Basic Concepts of Measurements Made on Superconducting RF Cavities

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Presented here is an introduction to some of the concepts used in the tests performed on superconducting RF cavities in the SRF group at Cornell. It is intended for the 'novice' to this field and by no means attempts a rigorous derivation. Assumed is some previous exposure to the physics of waveguides and cavities. Explained is the concept of the *Quality* (Q) of a cavity, the coupling parameters and how these quantities are determined by experiment. Furthermore a short overview of the results obtained for a typical cell is presented.

1. THE THEORETICAL CONCEPTS

1.1 THE QUALITY OF A CAVITY

The main task of the experiments is to determine the performance of superconducting niobium cavities. A possibility is to measure the so called unloaded Quality (Q_o) of the cavity as a function of the peak electric field within at a given resonance frequency. Q_o is defined by:

$$Q_o = 2\pi \frac{\text{Energy Stored in the Cavity}}{\text{Energy dissipated in the walls per cycle}}$$

If the angular frequency of the radiation is ω and we call the average dissipated power P_{diss} then assuming an $e^{i\omega t}$ dependence for the fields we find

$$Q_o = \omega \frac{U}{P_{diss}} \tag{1}$$

where U is the total energy stored in the cavity.

In practice though we somehow need to hook up the cavity to some source, to actually be able to 'inject' the microwave radiation. As we will show later we also need to measure the energy of radiation emitted through an output port in the cavity. Therefore input and output probes are an experimental necessity, through which invariably energy will be lost. We denote the power lost to the output probe by P_t and the power lost to the input probe by P_e. We can define the loaded Quality (Q_L) analogous to the unloaded Q_o as:

$$Q_L = \omega \frac{U}{P_{total}} \tag{2}$$

where

$$P_{total} = P_{diss} + P_e + P_t \tag{3}$$

is the total average Energy loss from the cavity per cycle. We now are in a position to obtain an expression for the stored energy as a function of time [$U(t)$] once the input power to the cavity is shut off.

Since in this case all energy lost from the cavity must come from the energy stored in the within, we have

$$P_{total} = -\frac{dU}{dt} \tag{4}$$

and we obtain $U(t)$ by combining (4) with (2) to give:

$$U(t) = U_o\, e^{-\omega t/Q_L} \tag{5}$$

(5) tells us that U falls to 1/e in a time $\tau = Q_L/\omega$ which can be measured experimentally to give the loaded Q. We will return to this later on.

1.2 THE WIDTH OF A RESONANCE

Usually, in the discussion of cavities and waveguides one of the simplifications made is that the walls are perfectly conducting. In the case of isolated cavities (ie. no ports) we have an infinite Q_o and the resonances of all modes are thus razor sharp δ-functions.

However in a lossy cavity wall losses result in a finite Q_o and energy transmission through any ports means that $Q_L <> Q_o$ which serve to broaden the resonances. Excitation of a mode is hence possible even if the frequency is not tuned perfectly, provided it at least lies within the line width of the resonance.

We know that the energy density scales as the electric field squared. Thus from (5) we can deduce that the electric field in an loaded cavity decays as:

$$E(t) = E_o\, e^{-\omega_o t/2Q_L}\, e^{-i(\omega_o+\Delta\omega)t} \tag{6}$$

where ω_o is the resonance frequency of the equivalent perfectly conducting cavity and $\Delta\omega$ is included to allow for a possible (small) frequency shift in the resonance frequency due to any losses. If we wish to determine the electric field as a function of frequency we simply need to Fourier transform (6):

$$E(\omega) = E_o \int_0^{\inf} e^{-\omega_o t/2Q_L}\, e^{i(\omega-\omega_o-\Delta\omega)t}\, dt \tag{7}$$

which gives

$$E(\omega) = \frac{E_o}{i\,(\omega-(\omega_o+\Delta\omega)) - \omega_o/2Q_o} \tag{8}$$

from which we see that the energy density per frequency interval scales as:

$$U(\omega) \;\alpha\; |E(\omega)|^2 \;\alpha\; \frac{1}{(\omega-\omega_o-\Delta\omega)^2 + (\omega_o/2Q_o)^2} \quad (9)$$

which is of the classic Breit-Wigner shape, shown in Figure 1.

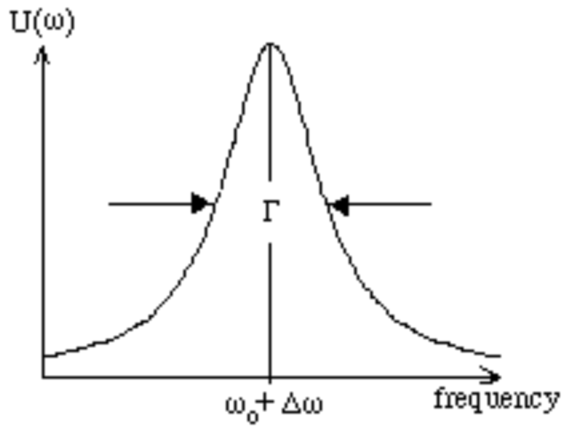


Figure 1

The curve falls to half its peak value at frequencies $\omega_o/2Q_L$ to the right and left of the central frequency ($\omega_o + \Delta\omega$). We identify this with the line width at half maximum $\Gamma = \omega_o/Q_L$. In the case of the superconducting rf cavities used in the SRF group, Q_o is typically around 10^9 and the width of the curve is only a tiny fraction of the resonance frequency.

1.3 RELATIONSHIP BETWEEN Q_L AND Q_o

Equation (5) is useful for determining the loaded Q of a cavity hooked up to input and output lines. However we usually wish to determine Q_o so that we can describe the characteristics of the cavity independent of the setup used to drive it.

To relate Q_L to Q_o we make two further definitions which are similar to (1):

$$Q_e = \frac{\omega U}{P_e} \quad (10)$$

$$Q_t = \frac{\omega U}{P_t} \quad (11)$$

combining (2) and (3) it is easy to see that:

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_e} + \frac{1}{Q_t} \quad (12)$$

If we identify the 'coupling parameters' as $\beta_e = Q_o/Q_e$ and $\beta_t = Q_o/Q_t$ then (12) can be recast as:

$$\frac{1}{Q_L} = \frac{1}{Q_o}(1 + \beta_e + \beta_t) \quad (13)$$

Clearly, if we can determine the coupling parameters and the loaded Q then Q_o can be calculated.

