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Moment Analysis of Inclusive Semileptonic B Meson Decay

CLEO Collaboration

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Abstract

We have measured the first and second moments of the hadronic mass-squared distribution in the inclusive decay $B \to X_c \ell \nu$. Additionally, we have made a preliminary determination of the first and second moments of the lepton energy distribution, using our previously published spectrum. We use these moments to obtain information about the HQET Operator Product Expansion parameters $\bar{\Lambda}$ and λ_1. There appear to be inconsistencies in the results, suggesting either experimental error or problems with the theory. A resolution is needed in order to obtain a precise determination of $|V_{cb}|$.

J. Bartelt,[1] S. E. Csorna,[1] K. W. McLean,[1] S. Marka,[1] Z. Xu,[1] R. Godang,[2] K. Kinoshita,[2] I. C. Lai,[2] P. Pomianowski,[2] S. Schrenk,[2] G. Bonvicini,[3] D. Cinabro,[3] R. Greene,[3] L. P. Perera,[3] G. J. Zhou,[3] S. Chan,[4] G. Eigen,[4] E. Lipeles,[4] J. S. Miller,[4] M. Schmidtler,[4] A. Shapiro,[4] W. M. Sun,[4] J. Urheim,[4] A. J. Weinstein,[4] F. Würthwein,[4] D. E. Jaffe,[5] G. Masek,[5] H. P. Paar,[5] E. M. Potter,[5] S. Prell,[5] V. Sharma,[5] D. M. Asner,[6] J. Gronberg,[6] T. S. Hill,[6] D. J. Lange,[6] R. J. Morrison,[6] H. N. Nelson,[6] T. K. Nelson,[6] D. Roberts,[6] B. H. Behrens,[7] W. T. Ford,[7] A. Gritsan,[7] H. Krieg,[7] J. Roy,[7] J. G. Smith,[7] J. P. Alexander,[8] R. Baker,[8] C. Bebek,[8] B. E. Berger,[8] K. Berkelman,[8] V. Boisvert,[8] D. G. Cassel,[8] D. S. Crowcroft,[8] M. Dickson,[8] S. von Dombrowski,[8] P. S. Drell,[8] K. M. Ecklund,[8] R. Ehrlich,[8] A. D. Foland,[8] P. Gaidarev,[8] R. S. Galik,[8] L. Gibbons,[8] B. Gittelman,[8] S. W. Gray,[8] D. L. Hartill,[8] B. K. Heltsley,[8] P. I. Hopman,[8] J. Kandaswamy,[8] D. L. Kreinick,[8] T. Lee,[8] Y. Liu,[8] N. B. Mistry,[8] C. R. Ng,[8] E. Nordberg,[8] M. Ogg,[8,*] J. R. Patterson,[8] D. Peterson,[8] D. Riley,[8] A. Soffer,[8] B. Valant-Spaight,[8] A. Warburton,[8] C. Ward,[8] M. Athanas,[9] P. Avery,[9] C. D. Jones,[9] M. Lohner,[9] C. Prescott,[9] A. I. Rubiera,[9] J. Yelton,[9] J. Zheng,[9] G. Brandenburg,[10] R. A. Briere,[10] A. Ershov,[10] Y. S. Gao,[10] D. Y.-J. Kim,[10] R. Wilson,[10] H. Yamamoto,[10] T. E. Browder,[11] Y. Li,[11] J. L. Rodriguez,[11] S. K. Sahu,[11] T. Bergfeld,[12] B. I. Eisenstein,[12] J. Ernst,[12] G. E. Gladding,[12] G. D. Gollin,[12] R. M. Hans,[12] E. Johnson,[12] I. Karliner,[12] M. A. Marsh,[12] M. Palmer,[12] M. Selen,[12] J. J. Thaler,[12] K. W. Edwards,[13] A. Bellerive,[14] R. Janicek,[14] P. M. Patel,[14] A. J. Sadoff,[15] R. Ammar,[16] P. Baringer,[16] A. Bean,[16] D. Besson,[16] D. Coppage,[16] C. Darling,[16] R. Davis,[16] S. Kotov,[16] I. Kravchenko,[16] N. Kwak,[16] L. Zhou,[16] S. Anderson,[17] Y. Kubota,[17] S. J. Lee,[17] R. Mahapatra,[17] J. J. O'Neill,[17] R. Poling,[17] T. Riehle,[17] A. Smith,[17] M. S. Alam,[18] S. B. Athar,[18] Z. Ling,[18] A. H. Mahmood,[18] S. Timm,[18] F. Wappler,[18] A. Anastassov,[19] J. E. Duboscq,[19] K. K. Gan,[19] T. Hart,[19] K. Honscheid,[19] H. Kagan,[19] R. Kass,[19] J. Lee,[19] H. Schwarthoff,[19] A. Wolf,[19] M. M. Zoeller,[19] S. J. Richichi,[20] H. Severini,[20] P. Skubic,[20] A. Undrus,[20] M. Bishai,[21] S. Chen,[21] J. Fast,[21] J. W. Hinson,[21] N. Menon,[21] D. H. Miller,[21] E. I. Shibata,[21] I. P. J. Shipsey,[21] S. Glenn,[22] Y. Kwon,[22,†] A.L. Lyon,[22] S. Roberts,[22] E. H. Thorndike,[22] C. P. Jessop,[23] K. Lingel,[23] H. Marsiske,[23] M. L. Perl,[23] V. Savinov,[23] D. Ugolini,[23] X. Zhou,[23] T. E. Coan,[24] V. Fadeyev,[24] I. Korolkov,[24] Y. Maravin,[24] I. Narsky,[24] R. Stroynowski,[24] J. Ye,[24] T. Wlodek,[24] M. Artuso,[25] E. Dambasuren,[25] S. Kopp,[25] G. C. Moneti,[25] R. Mountain,[25] S. Schuh,[25] T. Skwarnicki,[25] S. Stone,[25] A. Titov,[25] G. Viehhauser,[25] and J.C. Wang[25]

