Tuning of Multi-Cell Cavities using Bead Pull Measurements

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The equivalent electrical circuit of a single resonant cavity is an LC circuit with a series resistance R. In a multicell cavity with N cells, these resonance circuits are coupled. In the following I assume that the coupling is inductive (in superconducting cavities, capacitive coupling dominates, but the mathematical treatment is quite similar). Fig.1 is the equivalent electrical diagram of N inductively coupled cells. Initially we will assume that the multicell cavity has the ideal shape, so that the inductivities L_1, L_2,.. L_N are all equal: $L_1 = L_2 = L_3 = ... = L_N = L$. In the end cells, a small additional inductivity L' (L' << L) has to be introduced to obtain a 'flat' π mode, that is to obtain equal amplitudes in all cells for the π mode. The magnitude of L' will be computed in section 1.

1 Eigenmodes and eigenfrequencies of unperturbed cavity

In the ideal case, all inductivities are identical to L. The eigenfrequency and quality factor of a single resonance circuit are:

$$\omega_0 = 1/(LC)^{1/2}, \qquad Q = (\omega_0 L)/R \qquad (1)$$

The magnetic coupling between adjacent cells is given by the parameter κ. For a periodic time dependence $\exp(i\omega t)$ we get the following set of equations for the currents I_1,.....I_N in the cells 1,...N:

cell #1: $$[i\omega L + 1/(i\omega C) + R]\, I_1 + i\omega L'\, I_1 + i\omega\kappa L\, I_2 = 0$$

cell #j (1< j < N): $$[i\omega L + 1/(i\omega C) + R]\, I_j + i\omega\kappa L (I_{j-1} + I_{j+1}) = 0$$

cell #N: $$[i\omega L + 1/(i\omega C) + R]\, I_N + i\omega L'\, I_N + i\omega\kappa L\, I_{N-1} = 0$$

Dividing by $i\omega L$ and using (1) the following N equations result:

$$(1 - \Omega)\, I_1 + \gamma I_1 + \kappa I_2 = 0 \qquad (2) \qquad j = 1$$

$$(1 - \Omega)\, I_j + \kappa (I_{j-1} + I_{j+1}) = 0 \qquad (3) \qquad 1 < j < N$$

$$(1 - \Omega)\, I_N + \gamma I_N + \kappa I_{N-1} = 0 \qquad (4) \qquad j = N$$

where we have defined the quantities

$$\Omega = \Omega(\omega) = (\omega_0/\omega)^2 [1 + i\omega/(\omega_0 Q)], \qquad \gamma = L'/L \qquad (5)$$

To solve the equations the following ansatz is made:

$$I(m,j) = A_m \sin[m\pi(2j - 1)/(2N)] \qquad (6) \qquad m, j = 1,....N$$

Here m is the mode number (m=1,....N) and j the cell number (j=1,...N).

For a given mode m and for a cell number $1 < j < N$ one has:

$$I(m,j-1) + I(m,j+1) = 2\, I(m,j)\, \cos(m\pi/N) \qquad (7)$$

Inserting (6) into (3) and using (7) one obtains a relation for the eigenfrequencies ω_m of the modes m:

$$\Omega_m = \Omega(\omega_m) = 1 + 2\,\kappa \cos(m\pi/N) \qquad (8) \qquad 1 \leq m \leq N$$

Now in general $Q \gg 1$, so $\Omega_m \approx (\omega_0/\omega_m)^2$ and the frequency of mode # m is given in terms of the eigenfrequency ω_0 of a single cell and the coupling parameter κ by

$$\omega_m = \omega_0 / [1 + 2\,\kappa \cos(m\pi/N)\,]^{1/2} \qquad (9) \qquad m = 1,....N$$

This formula is valid with $\kappa > 0$ for capacitive coupling of the cells, as is realized in superconducting cavities. In that case, the "π mode", given by $m = N$, has the highest frequency. The eigenfrequencies are plotted in Fig.2 for cavities with 5 and 9 cells. (For inductive coupling, κ appears with a negative sign and the π mode has the lowest frequency). The π mode is used for acceleration, since it features the highest accelerating field for a given peak field at the superconductor surface. This is immediately evident from Fig.3 where the eigenmodes of a 9-cell cavity are plotted. In this mode, the electric field on the axis (corresponding to the current in our equivalent electrical circuit) has opposite sign in adjacent cells. The length of the cells is chosen such that the field has just inverted when the electron has travelled from cell j to cell j+1, so the field is in fact accelerating in each cell.

Normalized eigenmodes

In the perturbation treatment of section 2, the properly normalized eigenmodes are needed. They are also denoted by I(m,j) but this should not necessarily imply that they describe currents. In a multicell cavity that is excited in the mode m, the electric field in cell # j is proportional to I(m,j). The amplitudes of the normalized modes are:

$$A_m = [(2-\delta_{mN})/N]^{1/2} \qquad (10)$$

The modes are normalized and orthogonal in the sense that the Kronecker δ results when the product of two modes is summed over all cells:

$$\sum_{j=1}^{N} I(m,j)I(n,j) = \delta_{mn} \qquad (1 \text{ for } m = n, \quad 0 \text{ for } n \neq m) \qquad (11)$$

Detuning of end cells

The end cells are coupled to neighbours on one side only and have to be detuned to obtain a 'flat' π mode with equal amplitude in all cells. In the equivalent electrical circuit we achieve this detuning by introducing an additional inductivity L' (see Fig.1. One could modify the capacity as well). The required magnitude of L' can be computed by inserting I(N,1) and I(N,2) of equation (6) into (2):

$$(1 - \Omega_N + L'/L)\sin[N\pi/(2N)] + \kappa \sin[3N\pi/(2N)] = 0$$

Using (8), it follows: $\gamma = L'/L = - \kappa$ (12)

Matrix formalism

For a given mode number m , the function I(m,j) can be represented by a column vector V with component # j given by $V_j = I(m,j)$. The linear equations (2)....(4) can be written as an eigenvalue equation with the eigenvalues Ω_m given by (8):

$$G\,V = \Omega_m V \qquad (13)$$

The coefficient matrix, called G, is given in (14) for the general case and in (15) for N = 5

$$G_{jk} = \delta_{jk} + \kappa[\delta_{j+1,k} + \delta_{j,k+1} - \delta_{j1}\delta_{k1} - \delta_{jN}\delta_{kN}] \qquad (14)$$

$$G = \begin{pmatrix} 1-\kappa & \kappa & 0 & 0 & 0 \\ \kappa & 1 & \kappa & 0 & 0 \\ 0 & \kappa & 1 & \kappa & 0 \\ 0 & 0 & \kappa & 1 & \kappa \\ 0 & 0 & 0 & \kappa & 1-\kappa \end{pmatrix} \qquad (15)$$

