Magnetic Field Error Sensitivity in an Energy Recovery Linac

Changsheng Song, Georg Hoffstaetter
Laboratory of Elementary Particle Physics, Cornell University

March 18, 2007

Abstract

The three types of magnetic field errors we consider in this report are: dipole field errors, quadrupole field errors and combined dipole and quadrupole field errors. The version 1.2a of the Cornell 5GeV ERL lattice is used in our simulation. Only linear optics is analyzed and the conclusions should be valid for weakly nonlinear optics as well. Analytical formulas for magnetic field error sensitivities are presented. They are applied to the Cornell 5GeV ERL and compared to numerical simulations of this accelerator. An interesting observation is that the combined dipole and quadrupole field errors, although being a second order effect, have a significant impact on the requirement of the magnetic field stability when the first order effect of dipole field errors is eliminated in achromatic sections. If the tolerance of beam orbit errors at the insertion device (ID) sections is no more than one tenth of the designed beam size in those sections, we find out that the magnetic field in both dipoles and quadrupoles must be stabilized approximately to the level of 2×10^{-4}, which is feasible under current technology [1]. Moreover, we demonstrate that it is crucial for dipoles in all sections forming an achromat to be powered in series so that their field fluctuations are correlated.

1 Introduction

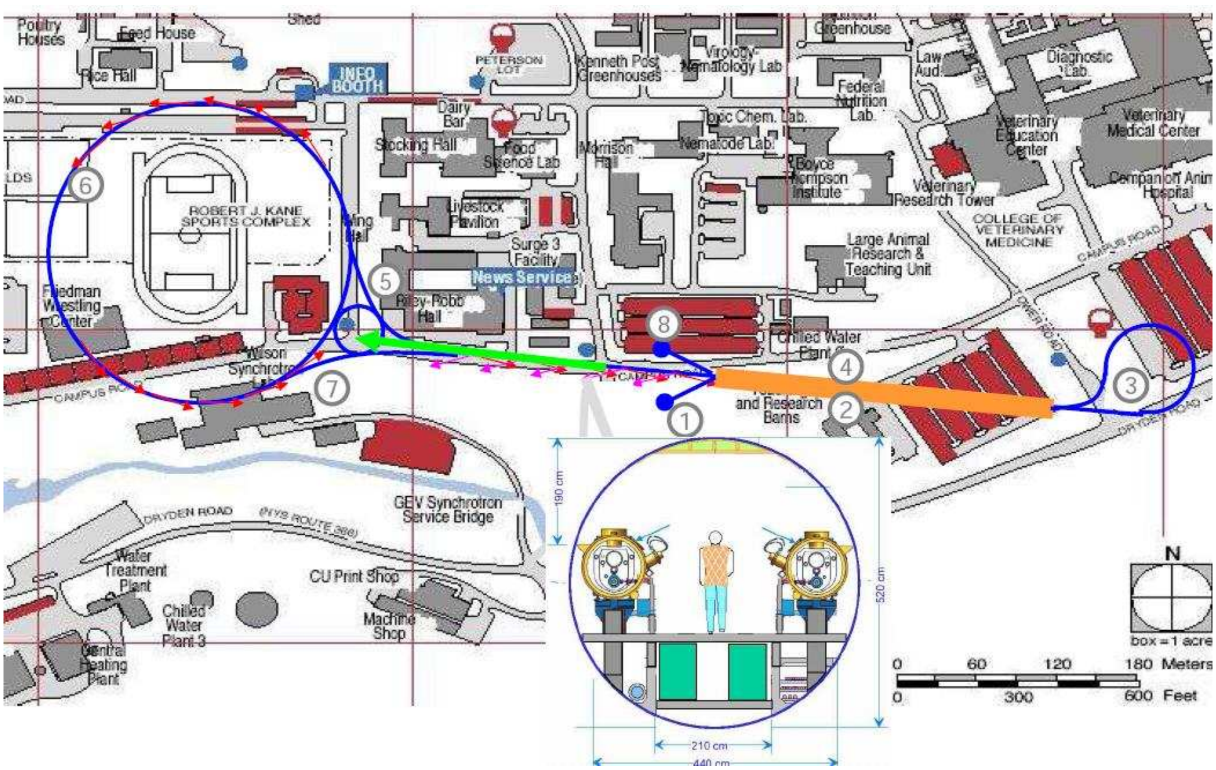

Figure 1: A Schematic Layout of Cornell 5GeV ERL

The usage of synchrotron X-ray facility for the study of solid state and biological physics has become more and more specialized and sophisticated as the sample size becomes smaller and smaller. The study of smaller samples requires an X-ray source of higher brilliance and micro-X-ray beams, which entails smaller beam emittance at the ID sections [2]. Thus it is important to keep the particle orbit errors within one tenth of the designed beam size in those sections. Sub-micron orbit stability of the electron beam has been achieved at many present light sources [3].

Figure 1 is the schematic layout of the ERL planned at Cornell university [4]. The orange lines ② and ④ represent the linac part for accelerating and decelerating the beam. The smaller blue circle ③ is the turn-around loop and the bigger blue circle ⑥ is the return loop. The two blue arcs ⑤ and ⑦ are the north and south arc where highly brilliant x-ray beams are generated. ⑧ is the injector and ① is the beam dump.

