Memo on chicane for ERL Phase I (Dec, 2001)

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Abstract
The purpose of this note is to provide basic analytical formulas of longitudinal phase space manipulations in 100 MeV ERL. The choice of the chicanes design is outlined.

Longitudinal phase space in ERL
For the sake of consistency I choose TRANSPORT notations for longitudinal phase space coordinates: l – the path length difference between the trajectory and the central trajectory as measured from the beginning of the system (positive l means that the particle is in the tail, front particles have $l < 0$); $\delta = \Delta P/P_0 \approx \Delta E/E_0$ is the momentum (energy) deviation of the trajectory from that of the central trajectory.
We envision two modes of operation in ERL: 1) small emittance; 2) short bunches. Chicanes are needed for the short bunch mode (Fig. 1).

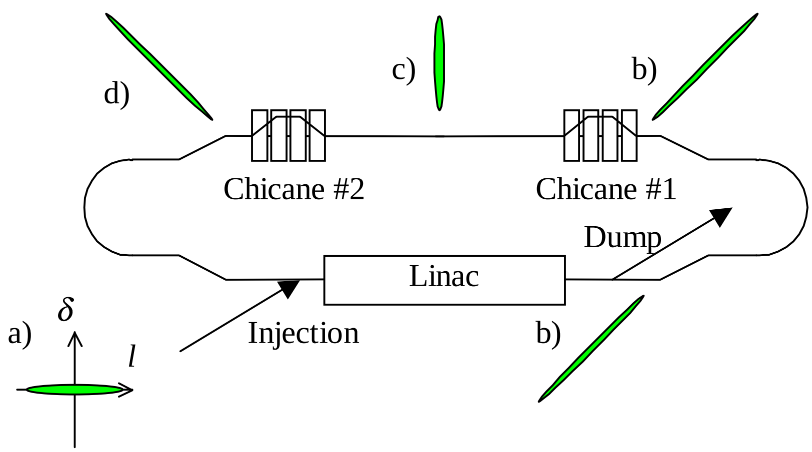

Fig.1. Short bunch mode: longitudinal phase space a) from the injector (small energy spread, no δ - l correlation); b) after the linac with off-crest operation (same bunch length, δ - l correlation is created); c) after Chicane #1 (upright ellipse, maximal bunch compression, same energy spread); d) after Chicane #2 (same as (b) but reversed δ - l correlation). Settings for the Chicanes are identical (i.e. same R_{56}, see below).

For small emittance mode (no bunch compression), the following options are possible:

1) Linac is run on-crest, Chicanes have the same settings (R_{56}) as for the short bunch mode. Since the linac produces no energy correlation, bunch length stays about the same all over the machine.
2) Chicanes have the possibility of varying momentum compaction, R_{56}. The bunch length stays the same if $R_{56} = 0$ independent of the RF phase offset in the main linac.
3) Chicanes with set R_{56} are removed and replaced by a straight section. Chicanes are used for the short bunch mode and removed for the low emittance mode.

Implications of each option, if chosen, are examined below.

Some definitions and simple relations

Matrix element R_{56} [m], or momentum compaction, is a transfer matrix element (6x6 TRANSPORT matrix mapping *x*, *x'*, *y*, *y'*, *l*, δ) mapping δ to *l*. It is found by integrating:

$$R_{56} = \int_1^2 \frac{\eta_x}{\rho} ds\,, \tag{1}$$

η_x is dispersion; $\rho = \rho(s)$ is bending radius; *s* is reference particle path length.

Longitudinal phase space coordinates satisfy the following relation (let's not worry about the order higher than 2 now):

$$\delta(l) = \delta_0 + \left.\frac{d\delta}{dl}\right|_{l=0} l + \frac{1}{2!}\left.\frac{d^2\delta}{dl^2}\right|_{l=0} l^2 + \ldots = \delta_0 + \alpha l + \frac{1}{2}\beta l^2\,, \tag{2}$$