1.4 EQUIVALENT CIRCUITS FOR CAVITIES

To help us understand how to determine the coupling parameters we turn to an equivalent circuit for the cavity as shown in Figure 2.

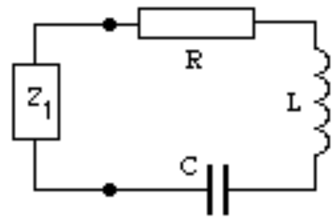


Figure 2

The two terminals represent the input to the cavity. Z_1 is meant to simulate the impedance of the circuitry leading to the input.

In the absence of any energy losses in the cavity walls (characterized by R in the circuit) and the input circuitry we would have a simple LC-circuit with resonance frequency:

$$\omega_o = \sqrt{\frac{1}{LC}} \quad (14)$$

The energy stored in the resonant circuit alone is simply $W_{stored} = 1/2LI^2$ where I is the peak current. Similarly the average power dissipated is $P_{diss} = 1/2RI^2$ so that the unloaded Q_o follows from (1) as:

$$Q_o = \omega \frac{1/2\; LI^2}{1/2\; RI^2} = \frac{\omega L}{R} \quad (15)$$

Similarly, in the presence of the input line the power dissipated in the load Z_1 is $P_e = 1/2\; Z_1 I^2$ so that the total power dissipated in the loaded circuit is $P_{total} = 1/2\; RI^2 + 1/2\; Z_1 I^2$. We see from (2) that Q_L in (2) takes on the form:

$$Q_L = \frac{\omega L}{R + Z_1} = \frac{\omega L/R}{1 + Z_1/R} \quad (16)$$

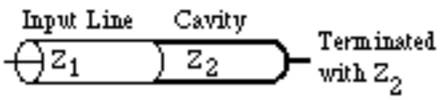
If we now identify β with Z_1/R then using (15) we can see that

$$\frac{1}{Q_L} = \frac{1}{Q_o}(1+\beta) \qquad (17)$$

which is of the same form as (13). We hence find that β in the case of the circuit is equivalent to β_e defined for the cavity.

Consider now the equivalent circuit being driven at a frequency ω via a transmission line of impedance Z_1. It will thus present an impedance of

$$Z_2 = \sqrt{R^2 - \left(\omega L - \frac{1}{\omega C}\right)^2}$$



to the transmission line. An electric wave travelling down the line is partially reflected at the interface in order to satisfy the following boundary conditions:

$$V_i + V_r = V_c$$
$$I_i - I_r = I_c$$

where V_i and V_r (I_i and I_r) are the voltages (currents) of the incident and reflected waves respectively. Similarly V_c (I_c) represents the voltage (current) of the wave transmitted across the boundary to our equivalent circuit. By definition the impedance is:

$$Z_1 = V_r/I_r = V_i/I_i, \; Z_2 = V_c/I_c$$

allowing us to solve the two equations to give:

$$\frac{V_r}{V_i} = \frac{1 - Z_1/Z_2}{1 + Z_1/Z_2} \qquad (18)$$

If the equivalent circuit is driven at ω_o (on resonance) then $Z_2 = R$ (the reactance is zero at ω_o). In this case, thus:

$$\frac{V_r}{V_i} = \frac{1-\beta}{1+\beta} \qquad (19)$$

Since power is proportional to $|V|^2$ we finally obtain the reflected power (P_r) as a function of the incident power (P_i):

$$P_r = \left(\frac{1-\beta}{1+\beta}\right)^2 P_i \qquad (20)$$

and by conservation of energy we have for the transmitted power (P_t):

$$P_t = P_i - P_r = \frac{4\beta}{(1+\beta)^2} P_i = P_{diss} \qquad (21)$$

which for continuous input power must be dissipated in R. We call this P_{diss}. Of course all these discussions here apply to the case where power has been supplied to the equivalent circuit for a sufficiently long time so that any transients which may occur due to the build up of fields in the inductor and capacitor have died down. In an actual cavity these transients would be due to the cavity filling with energy when power is switched on. This will be discussed later.

Imagine now, the input power were suddenly switched off. Immediately after this happens a current I_{inst} flows, given by:

$$I^2_{inst} = P_{diss}/R \qquad (22)$$

using $\beta = Z_1/R$ it follows from (21) that the power emitted instantaneously (P_e) back into the transmission line is:

$$P_e = I^2_{inst} Z_1 = \frac{4\beta^2}{(1+\beta)^2} P_i \qquad (23)$$

2.1 DETERMINING Q_o EXPERIMENTALLY

It is now up to us to put the results obtained for the equivalent circuit to use in determining the Q_o of the cavity.

Comparing (17) with (13) we see that provided we use $\beta_e = Q_o/Q_e$ for the coupling between the input probe and the cavity and if no output probe is present, then the reflected power is simply given by (20), which we repeat here:

$$P_r = \left(\frac{1-\beta_e}{1+\beta_e}\right)^2 P_i \qquad (24)$$

and in the same manner the instantaneously emitted power is given by (23):

$$P_e = \frac{4\beta_e^{\,2}}{(1+\beta_e)^2} P_i \qquad (25)$$

Solving these equations we obtain two expressions for β_e:

$$\beta_e = \frac{1 \pm \sqrt{\frac{P_r}{P_i}}}{1 \mp \sqrt{\frac{P_r}{P_i}}} \quad (26)$$

and

$$\beta_e = \frac{1}{2\sqrt{\frac{P_i}{P_e}} - 1} \quad (27)$$

These allow us determine the coupling between a transmission line and our cavity by simply measuring the incident power, the reflected power and, if we wish, the instantaneously emitted power P_e as P_i is turned off.