In the Cornell 5GeV ERL, a total number of 91 dipole magnets and 498 quadrupole magnets are installed in the two linacs, turn-around loop, south arc, north arc and return loop. Dipole magnetic field fluctuations can cause wrong beam steering leading to emittance growth and thus increase the beam size at the ID sections. Quadrupole field errors can change the beta functions resulting in beam size variations in the ID sections. For a third generation light source, the typical power supply ripple can be control within 100 ppm [1]. For an accelerator with reduced emittance, it is essential to estimate the requirement for the power supply stability and compare it with what the current technology could provide.

2 Dipole Field Errors in an ERL

The requirement for the dipole magnetic field accuracy can be estimated by evaluating the orbit response at the IDs.

For a linear system, a particle's orbit at element i can be described by

$$x_i = \sqrt{\frac{m_0 c}{P_i}} \sqrt{2J\beta_i} \sin(\psi_i + \phi_0) \tag{1}$$

where P_i is the nominal momentum of the particle at element i, β_i is the beta function and ψ_i is the phase advance [5].

If the designed orbit at element i is 0, then we have

$$\psi_i + \phi_0 = n\pi\,, \qquad n \in \text{integer} \tag{2}$$

If a beam steering magnet is installed at the same position and gives the particle a transverse kick $\Delta\vartheta_i$, the particle's transverse divergence x' can be written as

$$x' = \Delta\vartheta_i = \sqrt{\frac{m_0 c}{P_i}} \sqrt{\frac{2J}{\beta_i}} [\cos(\psi_i + \phi_0) - \alpha \sin(\psi_i + \phi_0)] \tag{3}$$

Thus we can write parameter J of the particle's subsequent motion as

$$J = \frac{1}{2} \frac{P_i}{m_0 c} \beta_i \Delta\vartheta_i^2 \tag{4}$$

After being kicked by the steering magnet, the particle's transverse coordinate x at a subsequent element j can be written as

$$x_j = \sqrt{\frac{m_0 c}{P_i}} \sqrt{2J\beta_j} \sin(\psi_j + \phi_0) = \Delta\vartheta_i \sqrt{\frac{P_i}{P_j}} \sqrt{\beta_j \beta_i} \sin(\psi_j - \psi_i) \tag{5}$$

Generally for a linear system, if multiple steering magnets are placed before one beam position monitor (BPM), the reading of such a BPM can be written as the sum of the contributions from those magnets:

$$x_j = x_{j0} + \sum_i \Delta\vartheta_i \sqrt{\frac{P_i}{P_j}} \sqrt{\beta_j \beta_i} \sin(\psi_j - \psi_i) \tag{6}$$

Because the effect of a dipole magnetic field error is producing an additional kick on the bunch, the orbit errors at the ID sections due to such a field error can be estimate by Eq. 6. Furthermore, we can simplify the formula by assuming the field error at each individual dipole is random and not correlated to errors at other dipoles. Later in the simulation, we will consider cases where the dipole field fluctuations are correlated, which is more realistic due to the fact that many dipoles share the same power supply and thus their magnetic fields fluctuate coherently. In

