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A Study of Jet Production Rates in the Four Flavor Continuum and a Test of QCD

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Abstract

We present a QCD analysis of jet production rates measured in e^+e^- annihilation into hadrons at about 10 GeV where only four flavors are produced in the continuum. All previous jet studies have been performed in the five flavor continuum. In the four flavor continuum QCD predictions are substantially different due to the different contributions from vacuum polarization diagrams. The measured jet rate distributions are compared to the predictions of $\mathcal{O}(\alpha_s^2)$ QCD computed for the four flavor continuum. From the fits of the theory to the data we obtain the final value of $\alpha_s(10.53)=0.164\pm0.004\pm0.014$ where the first error is due to the experimental uncertainties and the second error includes theoretical uncertainties due to hadronization, parton virtuality, renormalization scale and jet resolution parameter, all added in quadrature. This value of α_s extrapolated to M_{Z^0} in the five flavor continuum yields $\alpha_s(M_Z)=0.113\pm0.002\pm0.006$.

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I. INTRODUCTION

Within the framework of quantum chromodynamics (QCD), multihadron production in e^+e^- collisions is associated with the production of partons which subsequently materialize into collimated clusters of colorless hadrons called jets. The first observation of jets in e^+e^- collisions was reported in 1975 from SPEAR [1] at E_{cm}=6-8 GeV. Since then, jets have emerged as powerful tools to perform tests of perturbative QCD [2–5]. The measurements of relative production rates of multijet events in the e^+e^- continuum and around the Z^0 peak have been found in good agreement with the theoretical predictions and the values of α_s determined from these measurements at different energies have been found consistent with each other [4,6]. All these studies have been performed in the regime of five flavors. However, in the four flavor regime, the QCD predictions of the next to leading order coefficients are substantially different due to the different contributions from vacuum polarization diagrams. Therefore, from the agreement of theory and data in the five flavor regime, it does not follow at all that an agreement would also be found in the four flavor continuum. Thus, the study of jet rates in the four flavor continuum constitutes a new test of the validity of QCD. Moreover, QCD has never been tested with the same process on both sides of a threshold. Comparison of the results obtained from this study with those in the five flavor continuum from the same observable (jet rates in our case) provides a unique test of the renormalization group equation used to extrapolate the results from one energy point to another.

The Cornell Electron Storage Ring (CESR) has produced bulks of e^+e^- continuum data at the highest possible energy (E_{cm} = 10.53 GeV) in the four flavor continuum. In this study, we analyze these data.

II. EVENT SELECTION

The data used in this analysis were collected with the CLEO-II detector, described elsewhere [7], at CESR at an average energy of E_{cm} = 10.53 GeV. Hadronic events are selected by requiring at least five charged tracks in the central detector and a total energy in the calorimeters greater than 15 % of the center-of-mass energy. This largely suppresses QED and two-photon backgrounds. Background from beam gas and beam wall interactions was further suppressed by vetoing events which had a large momentum imbalance. We require that the fitted event vertex be within $\pm$ 5 cm of the beam interaction point along the beam axis. Any event having a track with momentum greater than 6 GeV was rejected. The thrust axis of the selected events was required to point to inside the central tracking chamber. This event selection criteria had an efficiency of about 70 % Electromagnetic clusters which were unmatched with any charged track were used in the analysis. The clusters were required to have a minimum energy of 50 MeV in the endcaps and 30 MeV in the barrel.

III. DEFINITION OF JETS IN THEORY AND EXPERIMENT

The criteria of a good jet definition [8] are:

1. It be defined at any order of perturbation theory;

2. It yields finite cross sections at any order of perturbation theory;
3. It be simple to implement in both theoretical calculations and experimental analyses; and
4. It be insensitive to hadronization.