[1]Vanderbilt University, Nashville, Tennessee 37235
[2]Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
[3]Wayne State University, Detroit, Michigan 48202
[4]California Institute of Technology, Pasadena, California 91125
[5]University of California, San Diego, La Jolla, California 92093
[6]University of California, Santa Barbara, California 93106

[*]Permanent address: University of Texas, Austin TX 78712.

[†]Permanent address: Yonsei University, Seoul 120-749, Korea.

[7]University of Colorado, Boulder, Colorado 80309-0390
[8]Cornell University, Ithaca, New York 14853
[9]University of Florida, Gainesville, Florida 32611
[10]Harvard University, Cambridge, Massachusetts 02138
[11]University of Hawaii at Manoa, Honolulu, Hawaii 96822
[12]University of Illinois, Urbana-Champaign, Illinois 61801
[13]Carleton University, Ottawa, Ontario, Canada K1S 5B6
and the Institute of Particle Physics, Canada
[14]McGill University, Montréal, Québec, Canada H3A 2T8
and the Institute of Particle Physics, Canada
[15]Ithaca College, Ithaca, New York 14850
[16]University of Kansas, Lawrence, Kansas 66045
[17]University of Minnesota, Minneapolis, Minnesota 55455
[18]State University of New York at Albany, Albany, New York 12222
[19]Ohio State University, Columbus, Ohio 43210
[20]University of Oklahoma, Norman, Oklahoma 73019
[21]Purdue University, West Lafayette, Indiana 47907
[22]University of Rochester, Rochester, New York 14627
[23]Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
[24]Southern Methodist University, Dallas, Texas 75275
[25]Syracuse University, Syracuse, New York 13244

