

TRANSVERSE HYBRID MODE IN A FERRITE-LINED PILLBOX CAVITY. COMPARISON OF THEORY AND CODES: CLANS2, MWS et al.

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INTRODUCTION

For cavity calculations 2D codes of the SLANS family are used in our lab. There are 4 codes: SLANS, SLANS2, CLANS, and CLANS2 [1], all of them are for axially symmetric geometries. Two first codes calculate cavities without losses; perturbation method is only used for Q -factor calculation associated with losses in the cavity envelope. Last two codes deal with losses in ferromagnetic materials, “C” stands for complex ϵ and μ . Codes SLANS and CLANS deal with axially symmetric modes, the other pair can calculate modes with azimuthal variations. For 3D calculations with electric and magnetic losses in the materials we use Microwave Studio code by CST [2].

In the process of calculations of dipole Higher Order Modes in a multicell cavity, a significant difference in results obtained with CLANS2 and Microwave Studio (MWS) was revealed. When the authors of the codes were asked about the possibility of bugs in the codes both sides readily agreed: yes, there is a bug. However, it was not so easy to find the bug because it turned out that no available

explicit analytical solutions for the transverse mode were at hand. This failure of CLANS2 was very disappointing because the versions for calculations of axially symmetric modes (SLANS, CLANS) showed very accurate results [3] compared to any other code. Obviously we need to make an analogous accuracy test for CLANS2. Later the CST company reported that no significant errors in the code were found and their 3D code MWS can give accurate enough results but requires for this purpose a denser mesh. We can add that this means a lot of time that is not good for optimization when you have to calculate small changes of Figures of Merit versus changes of geometrical parameters. The CLANS2 is a much faster code because it is dedicated to calculate an *axially symmetric (2D) geometry*. At the same time, as it is said above, it calculates *axially asymmetric modes* and takes into account *lossy materials*: dielectrics and ferromagnetics. In any case, a bench test is needed to guess how far from the exact solution we are. This is why this technical note is written.