2 Perturbed Cells

Let the inductivities of the individual cells deviate by a small amount ε from the ideal value

$$L_j = L(1 - \varepsilon_j)\ ,\ |\varepsilon_j| << 1 \qquad (16)$$

The coupling between cell j and cell j+1 is now given by

$$\kappa(L_j L_{j+1})^{1/2} \sim \kappa L[1 - 1/2(\varepsilon_j + \varepsilon_{j+1})]$$

and the generalizations of equations (2,3) read

$$[1 - \Omega\,(1 + \varepsilon_1)]\ I_1 + \gamma I_1 + \kappa[1 - 1/2(\varepsilon_2 - \varepsilon_1)]\ I_2 = 0 \qquad (17)$$

$$[1 - \Omega\,(1 + \varepsilon_j)]\ I_j + \kappa(I_{j-1} + I_{j+1}) + \kappa/2[\,(\varepsilon_j - \varepsilon_{j-1})\,I_{j-1} + (\varepsilon_j - \varepsilon_{j+1})\,I_{j+1}] = 0 \qquad (18)$$

The coefficient matrix of this set of linear equations is G + δG with

$$\delta G = - \Omega \cdot g + \kappa/2 \cdot h \qquad (19)$$

$$g_{ij} = \varepsilon_i \, \delta_{ij} \, , \quad h_{ij} = \sum_{k=1}^{N} [\delta_{ik} \, \delta_{j-1,k} - \delta_{i+1,k} \, \delta_{jk} + \delta_{ik} \, \delta_{j+1,k} - \delta_{i-1,k} \, \delta_{jk}] \, \varepsilon_k \qquad (20)$$

Explicitely, for N = 5 the correction matrices g and h are:

$$g = \begin{pmatrix} \varepsilon_1 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_2 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_3 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_4 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_5 \end{pmatrix} \qquad h = \begin{pmatrix} 0 & \varepsilon_1 - \varepsilon_2 & 0 & 0 & 0 \\ \varepsilon_2 - \varepsilon_1 & 0 & \varepsilon_2 - \varepsilon_3 & 0 & 0 \\ 0 & \varepsilon_3 - \varepsilon_2 & 0 & \varepsilon_3 - \varepsilon_4 & 0 \\ 0 & 0 & \varepsilon_4 - \varepsilon_3 & 0 & \varepsilon_4 - \varepsilon_5 \\ 0 & 0 & 0 & \varepsilon_5 - \varepsilon_4 & 0 \end{pmatrix}$$

Due to the perturbation both eigenfunctions and eigenvalues will change. These changes are denoted by $\delta I(m,j)$ and $\delta\Omega_m$. The eigenvalue equation is now

$$[\, G - \Omega_m \, g + \kappa/2 \, h] \, J(m,j) = [\Omega_m + \delta\Omega_m] \, J(m,j) \qquad (21)$$

with $J(m,j) = I(m,j) + \delta I(m,j)$

First order perturbation theory is applied to compute the changes. The modifications of the eigenvalues are obtained by calculating the expectation values of the perturbing matrix δG for unperturbed eigenstates:

$$\delta\Omega_m = < m|\delta G|m > = \sum_{j=1}^{N} I(m,j)\{ \, -\Omega_m \cdot g + \kappa/2 \cdot h \, \} \, I(m,j) \qquad (22)$$

The expectation value of the matrix h vanishes, so

$$\delta\Omega_m = -\Omega_m \sum_{j=1}^{N} (A_m)^2 \sin^2[m\pi(2j-1)/(2N)] \;\; \varepsilon_j \qquad (23)$$

For $Q \gg 1$ one has

$$\Omega_m \approx (\omega_0/\omega_m)^2 \, , \quad \delta\omega_m/\omega_m = -1/2 \; \delta\Omega_m/\Omega_m$$

For the π mode m = N this yields the frequency shift

$$\delta\omega_N/\omega_N = 1/2 \sum_{j=1}^{N} (A_N)^2 \varepsilon_j \qquad (24a)$$

$$= 1/2 < \varepsilon > \qquad (\text{since } A_N = 1/N^{1/2}) \qquad (24b)$$

$$\delta f_\pi = 1/2 \; f_\pi < \varepsilon > \qquad (24c)$$

The changes in the eigenmodes are computed in the following manner:

$$\delta I(m,j) = \sum_{q=1\ (q\neq m)}^{N} b_{mq} I(q,j) \qquad (25)$$

$$b_{mq} = < m|\delta G|q >/(\Omega_m - \Omega_q) = \sum_{i,j=1}^{N} I(m,i)\ \delta G_{ij}\ I(q,j)/(\Omega_m - \Omega_q) \qquad (26)$$

Equation (25) can be used to calculate the modification of the eigenfunctions resulting from known perturbations ε_j in the cells. The task in tuning a cavity, however, is just the opposite: from the bead-pull measurement the amplitude changes $\delta I(m,j)$ are derived (see sect. 3) and then the ε_j have to be deduced to permit tuning of the cells. Since the ε_j are contained in the matrix δG, (25) is not directly suitable for performing this task. For this reason, expression (25) is rewritten in terms of a matrix H that is independent of the perturbation, multiplied with a vector whose components are the ε_j. To this end, expressions (19) and (20) are inserted into (25) and (26) and the summation over the ε's is done last.

After some reshuffling one finds

$$\delta I(m,j) = \sum_{k=1}^{N} H_{jk}\varepsilon_k = H\,\varepsilon \qquad (27)$$

The m-dependent matrix H is given by

$$H_{jk} = 1/2 \sum_{q=1\ (q\neq m)}^{N} [\Omega_q - 3\Omega_m]/[\Omega_m - \Omega_q]\ I(q,j)I(q,k)I(m,k) \qquad (28)$$

3 Bead pull measurements

A small copper tube is pulled along the axis of the multi-cell cavity and the resulting frequency shift Δf_j is recorded for all cells j = 1...N (a capital 'Δ' is used instead of a small 'δ' to distinguish the changes caused by the bead from those due to individual cell errors). The bead introduces a constant perturbation ε_{bead} in one cell after the other. If all cells were identical, the Δf_j would all be equal and (for the π mode with frequency f_π) would be given by expression (24)

$$\Delta f_j = <\Delta f> = 1/2(A_N)^2\ f_\pi\ \varepsilon_{bead} = 1/(2N)\ f_\pi \varepsilon_{bead} \qquad (j = 1,....N) \qquad (29)$$