I. INTRODUCTION

A rigorous theoretical treatment of inclusive semileptonic decays $B \to X\ell\nu$ is now possible [1]. Using the ideas of heavy quark effective theory (HQET) and the operator product expansion (OPE), one can express many of the observables of the inclusive decay as a double expansion in powers of α_s and Λ_{QCD}/m_b [2], [3]. The first few terms look very much like the spectator model [4]. To order $1/m_b^2$, the expansion parameters are the matrix elements

$$\lambda_1 = -\langle B|\bar{b}_v(i\vec{D})^2 b_v|B\rangle, \tag{1}$$

and

$$\lambda_2 = (1/3)\langle B|\bar{b}_v \sigma_{\mu\nu} G^{\mu\nu} b_v|B\rangle. \tag{2}$$

Here b_v is the heavy quark field and $G^{\mu\nu}$ is the chromomagnetic field operator. Intuitively, λ_1 may be thought of as the negative of the average momentum-squared of the b quark, and λ_2 as the energy of the hyperfine interaction of the spin of the b with the light degrees of freedom. To order $1/m_b^2$, the b quark mass m_b may be eliminated in favor of the B meson mass M_B with the addition of a third parameter $\bar{\Lambda}$ satisfying

$$M_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b}. \tag{3}$$

According to HQET, these expressions are unchanged, to order $1/m^2$, if the B meson and b quark are replaced by the D meson and c quark.

It is clearly important to determine the expansion parameters, as they allow information about the quark-level process $b \to c\ell\nu$ (for example m_b and V_{cb}) to be extracted from the hadron-level inclusive process $B \to X_c\ell\nu$, and also for the extraction of V_{cb} from the exclusive process $B \to D^*\ell\nu$. One readily finds $\lambda_2 = 0.12$ GeV^2 from the $B^* - B$ mass difference. $\bar{\Lambda}$ and λ_1 are more challenging. This paper describes a study of the inclusive semileptonic decay $B \to X_c\ell\nu$ in which the distribution in the mass-squared of the charmed hadronic system X_c is measured, and the first and second moments of the distribution obtained. In addition, we have performed a *PRELIMINARY* analysis of our previously published [5] lepton momentum spectrum to obtain first and second moments of that distribution. Each of these four moments can be expressed by a $1/m$ expansion and so, to order $1/m^2$, defines a curve in $\lambda_1 - \bar{\Lambda}$ space (α_s and λ_2 being treated as known). Each measured moment, with its error, defines a band in the $\lambda_1 - \bar{\Lambda}$ plane, and the intersection of the bands provides a determination of λ_1 and $\bar{\Lambda}$. The extent to which all bands intersect at a common point provides a check on the theory.

II. MOMENTS OF THE HADRONIC MASS DISTRIBUTION

The data used in this analysis were taken with the CLEO detector [6] at the Cornell Electron Storage Ring (CESR), and consist of 3.2 fb^{-1} at the $\Upsilon(4S)$ resonance and 1.6 fb^{-1} at a center-of-mass energy 60 MeV below the resonance. The sample contains 3.4 million $B\bar{B}$ pairs. We select events containing a lepton – μ or e – with momentum between 1.5

and 2.5 GeV/c. We "reconstruct" the neutrino in the event, taking its momentum to be the negative of the vector sum of the momenta of all detected particles, and its energy to be the difference between twice the beam energy and the sum of the energies of all detected particles. Considerable effort was expended to differentiate between poorly measured charged particles (kept in the sum) and faked charged particles (discarded), and between clusters in the electromagnetic calorimeter caused by photons (kept) and by charged particles already included in the sum (discarded). To assure a well-measured neutrino, we require: a neutrino mass consistent with zero; no additional leptons in the event; a net charge zero for the event. The "neutrino reconstruction" aspect of this analysis is similar to that of Ref [7], and is described in detail in Ref [8]. Continuum background – events other than $B\bar{B}$ – is suppressed by cuts on shape variables, and measured with off-resonance data.