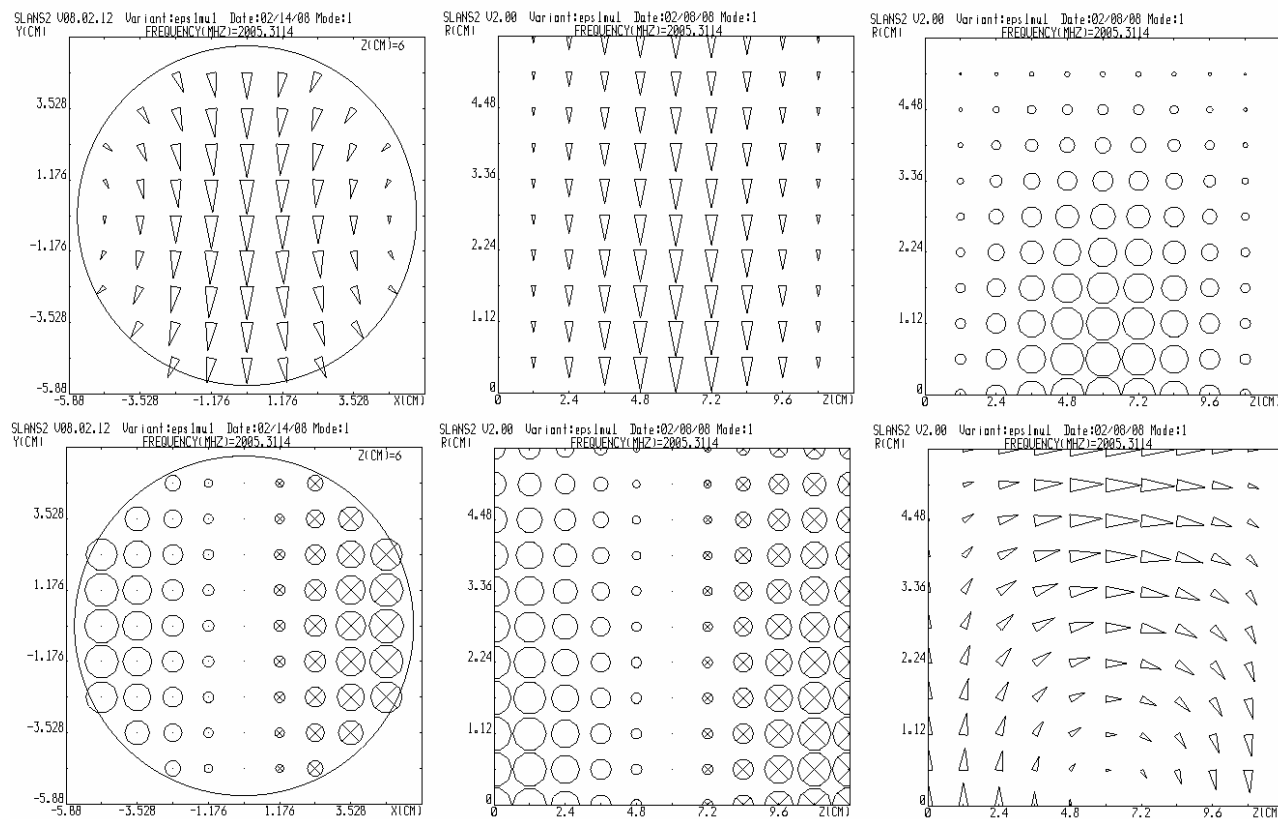


Fig. 1. Electric (upper row) and magnetic fields in an empty cylindrical cavity with the transverse mode TE₁₁₁.
SLANS2.

THE SIMPLEST CASE

Fields in an empty cylindrical cavity (pill-box) with a TE₁₁₁ mode have a form (see, for example, [4]):

$$\begin{aligned} H_r &= i \frac{\beta}{\chi} J_1'(\chi r) \cos \varphi \cos \beta z, \\ H_\varphi &= -i \frac{\beta}{\chi^2 r} J_1(\chi r) \sin \varphi \cos \beta z, \\ H_z &= i J_1(\chi r) \cos \varphi \sin \beta z, \\ E_r &= -\frac{\omega \mu_0}{\chi^2 r} J_1(\chi r) \sin \varphi \sin \beta z, \\ E_\varphi &= -\frac{\omega \mu_0}{\chi} J_1'(\chi r) \cos \varphi \sin \beta z, \\ E_z &= 0. \end{aligned} \quad (1)$$

Here, β and χ are the longitudinal and transverse wave number, respectively, $J_1(x)$ is the first type Bessel function, φ, r, z are cylindrical coordinates, the time factor $e^{i\omega t}$ is leaved out. From the boundary conditions $\beta = \pi/L$ for the mode with 1 variation along z where L is the length of the cavity.

Fields of this mode calculated with SLANS2 are presented in Fig. 1.

THE HYBRID MODE

For calculations of the hybrid mode, a simple geometry was chosen; a hollow metal cylinder with length L and radius b , partially, from radius a to b , filled with a material having permeability ε and permittivity μ , Fig. 2. The TE₁₁₁-like mode is excited in the cylinder. This is a hybrid mode in the presence of a magnetodielectric but it is a pure TE₁₁₁ mode when the cavity is empty: $\varepsilon = \mu = 1$, see above.

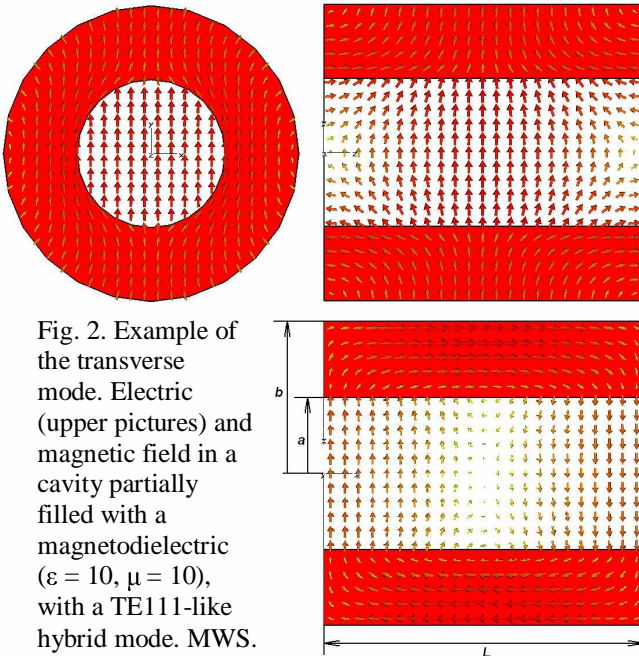


Fig. 2. Example of the transverse mode. Electric (upper pictures) and magnetic field in a cavity partially filled with a magnetodielectric ($\varepsilon = 10$, $\mu = 10$), with a TE₁₁₁-like hybrid mode. MWS.