Table 1: Orbit errors as a result of all dipoles having the same error $\pm\Delta B/B$. (absolute in micrometer and relative to the rms beam size)

$\Delta B/B$	+	-
10^{-5}	$-1.42\times10^{-5}(0.00007\%)$	$-1.46\times10^{-5}(0.00007\%)$
5×10^{-5}	$-3.59\times10^{-4}(0.0002\%)$	$-3.60\times10^{-4}(0.0002\%)$
10^{-4}	$-1.44\times10^{-3}(0.007\%)$	$-1.44\times10^{-3}(0.007\%)$
5×10^{-4}	$-3.62\times10^{-2}(0.02\%)$	$-3.58\times10^{-2}(0.02\%)$
10^{-3}	$-1.46\times10^{-1}(0.7\%)$	$-1.42\times10^{-1}(0.7\%)$

the simulation, we simplify this scenario as assigning the same relative field error to all dipoles. The orbit error due to random dipole field errors can be written as:

$$\Delta x=\sum_{n=1}^{N_d} r_\theta \xi_n \theta_n \sqrt{\beta\beta_n}\sqrt{\frac{P_n}{P}}\sin(\psi-\psi_n) \tag{7}$$

where ξ_n is a Gaussian random variable and r_θ represents the rms relative field error.

For the Gaussian random variable ξ_n, we have

$$\langle\xi_n\rangle=0 \qquad \langle\xi_n\xi_m\rangle=\delta_{nm} \tag{8}$$

where the average is taken over many bunches. As a result, the rms orbit error at a ID section can be written as

$$\langle\Delta x^2\rangle=\sum_{n=1}^{N_d} r_\theta^2\theta_n^2\frac{P_n}{P}\beta_n\beta\sin^2(\psi-\psi_n) \tag{9}$$

In the Cornell 5GeV ERL, the designed normalized beam emittance is $\varepsilon=3\times10^{-7}$m and the orbit error is required to be less than one tenth of the beam size at the ID sections. Thus the requirement placed on the dipole error sensitivity factor r_θ is

$$\langle\Delta x^2\rangle=\sum_{n=1}^{N} r_\theta^2\theta_n^2\frac{P_n}{P}\beta_n\beta\sin^2(\psi-\psi_n)\le\left(\frac{\sigma_x}{10}\right)^2=\frac{\beta\varepsilon}{100\gamma} \tag{10}$$

$$r_\theta\le\frac{\sqrt{\varepsilon}}{10\sqrt{\gamma\sum_{n=1}^{N}\theta_n^2\frac{P_n}{P}\beta_n\sin^2(\psi-\psi_n)}} \tag{11}$$

According to Eq. 11, the sensitivity requirement for the Cornell 5GeV ERL is about 2×10^{-7}.

With a global feedback system for the orbit correction, the requirement for the orbit error at the IDs can be relaxed to about 100μm. Thus the new sensitivity requirement can be calculated accordingly as

$$\langle\Delta x^2\rangle=\sum_{n=1}^{N} r_\theta^2\theta_n^2\frac{P_n}{P}\beta_n\beta\sin^2(\psi-\psi_n)\le(100\times10^{-6}\mathrm{m})^2 \tag{12}$$

$$r_\theta\le\frac{10^{-4}\mathrm{m}}{\sqrt{\beta\sum_{n=1}^{N}\theta_n^2\frac{P_n}{P}\beta_n\sin^2(\psi-\psi_n)}} \tag{13}$$

From Eq. 13, the sensitivity requirement in the ERL@CESR is about 7×10^{-6}, which is extremely difficult to achieve in a real accelerator. Thus we have to have a common power supply for all dipoles within each achromat to correlated their field errors.

In Tab. 1 and Tab. 2, both the results for correlated and random dipole errors are listed in terms of orbit error(μm) and percentage of the beam size. For the random dipole errors, five different random seeds are tried and the worst orbit errors are listed. In the correlated case, where all dipoles have the same relative field error, we can see that changing the signs of the dipole errors does not change the signs of the orbit errors because it's a second order effect.