The frequency shifts are negative, so $\varepsilon_{bead} < 0$. In the following, I consider the π mode only. The amplitude of the unperturbed eigenfunction I(N,j) will be denoted by A, omitting the subscript "N". If a cavity is distorted, the yet unknown pertubations ε_j induce changes $\delta I(N,j)$ of the eigenmode and

corresponding changes δA_j of the amplitudes. Equation (24) tells us that the frequency shift Δf_j, that is measured when the bead is inside cell j, is proportional to the square of the amplitude A_j of the perturbed π eigenmode (this mode has in general different amplitudes $A_j = A + \delta A_j$ in all cells, but it must still be normalized according to (11)).
In that case, the frequency shifts caused by the bead are:

$$\Delta f_j = 1/2(A_j)^2 f_\pi \varepsilon_{bead} = 1/(2N)\, f_\pi \varepsilon_{bead}(1 + \delta A_j/A)^2 \qquad (30a)$$

$$\Delta f_j \sim 1/(2N)\, f_\pi \varepsilon_{bead}(1 + 2\delta A_j/A) \qquad (30b)$$

In the following the perturbation is consequently treated in the first order, so equ. (30b) is used to compute the frequency shifts. Now I assume that (in spite of the perturbations ε_j) the measured π mode frequency f_π is identical to the desired frequency. (The generalization to an overall frequency shift is straightforward and will be done later).
Under this assumption, we have to first order in $\delta A/A$

$$<\Delta f_j> = 1/N \sum_{j=1}^{N} \Delta f_j \equiv <\Delta f>, \quad \text{therefore} \quad \varepsilon_{bead} = 2<\Delta f>/f_\pi \qquad (31)$$

So from the average frequency shift $<\Delta f>$ we can deduce the unknown perturbation ε_{bead} caused by the bead. Once this is known we can use (30b) to calculate the changes of the π eigenmode in cells 1,..N.

Using $\quad \delta I(N,j)/I(N,j) = \delta A_j/A$

we get $\quad \delta I(N,j) = [\, \Delta f_j/<\Delta f> - 1]\; I(N,j)/2 \qquad (32)$

Recall equ. (27): $\quad \delta I(N,j) = \sum_{k=1}^{N} H_{jk}\varepsilon_k = H\cdot\vec{\varepsilon}$

Here H is the matrix which computes the amplitude changes from the individual cell perturbations (here for the π mode m=N). It might appear easy to invert (27) in order to compute the searched-for ε's from the measured mode shifts $\delta I(N,j)$. Unfortunately this is not possible because the matrix H has a vanishing determinant since among the N equations (27), only (N-1) are independent and there exist infinitely many sets of solutions (ε_j). The physical reason for this is obvious: suppose we have a 5-cell cavity in which cells 1 to 4 are identical but cell 5 is different. There is no way to tell from the bead-pull measurement whether the cells 1 to 4 are 'correct' and cell 5 is 'wrong' or vice versa. It may even happen that the cells are all 'wrong' and that an overall adjustment is needed to achieve the desired π mode frequency.
This last observation leads the way out of the difficulty. We assume that an arbitrary cell, say number N, is 'correct'. Then $\varepsilon_N = 0$ and the N-th column of matrix H is of no importance. We define a 'reduced' matrix of (N-1) dimensions as well as 'reduced' ε- and δI-vectors:

$$Hr_{jk} = H_{jk}\,, \quad \varepsilon r_j = \varepsilon_j\,, \quad \delta I\, r_j = \delta I(N,j)\,, \quad j, k = 1,.....(N-1) \qquad (33)$$

Then of course $\delta I\, r_j = \sum_{k=1}^{N-1} Hr_{jk} \varepsilon r_k$ (34)

The reduced matrix, fortunately, can be inverted. Calling R the inverse matrix we have

$$\varepsilon r_j = \sum_{k=1}^{N-1} R_{jk} \delta I\, r_k \quad \text{with} \quad R = (Hr)^{-1} \quad \text{and} \quad j=1,...(N-1) \qquad (35)$$

The assumption that cell N happens to be just correct is of course a very artificial one and might lead to the unpleasant situation that all remaining cells have to be tuned to compensate for a geometry error of this cell. There is a way out, however, permitting to treat all N cells on the same footing. If the measured π mode frequency (without bead in the cavity) coincides with the desired frequency, as was assumed above, then from (24b) it is obvious that the average value of all ε_j (j=1,..N) vanishes.

So $\quad <\varepsilon> = (\varepsilon_1 + \varepsilon_2 + ... + \varepsilon_N)/N = 0$

but in general $\varepsilon_N \neq 0$, so $<\varepsilon r> = (\varepsilon r_1 + \varepsilon r_2 + ... + \varepsilon r_{N-1})/N \neq 0$ (36)

If we took the 'reduced' ε-vector $\vec{\varepsilon r}$ for tuning we would not only fail to correct a possible error in cell N but would in addition change the π mode frequency. Both can be avoided if we shift all εr's by their average value, defining an N-dimensional 'corrected' εc-vector by (37). This vector is defined with a negative sign with respect to $\vec{\varepsilon r}$ since the existing cell errors shall be cancelled.

$$\varepsilon c_j = -(\varepsilon r_j - <\varepsilon r>) \quad \text{for } j = 1,.....(N-1) \quad \text{and} \quad \varepsilon c_N = <\varepsilon r> \qquad (37)$$

Since $<\varepsilon c> = 0$, the frequency f_π is not affected by the tuning, and all cells, including number N, are tuned according to their respective errors.
The frequency correction $fcorr_j$ that is needed to remove the error in cell number j is finally:

$$fcorr_j = \varepsilon c_j\, f_\pi/(2N) \qquad (38)$$

The last step is to apply a constant frequency shift to all cells if one wants to move the π mode frequency to another value:

$$df = (f_{desired} - f_{measured})/N \qquad (39)$$

A MathCAD program Multicell-Tuning has been written based on the above formalism. Its results agree quite well with the TUNE2 program for small errors. Slight discrepancies may be due to the fact that in TUNE2 some approximations are not strictly kept to first order.

4 Practical example: tuning of the 5-cell nobium cavity LDP5-2

The 1300 MHz cavity LDP5-2 had a rather non-uniform initial field profile, shown in Fig.4a. Two reasons may be responsible for this unusual property: the material used for the fourth cell has different elastic properties and during the electron-beam welding a hole was burned into cell 3 due to a failure of the

electron gun. Having such a large deviation from field flatness, this cavity is of course particularly well suited to check the performance of the Multicell-Tuning program. In the first tuning sequence, the π mode frequency f_π was kept fixed, so equ. (38) was used to compute the frequency corrections of the cells 1-5. Table 1 lists the computed values of the frequency corrections as well as the actually applied corrections. Note that in order to achieve a negative frequency shift, the cell in question must be squeezed, for a positive frequency shift it must be lengthened. This squeezing or lengthening must be extended into the plastic regime of niobium to get a permanent deformation. Typically, during the squeeze the frequency shift must exceed the target value by some 300 kHz, but in the stiffer cell 4 more than 400 kHz were required. This illustrates the difficulty of tuning to the desired accuracy of 1 kHz. The field profile of the π mode after the first tuning is shown in Fig.4b. It is considerably better than the initial profile but still a field non-uniformity of about 15% is present. The second tuning was made allowing the frequency f_π to change according to (39) in order to save time. The computed and the applied frequency corrections are again listed in Table 1. The resulting bead-pull data are displayed in Fig.4c. The field flatness has again improved, the largest cell-to-cell difference in electric field being 5%. To check the capability of the method, a final tune was made with quite small frequency corrections (see Table 1). Fig.4d demonstrates that the cavity tuning is almost perfect after this step.
The very precise frequency corrections that are needed for obtaining a perfectly tuned cavity could be more readily achieved if a set-up was designed that permits squeezing or expanding of one cell after the other without having to remount the cavity in between the tuning steps.