Furthermore, (10) indicates that if the input power is turned off, the power emitted from the cavity as a function of time is proportional to the present energy U stored within. We know from (5) that U decays as:

$$U(t) = U_o \, e^{-\omega_o t/Q_L} \quad (28)$$

Note that it is Q_L which appears in this expression, not Q_o. Clearly the emitted power should therefore also decay with the same time constant

$$\tau = \frac{Q_L}{\omega_o} \quad (29)$$

which we can measure directly. Once we have determined β_e from either (26) or (27) we finally obtain the unloaded Q_o:

$$Q_o = Q_L (1 + \beta_e) = \omega_o \tau (1 + \beta_e) \quad (30)$$

To obtain the peak electric field (E_{peak}) in the cavity we recall that energy density is related to the electric field by:

$$E_{peak} = \kappa_e \sqrt{U} \quad (31)$$

where κ_e is a constant dependent on the geometry of the cavity. For a pill box cavity this factor can be calculated analytically, however for the geometries we are using it needs to be determined numerically using computer codes. Once this factor has been obtained we are able to determine E_{peak} as follows.

A second probe is added to the other end of the cavity. Provided the coupling (β_t) is <<1, ie. $Q_t >> Q_o$, only very little power will be dissipated via this output. In this limit the previous discussion and the expressions for P_e and P_r are still valid. Since we know that $P_t \propto U$ by (11) we obtain the relation:

$$E_{peak} \propto \sqrt{P_t} \quad (32)$$

A measurement of β and τ allows us to calculate Q_o using (29) and (30). This is done at low field levels for reasons which will become clear later. Furthermore we know P_{diss} from (21) which allows us to calculate U using (1). It is then trivial to calculate E_{peak} from (31) assuming we have previously determined κ_e. The proportionality constant in (32) then follows from a measurement of P_t. At the same time this process also yields the factor relating P_t to U.

One could argue that all this is not necessary since we can determine Q_o and E_{peak} directly from (30) and (31). This is true. However, this relies critically on our ability to measure τ. At high field strengths processes such as field emission and thermal breakdown degrade the Q_o of the cavity. If we were to measure τ from the decay of P_e or P_t as P_i is pulsed we would observe that τ varies. At the start of the decay τ is small (small Q_o) until the fields within the cavity drop below the threshold value for these processes. Q_o then changes and correspondingly τ. To avoid the difficulties arising from this we determine the factors relating P_t to U and E^2_{peak} at low field strengths, where field emission etc. isn't possible and τ is constant throughout a decay. At the critical high fields our value for P_t then directly yields U and E_{peak}. From the former we then calculate Q_o via (1) and (21).

2.2 EXPERIMENTAL TECHNIQUES

2.2.1 General Setup

To couple the power into the cavity a setup as shown in Figure 3 is used. The power from the source is carried to the cavity via a coaxial cable as a TEM mode. The central conductor protrudes into the beam tube of the cavity which forms the outer conductor.

Since TEM modes cannot be supported in a hollow waveguide, we obtain a mixing of TE and TM modes once the end of the central conductor is reached. The diameter of the beam tube is arranged in such a manner, that the lowest cutoff frequency lies above the frequency we operate at.

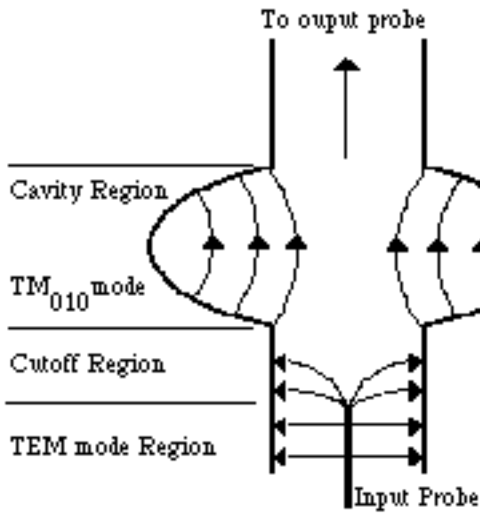


Figure 3: behavior of the fields in various parts of the cavity

Hence the fields decay exponentially until they reach the actual cavity.

The Q_o of a niobium cavity will generally be around 10^9 to 10^{10} at 1.5K. However, as field emission, multipacting and other processes begin to set in at higher electric fields, the Q_o begins to degrade, rapidly falling to 10^8 and lower. From (21) we see that the power transmitted into the cavity strongly depends on β_e. If the match between Q_e and Q_o is bad (ie β_e either <<1 or >>1) then a lot of power is required to achieve sizeable fields. It is hence desirable to maintain unity coupling. Since Q_o changes we have to be able to vary Q_e as well. To facilitate this the cavity is mounted on bellows. A motor drive allows the operator to move the cavity up and down over the input probe which remains fixed. Since the fields between the tip of the input coupler and the iris of the cavity decay exponentially only small movements are required to achieve changes in Q_e of several orders in magnitude.

A transmission probe is also added to the other end of the cavity. It is arranged in such that $Q_t \sim 10^{13}$, ie β_t <<1. As explained previously, this has a negligible effect on the cavity which hardly 'sees' the probe.

The general outline of the experimental apparatus used to test cavities is shown in Figure 4. For a more detailed description refer to Appendix A[1].

An oscillator produces the initial 1.5 GHz signal, which is the resonant frequency of the TM_{010} mode in the cavity. A modulator controls the amplitude of the signal via a Pin Attenuator (not shown here), allowing us to pulse the signal. The resulting waveform is amplified by a 100W amplifier and fed into the cavity from the bottom. This is simply to avoid dirt falling into the cavity from potential sources of dust, such as the bellows and the coupler.

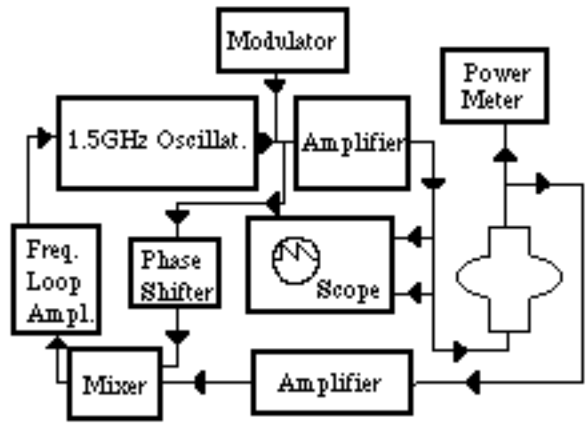


Figure 4: Outline of the experimental setup

Using directional couplers (also not shown) part of the signal is diverted to an oscilloscope or power meter, before it reaches the cavity. Another directional coupler also diverts part of the signal reflected at the cavity input into the same scope, allowing us to measure P_i and P_r. Similarly part of the signal from the output probe is routed into a power meter yielding P_t. The rest of the signal is amplified and fed into a mixer. A signal coming from the oscillator via a phase shifter forms the second input for the mixer. The output dc signal is dependent on the phase between the two inputs and is transmitted to a frequency loop amplifier. It provides the feedback for the oscillator which stabilizes the oscillator's frequency and ensures that the microwave signal stays 'in step' with the oscillations in the cavity. As an analogy consider a parent pushing their kid on a swing. To be able to maintain the oscillations, pushes are needed at the resonant frequency of the swing. But in addition to that they need to be in phase with the kid on the swing.

