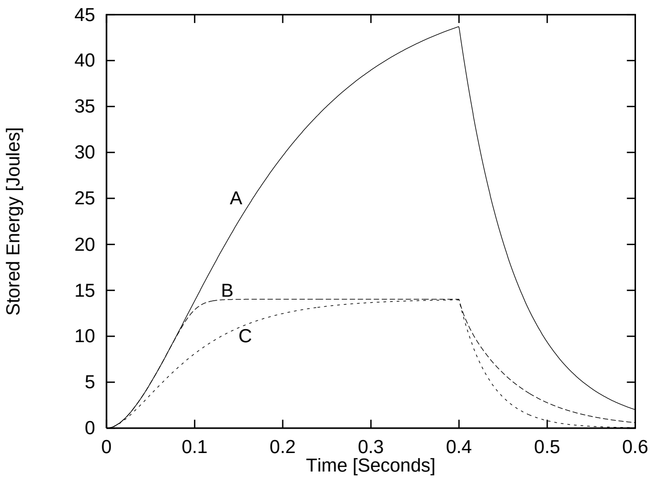

Figure 2: Stored energy in a cavity in response to a 0.4 second rectangular pulse of 200 Watts. $Q_{ext} = 5.74 \times 10^8$ for all three cases. (A) Over-coupled cavity with constant $Q_0 = 7 \times 10^9$. (B) Field emission loaded cavity that changes its Q_0 from 7×10^9 at low field to 5.74×10^8 at high field during this pulse(see figure 3). (C) Unity-coupled cavity with constant $Q_0 = 5.74 \times 10^8$.

3 Simulating Cavity Response to Pulses

By numerically integrating (9) and choosing a realistic $Q(E)$, one can easily simulate a cavity's behavior. Among other things, one can predict the maximum achievable cavity fields (in the absence of thermal breakdown) for a given $P_f(t)$ and Q_{ext}. In addition, (2) can be used to predict the the reverse power as a function of time.

To illustrate the importance of incorporating $Q(E)$ in the simulation, figure 2 shows the response of three cavities who differ in their $Q(E)$ but have the same $Q_{ext} = 5.74 \times 10^8$. The same rectangular pulse of 200 Watts for 0.4 seconds was sent to each. In case (B) the $Q(E)$ behavior of figure 3 is used [3]. In the other two cases the cavities have constant Q_0's representing the low field and high field Q_0's of case (B).

The constant Q_0 cavities have the predicted[1, 2] form

$$U(t) = U_0[1 - \exp(-t/2\tau_L)]^2 \tag{12}$$

for the rise and

$$U(t) = U_1 \exp(-t/\tau_L) \tag{13}$$

for the decay. Here U_1 is the stored energy reached at the end of the pulse. Even though (B) achieves the same steady state field as (C) it fills essentially at the same rate as (A).

With this simulation tool, the peak field reachable during high-power pulsed processing for a given incident power can be predicted more precisely. In the past, for simplicity, the Q_0 was taken to be constant at some pessimistic value and the cavity was assumed to fill like (12) until the RF

[3]For this cavity shape $\kappa_e = 14.62\mathrm{MV/(m\sqrt{J})}$.

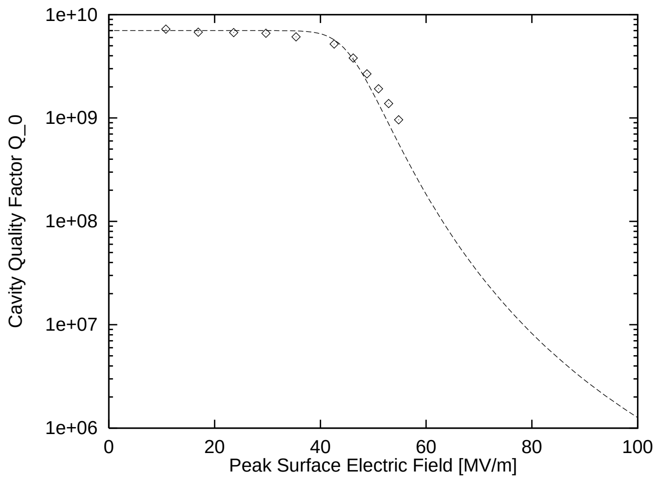

Figure 3: Q_0 vs E data and a fitted curve for a single cell 1.3 GHz superconducting cavity with field emission loading. The fields in the cavity had been pulsed to 86 MV/m peak surface electric field before this curve was made. The functional form of the curve comes from a model of BCS and field emission (Fowlwer-Nordheim) losses.