We make no effort to sort the detected particles into those from the semileptonically decaying B and those from the other B. Instead we determine the mass of the hadronic system X in $B \to X\ell\nu$ from the lepton and neutrino momentum vectors alone:

$$\begin{aligned} M_X^2 &= (E_B - E_\ell - E_\nu)^2 - (\vec{P}_B - \vec{P}_\ell - \vec{P}_\nu)^2 \\ &= M_B^2 + M_{\ell\nu}^2 - 2E_B E_{\ell\nu} + 2|\vec{P}_B||\vec{P}_{\ell\nu}|\cos\theta_{\ell\nu,B} \end{aligned} \tag{4}$$

For B mesons produced at the $\Upsilon(4S)$, E_B and $|\vec{P}_B|$ are known and constant, but the angle between B and $\ell\nu$ system varies from event to event, and is not known. Since $|\vec{P}_B|$ is small (300 MeV/c), we approximate M_X^2 by dropping the final term in Eq. 4, writing

$$\widetilde{M_X^2} = M_B^2 + M_{\ell\nu}^2 - 2E_B E_{\ell\nu}. \tag{5}$$

The measured $\widetilde{M_X^2}$ distributions, for on-resonance and scaled off-resonance data, are shown in Fig. 1. (The true M_X^2 distribution has spikes at the D and D^* masses, distinct peaks for the narrow D^{**}'s, and a continuum from $D\pi$ and $D^*\pi$. For $\widetilde{M_X^2}$, these features are smeared out by the neglect of the last term in Eq. 4. There is further smearing from the imperfect neutrino resolution.)

We extract the first and second moments of the M_X^2 distribution from the measured $\widetilde{M_X^2}$ distribution, with two different methods. In both, we subtract the scaled off-resonance distribution from the on-resonance distribution to obtain a distribution for $B\bar{B}$ events. It contains a 1% component from $b \to u\ell\nu$ decays, a 3% component from secondary decays $b \to c \to s\ell\nu$ and leptons from ψ and ψ', and a dominant (96%) component from $b \to c\ell\nu$. We make Monte Carlo samples to represent these three components. The dominant $b \to c\ell\nu$ piece is further divided into three subcomponents: $B \to D\ell\nu$, $B \to D^*\ell\nu$, and "$B \to D^{**}\ell\nu$", where D^{**} is a representation of all charmed hadronic systems heavier than D^*.

In the first method of extracting moments, we fit the $\widetilde{M_X^2}$ distribution with the $\widetilde{M_X^2}$ distributions given by the five Monte Carlo components. We fix the $b \to u\ell\nu$ and $b \to c \to s\ell\nu$ scales at their expected values, and let the $B \to D\ell\nu$, $B \to D^*\ell\nu$, and $B \to D^{**}\ell\nu$ scales float so as to optimize the fit. Having determined that mix of $D/D^*/D^{**}$ that gives the best fit to the $\widetilde{M_X^2}$ distribution, we can readily compute what first and second moments that mix implies for the $b \to c\ell\nu$ part of the true M_X^2 distribution.

In the second method of extracting moments, we directly compute the first and second moments of the measured $\widetilde{M_X^2}$ distribution. We apply (small) subtractions to these moments for $b \to u\ell\nu$ and $b \to c \to s\ell\nu$ backgrounds, determining these subtractions with Monte Carlo. We are left with first and second moments of the $b \to c\ell\nu$ component of the $\widetilde{M_X^2}$ distribution. These moments differ from the moments of the true M_X^2 distribution by a bias term, due to the difference between $\widetilde{M_X^2}$ and M_X^2, and due to our imperfect (and asymmetric) neutrino momentum resolution. We determine the bias term, separately for D, D^*, and D^{**}, with Monte Carlo. The bias term does vary from mode to mode, and we include in our systematic error allowance for our imperfect modelling of each mode and for our imperfect knowledge of the mix of the three modes.