In earlier studies, jet finding algorithms were based on event shape parameters [2]. Soon, it was realized that a jet definition of a more quantitative nature was required in order to make a direct comparison between experiment and theory. This gave rise to the so called cluster jet algorithms. In the cluster algorithm, more compatible with QCD calculations, one begins by considering all particles in an event as clusters. A jet resolution variable y_{ij}, defined in general

$$y_{ij} = \frac{M_{ij}^2}{E_{vis}^2}, \tag{1}$$

is computed for all possible pairs (i,j); where M_{ij} is the combined "mass" of clusters i and j, and E_{vis} is the visible energy of the event. The pair (i,j) with the smallest value of y_{ij} is combined into a single cluster with 4-momentum p_k, determined by some recombination scheme, for example,

$$\begin{aligned} \vec{p_k} &= \vec{p_i} + \vec{p_j}, \\ E_k &= P_k, \end{aligned} \tag{2}$$

called the P-scheme [4,9], or

$$\begin{aligned} \vec{P_k} &= \frac{E_k}{|\vec{p_i} + \vec{p_j}|}(\vec{p_i} + \vec{p_j}), \\ E_k &= E_i + E_j, \end{aligned} \tag{3}$$

called the E_0-scheme [4,9]. This process is repeated until the value of y_{ij} for all the possible pairs of the remaining clusters exceeds a prescribed threshold value, y_{cut}. The remaining number of clusters at this stage are counted as the number of jets in the event corresponding to the given value of y_{cut}. We use the following two definitions of M_{ij}^2:

$$M_{ij}^2 = (p_i + p_j)^2 \tag{4}$$

first presented by Z. Kunszt in the context of higher order QCD calculations [10]. This was implemented by the JADE collaboration using E_0 scheme in their analysis giving rise to the popular JADE-algorithm [11] which we shall call JJ-algorithm; and

$$M_{ij}^2 = Min(E_i^2, E_j^2) \tag{5}$$

first suggested by Dokshitzer [12]. These two definitions of M_{ij}^2, combined with the "P" recombination scheme constitute the JADE-algorithm and Durham-algorithm in the P-scheme respectively which we shall refer to as the JP and DP algorithms. There are other recombination schemes which can be used to vary the JADE and Durham algorithms [4,9]. However, we use only the P-scheme as, based on a Monte Carlo (MC) study [13] in the energy range

from 10 - 91 GeV, we have found that the JADE and the Durham algorithms in the P-scheme are relatively less sensitive to hadronization, especially at the lower energy end. However, we also use JJ-algorithm just for a consistency check.

These jet algorithms were implemented consistently in computing the theoretical predictions and in the experimental measurements.

IV. MEASUREMENT OF JET PRODUCTION RATES

Using the jet finding algorithms, JJ, JP and DP described in the previous section, we measure the jet production rates in the hadronic event sample in the four flavor continuum collected at an average energy of 10.53 GeV. To unfold the data for the effects of limited detector resolution and acceptance, and to correct for initial state radiation, we have used JETSET shower MC [14]. We use the standard values of the parameters which describe the experimental data well in the range of 30 - 91 GeV [15], as we have found in a separate study that they also give a good description of the global features of our multihadron data. To determine the correction factors, jet rate distributions were produced from two MC samples: (I) without detector simulation and initial state radiation, (II) with detector simulation and initial state radiation and subject to the same reconstruction procedure and event selection criteria as the experimental data. The bin-by-bin correction factor, C_i was calculated as

$$C_i = \frac{(U_i^{MC}/N_U^{MC})}{(D_i^{MC}/N_D^{MC})}; \tag{6}$$

where U_i^{MC} is the number of entries in bin i of the distribution from sample (I) which contained a total of N_U^{MC} events, and D_i^{MC} is the number of entries in bin i of the distribution generated from sample (II) which contains N_D^{MC} events that survived the reconstruction and selection process. The unfolded experimental value for bin i, U_i, is given by

$$U_i = C_i D_i; \tag{7}$$

where D_i is the uncorrected experimental measurement.

The unfolded data is compared to the predictions of the JETSET shower MC in Figures 1b, 2b and 3b for the JJ, JP and DP algorithms respectively. We have also checked that the data and the MC also agree well at the uncorrected level. The detector correction factors are presented in Figures 1c, 2c and 3c. In the y_{cut} region where the comparison of theory and data will be made, the detector corrections are less than 30 %. Figures 1a, 2a, and 3a present the jet rates at the parton and hadron level as predicted by JETSET.