In the simplest case, the z -component of the electric field is equal to zero, see (1) and Fig. 1. For the more general case we will invoke the results of [5]. This very useful paper has no explicit expressions for φ and r components and we will derive them here. From [5] we have for the longitudinal fields:

$$\begin{aligned} E_z &= A_1 \chi_1^2 J_1(\chi_1 r) \sin \varphi \cos \beta z, \\ H_z &= C_1 \chi_1^2 J_1(\chi_1 r) \cos \varphi \sin \beta z \end{aligned} \quad (2)$$

- for the inner area and

$$\begin{aligned} E_z &= A_2 \chi_2^2 F_1(\chi_2 r) \sin \varphi \cos \beta z, \\ H_z &= C_2 \chi_2^2 G_1(\chi_2 r) \cos \varphi \sin \beta z \end{aligned} \quad (3)$$

- for the area of the magnetodielectric, where χ_1 and χ_2 are transverse wave numbers for the inner and the dielectric area, respectively, A_1, A_2, C_1, C_2 are field amplitude coefficients,

$$\begin{aligned} F_1(\chi_2 r) &= J_1(\chi_2 r) - \frac{J_1(\chi_2 b)}{Y_1(\chi_2 b)} Y_1(\chi_2 r), \\ G_1(\chi_2 r) &= J_1(\chi_2 r) - \frac{J_1'(\chi_2 b)}{Y_1'(\chi_2 b)} Y_1(\chi_2 r), \end{aligned} \quad (4)$$

$Y_1(x)$ is the second type Bessel function.

Using [6] Maxwell's equations for the curl components:

$$E_\varphi = \frac{1}{i\omega\varepsilon\varepsilon_0} \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) \quad \text{and}$$

$$H_\varphi = \frac{i}{\omega\mu\mu_0} \left(\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right), \quad \text{where } i\omega = \frac{\partial}{\partial t}, \quad \varepsilon = \mu = 1$$

for the inner area, and taking into account that

$k^2 = \beta^2 + \chi_1^2$ and $\varepsilon\mu k^2 = \beta^2 + \chi_2^2$ we can express the φ -components in terms of functions in (2) and (3):

$$E_\varphi = \left[-(\beta/r) A_1 J_1(\chi_1 r) + i\omega\mu_0 C_1 \chi_1 J_1'(\chi_1 r) \right] \cdot \cos \varphi \sin \beta z,$$

$$H_\varphi = \left[-(\beta/r) C_1 J_1(\chi_1 r) - i\omega\varepsilon_0 A_1 \chi_1 J_1'(\chi_1 r) \right] \cdot \sin \varphi \cos \beta z, \quad (5)$$

$$E_\varphi = \left[-(\beta/r) A_2 F_1(\chi_2 r) + i\omega\mu\mu_0 C_2 \chi_2 G_1'(\chi_2 r) \right] \cdot \cos \varphi \sin \beta z,$$

$$H_\varphi = \left[-(\beta/r) C_2 G_1(\chi_2 r) - i\omega\varepsilon\varepsilon_0 A_2 \chi_2 F_1'(\chi_2 r) \right] \cdot \sin \varphi \cos \beta z. \quad (6)$$

Equations (5) belong to the inner area and (6) – to the area of magnetodielectric.

Finally, from

$$E_r = \frac{1}{i\omega\varepsilon\varepsilon_0 r} \left(\frac{\partial H_z}{\partial \varphi} - r \frac{\partial H_\varphi}{\partial z} \right) \quad \text{and}$$

$$H_r = \frac{i}{\omega\mu\mu_0 r} \left(\frac{\partial E_z}{\partial \varphi} - r \frac{\partial H_\varphi}{\partial z} \right),$$

we can find

$$E_r = \left[\frac{i\omega\mu_0}{r} C_1 J_1(\chi_1 r) - \beta\chi_1 A_1 J_1'(\chi_1 r) \right] \sin \varphi \sin \beta z,$$

$$H_r = \left[\frac{i\omega\varepsilon_0}{r} A_1 J_1(\chi_1 r) - \beta\chi_1 C_1 J_1'(\chi_1 r) \right] \cdot \cos\varphi \cos\beta z, \quad (7)$$

$$E_r = \left[\frac{i\omega\mu\mu_0}{r} C_2 G_1(\chi_2 r) - \beta\chi_2 A_2 F_1'(\chi_2 r) \right] \cdot \sin\varphi \sin\beta z,$$

$$H_r = \left[\frac{i\omega\varepsilon_0}{\mu r} A_2 F_1(\chi_2 r) - \beta\chi_2 C_2 G_1'(\chi_2 r) \right] \cdot \cos\varphi \cos\beta z. \quad (8)$$

where, again, two equations (7) are for the vacuum area and the last pair (8) is for the filled area.