For both methods we need a model for $b \to c\ell\nu$. We have detailed models of the $D\ell\nu$ and $D^*\ell\nu$ components, including form factor parameters measured by CLEO. For $D^{**}\ell\nu$, we use ISGW1 and ISGW2 shapes. For nonresonant $D^{(*)}\pi\ell\nu$ shapes, we use the model of Goity and Roberts [9], and also a home-brew model with a broad resonance decaying into $D^{(*)}\pi$ via phase space. We vary the less-well-known components of this ($D^{**}\ell\nu$, non-resonant) over a broad range, to reflect our limited knowledge.

Results for the first and second moments of the M_X^2 distribution, as obtained by the two methods just described, are given in Table 1. There we have taken the moments about $\bar{M}_D^2 = (1.975\mathrm{GeV})^2$, the square of the D/D^* spin-averaged mass.

We have considered systematic errors from several sources, including detector simulation, modeling of $b \to c\ell\nu$, B mass and beam energy uncertainties, and uncertainties in charged particle momentum scale and photon energy scale. The first two dominate. The systematic errors are combined in quadrature, and the combined error given in Table 1. One notes that systematic errors dominate over statistical errors, for both moments.

The two methods of determining moments have comparable errors. We combine them by taking a straight averge, with equal weighting. Those averages, with their statistical and systematic errors, are listed in Table 1. We have investigated the correlation in the errors between the first and second moments. We find the statistical errors to be essentially uncorrelated, and the systematic errors to have a small positive correlation ($\rho = 0.16$). Note that the results for moments of the hadronic mass distribution are *subject to the restriction that the lepton momentum lie in the interval 1.5 – 2.5 GeV/c.*

III. MOMENTS OF THE LEPTON MOMENTUM SPECTRUM

The lepton (electron) momentum spectrum given in Ref [5] is as measured, in the lab frame. Moments calculated from it require a few corrections before they can be compared with OPE predictions. The largest of these are for electromagnetic radiative corrections (not included in the OPE prediction), for experimental resolution (including bremsstrahlung by the electron) and for the boost from lab frame to B rest frame. Also, the spectrum is measured only down to 600 MeV/c, and a model is needed for the extrapolation to zero. Corrections are obtained from Monte Carlo samples, where we turn each effect (radiation, smearing, boost) on or off, and note the change in calculated moments. The raw moments, corrections, and corrected moments are given in Table 2.

We have considered systematic errors from several sources, including: leptons from D_s, D^0, and $D^{\pm}$ produced via $b \to c\bar{c}s$, $\bar{c} \to \bar{D}$; leptons from ψ, ψ'; leptons from taus from $b \to c\tau\nu$; Λ_c semileptonic decays; π^0 and η Dalitz decays; γ conversions; hadrons misidentified as electrons; electrons from $b \to cW$, $c \to se\nu$; uncertainty in mixing correction; momentum dependence to electron-finding efficiency; uncertainties in corrections given in Table 2. The largest is the first-mentioned. We combine all systematic errors in quadrature, obtaining the *PRELIMINARY* results:

$$\langle E_\ell \rangle = 1.36 \pm 0.01 \pm 0.02 \text{ GeV};$$

$$\langle E_\ell^2 \rangle = 2.04 \pm 0.02 \pm 0.04 \text{ GeV}^2.$$

There are substantial correlations between the errors on $\langle E_\ell \rangle$ and $\langle E_\ell^2 \rangle$. These are reduced, but not eliminated, by taking the second moment about the mean, i.e., by considering $\langle (E_\ell - \langle E_\ell \rangle)^2 \rangle$. For that, we find

$$\langle (E_\ell - \langle E_\ell \rangle)^2 \rangle = 0.19 \pm 0.004 \pm 0.005 \text{ GeV}^2.$$