Table 2: Orbit errors as a result of all dipoles having Gaussian random errors with $\sigma(\Delta B/B) = 2 \times 10^{-7}$.(absolute in micrometer and relative to the rms beam size)

Random Seed ($\Delta B/B = 2 \times 10^{-7}$)	Worst Orbit Error (μm)
1	3.5 (18%)
2	-2.0 (7%)
3	-2.8 (14%)
4	-7.3 (37%)
5	-3.5 (18%)

3 Quadrupole Errors in an ERL

In addition to the effect of dipole field errors on the particle orbit, quadrupole field errors can cause changes in beta functions and betatron phase advances. The change in the beta function at an ID section will change the beam size and thus is detrimental to high quality X-rays. The effect of such quadrupole errors can be estimated by adding the perturbing force into the equation of motion and deriving the variation of the beta function accordingly.

A quadrupole field error causes an error in the force exerted on the bunch. The equation of motion under such a force can be written as

$$\vec{z}' = \mathbf{L}(s)\vec{z} + \Delta\vec{f}(\vec{z}, s) \tag{14}$$

where $\mathbf{L}(s)$ is the linear part of the total force and $\Delta\vec{f}$ describes the quadrupole field error.

$$\begin{aligned} \vec{z}(s) &= \vec{z}_H + \int_0^s \mathbf{M}(s,\hat{s})\Delta\vec{f}(\hat{\vec{z}},\hat{s})d\hat{s} \\ &\approx \vec{z}_H + \int_0^s \mathbf{M}(s,\hat{s})\Delta\vec{f}(\hat{\vec{z}}_H,\hat{s})d\hat{s} \end{aligned}$$

The quadrupole field error can also be expressed in the form of a matrix:

$$x'' = -(\kappa^2 + k)x - \Delta k(s)x \qquad \Longrightarrow \qquad \begin{pmatrix} x' \\ x'' \end{pmatrix} = \begin{pmatrix} a \\ -(\kappa^2+k)x \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ \Delta k(s) & 0 \end{pmatrix}\begin{pmatrix} x \\ x' \end{pmatrix} \tag{15}$$

Thus we can write the perturbing force resulting from quadrupole errors as

$$\Delta\vec{f}(\hat{\vec{z}}_H,\hat{s}) = -\begin{pmatrix} 0 & 0 \\ \Delta k(\hat{s}) & 0 \end{pmatrix}\hat{\vec{z}}_H \tag{16}$$

A bunch's phase space vector at position s is related to its initial value at position s_0 as

$$\vec{z}(s) = \mathbf{M}(s)\vec{z}_0 - \int_0^s \mathbf{M}(s,\hat{s})\begin{pmatrix} 0 & 0 \\ \Delta k(\hat{s}) & 0 \end{pmatrix}\mathbf{M}(\hat{s})\vec{z}_0 d\hat{s} \tag{17}$$

where $\mathbf{M}$ is the transfer matrix for the unperturbed system and the action of the force error is added as a perturbation. If we start from a quadrupole with an integrated field error ΔkL and assume that it acts on the bunch as an instantaneous kick, then the force error can be approximated as a delta function $\Delta kL\delta(s - s_0)$ and the orbit at s can be written as

$$\vec{z}(s) = \left\{\mathbf{M}(s) - \mathbf{M}(s)\begin{pmatrix} 0 & 0 \\ \Delta kL & 0 \end{pmatrix}\right\}\vec{z}_0 \tag{18}$$

The transfer matrix has the general form:

$$\mathbf{M} = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos\psi + \alpha_0\sin\psi) & \sqrt{\beta\beta_0}\sin\psi \\ \frac{1}{\sqrt{\beta\beta_0}}\{(\alpha_0 - \alpha)\cos\psi - (1 + \alpha_0\alpha)\sin\psi\} & \sqrt{\frac{\beta_0}{\beta}}(\cos\psi - \alpha\sin\psi) \end{pmatrix} \tag{19}$$

Table 3: Beam size variations at the ID sections due to Gaussian random quadrupole field errors with different $\sigma(\Delta k1/k1)$

$\sigma(\Delta k1/k1)$	$\Delta\sigma_x/\sigma_x$	$\Delta\sigma_y/\sigma_y$
10^{-2}	392%	248%
5×10^{-3}	108%	45.8%
10^{-3}	10.6%	5.48%
10^{-4}	1.1%	0.52%