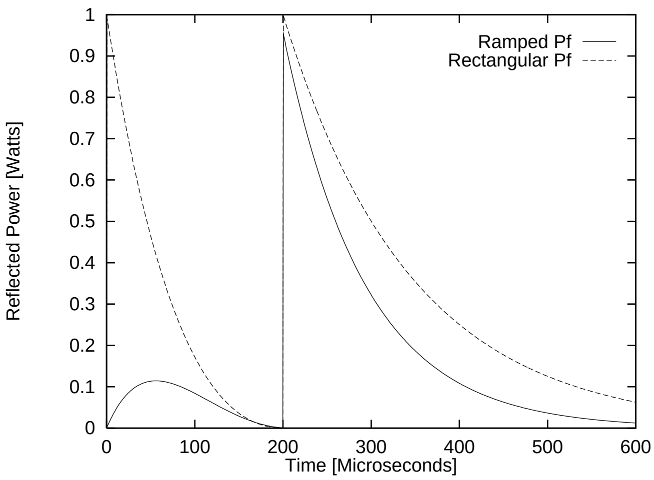

Figure 4: The effect of a ramped incident power pulse compared to a rectangular incident power pulse for a grossly over-coupled cavity (not reaching equilibrium). The "fill peak" of the cavity is only 11% that of the rectangular pulse. The stored energy (not shown) reached by the ramp is 61% of that achieved with the rectangular pulse. Here $Q_0 = 7 \times 10^9$, $\beta = 9333$ for the ramped pulse, and $\beta = 5932$ for the rectangular pulse.

was turned off.[3] Since the cavity does not generally reach a steady state with the short pulses used, the fill rate is significant. In addition, the shape of the high power pulse need not be a simple rectangular pulse. Real pulsed high-power sources rarely produce ideal rectangular pulses. Variable pulse shapes can be modeled too.

4 Effect of Linearly Ramped Pulse

It has been found that standing wave patterns in the Cornell 1.3 GHz high power test stand can cause breakdown phenomena at a ceramic RF window. We are thus motivated to eliminate the reverse power coming back from the cavity which would combine with the forward power. By adjusting the coupling and the pulse length, the reverse power can be made to have only a "fill peak" due to the reflection off of the input coupler[3]. The pulse is not on long enough for the emitted wave to dominate and change the phase of the reverse wave.[2]

Further reduction of standing waves can be accomplished through shaping the input power pulse. The trade-off is that this results in a reduction of the peak stored energy compared to using a rectangular $P_f(t)$ with the same peak power and pulse length. Figure 4 illustrates an example of this using a linearly ramped pulse. In general, the Q_{ext} needed when using a ramp is less than that of a rectangular pulse. The effectiveness of this ramp compared to a rectangular pulse varies with the τ_L and the pulse length. In general, if τ_L is small relative to the pulse length then the cavity is near equilibrium at any instant resulting in a very small fill peak and a high peak stored energy.

5 Reflectionless Cavity Fill

The ultimate solution to a totally reflectionless cavity fill for an arbitrary incident pulse can be obtained with a time dependent Q_{ext}. From (2), one sets $P_f = P_e$ to find that the condition

$$Q_{ext}(t) = \frac{\omega U(t)}{P_f(t)} \tag{14}$$

allows the cavity to absorb all the incident power.

In practice, Q_{ext} could probably not change the 10 or so orders of magnitude required to effectively satisfy this equation for a superconducting cavity and a rectangular incident pulse, but in combination with a ramped pulse, a finite range of Q_{ext} can come very close to a reflectionless fill. Note that this would also mean that we are making the most efficient use of the available power, since all of it goes into the cavity.