IV. INTERPRETATION OF MOMENTS

Each of the four moments (lepton energy, hadronic mass-squared; first moment, second moment) can be written as a function of $\bar{\Lambda}$ and λ_1. In particular [2]:

$$\begin{aligned}\langle M_X^2 - \bar{M}_D^2 \rangle = M_B^2 [0.0272 \tfrac{\alpha_s}{\pi} + 0.207 \tfrac{\bar{\Lambda}}{M_B}(1 + 0.43 \tfrac{\alpha_s}{\pi}) + 0.193 \tfrac{\bar{\Lambda}^2}{M_B^2} \\ + 1.38 \tfrac{\lambda_1}{M_B^2} + 0.203 \tfrac{\lambda_2}{M_B^2}],\end{aligned} \tag{6}$$

$$\langle (M_X^2 - \bar{M}_D^2)^2 \rangle = M_B^4 [0.00148 \frac{\alpha_s}{\pi} + 0.038 \frac{\bar{\Lambda}}{M_B} \frac{\alpha_s}{\pi} + 0.0535 \frac{\bar{\Lambda}^2}{M_B^2} - 0.12 \frac{\lambda_1}{M_B^2}], \tag{7}$$

where these expressions are derived subject to the requirement that the lepton energy is above 1.5 GeV. Similar equations hold for $\langle E_\ell \rangle$ and $\langle (E_\ell - \langle E_\ell \rangle)^2 \rangle$ [3]. Consequently, each of the four measurements of a moment determines a band in $\bar{\Lambda} - \lambda_1$ space. Those bands are shown in Fig. 2.

The figure, of course, cannot depict the correlations among the four measurements. We have not yet considered correlations between the errors on hadronic mass moments and lepton energy moments, but believe them to be small. Correlations between the hadronic mass moments are small, while those between the lepton energy moments are not small.

The four bands of Figure 2 do not intersect at a common point, and on the face of it, the disagreement is outside the errors. Possible explanations include experimental error (not unlikely considering the preliminary state of the analysis); need to go beyond $1/m^2$ in the expansions; or a more fundamental problem in the theory. A resolution of the discrepancy is needed before one can obtain a precision determination of $|V_{cb}|$, either from inclusive or exclusive decays.

For illustrative purposes, we ignore the discrepancy of Fig. 2, use only the hadronic mass moments, and solve for $\bar{\Lambda}$ and λ_1. We find:

$$\bar{\Lambda} = 0.33 \pm 0.02 \pm 0.08 \quad \text{GeV},$$

$$\lambda_1 = -0.13 \pm 0.01 \pm 0.06 \quad (\text{GeV/c})^2.$$

Given $\bar{\Lambda}$ and λ_1, one can determine $|V_{cb}|$ from measurements of the B meson lifetime and semileptonic decay branching fraction. The precision to which we can determine $\bar{\Lambda}$ and λ_1 is such that their uncertainty would contribute less than a 2% error to $|V_{cb}|$. One can also determine the b quark mass, using Eq. 3. We find $m_b = 4.97 \pm 0.10$ GeV, where this is the pole mass in $\overline{MS}$ at one loop.

If instead of using the values of $\bar{\Lambda}$ and λ_1 determined by the intersection of the bands for hadronic mass moments, we were to use the values determined by the intersection of the bands for the lepton energy moments, the value obtained for $|V_{cb}|$ from the exclusive decay $B \to D^* \ell \nu$ would be 4.5% larger, the value obtained for $|V_{cb}|$ from the inclusive decay would be more than 10% larger, and the value obtained for m_b would be lower by 0.7 GeV.

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TABLE I. First and second moments of M_X^2 distribution, as determined by the fitting method, the direct method, and their equal-weight average. These moments are subject to the requirement $E_\ell > 1.5$ GeV.