Thus the change of the transfer matrix is

$$\Delta\mathbf{M} = -\mathbf{M}\begin{pmatrix} 0 & 0 \\ \Delta kL & 0 \end{pmatrix} = \begin{pmatrix} -\Delta kL\sqrt{\beta\beta_0}\sin\psi & 0 \\ -\Delta kL\sqrt{\dfrac{\beta_0}{\beta}}(\cos\psi - \alpha\sin\psi) & 0 \end{pmatrix} \tag{20}$$

We can also write the change of the transfer matrix in terms of variations in the beta function and phase advance, which gives

$$\begin{aligned} \Delta M_{11} &= \frac{1}{2}\frac{\Delta\beta}{\sqrt{\beta\beta_0}}(\cos\psi + \alpha_0\sin\psi) + \sqrt{\frac{\beta}{\beta_0}}(-\sin\psi + \alpha_0\cos\psi)\Delta\psi \\ \Delta M_{12} &= \frac{1}{2}\frac{\Delta\beta}{\sqrt{\beta\beta_0}}\sin\psi + \sqrt{\frac{\beta}{\beta_0}}\cos\psi\Delta\psi \end{aligned} \tag{21}$$

Comparing both results we can infer the changes in the beta function and the phase advance:

$$\begin{aligned} \Delta\beta(\cos\psi + \alpha_0\sin\psi) + 2\beta(-\sin\psi + \alpha_0\cos\psi)\Delta\psi = -2\Delta kL\beta_0\beta\sin\psi \\ \Delta\beta\sin\psi + 2\beta\cos\psi\Delta\psi = 0 \end{aligned} \tag{22}$$

The relative beta function error as a result of a quadrupole error is

$$\frac{\Delta\beta}{\beta} = -\Delta kL\beta_0\sin 2\psi \tag{23}$$

The error of the phase advance is

$$\Delta\psi = \Delta kL\beta_0\sin^2\psi \tag{24}$$

Thus the beta function error at the n-th element as a result of preceding quadrupole errors is

$$\frac{\Delta\beta_n}{\beta_n} = -\sum_{m=1}^{N_q}(\Delta kL)_m\beta_m\sin 2(\psi_n - \psi_m) \tag{25}$$

Because the beam size at the ID section can be written as

$$\sigma_x^2 = \beta_x\varepsilon \qquad \sigma_y^2 = \beta_y\varepsilon \tag{26}$$

we have the relative change in the beam size is

$$\frac{\Delta\sigma_x}{\sigma_x} = \frac{1}{2}\frac{\Delta\beta_x}{\beta_x} \qquad \frac{\Delta\sigma_y}{\sigma_y} = \frac{1}{2}\frac{\Delta\beta_y}{\beta_y} \tag{27}$$

The horizontal and vertical beam size variations as a function of random quadrupole field errors are listed in Tab. 3. In order to make sure that the beam size variation does not exceed one tenth of the designed value, the field stability must reach the level of 10^{-3}.

Table 4: Orbit errors (absolute in micrometer and relative to the rms beam size) at the ID sections due to combined correlated dipole field errors ($\Delta B/B$) and Gaussian random quadrupole field errors ($\Delta k1/k1$).

$\Delta B/B$	$\sigma(\Delta k1/k1)$	Orbit Error (μm)
10^{-3}	10^{-3}	146 (521%)
10^{-3}	10^{-5}	1.52 (5.4%)
10^{-4}	5×10^{-4}	7.45 (26%)
2.0×10^{-4}	2.0×10^{-4}	5.98 (21%)
10^{-4}	10^{-4}	1.50 (5.4%)

4 Combined Magnetic Field Error

In a real accelerator where dipole field errors and quadrupole field errors coexist, an orbit error due to the combination of these two kinds of errors, although a second order effect, can have significant impact on the stability requirement of the power supply system when the first order effect of dipole field errors is eliminated by correlating them in achromatic, i.e. dispersion free sections. A description of the physical process is that the dipole field errors can create orbit errors at subsequent quadrupoles, whose field errors combined with such orbit errors can produce an effective beam steering, which can affect the particle orbit at following IDs. Such an effect can be estimated similarly to that of dipole field errors.