6 Extracting $Q(t)$ and $Q(E)$ from a pulsed cavity

As was mentioned before, (11) can be used to extract the Q_0 of a cavity from stored energy, the forward power, and Q_{ext}. Previously, a form of this technique has been used to analyze the fields' decay at the end of a pulse (no forward power)[4]. In practice, the $Q(E)$ curve of a cavity can be obtained in CW. The measurement is limited by available CW power. It is during high power pulsed operation that the extraction of $Q(E)$ from the time-dependent cavity response could provide new information, since higher cavity fields can only be obtained in pulsed mode. In addition,

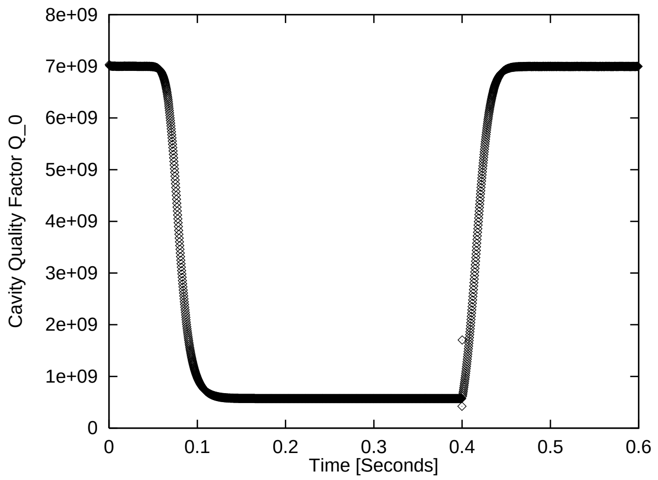

Figure 5: Q_0 vs time is extracted from a high precision simulation of a cavity response. The stored energy analyzed is curve (A) in figure 2. Note that the discontinuous slope in the stored energy curve when the P_f is turned off shows up as bogus points on this curve.

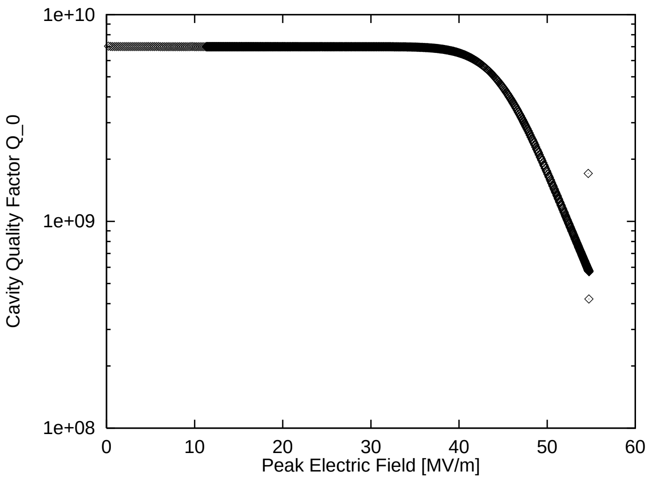

Figure 6: This Q_0 vs E curve is another way of presenting the Q_0 information shown in figure 5. The data extracted from the fill and the decay are overlayed.

at high fields, one is likely to encounter thermal breakdown phenomena in the cavity so that the decay of the stored energy is determined by more than just the field emission losses. In that case the $Q(E)$ must be obtained from the cavity fill (forward power present) before breakdown occurs.

A particular implementation of (11) was coded into a computer and used to analyze data created by a separate cavity simulation program. The simulation data was created using the $Q(E)$ of figure 3 and written to a file as a list of numbers of a chosen precision, which were then read by the analysis program. An example of the technique using very high precision data of a cavity pulsed with 200 Watts is shown in figure 5. One can extract a $Q(E)$ curve from the fill or the decay in this case. Figure 6 shows the resulting $Q(E)$ to be compared with figure 3.

More work needs to be done to determine how best to apply this method to experimental data and deal with limited precision and experimental error. It turns out, not surprisingly, that one needs to have rather high precision in the stored energy information in order to extract Q_0 properly. For instance, in greatly over-coupled situations τ_L is dominated by Q_{ext}, and Q_0 plays only a small role in determining the cavity's response.

7 Acknowledgements

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References

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[4] D. W. Reschke and R. W. Roth *Fundamentally Fastest Method Measuring $Q_a(E_{acc})$ - Performance of S.C. Cavities* Fachbereich Physik, Bergische Universitat Wuppertal, Germany. Presented at the 6th Workshop on RF Superconductivity at CEBAF in Newport News, VA, USA.