δ_0 is uncorrelated energy spread (i.e. $\langle \delta_0 l \rangle = 0$), which comes mostly from the injector; $\alpha = d\delta / dl$ is energy-length correlation [m^{-1}]; $\beta = d^2\delta / dl^2$ is second order energy-length correlation (curvature) [m^{-2}] (not to be confused with Twiss parameters). Uncorrelated energy spread from the injector for the new bunching scheme is very small, rms value is less than 10 keV, so, for our purposes uncorrelated energy spread $\sqrt{\langle \delta_0{}^2 \rangle}$ < 10 keV/100 MeV = 10^{-4} is essentially zero. Longitudinal phase space dynamics is dominated by δ - *l* correlation, α, second order δ - *l* (curvature), β, and higher order terms. α is created by RF sections for off-crest running and modified by the optics in bends (R_{56}). β is created in RF sections each time reference particle energy is changed and can be modified through second-order optics (sextupoles) or higher harmonic RF (e.g. linearizer, which uses three times the fundamental RF frequency). Other effects (wakes, CSR) also change the curvature, β, but for the purpose of this memo we neglect them totally. Also note that the case $1/\alpha = 0$ represents erect phase space "ellipse" or maximum bunch compression, since α is slope of δ - *l* distribution.

Emittance is defined as always:

$$\varepsilon_z = \sqrt{\langle \delta^2 \rangle \langle l^2 \rangle - \langle \delta l \rangle^2}\,. \tag{3}$$

Note that this quantity is conserved only for linear transformations (i.e. first order optics) but can be modified by nonlinear elements (i.e. cavities, sextupoles).

Assuming normal longitudinal particle distribution we can write rms values of relative energy spread and emittance in terms of rms bunch length, σ_l, uncorrelated energy spread, σ_{δ_0}, α and β:

$$\sigma_\delta = \sqrt{\sigma_{\delta_0}^2 + \alpha^2\sigma_l^2 + \frac{1}{2}\beta^2\sigma_l^4}, \tag{4}$$

$$\varepsilon_z = \sigma_l\sqrt{\sigma_{\delta_0}^2 + \frac{1}{2}\beta^2\sigma_l^4}. \tag{5}$$

Energy change for RF section is:

$$\Delta E = E_{\mathrm{cav}} \cos\left(\varphi + l/\lambdabar_{\mathrm{RF}}\right), \tag{6}$$

φ is off-crest RF phase, E_{cav} is RF maximal energy gain, $\lambdabar_{\mathrm{RF}} = \lambda_{\mathrm{RF}}/2\pi$, where λ_{RF} is RF wavelength.

Thus, α and β are given by:

$$\begin{aligned} \alpha &= -\frac{E_{\mathrm{cav}}\sin\varphi}{E_0}\frac{1}{\lambdabar_{\mathrm{RF}}}, \\ \beta &= -\frac{E_{\mathrm{cav}}\cos\varphi}{E_0}\frac{1}{\lambdabar_{\mathrm{RF}}^2}. \end{aligned} \tag{7}$$

Longitudinal phase space looks like in Fig. 2. For $\varphi = -11°$, we have $\alpha = 5.0$ m^{-1}, and $\beta = -705.2$ m^{-2}.

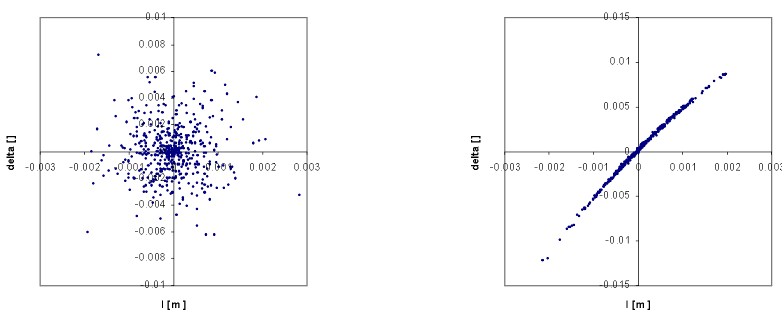

Fig. 2. Longitudinal phase space distribution before (left) and after the main linac at 100 MeV (right). $\varphi = -11°$, rms bunch length is 0.6 mm. Initial uncorrelated rms energy spread is 10 keV at 5 MeV (left) and particle distribution is Gaussian.