Similar to Eq. 7, we can replace the dipole field error $\Delta\theta_n$ with the effective beam steering $\xi_n r_q (kL)_n \Delta x_n$ from the quadrupole field error on a displaced bunch, where ξ_n is a random number, r_q is the quadrupole field sensitivity factor and Δx_n is the bunch displacement.

$$\Delta x = \sum_{n=1}^{N_q} r_q \xi_n (kL)_n \Delta x_n \sqrt{\frac{P_n}{P}} \sqrt{\beta_n \beta} \sin(\psi - \psi_n) \tag{28}$$

Thus the rms orbit at the IDs is

$$\langle \Delta x^2 \rangle = \sum_{n=1}^{N_q} r_q^2 (kL)_n^2 \Delta x_n^2 \frac{P_n}{P} \beta_n \beta \sin^2(\psi - \psi_n) \tag{29}$$

The requirement for the quadrupole field sensitivity r_q can be calculated as

$$r_q \le \frac{\sqrt{\varepsilon}}{10\sqrt{\gamma \sum_{n=1}^{N_q} (kL)_n^2 \Delta x_n^2 \frac{P_n}{P} \beta_n \sin^2(\psi - \psi_n)}} \tag{30}$$

According to Eq. 30, with 0.1% correlated dipole magnetic field error, the required accuracy of the quadrupole field is about 5×10^{-5}. If we increase the accuracy of the dipole field to 0.01%, the required accuracy of the quadrupole field will be reduced to 5×10^{-4}. We can also include magnetic field errors in our simulation and compare the result with our analytic formula. The simulation results are shown in Tab. 4, which summarizes the worst orbit error of the 27 ID sections in our ERL lattice with different combinations of dipole field errors and quadrupole field errors.

5 Conclusion

In the Cornell ERL the tolerance of beam orbit errors at the insertion device (ID) sections are no more than one tenth of the beam size in those sections. In this report, we are only concerned with different types of magnetic field errors. Other types of errors, such as magnet rotations, which can lead to dispersion in the vertical plane, or quadrupole displacements, which can lead to orbit shifts, etc. are not discussed here, but will be the subject of an upcoming report.

If individual power supplies were assigned to the dipole magnets, the field fluctuation in each dipole would be random and uncorrelated to each other. Thus Eq. 12 shows that the stability requirement would be at least 7×10^{-6}. Such a requirement, however, is extremely difficult to achieve in a real accelerator and thus we have to resort to a common power supply for all dipoles within each achromat to correlate their field errors. With all dipoles in one achromatic section sharing the same power supply, the dipole field fluctuations in different dipoles become correlated and the stability requirement can be reduced to about 10^{-3} due to our lattice design.

For Gaussian random quadrupole field errors, the rms relative field stability must reach the level of 10^{-3} to keep the beam size variation within one tenth of the designed value.

From the simulation for combined magnetic field errors, we conclude that the stability of power supplies for both type of magnets must at least reach the level of 2×10^{-4}. Similar power supply stability has been achieved in the Swiss Light Source [1]. One caveat that should be mentioned here is that such a conclusion is based on the assumption that, by sharing the same power supply, the dipole magnets in one achromatic section have completely correlated field fluctuation. If there are extra sources of random field fluctuation not from the power supply, the field stability requirement would have to be limited to 7×10^{-6}.

References

[1] G. Irminger, M. Horvat, F. Jenni, H.U. Boksberger, *A 3Hz, 1MW(peak) Bending Magnet Power Supply for the Swiss Light Source*, SLS-PRE-TA-1998-0110, 1998

[2] S.M. Gruner, D.H.Bilderback, *Energy recovery linacs as synchrotron light sources*, Nucl. Instr. and Meth. A 500 (2003) 25-32

[3] M. Böge, *Achieving Sub-micron Stability in Light Sources*, Proceedings of EPAC'04, Lucerne, Switzerland, 2004

[4] G.H. Hoffstaetter, I.V. Bazarov, etc. *Status of a Plan for an ERL Extension to CESR*, Proceedings PAC05, Knoxville/TN (2005)

[5] A. Chao, M. Tigner, *Handbook of Accelerator Physics and Engineer*, P.65-66, World Scientific, 2006