$E_z = 0$ and $E_\varphi = 0$ at $r = b$ due to definition of functions $F_1(\chi_2 r)$ and $G_1(\chi_2 r)$ (4). Boundary conditions require equality of fields E_z, E_φ, H_z and H_φ at $r = a$.

We get

$$\begin{aligned} A_1 x^2 J_1(x) - A_2 y^2 F_1(y) &= 0, \\ -A_1 \beta J_1(x) + A_2 \beta F_1(y) + iC_1 \omega \mu_0 x J_1'(x) - iC_2 \omega \mu_0 y G_1'(y) &= 0, \\ C_1 x^2 J_1(x) - C_2 y^2 G_1(y) &= 0, \\ C_1 \beta J_1(x) - C_2 \beta G_1(y) + iA_1 \omega \varepsilon_0 x J_1'(x) - iA_2 \omega \varepsilon_0 y F_1'(y) &= 0, \end{aligned} \quad (9)$$

where $x = \chi_1 a$, $y = \chi_2 a$.

The determinant of the coefficient matrix of the system (9) must be zero to obtain nonzero roots, and then the dispersion equation is

$$\left[y J_1'(x) G_1(y) - \mu x J_1(x) G_1'(y) \right] \cdot \left[y J_1'(x) F_1(y) - \varepsilon x J_1(x) F_1'(y) \right] - \frac{\beta^2 (y^2 - x^2)^2 J_1^2(x) F_1(y) G_1(y)}{k^2 x^2 y^2} = 0. \quad (10)$$

If we rewrite the system (9) in the form

$$\begin{aligned} a_{11} A_1 + a_{12} A_2 &= 0, \\ a_{21} A_1 + a_{22} A_2 + a_{23} C_1 + a_{24} C_2 &= 0, \\ a_{33} C_1 + a_{34} C_2 &= 0, \\ a_{41} A_1 + a_{42} A_2 + a_{43} C_1 + a_{44} C_2 &= 0, \end{aligned}$$

then we can find all the coefficients for (2), (3), (5) – (8). The solution can be expressed in two ways because of zero value of determinant of the above system; both versions give the same values:

$$\begin{aligned} A_2 = -\frac{a_{11}}{a_{12}} A_1, \quad C_1 = -\frac{a_{21} - \frac{a_{11} a_{22}}{a_{12}}}{a_{23} - \frac{a_{24} a_{33}}{a_{34}}} A_1, \quad C_2 = -\frac{a_{21} - \frac{a_{11} a_{22}}{a_{12}}}{a_{24} - \frac{a_{23} a_{34}}{a_{33}}} A_1 \quad \text{or} \\ A_2 = -\frac{a_{11}}{a_{12}} A_1, \quad C_1 = -\frac{a_{41} - \frac{a_{11} a_{42}}{a_{12}}}{a_{43} - \frac{a_{44} a_{33}}{a_{34}}} A_1, \quad C_2 = -\frac{a_{41} - \frac{a_{11} a_{42}}{a_{12}}}{a_{44} - \frac{a_{43} a_{34}}{a_{33}}} A_1. \end{aligned}$$

For complex ε and μ we have to solve equation (10) together with

$$x = a\sqrt{k^2 - \beta^2}, \quad y = a\sqrt{\varepsilon\mu k^2 - \beta^2} \quad \text{and} \quad z = b\sqrt{\varepsilon\mu k^2 - \beta^2}, \quad \text{where } z = \chi_2 b \text{ is used for calculation of functions (4).}$$

When this system of 4 equation is solved relative to x, y, z , and k – we can find the resonant frequency $f = \text{Re}(kc/2\pi)$ and $Q = \text{Re}(k)/2\text{Im}(k)$. The system can be solved, for example, in MathCAD, however, this software has no built-in Bessel functions of complex arguments. This was done with help of known expansions generalized for complex arguments. Complex character of roots of (10) should be taken into account when signs for coefficient are being chosen. Some results of calculations with several codes and their comparison with theoretical values are presented in Appendix.