The phase shifter in the line between the oscillator and the mixer is required to compensate for a difference in distance travelled by the two input signals at the mixer and to compensate for any offsets the mixer might have. Note that at 1.5 GHz the wavelength in vacuo is 20 cm.

[1]For a detailed explanation of the individual microwave components used in our setup refer to Kevin Green: *An Introduction to Coaxial Microwave System*, Cornell University, June 1989

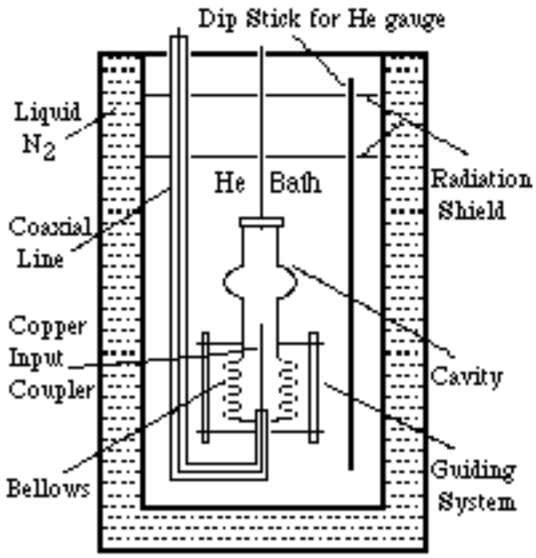


Figure 5

The cavity itself is housed in a dewar filled with liquid helium, surrounded by a liquid nitrogen bath (see Figure 5). The entire inner dewar is vacuum sealed so that the interior can be evacuated using the large pumps in the pump room. This lowers the vapor pressure allowing us to vary the temperature of the bath between 4.2K and 1.5K. A dip stick, which goes superconducting at liquid helium temperatures is connected to a meter measuring its resistance. From this one obtains a value for the amount of helium remaining in the dewar.

2.2.2 MAKING THE MEASUREMENTS

First measurements are usually made at 4.2K after the He transfer. At this temperature the surface resistance of the cavity is still relatively high giving rise to a large P_{diss}. As a result Q_o is low and we expect a wide resonance, see (9). This makes it easier to tune the oscillator. Initially the oscillator is set to about 1.5 GHz and the modulator is on cw-mode. The input coupler needs to be fairly far in, so that P_e is high and matches P_{diss}. This ensures near unity coupling. Viewing the reflected signal on the oscilloscope just gives a straight line which should be the same amplitude as the input signal, assuming we aren't locked in on the resonance by chance. The frequency is varied slowly until the resonance is found. Barring exceptional circumstances it should lie within a few tens of MHz of 1.5GHz. The reflected signal will oscillate violently once the resonance is found and the system tries to lock on. Usually the phase shifter now needs to be adjusted. It helps to look at the signal from the mixer in this case. When the phase is correctly set the output should be at a minimum.

Once the phase is set the modulator is switched to pulsed with a rep-rate of about 50 ms. Often it is convenient to look at the reflected signal on the scope which should look like one of the traces in Figure 6. If the system is still not locked in on the resonance, we expect all of the signal to be reflected, yielding square wave pulses as in Figure 5a). In that case the frequency and/or phase needs further adjustment. There is no set recipe for the adjustment procedure and a bit of luck is involved.

Once we do achieve a lock on, the trace will resemble one of the other three in Figure 6. In general the coupling won't be perfect (ie. $\beta <> 1$). Look at fig 6d) first. In this case the cavity is undercoupled, that is $\beta < 1$ and $Q_o < Q_e$. Initially, as the incident power is turned on, no energy is stored in the cavity and we have not yet established an equilibrium condition. This results in a mismatch between the cavity and the transmission line and hence all power is reflected leading to the large spike. As the cavity fills, the matching improves and the reflected power drops[2] . However, since we are undercoupled we still get reflected power, even when the cavity is filled completely (ie the power dissipated equals the power entering the cavity). In such a case, we use the motor drive to lower the cavity in the dewar thereby increasing the distance by which the input probe

One usually finds that at 4.2K the system is undercoupled because the cavity Q is so low. Note also when the input power is cut, the reflected signal does not drop to zero instantaneously. What we in effect are then viewing is the power emitted from the cavity opening (P_e) as the energy in the cavity empties. The decay is exponential with the decay constant given by (29). Thus we can measure τ directly from the oscilloscope. At lower temperatures and small input powers especially, one may find that the system is overcoupled, giving rise to a trace as in Figure 6c). In this case perfect coupling is achieved before the cavity is completely filled and the reflected signal dips to zero at that point. However as energy continues to 'pour' in, we again have a mismatch and some energy is reflected, resulting in a rise in P_r. In such a situation we would raise the cavity to extract the input probe from the beam tube in order to reduce the coupling.

protrudes into the beam tube.Ideally we wish to adjust the coupling until $\beta = 1$ and we obtain a trace just like Figure 6b). In practice that may not always be

[2] For a discussion of the reflected power during this transient period (ie. while equilibrium has not yet been achieved) refer to Appendix B.

possible, especially when the cavity Q has degraded a lot due to field emission, multipacting etc.