Since energy spread σ_δ does not change in the ring, ratio $\varepsilon_z / \sigma_\delta$ gives us minimal bunch length achievable with compression when longitudinal phase space ellipse is upright.

Neglecting uncorrelated energy spread from the injector we arrive at a simple formula for minimum bunch length achievable when compressing without correction of β (sextupole correction):

$$\sigma_l^{\min} = \varepsilon_z / \sigma_\delta \approx \frac{\sigma_l^2}{\sqrt{2\bar{\lambda}_{\mathrm{RF}}^2 \tan^2\varphi + \sigma_l^2}}. \tag{8}$$

Neglecting uncorrelated energy spread from the injector (on the order of 10 keV) is well justified for our parameters, as it can be seen in Fig. 3. $\sigma_l^{\min}$ is plotted as a function of RF phase offset for two values of σ_l in the main linac. Curves are plotted using expression (8), whereas for data pointes uncorrelated energy spread is also taken into account.

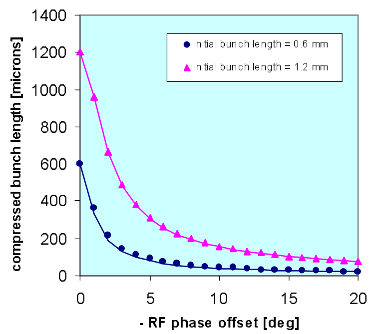

Fig. 3. Minimal achievable bunch length after compression without curvature correction. Uncorrelated energy spread is taken into account for the data points, the curves are plotted using (8). Final beam energy was set to 100 MeV for all RF phase offset values.

Accordingly, the bunch can be compressed down to 30 μm (our goal of 100 fs) at $\varphi = -15°$ for rms bunch length in the main linac of 0.6 mm, and $\varphi = -43°$ for initial bunch length of 1.2 mm (see Fig. 4). Of course, there are too many assumptions built in that result, and CSR, wakes, non-Gaussian distribution from the injector to begin with will most likely prevent bunch compression down to 100 fs without curvature correction. However, this should provide us with some rough estimates of what to expect.

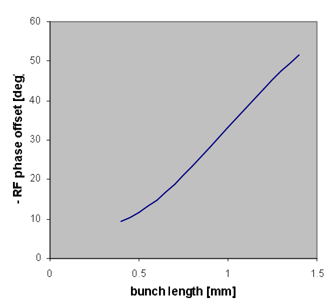

Fig. 4. Estimated RF phase offset to produce 100 fs bunches after compression vs. bunch length in the main linac.

Optics transforms

In the optics, after an achromat, we have the following transform:

$$l^* = l + R_{56}\delta + T_{566}\delta^2 ,$$
$$\delta^* = \delta . \qquad (9)$$

For not strictly ultrarelativistic particles the transform for l should also include term $L(\Delta v / v_0)$ (here v_0 and Δv are the central particle velocity and velocity spread respectively, L is the length of the transport line). At 100 MeV for 40 m of the transport line $L\sigma_\delta / \gamma^2 \cong 3$ µm, so the effect is negligible.

One can still represent the longitudinal phase space distribution as $\delta^* = \delta_0^* + \alpha^* l^* + \frac{1}{2}\beta^* l^{*2}$, and figure out new α^* and β^* after the transform as functions of old α and β. Although mathematically straightforward, this is laborious, so let's limit ourselves to the first order expansion coefficient only (i.e. α), then:

$$\alpha^* = \frac{\alpha}{1 + R_{56}\alpha} . \qquad (10)$$

Recalling that the maximal compression case corresponds to $1/\alpha^* = 0$, we find an expression for the required R_{56}:

$$R_{56} = -\frac{1}{\alpha} = \frac{E_0}{E_{\mathrm{cav}} \sin\varphi} \lambda\!\!\!^{-}_{RF} . \qquad (11)$$

For $\varphi = -11°$, R_{56} = –20 cm. This momentum compaction can come from two sources: chicane and Bates' magnets. We will assume that Bates' system is isochronous and all of R_{56} has to be from the chicane (i.e. both chicanes capable of producing R_{56} up to –20 ÷ –25 cm).