CONCLUSIONS

Theoretical formulas for electric and magnetic field components of a TE₁₁₁-like mode in a ferrite-lined pill-box cavity are derived. Comparison with results obtained with CLANS2 showed that in the regime of E primary field this code gives results that are in a very good

agreement with theory. Possible error in the CLANS2(H) code is in calculation of losses in the metal envelope and we offer to the developer to check this regime.

Results with MWS show a too low quality factor and this was not improved even after increase of mesh cells number up to 10^6 with 3 planes of symmetry. Time of

calculation was 4 hours (Dell Precision PWS670, Intel®, Xeon™, CPU 3.40 GHz, 11.9 GB of RAM). Possible explanation of this low Q can be an impossibility to increase the mesh density in the thin circumferential layer only without a big increase of the whole number of mesh cells that can be done in a 2D case of CLANS2. We also address to the CST to help us to find out a possible way to overcome this problem.

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APPENDIX. RESULTS OF CALCULATIONS OF THE TE111-LIKE MODE IN A PILL-BOX FERRITE-LINED CAVITY WITH THEORY AND DIFFERENT CODES.

Calculations were done with a thin material layer that corresponds to an ideal lossless in comparison with a real situation when lossy material is lining the beam pipe (Table 1), and with a thick layer to better understand dependences on different values of ε and μ (Table 2). Also, the situation close to the real usage of the lossy materials is presented (Table3) with measured parameters of the materials. In the last case very low Q -factors are obtained for the geometry of Fig. 2 with a 3 mm thick ferrite layer. However, in reality, very low coupling with the lossy area can exist so that the Q -factor of the mode can be high.

Codes SLANS2 and CLANS2 have two modes of calculation: with electric E or magnetic H field used as a primary field: CLANS2(E) or CLANS2(H). For multipole modes in this geometry there are no pure H (TE) or E (TM) modes as in the case of monopole modes. So, the problem for eigenvalues can be solved either for E or H ,

with two only components of them: r and z . It is supposed that these two components are calculated with better accuracy because others are expressed through them.

In the left part of Table 1, $Q = Q_e$ is due only to losses in the copper envelope. In the right part of the Table 1 the upper value of Q in the “Theory” column relates to the quality of the magnetodielectric material, Q_m , and the lower value is calculated from these quality and quality of the copper envelope taken from the left-side: $Q = Q_e Q_m / (Q_e + Q_m)$. Calculations with the codes include both losses in the copper walls and in the dielectric (if it is lossy) in the lower line. For MWS and CLANS2 (from the subroutine CLANSI2), separate losses in the material are also available; they are shown in the upper line for Q . One can see a perfect result of CLANS2(E) compared to theory.

Table 1. Results for $a = 55$, $b = 56$ mm, $L = 120$ mm (for all Tables), $\tan \delta_\mu = 0$, a , b , L – see designations in Fig. 2.

	Theory	MWS 2008*	CLANS2 (H)**	CLANS2 (E)**	HFSS	Theory	MWS 2008*	CLANS2 (H)**	CLANS2 (E)**	HFSS
ε	20					20				
μ	1					1				
$\tan \delta_\varepsilon$	0					0.5				
f	1990.307	1986.63	1990.307	1990.307	1990.69	1990.153	1988.3	1990.153	1990.153	1990.54
Q		26244	41624	25994	26170	1864 1739	1288 1227	1864 1784	1864 1740	1745

*For MWS: meshcell number = 973359, three planes of symmetry.

**For CLANS2: entire mesh is 60×60 , mesh in the dielectric region is 60×5 , no symmetry planes used.

Pictures of fields in Fig. A1 are obtained from CLANS2(E), left picture, and from formulas for fields from the text above for $\varepsilon = 20$, $\mu = 1$. One can see qualitative agreement between all fields (transverse only magnetic field is outputted from CLANS2). Same fields obtained with CLANS2(H) (not shown here) have all values of fields 1.265 times smaller with the same pattern of curves. One can see that values of Q for (E) and (H) versions in Table 1 differ by the factor of $\frac{42137}{25968} = 1.600 = 1.265^2$. So, one of the version has a wrong scale of the fields. Possibly, this mistake is

done at the very end of calculations and gives wrong losses in the walls. Frequencies and Q 's of the lossy magnetodielectric (Table 3) look more consistent because losses in the lining material grossly prevail over losses in the walls.