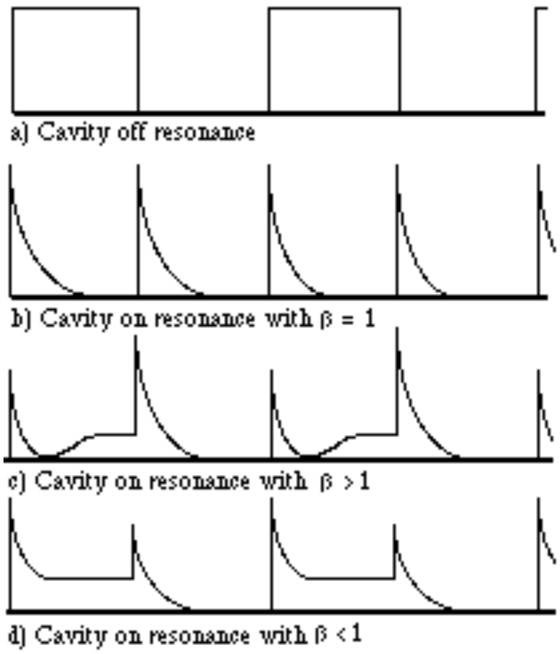


Figure 6

Once things are optimized we are in a position to measure Q_o. As mentioned earlier, τ is measured directly from the decay of the emitted power after the input power is switched off.

For our future measurements we require P_i , this being the power incident on the cavity. However note that the forward power is measured at a directional coupler located outside of the dewar. We must remember that the cables in the dewar leading to the cavity do have some attenuation α. We need to therefore first calibrate this. Denoting the forward power measured at the directional coupler by P_f we have $P_i = \alpha\ P_f$. Furthermore if the cavity is out of lock then all power will be reflected. Thus the reflected power we measure at the directional coupler (P_r) will be given by $P_r = \alpha\ P_i$.. Hence we can solve these two equations to give:

$$\alpha = \sqrt{\frac{P_r}{P_f}} \quad \text{(Cavity out of lock)}$$

so that in future we can always obtain P_i by measuring P_f once α has been determined.

Measurement of P_i and the instantaneously emitted power P_e yields β directly from (27). P_e is obtained from the height of the peak produced as the incident power is switched off (see Figure 6). On the other hand P_f is given by the height of the peak as the input power is switched on or, alternatively, by the height of the square pulses when the system is taken out of lock. Again one should note that these powers are measured at the directional coupler. However if we determine both P_i and P_e from the reflected power trace both signals will suffer the same attenuation on their way from the cavity to the coupler, so that the ratio P_i to the true P_e will be the same as that of P_f to the measured value of P_e . Furthermore since only the ratio of the two is required we don't need to calibrate the scope to obtain absolute units.

Finally we also measure the transmitted power, P_t and P_i, this time in proper units. Note that this needs to be done with the modulator set to cw. The powers are measured on calibrated power meters.

Once these preliminary measurements have been completed we can calibrate the system as explained in section 2.1. We are now in a position to measure Q versus E_{peak} as the input power is increased. Once the calibration has been done measurements of τ are no longer required.

For moderately high fields the procedure remains unchanged w.r.t. to the one described above. However as E_{peak} is increased to perhaps 15 - 20 MV/m it becomes increasingly likely that breakdown and other processes occur.

Often one first observes multipacting. This is a result of stray electrons in the cavity, which can for example be a result of cosmic rays or emission from the surface. At certain field strengths these electrons will follow a closed orbit trajectory, and may impact on the surface. Near the surface of the cavity the magnetic fields are strong and electrons in this region will follow cyclotron orbits of frequency

$$\omega_c = \frac{eB}{m} \qquad (32)$$

where e is the electron's charge, B is the applied field strength and m is the electron's mass.

Thus if $n\omega_c$ (n = integer) is equal to the driving frequency ω the electrons will return to the same point in phase with the field and impact on the surface.

If the secondary emission coefficient (δ) of that surface is greater than 1, further electrons are emitted usually with a few electron volts of energy. They too follow the closed orbit, gaining energy from the perpendicular electric field and impact at the same point. We therefore arrive at an avalanche situation. If the input power is increased it no longer serves to increase the stored energy in the cavity, but simply enhances the multipacting process. It is as if we have reached a barrier beyond which E_{peak} no longer can be increased.

Correspondingly, if we look at the transmitted power P_t which is a measure of the energy stored in the cavity, we see that once the fields within are high enough for multipacting, clipping occurs. An increased P_i no longer increases U and thus P_t remains constant. Hence we never achieve unity coupling, and P_r remains finite.

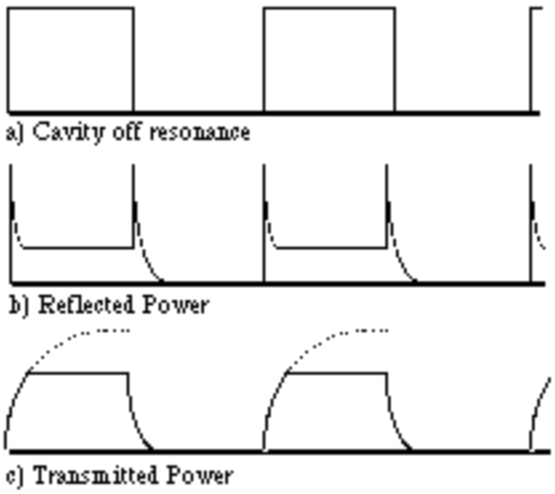


Figure 7

It is possible to break through this multipacting by increasing the magnetic field levels so that $\omega_c = \omega$ cannot be satisfied. Furthermore, δ is a function of energy, being greater than one in only a narrow energy interval (E_1 to E_2) and hence it is only necessary to break through a finite number barriers in this energy range. One also finds that it is possible to alter the surface characteristics due to the severe electron bombardment and lower the secondary emission coefficient to values below one by allowing multipacting to proceed for some time (perhaps 20 minutes or so). This method is very effective and is termed 'processing'. Multipacting barriers which can be processed away are referred to as 'soft barriers'.

In fact the cavity design is in part such that multipacting is reduced. For the best shapes used, stable electron trajectories are not possible and any orbiting electrons quickly drift to the equator of the cavity where the perpendicular electric field is zero. Thus multipacting electrons no longer can gain energy and the process is halted. It is therefore fairly simple to process through any multipacting barriers which may still exist. Cavity cleanliness is also important in avoiding multipacting. One finds that δ depends on surface composition, clean surfaces in general displaying a lower δ and a narrower window (E_1 - E_2) for which $\delta > 1$.