Chicane

Chicane is made of four identical rectangular magnets. The geometry of the chicane is all figured out in Klaus Steffen's book (Fig. 5). One may notice that the distance, *d*, between the second and the third magnets is irrelevant for momentum compaction. The amplitude in the symmetry midplane for the central particle trajectory ("vacuum chamber size") is

$$\xi_m = 2\rho(1 - \cos\vartheta) + \lambda \tan\vartheta , \qquad (12)$$

where ϑ is the bending angle in the dipoles: $\sin\vartheta = L/\rho$; ρ is bending radius; L is magnet length; λ is spacing between the first and the second, the third and the fourth. Path length, *s*, is given by

$$s = 4\rho\vartheta + \frac{2\lambda}{\cos\vartheta} + d\ . \tag{13}$$

Since $\rho(\delta) = \rho(1+\delta)$, we find R_{56} to be

$$R_{56} = \left.\frac{ds}{d\delta}\right|_{\delta=0} = -\frac{4L}{\cos\vartheta} - \frac{2L^2\lambda}{\rho\cos^3\vartheta} + 4\rho\vartheta\ . \tag{14}$$

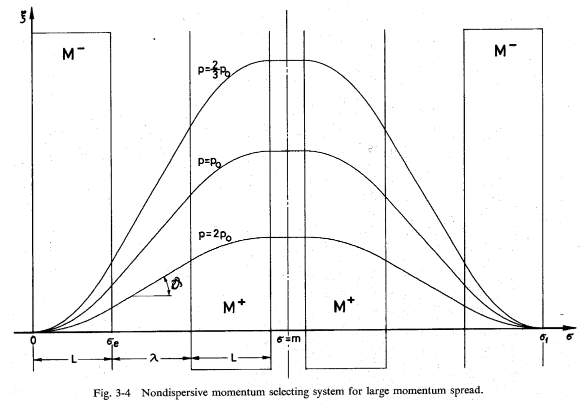

Fig. 5. Chicane geometry.

Naturally, one would like to have a chicane with R_{56} adjustable from 0 to –25 cm. As far as the bending radius for the chicanes: assuming that we tolerate 1 T, the minimum bending radius is 33 cm. Shorter radii are required for larger R_{56} (absolute value) at some fixed magnet length, and that has its own implications for CSR.

Here is an example of chicane that will do the job for us:

L = 40 cm, λ = 40 cm, d = 10 cm.

Total length 2.5 m. We have about 17 m in the straight section of the ring opposite to the linac. For ρ =1 (magnetic field of 0.334 T), R_{56} is –26.6 cm. Maximum deviation of the central trajectory in the chicane, ξ_m, is 34 cm. If we will vote for adjustable R_{56} then horizontal vacuum chamber size will have to be more than 34 cm as well as magnets width. Alternatively, we can use smaller magnets but will have R_{56} to some smaller range (e.g. ξ_m between 24 and 34 cm corresponds to R_{56} adjustable from –15.1 to –26.6 cm). Chicanes can be made of either 4 identical magnets or 3 magnets (d =0), with the middle one twice as long. However, for the cost optimization purposes, it seems more practical to use 4×2 identical magnets.

Pros and cons of different chicane designs

1) *Chicanes with $R_{56} \in [0, -26.6]$ cm.*

Pros: the widest range of possible R_{56}.
Cons: Large horizontal dimensions of the vacuum chamber and magnets (or at least middle magnets). Vacuum implications are not clear to me.

2) *Chicanes with set R_{56}*
   Pros: compact magnets; cheaper than 1).
   Cons: R_{56} is limited to a more narrow range. Both modes with and without bunch compression should be achievable by operating linac with different off-crest phase, however.
3) *Chicanes with set R_{56} and straight tube option.*
   Pros: allows to run with R_{56} = 0 by substituting the chicanes by pieces of straight line. Otherwise, the chicanes of type 2) are used.
   Cons: moving pieces of optics is not elegant.

P.S. The 4th option is not to use chicanes but change the momentum compaction with the Bates magnets. That is something that we decided to do at the end (as of Dec 01).