Table 2. Results for $a = 28, b = 56$ mm, $\tan \delta_\epsilon = \tan \delta_\mu = 0$. (Q in parentheses, for copper envelope).

	ϵ	1	2	10	1	2	10
	μ	1	1	1	2	2	10
Theory	f , MHz	2005.311	1634.236	891.486	1507.439	1205.195	291.616
	Q	27047*					
SLANS2 (E)	f , MHz	2005.311	1634.236	891.486	1507.829	1205.347	291.614
	Q	27027		10961			
SLANS2 (H)	f , MHz	2005.311	1634.154	891.288	1507.439	1205.195	291.612
	Q	27017		11918			
CLANS2 (E)	f , MHz	2005.311	1634.236	891.486	1507.440	1205.195	291.616
	Q	27054	20779	11864		41257	176947
CLANS2 (H)	f , MHz	2005.311				1205.195	291.616
	Q	27048				48914	333744
MWS 2008	f , MHz	2005.260					291.396
	Q	27241					60914

*In the SLANS family $\sigma = \frac{10^7}{0.175} \approx 57.14 \cdot 10^6$ Cm/m is taken for conductivity of copper, in MWS $\sigma = 58 \cdot 10^6$ Cm/m.

Table 3. Results for $a = 53, b = 56$ mm. Measured data for ferrites used for HOM loads are taken.

	Material	TT2-111R		Co2Z	
	Temperature	20°C	-190°C	20°C	-190°C
	ϵ	12-0.6i	11-0.3i	30-20i	20-1i
	μ	3-12i	4-5i	7-9i	3-0.7i
CLANS2(H)	f	1052.218	1497.767	853.680	1691.462
	Q	0.7783	1.7715	0.7228	9.752
CLANS2(E)	f	1052.218	1497.767	853.680	1691.462
	Q	0.7783	1.7715	0.7228	9.747
Theory	f	1052.222	1497.767	853.680	1691.462
	Q	0.7783	1.7716	0.7228	9.752

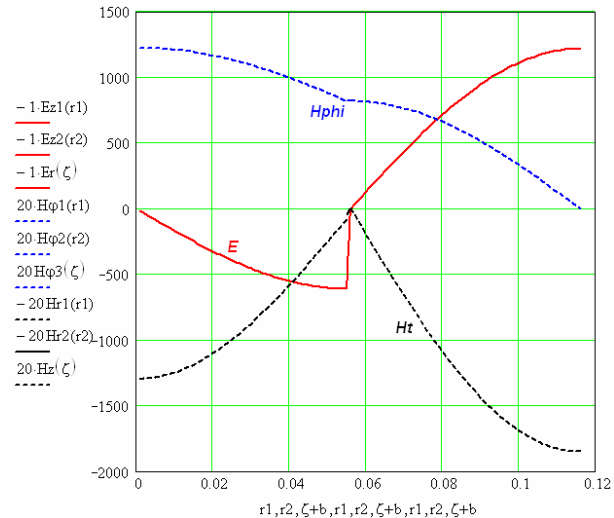
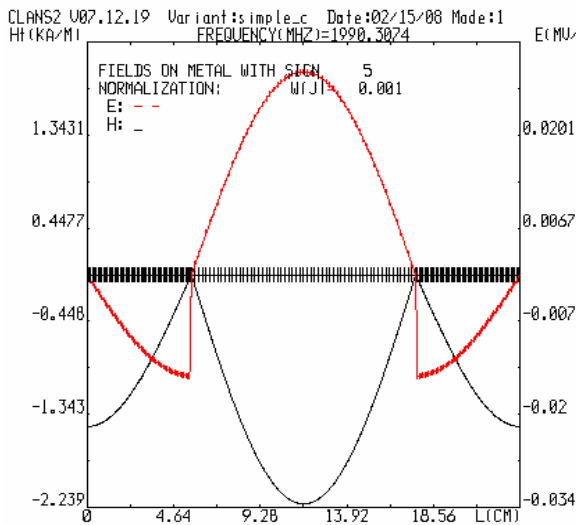


Fig. A1. Comparison of fields from CLANS2 and theory.