At higher field strengths it is common to run into a combination of field emission and thermal breakdown. At present, the mechanisms leading to these are still not fully understood and the reader is referred to literature for the latest theory on these. Here we simply present a short overview.

Field emission[3] occurs when the probability of tunnelling through the potential barrier presented to the electrons in the metal (the work function) becomes large enough to produce a significant current. This in turn draws energy from the incident power thereby spoiling the cavity Q. Typical of barrier penetration is the electron transmission probability's exponential dependence on field strength, which serves to lower the barrier height. Fowler and Nordheim treated this problem quantum mechanically and found that[4]:

$$j(E) = \frac{AE^2}{\phi} \exp\left(-B \frac{\phi^{3/2}}{E}\right) \tag{33}$$

where j is the current, ϕ is the workfunction of the metal, A and B are constants given below and E is the electric field strength. We have

$$A = 1.54 \times 10^{-6} \frac{\text{A eV}}{\text{Vcm}^4}$$

and

$$B = 6.83 \times 10^{7} \frac{\text{V eV}^{-3/2}}{\text{cm}}$$

provided $j(E)$ is given in A/cm^2, E in V/cm and ϕ in eV. Field strengths in typical cavities are of the order of 10 MV/m. Using such a field in (33) we find that the current is utterly negligible. However it is often observed that field emission does limit the cavity's performance and one finds empirically that field emission still displays Fowler-Nordheim behavior provided we scale the electric field in (33) by a factor of β_{FN}. It is as if locally the field is enhanced, sometimes by factors up several hundred. The reasons for this are by no means understood properly, yet.

This observation implies that (33) needs to be modified to give:

$$j(E) = \frac{A(\beta_{FN}E)^2}{\phi} \exp\left(B \frac{\phi^{3/2}}{\beta_{FN}E}\right) \tag{34}$$

The field emitted electrons are accelerated by the electric field and smash into the cavity walls at other

[3] Field emission, for example, is discussed in H. Padamsee: *Superconducting RF*, AIP Conference Proceedings: The Physics of Particle Accelerators.

sites. Due to Bremsstrahlung we thus get very energetic x-rays, which are a signature for field emission. These x-rays, sometimes, are even capable of penetrating the ~2 foot sheetrock blocks used for shielding. The number of emission sites increases exponentially with the field strength and we observe a steep decrease in the Q_o versus E_{peak} curve obtained once field emission occurs (see Figure 9).

Field emission currents are spread out over areas as small as 10^{-9} cm^2 and typically current densities are around 100 A/cm^2 although they can reach densities as high as 10^9 A/cm^2. Due to the finite resistance of the niobium and the fast time scales involved we therefore can have regions of severe heating. These drive the niobium into the normal state and ultimately are capable of melting the material. Sometimes emitters will process away leading to what is believed are small 'explosions'. Subsequent examination of emission sites under an electron microscope frequently reveals characteristic 'star bursts', so called because of the star like area (perhaps 50 - 100μm across) of reduced secondary emission coefficient around the emission site. At the center usually lies a lump of molten niobium.

Different methods have been developed to reduce field emission and to push the fields at which it occurs to higher limits. They include a) extended rinsing, b) heat treatment c) He processing and d) RF processing. For a discussion of these the reader is referred to reference 3. The last one on the list (d) is currently being studied extensively[4] and involves 'blasting' the cavity at high power levels (often pulsed with Powers ~10's of kW and pulse times ~100 μs). In certain instances, following this procedure one finds that the emitter has processed away.

Figure 8: Trace of P_t with thermal breakdown occurring

Field emission as well as imperfections and impurities as small as 0.1 mm across can also lead to a breakdown behavior called thermal breakdown. Tiny regions on the surface of the cavity which are not superconducting, or only become superconducting at temperatures lower than the bulk superconductor will dissipate far more energy from the rf-field than the rest of the surface. This causes heating of the surrounding areas, driving these normal as well once the temperature of the defect is above 9.2K (T_c for Nb). Because the normal state is far more resistive (by many orders of magnitude) a substantial amount of power dissipation will occur in this region. Q_o drops dramatically and hence τ too, typically from about 0.5s (Q_o ~ 10^{10}) to only a ~ milliseconds. A trace of P_t looks somewhat like Figure 8.

As the cavity starts filling the fields inside cause thermal breakdown. The Q then suddenly drops yielding a tiny τ and resulting in a mismatch between the input coupler and the cavity. No more energy is input in the cavity. The remaining fields within decay over a period ~ τ, yielding the spikes in fig. 8. During the decay the normal regions are cooled by the helium bath and go superconducting again. The original Q thus is restored, and we once again observe transmitted power and the process described above repeats once more. Increasing the input power thus only serves to increase the frequency of these breakdowns.

To reduce thermal breakdown and field emission it is imperative to go through extensive clean room procedures in order to minimize any foreign particles. Very effective in improving field strengths at which thermal breakdown first occurs, is an increased thermal conductivity of niobium. Any heat produced by the tiny impurities is quickly carried away by the surrounding regions, thereby reducing the likelihood that surrounding niobium is also driven normal. The thermal conductivity is strongly related to the purity of the niobium. Often quoted is the RRR or the 'residual resistance ratio' which is the ratio of the resistance at room temperature to that at 4.2K with the niobium in the normal state. (At 4.2K this is done by driving the material normal with a sufficiently high magnetic field, $H > H_c$ = critical field strength). Since the RRR is directly related to the impurity concentration, one finds that high RRR materials are not as susceptible to breakdown as low RRR ones. Typically the cavities we use have a RRR of the order of 100 - 500 at present. However we still need to contend with field emission which proves to be the limiting process.

3. TYPICAL RESULTS

Figure 9 presents a Q_o versus E_{peak} curve somewhat typical of the results usually obtained. Initially the Q_o is high (10^{10}) and remains constant as the power is increased. In this region multipacting is frequently observed but as mentioned previously it usually is not difficult to process through this.