Sections 1, 2 follow the paper "Standing Wave Modes in a Superconducting Linear Accelerator" by T.I.Smith (Stanford HEPL note 437, 1966) with some modifications. Sections 3, 4 are new.

Table 1: Computed and actually applied frequency corrections df in kHz for the three tuning steps of the 5-cell 1300 MHz cavity LDP5-2

	first tune		second tune		third tune	
cell #	df_{comp}	df_{appl}	df_{comp}	df_{appl}	df_{comp}	df_{appl}
1	103	90	0	0	0	0
2	199	200	-80	-85	-19	-18
3	190	208	-216	-216	-4	-3
4	-365	-369	-112	-111	-32	-31
5	-127	-109	-51	-43	-30	-32

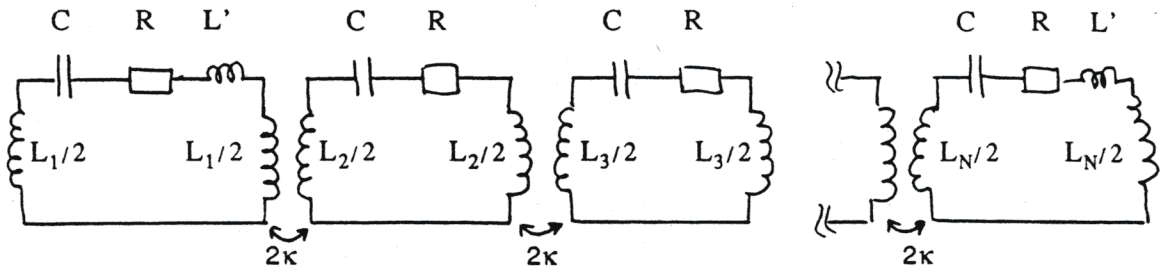


Fig.1: Equivalent electrical diagram for a multicell-cavity with inductive coupling. The small additional inductivity L' in the end cells is needed to obtain a 'flat' π mode with equal amplitudes in all cells. In principle, L' would also influence the coupling to the adjacent cell. This is neglected here since $L'/L << 1$.

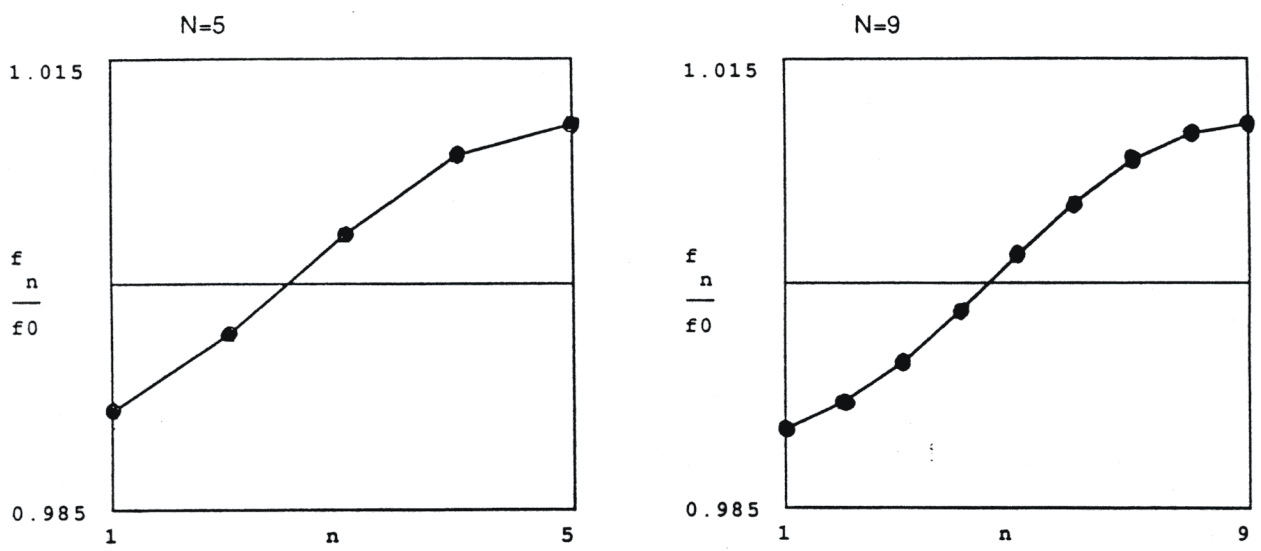
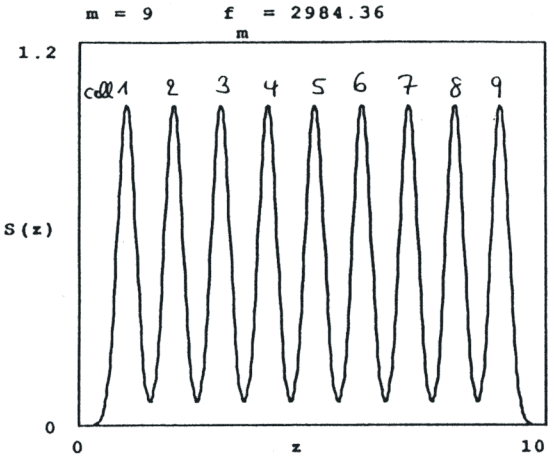
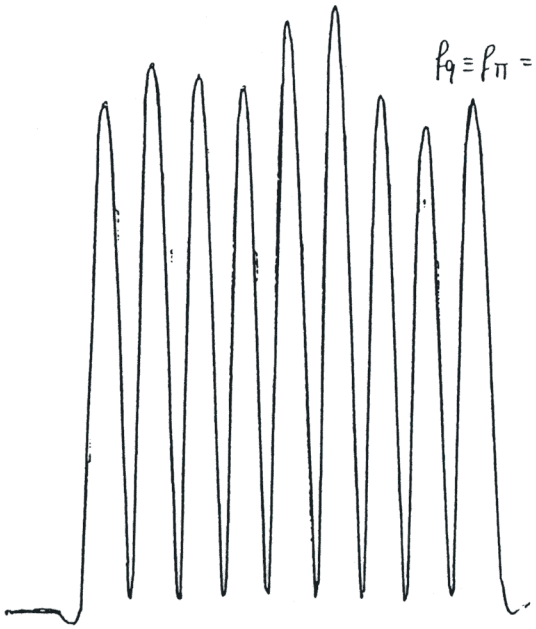
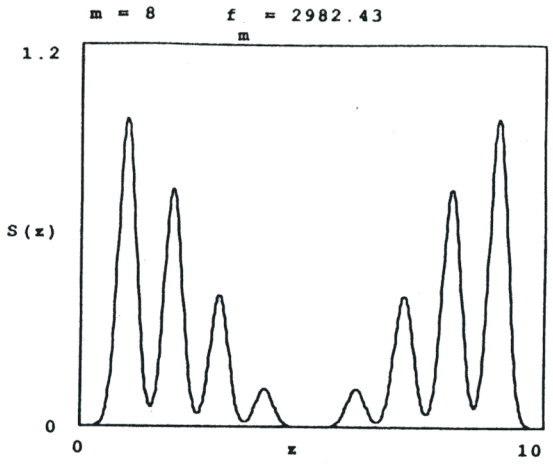