	$\langle M_X^2 - \bar{M}_D^2 \rangle$ (GeV^2)	$\langle (M_X^2 - \bar{M}_D^2)^2 \rangle$ (GeV^4)
fitting method	$0.286 \pm 0.026 \pm 0.103$	$0.721 \pm 0.054 \pm 0.304$
direct method	$0.286 \pm 0.026 \pm 0.090$	$1.101 \pm 0.117 \pm 0.379$
average	$0.286 \pm 0.023 \pm 0.080$	$0.911 \pm 0.066 \pm 0.309$

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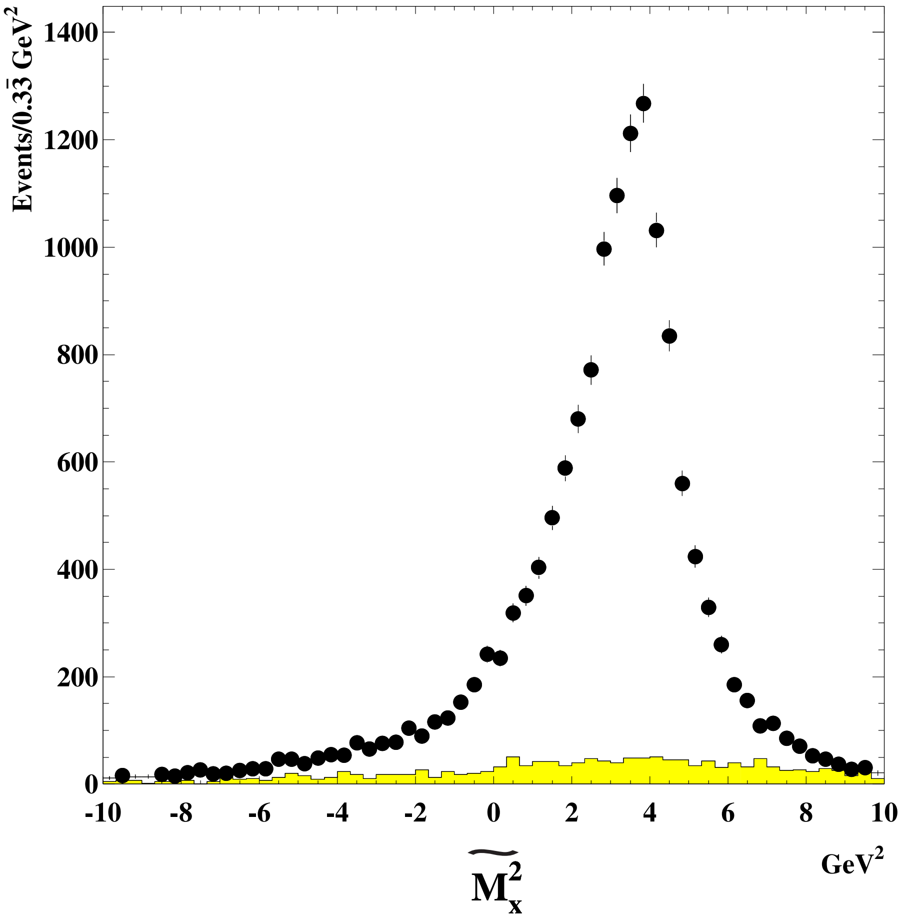

FIG. 1. Measured $\widetilde{M_X^2}$ distributions, for on-resonance data (points) and scaled off-resonance data (shaded histogram).

TABLE II. Raw lepton energy moments, corrections, and corrected moments with statistical errors.

	$\langle E_l \rangle$ (GeV)	$\langle E_l^2 \rangle$ (GeV^2)	$\langle (E_l - \langle E_l \rangle)^2 \rangle$ (GeV^2)
Raw	1.39 ± 0.01	2.08 ± 0.02	0.16 ± 0.004
$b \to u \ell \nu$ Correction	-3.67×10^{-3}	-1.22×10^{-2}	-2.04×10^{-3}
Smear Correction	2.28×10^{-2}	6.40×10^{-2}	3.60×10^{-5}
Boost Correction	-2.08×10^{-3}	-0.90×10^{-2}	-3.07×10^{-3}
Radiation Correction	0.99×10^{-2}	2.81×10^{-2}	-2.02×10^{-5}
$p < 0.6$ GeV Correction	-5.21×10^{-2}	-1.04×10^{-1}	4.12×10^{-2}
Total Correction	-2.52×10^{-2}	-3.34×10^{-2}	3.61×10^{-2}
Corrected	1.36 ± 0.01	2.04 ± 0.02	0.19 ± 0.004