[4] Joel Graber *'An Apparatus for High Power Processing of Field Emitters in Superconducting RF Cavities'* (MS Thesis), Cornell University, May 1990

At higher field levels (~ 15 MV/m) the first X-rays being produced in the cavity were detected, an indication that field emission was occurring. We see that from that point on the Q_o rapidly falls off reaching the low 10^9 at field levels around 35 MV/m. Note however the point at about 31 MV/m. During the experiment we found that the X-rays suddenly were reduced as that point was being measured. We also see that its Q_o is significantly higher than would be expected from the extrapolation of the curve. This is probably an example of an emitter being processed away, yielding an improved quality. However, as the field is increased to even greater values, new emitters become active and the Q_o drops further.

For other cavities other features in such a plot may exist. The constant Q_o at low field levels and the rapid fall off at higher ones, though, is typical for all results.

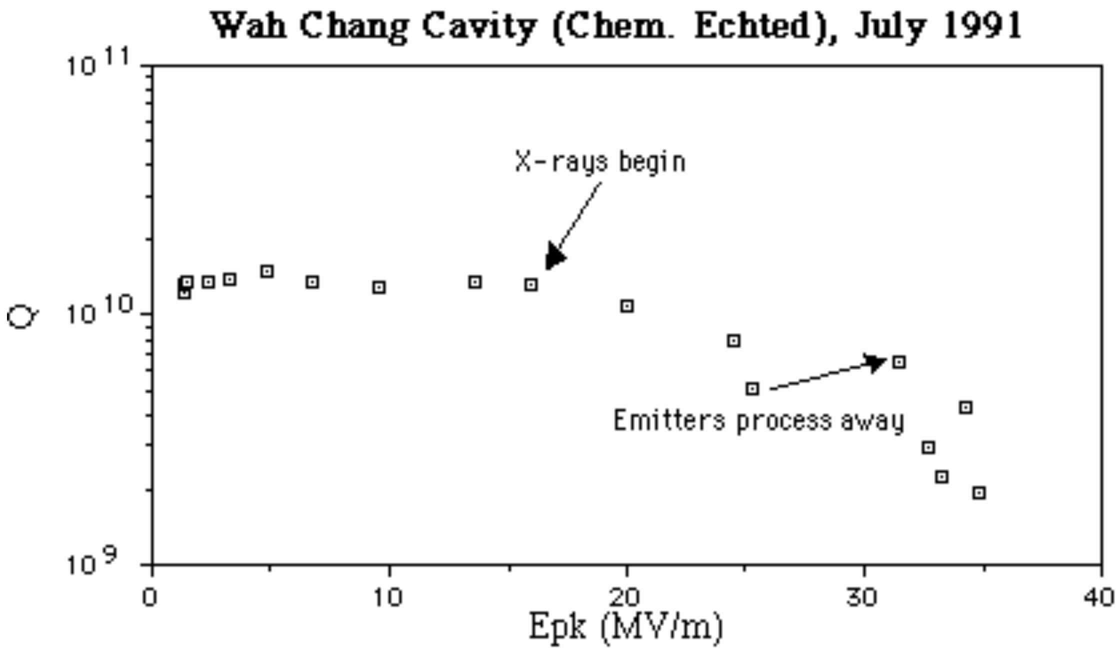


Figure 9

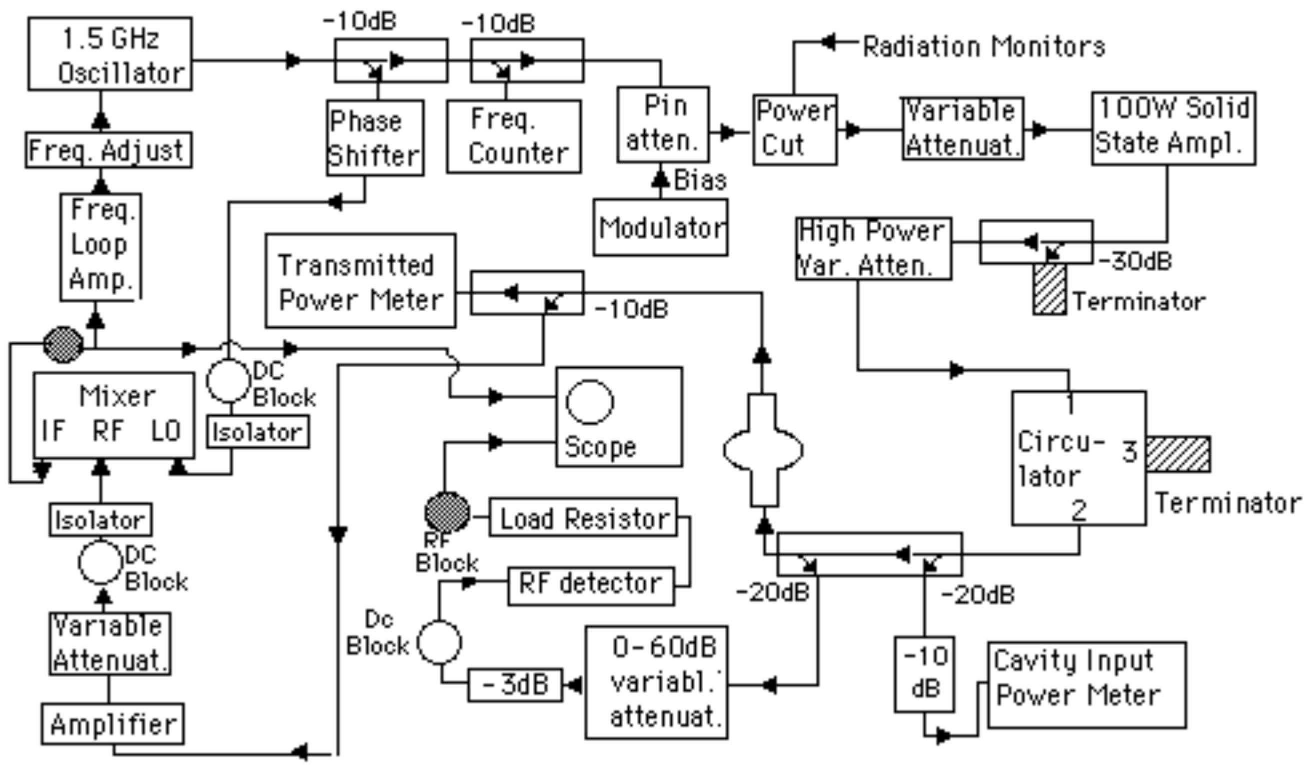
4. OTHER CAVITIES

In addition to the single cell cavities described here, the SRF group also tests multicell cavities (in effect several single cell cavities welded together) and mushroom cavities. The latter is actually only half a cavity with a flat baseplate. It is constructed in such a way that the plate can be removed after a test allowing us to examine the surface under an scanning electron microscope (SEM). A dimple about 3/4 cm in diameter ensures that the highest field strengths within the cavity are in this region thereby reducing the area that needs to be searched in the SEM.