Fig.2: The frequencies of the N eigenmodes in cavities with N = 5 and 9 cells. The cell-to-cell coupling is $2\kappa = 0.02$.



π mode





$\frac{8}{9}\pi$ mode

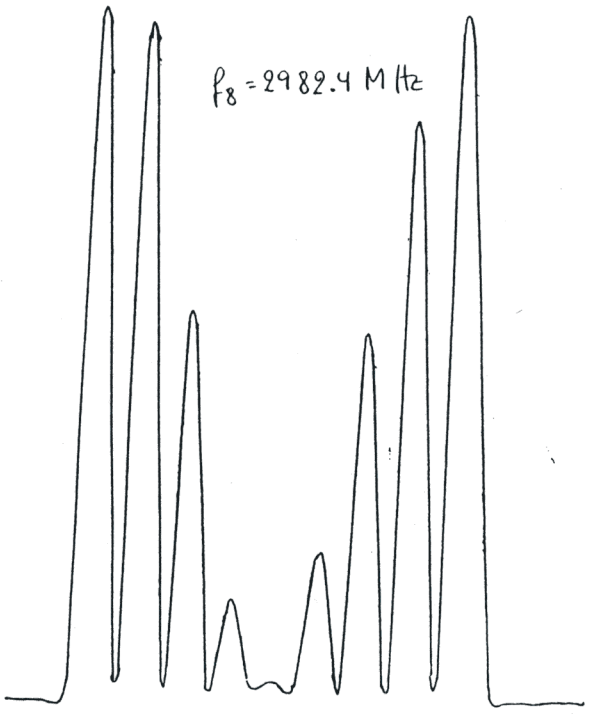
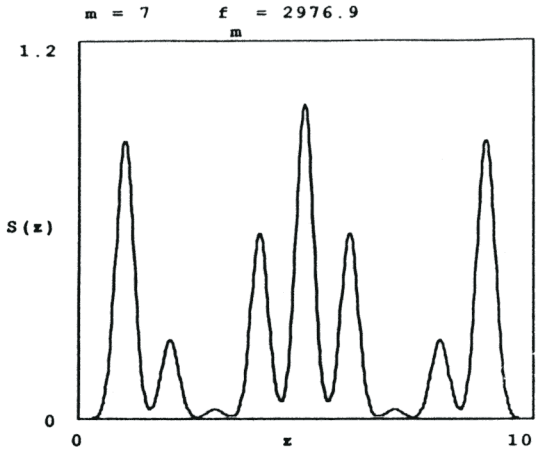
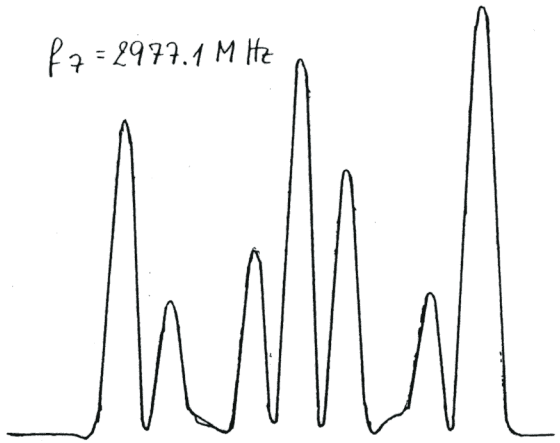
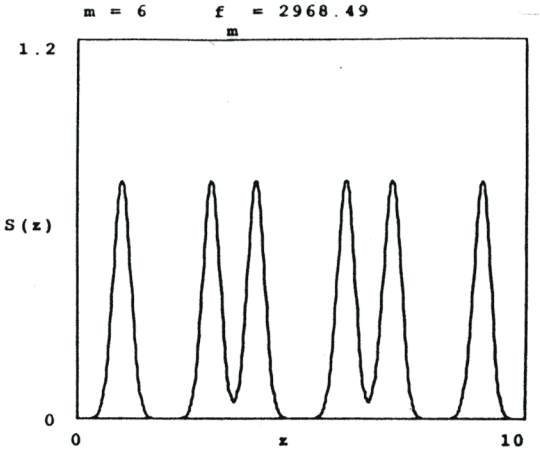


Fig.3: Computed and measured eigenmodes in the 3 GHz 9-cell cavity S3C9-5 in the normal-conducting state. The top graphs show the square of the electric field on the axis, folded with a gaussian to simulate the resolution of the bead-pull measurement. The bottom graphs are the result of the bead- pull measurement. The cavity was not optimally tuned to get a 'flat' π mode. For the two lowest modes (π/9 and 2π/9) the electric field in the end cells is very small and it turned out to be difficult to achieve stable operation of the phase loop circuit.