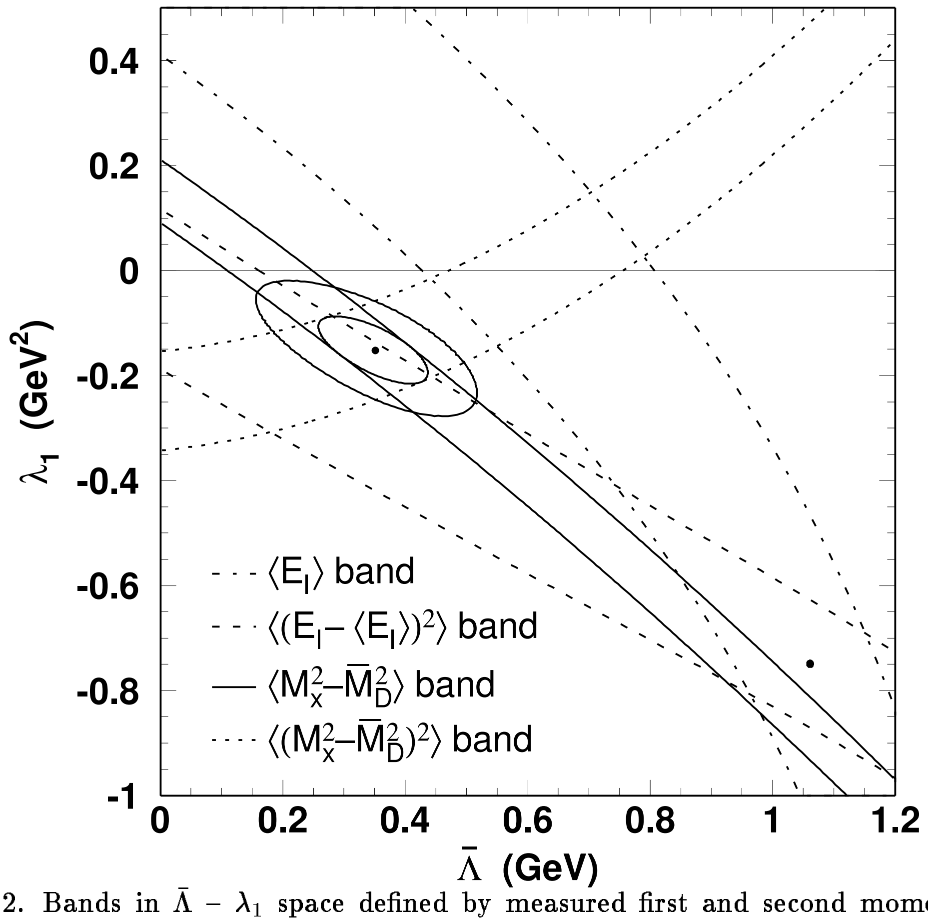

FIG. 2. Bands in $\bar{\Lambda} - \lambda_1$ space defined by measured first and second moments of hadronic mass-squared and lepton energy distributions, $\langle M_X^2 - \bar{M}_D^2 \rangle$ (solid), $\langle (M_X^2 - \bar{M}_D^2)^2 \rangle$ (dotted), $\langle E_l \rangle$ (dash-dot), and $\langle (E_l - \langle E_l \rangle)^2 \rangle$ (dash). The intersection of the centerlines of the two hadronic mass moment bands is shown as a dot, as is the intersection of the centerlines of the two lepton energy moment bands. One- and two-sigma error ellipses are shown for the hadronic mass moment case.