Due to different sizes and geometries the apparatus used for the multicell and mushroom cavities is somewhat different to the one described here. In addition to this the multicell cavities require some additional 'attention', for example the individual cells need to be tuned (by varying their size) so that all resonant frequencies are the same. This process is called 'bead pulling'. Despite these differences though, the methods used to obtain the final Q_o versus E_{peak} curve are very similar to the ones described here and the theoretical discussion still applies.

APPENDIX A)

Detailed schematic diagram of the setup used for single cell, 1.5 GHz Niobium cavities.



APPENDIX B)[5]

Presented here is a short discussion of the behavior of reflected power as a function of time during the transient periods just after the input power is switched on. In this case the the cavity is still filling with energy, ie. dU/dt is not zero, as would be the case in the steady state. By conservation of energy we can write down the following relation:

$$P_r = P_i - P_{diss} - \frac{dU}{dt} \qquad \text{(B1)}$$

where P_{diss} is given by (21) *for the steady state*. Now, from (5) we recall that U decays exponentially as $e^{-t/\tau}$ once the input power is turned off. This implies that the electric field decays as $e^{-t/2\tau}$. Similarly, when the input power is initially switched on, we expect the electric field to rise as:

$$E(t) = E_o\,(1 - e^{-t/2\tau}) \qquad \text{(B2)}$$

where E_o is the steady state E-field as t tends towards infinity. We can thus conclude that the energy density rises as:

$$U(t) = U_o\,(1 - e^{-t/2\tau})^2 \qquad \text{(B3)}$$

One could raise the objection that we should expect $U(t)$ to increase according to:

$$U(t) = U_o\,(1 - e^{-t/\tau}) \qquad \text{(B4)}$$

which would lead us to believe that $E(t)$ behaves as:

$$E(t) = E_o\sqrt{1 - e^{-t/\tau}} \qquad \text{(B5)}$$

which differs from (B2). To resolve this problem, we appeal to the equivalent circuit again. Recall that when calculating the resonant frequency, as well as the time dependence of the fields in for example the capacitor, we use the fields as the fundamental quantity, not the energy, in which case we obtain a $(1 - e^{-t/\tau})$ dependence for rising fields. In an analogous manner, if we consider the electric fields in the cavity it is actually quite straight forward to derive expression (B2) for $E(t)$. We will thus use (B2) and (B3) in the following analysis. If indeed one instead proceeds to use (B4) and (B5) one finds that the following analysis yields unphysical results.

Now back to our original problem. By (1) we know that the power dissipated in the cavity $[P_d(t)]$ scales linearly with $U(t)$, ie:

$$P_d(t) = \frac{\omega U(t)}{Q_o} = P_{diss}\,(1 - e^{-t/2\tau})^2 \qquad \text{(B6)}$$

which we can recast using (21) as:

$$P_d(t) = \frac{4\beta}{(1+\beta)^2}\,(1 - e^{-t/2\tau})^2\,P_i \qquad \text{(B7)}$$

Furthermore we recall that from (1)

$$U_o = \frac{P_{diss}Q_o}{\omega} = \frac{4\beta}{(1+\beta)^2}\frac{P_i\,Q_o}{\omega} \qquad \text{(B8)}$$

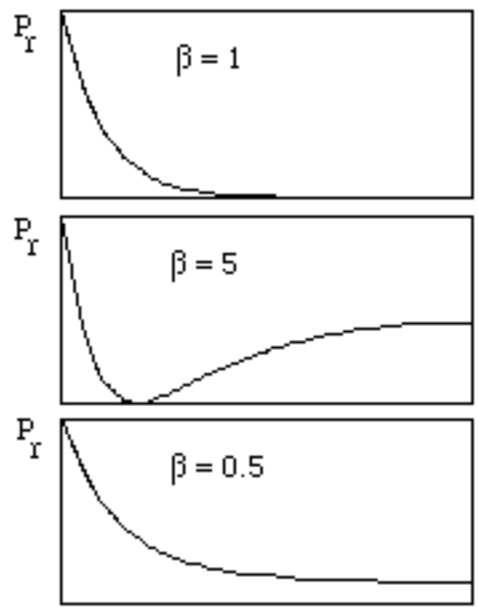


Figure B1: Plot of the reflected power for a) perfect coupling, b) overcoupling and c) undercoupling.

We can therefore rewrite (B1) as:

$$P_r(t) = P_i\left(1 - \frac{4\beta}{(1+\beta)^2}\,(1-e^{-t/2\tau})^2\right) - P_i\,\frac{4\beta\,Q_o}{\omega(1+\beta)^2}\,\frac{d}{dt}\Big((1 - e^{-t/2\tau})^2\Big) \qquad \text{(B9)}$$

x

[5] From private communications with Joel Graber, Cornell University.

which can easily be simplified, noting that $Q_0/(\omega(1+\beta)) = \tau$, giving:

$$P_r(t) = P_i \times$$
$$\times \left(1 - \frac{4\beta}{(1+\beta)^2}(1-e^{-t/2\tau})^2 - \frac{4\beta}{(1+\beta)}(1-e^{-t/2\tau})e^{-t/2\tau}\right)$$

This equation describes the traces in Figure 6 for various couplings very well. Some typical plots are given in Figure B1. Note that the third term in the above equation goes to zero as t goes to infinity. It represents the transients. On the other hand, the second term asymptotically approaches $4\beta/(1+\beta)^2$, so that the steady state value for $P_r(t)$ is given by:

$$P_r = \left(\frac{1-\beta}{1+\beta}\right)^2 P_i$$

in agreement with (20).