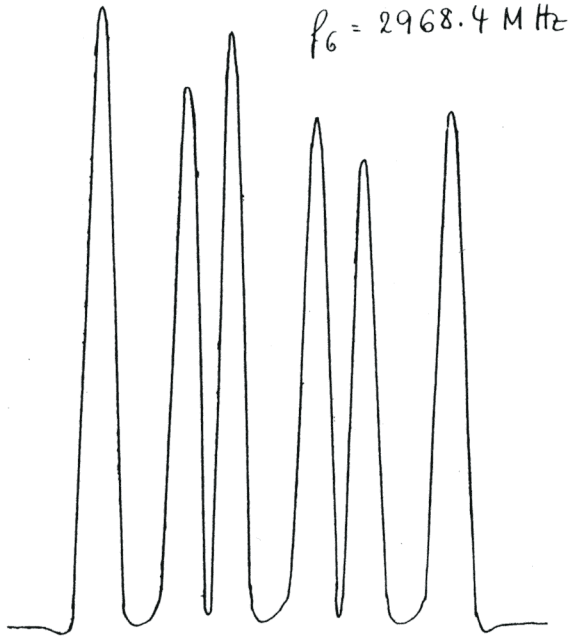


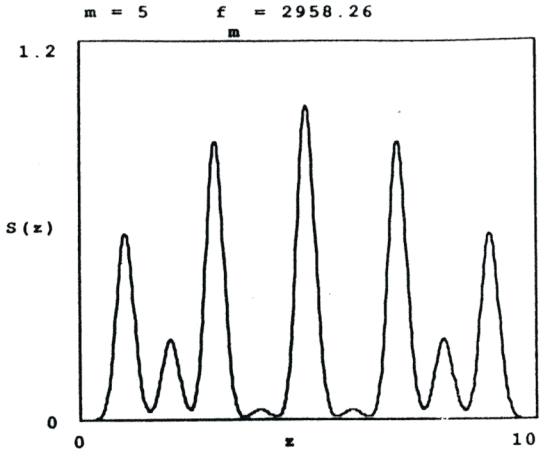
$\frac{7}{9}\pi$ mode





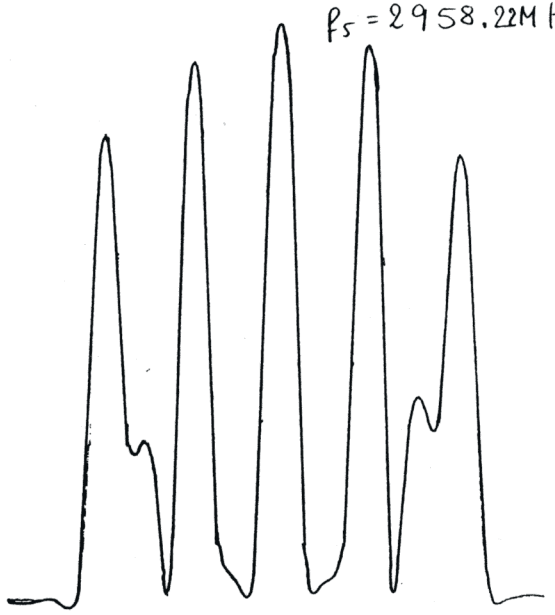
$\frac{6}{9}\pi$ mode

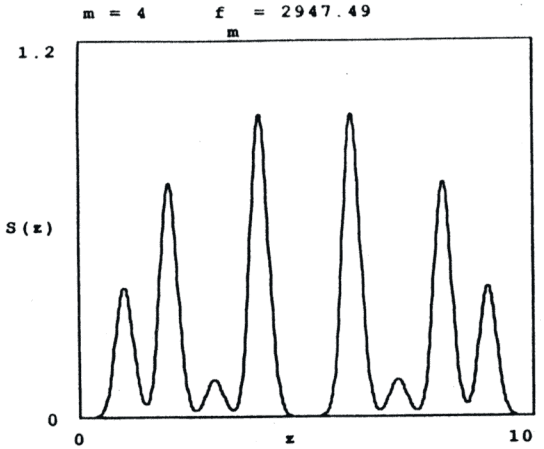




$\frac{5}{9}\pi$ mode

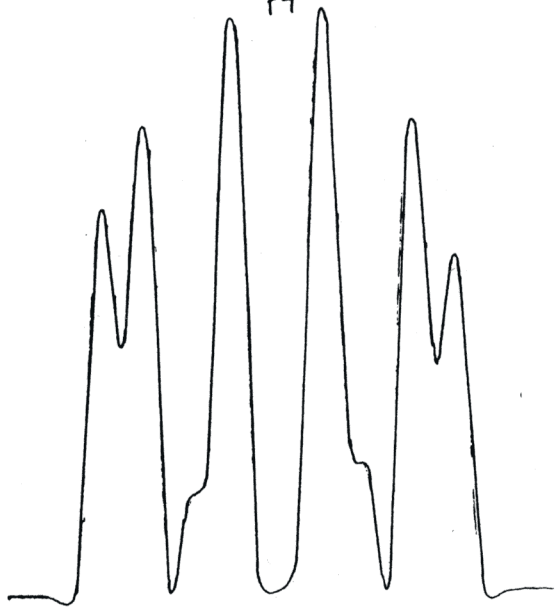
$f_5 = 2958.22\,MHz$

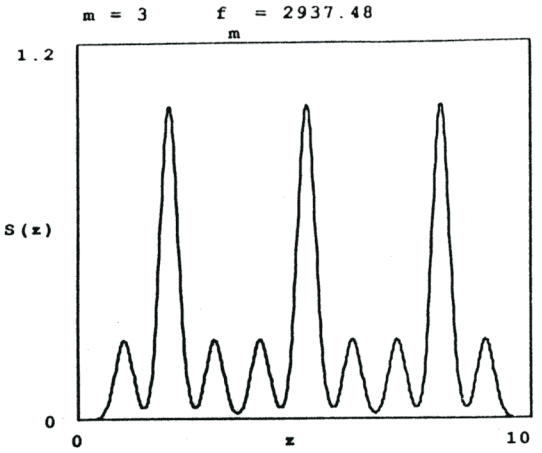




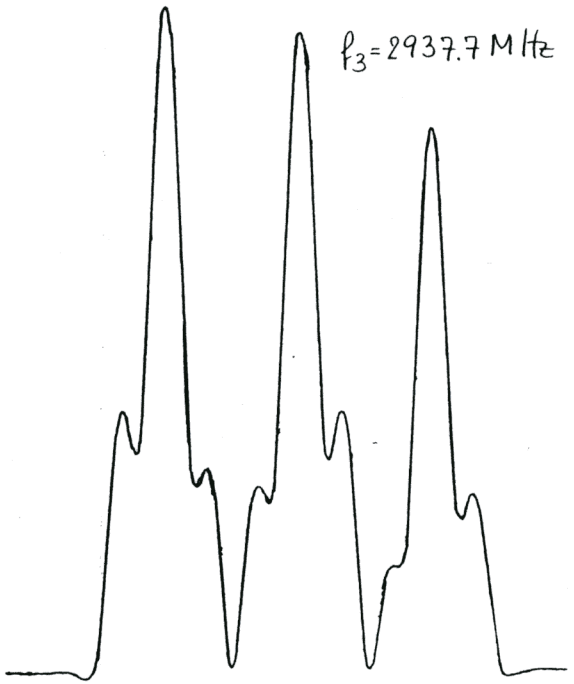
$\frac{4}{9}\pi$ mode

$f_4 = 2947.51\,MHz$

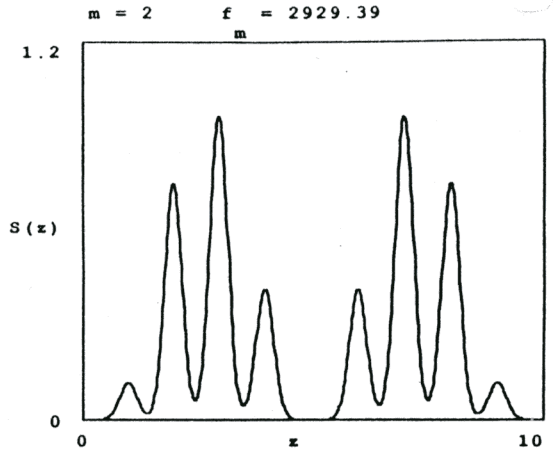




$\frac{3}{9}\pi$ mode

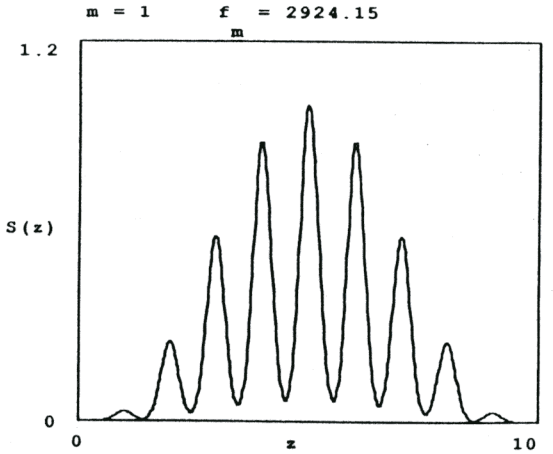


$f_3 = 2937.7$ MHz



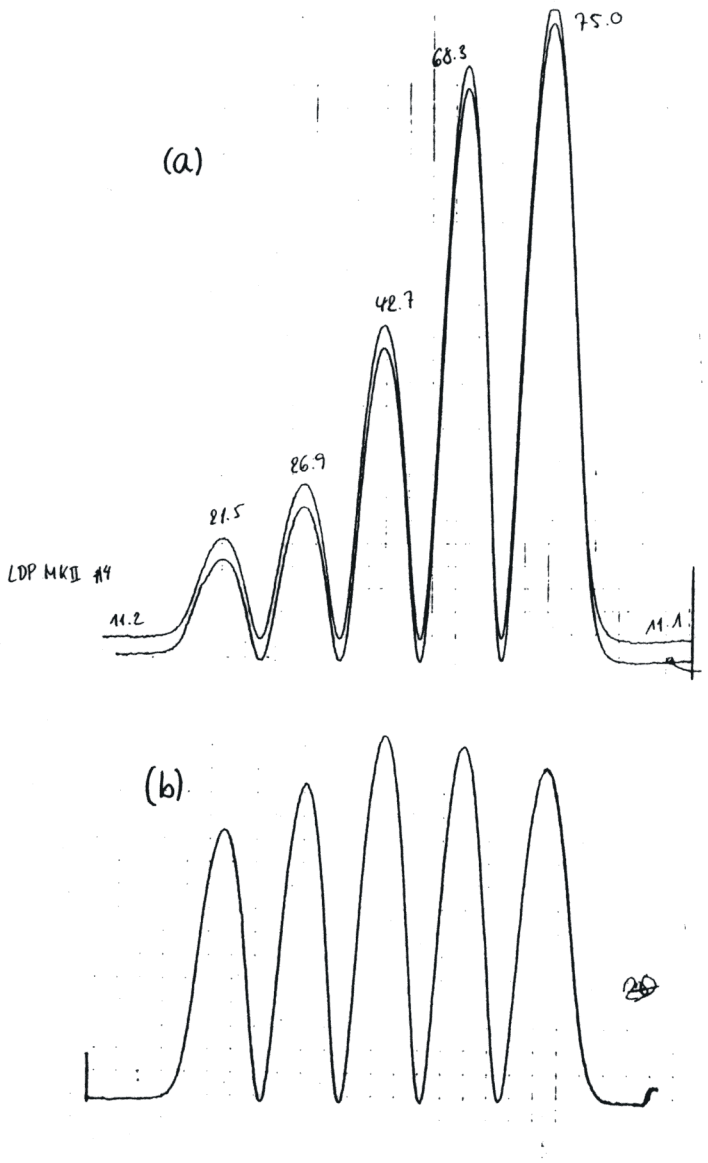
$\frac{2}{9}\pi$ mode

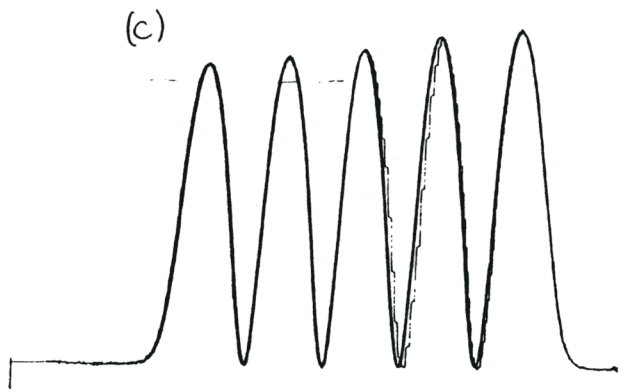
$f_2 = 2928.1$ MHz measured



$\frac{1}{9}\pi$ mode

$f_1 = 2923.2$ MHz measured





LDP 5 - 2
after third tuning

f_π = 1306.481.9
$\frac{4}{5}\pi$ 1303.916.9
$\frac{3}{5}\pi$ 1297.358.4
$\frac{2}{5}\pi$ 1289.369.1
$\frac{1}{5}\pi$ 1281.911.9

Fig.4: Bead-pull measurements on the 5-cell cavity LDP5-2.
a) after welding, b) after first tuning, c) after second tuning, d) after third tuning

Program "Multicell-Tuning" for tuning of multicell cavities using bead pull data

For data input move to section (B)

general program

(A) Computation of important quantities
compute coupling parameter

$$\kappa := \frac{fN^2 - f1^2}{2 \cdot \left[fN^2 + f1^2 \cdot \cos\left[\frac{\pi}{N} \right] \right]}$$

$200 \cdot \kappa = 2.001$
coupling in %

normalized eigenmodes I(m,j) and eigenvalues Omega(m)
m = mode number, j = cell number

$$I(m,j) := \sqrt{\frac{2 - \delta(m,N)}{N}} \cdot \sin\left[\frac{m \cdot \pi}{2 \cdot N} \cdot (2 \cdot j - 1) \right]$$

$$\Omega(m) := \left[1 + 2 \cdot \kappa \cdot \cos\left[m \cdot \frac{\pi}{N} \right] \right]$$

ORIGIN ≡ 1

Correction matrix Hr for pi mode (reduced to N-1 dimensions)

$q := 1 \,..\, (N - 1)$ $\quad j := 1 \,..\, (N - 1)$ $\quad k := 1 \,..\, (N - 1)$

$$Hr_{j,k} := \frac{1}{2} \cdot \sum_{q} \frac{\Omega(q) - 3 \cdot \Omega(N)}{\Omega(N) - \Omega(q)} \cdot I(q,j) \cdot I(q,k) \cdot I(N,k)$$

$R := Hr^{-1}$ inverse matrix for N-1 dimensions

(B) Input of cavity parameters. Name of cavity: LDP5-1

$N \equiv 5$ number of cells (max. N=9)

$fN \equiv 1307.18$ frequency of pi mode (m=N) in MHz

$f1 \equiv 1283.69$ frequency of pi/N mode (m=1) in MHz

$\Delta f0 := 0$ frequency offset in kHz when bead is outside the cavity

$j := 1 \,..N$

$\Delta f_j :=$ measured frequency shifts in kHz for cells 1,...N
(frequency offset df0 is included)
insert as many df's as there are cells in the cavity
the remaining dfj are left with the default value of 1

j
1
1
1
1
1
1
1
1
1

$f\pi meas := fN$ measured pi mode frequency (MHz)

$f\pi des := fN$ desired pi mode frequency
(insert here different value if you like)

$\Delta f_j := \Delta f_j - \Delta f0$ net frequency shifts

$$\Delta fav := \frac{1}{N} \cdot \sum_j \Delta f_j$$ average frequency shift in bead pull

(C) Calculation of frequency corrections

$j := 1 \,..(N - 1)$ $k := 1 \,..(N - 1)$

$$\delta Ir_j := \left[\frac{\Delta f_j}{\Delta fav} - 1 \right] \cdot \frac{I(N, j)}{2}$$

$$\varepsilon r_j := \sum_k R_{(j,k)} \cdot \delta Ir_k$$

$$\varepsilon rav := \frac{1}{N} \cdot \sum_j \varepsilon r_j \qquad \varepsilon c_j := \varepsilon rav - \varepsilon r_j \qquad \varepsilon c_N := \varepsilon rav$$

$$j := 1 \,..N$$

$$fcorr_j := \varepsilon c_j \cdot \frac{f\pi meas}{2 \cdot N} - \frac{f\pi des - f\pi meas}{N} \qquad fcorr_j := fcorr_j \cdot 1000$$

(D) Table of frequency corrections for all cells j=1,....N
A positive value of fcorr in cell #j means that this cell must be lengthened
For negative fcorr it must be shortened

cell # j	frequency correction in kHz fcorr j
1	0
2	0
3	0
4	0
5	0

Program "Multicell-Tuning" for tuning of multicell cavities using bead pull data

For data input move to section (B)

example : LDP5-2

(A) Computation of important quantities
compute coupling parameter

$$\kappa := \frac{fN^2 - f1^2}{2 \cdot \left[fN^2 + f1^2 \cdot \cos\left[\frac{\pi}{N} \right] \right]}$$

$200 \cdot \kappa = 1.987$
coupling in %

normalized eigenmodes I(m,j) and eigenvalues Omega(m)
m = mode number, j = cell number

$$I(m,j) := \sqrt{\frac{2 - \delta(m,N)}{N}} \cdot \sin\left[\frac{m \cdot \pi}{2 \cdot N} \cdot (2 \cdot j - 1) \right]$$

$$\Omega(m) := \left[1 + 2 \cdot \kappa \cdot \cos\left[m \cdot \frac{\pi}{N} \right] \right]$$

ORIGIN ≡ 1

Correction matrix Hr for pi mode (reduced to N-1 dimensions)

$q := 1 \,..\, (N - 1)$ $\quad j := 1 \,..\, (N - 1)$ $\quad k := 1 \,..\, (N - 1)$

$$Hr_{j,k} := \frac{1}{2} \cdot \sum_{q} \frac{\Omega(q) - 3 \cdot \Omega(N)}{\Omega(N) - \Omega(q)} \cdot I(q,j) \cdot I(q,k) \cdot I(N,k)$$

$R := Hr^{-1}$ inverse matrix for N-1 dimensions

(B) Input of cavity parameters. Name of cavity: LDP5-2

$N \equiv 5$ number of cells (max. N=9)

$fN \equiv 1306.778$ frequency of pi mode (m=N) in MHz

$f1 \equiv 1283.459$ frequency of pi/N mode (m=1) in MHz

$\Delta f0 := 12.3$ frequency offset in kHz when bead is outside the cavity

$j := 1 \,..\, N$

$\Delta f_j :=$ measured frequency shifts in kHz for cells 1,...N
(frequency offset df0 is included)
insert as many df's as there are cells in the cavity
the remaining dfj are left with the default value of 1

Δf_j
47.2
47.4
46.8
48.5
48.5
1
1
1
1

$f\pi meas := fN$ measured pi mode frequency (MHz)

$f\pi des := fN$ desired pi mode frequency
(insert here different value if you like)

$\Delta f_j := \Delta f_j - \Delta f0$ net frequency shifts

$$\Delta fav := \frac{1}{N} \cdot \sum_j \Delta f_j$$ average frequency shift in bead pull

(C) Calculation of frequency corrections

$j := 1 \,..\, (N - 1)$ $k := 1 \,..\, (N - 1)$

$$\delta Ir_j := \left[\frac{\Delta f_j}{\Delta fav} - 1 \right] \cdot \frac{I(N, j)}{2}$$

$$\varepsilon r_j := \sum_k R_{(j,k)} \cdot \delta Ir_k$$

$$\varepsilon rav := \frac{1}{N} \cdot \sum_j \varepsilon r_j \qquad \varepsilon c_j := \varepsilon rav - \varepsilon r_j \qquad \varepsilon c_N := \varepsilon rav$$

$$j := 1 \,..N$$

$$fcorr_j := \varepsilon c_j \cdot \frac{f\pi meas}{(2 \cdot N)} - \frac{(f\pi des - f\pi meas)}{N} \qquad fcorr_j := fcorr_j \cdot 1000$$

(D) Table of frequency corrections for all cells j=1,....N
A positive value of fcorr in cell #j means that this cell must be lengthened
For negative fcorr it must be shortened

cell # j	frequency correction in kHz fcorr_j
1	3.841
2	-15.37
3	43.733
4	-32.369
5	0.165