V. DETERMINATION OF α_s

To $\mathcal{O}(\alpha_s^2)$, the strong coupling constant may be written [16]:

$$\alpha_s(\mu) = \frac{12\pi}{(33-2n_f)\ln(\mu^2/\Lambda^2_{\overline{MS}})}\left[1 - 6\frac{(153-19n_f)}{(33-2n_f)^2}\frac{\ln(\ln(\mu^2/\Lambda^2_{\overline{MS}}))}{\ln(\mu^2/\Lambda^2_{\overline{MS}})}\right] \tag{8}$$

where n_f is the number of active quark flavors (taken to be 4 in our continuum data), $\Lambda_{\overline{MS}}$ is the QCD parameter referring to the $\overline{MS}$ renormalization scheme, and μ is the renormalization scale which may be related to the e^+e^- c.m. energy by

$$\mu = \mathbf{f}_\mu \cdot E_{cm} \tag{9}$$

The relative rates of 3- and 4-jet events, R_3 and R_4, have been calculated in $\mathcal{O}(\alpha_s^2)$ QCD [17] perturbation theory and parametrized [9] as below:

$$R_3 \equiv \frac{\sigma_3}{\sigma_0} = \frac{\alpha_s(\mu)}{2\pi} A_3(y_{cut}) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 [A_3(y_{cut}) 2\pi b_0 \ln(\frac{\mu^2}{E_{cm}^2}) + B_3(y_{cut})]; \tag{10}$$

$$R_4 \equiv \frac{\sigma_4}{\sigma_0} = B_4(y_{cut}) \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2; \tag{11}$$

where

$$b_0 = \frac{33 - 2n_f}{12\pi}; \tag{12}$$

σ_0 is the leading order cross-section for e^+e^- annihilation into hadrons and has to be replaced with $\sigma_{tot} = \sigma_0(1 + \alpha_s/\pi + 1.41(\alpha_s/\pi)^2)$ for comparing the above equations with the measurements. The 2-jet rate, R_2, may be computed by the condition $R_2 + R_3 + R_4 = 1$. Values for the coefficients A and B have been derived [9] by integrating the $\mathcal{O}(\alpha_s^2)$ matrix elements [17] using a FORTRAN program supplied by P. Nason [18].

All the points in the R_2 distribution are correlated because they have been determined from the same data sample. We compare the above equations with the measurements corresponding to a y_{cut} value of 0.06, which has small hadronization and detector corrections and still large statistics, and determine the value of $\Lambda_{\overline{MS}}$. Taking this value as input into the above equations, the agreement (or disagreement) between the theoretical predictions and the measurements on other y_{cut} points will be taken as a test of the theory. Figures 4a, 5a and 6a show the comparison of the theoretical predictions to the data for JJ, JP and DP algorithms respectively. The theory agrees well with the data except at the low end of y_{cut} values where 4-jets dominate. This is not surprising as the 4-jet rates are only calculated to leading order.

To determine α_s, we define the differential 2-jet distribution, $D_2(y)$ (where y is used for y_{cut} for brevity), as

$$D_2(y) = \frac{R_2(y) - R_2(y - \Delta y)}{\Delta y}, \tag{13}$$

which is the distribution of y_{cut} values for which the jet multiplicity of events changes from 3 to 2. In this distribution, all the points are statistically independent of each other. The QCD parameter $\Lambda_{\overline{MS}}$ is determined for each algorithm by fitting the predicted D_2 distribution to the measured distribution for two ways of treating the renormalization scale: first setting $\mathbf{f}_\mu$=1, and second taking $\mathbf{f}_\mu$ as a second free parameter in the fit. The y_{cut} regions in which the fits are made are chosen by the following requirements: 1) The measured 5-jet

rates should be less than 1 % as theoretical calculations for 5-jet rates are not available, and 2)The measured 4-jet rates should be less than 1 % as theoretical calculations for 4-jet rates are only available to leading order. The second condition is removed in the fits where $\mathbf{f}_\mu$ is taken as a free parameter and is expected to optimize the 4-jet prediction, but we avoid the low y region where detector corrections are larger (ie greater than 30 %). The $\Lambda_{\overline{MS}}$ values thus obtained may then be translated to α_s by using equation 8. The comparison of theoretical predictions with the data emerging from these fits is presented in figures 4b, 5b and 6b. The results of the fits are listed in Table 1.

The procedures we use to estimate the uncertainties on the determination of α_s from several sources are described below:

1. **Experimental uncertainties:** The experimental uncertainties on the final results come mainly from the detector correction factors which may depend upon the MC and upon the detector simulation procedure. We have used the JETSET MC with parameters which are known to describe the e^+e^- data well from 30 to 90 GeV and we have checked that they also describe well the global features of our data. We estimate the errors due to the possible remaining problems in our simulation procedure by performing the analysis in three different ways: 1) using both the charged tracks and clusters from the calorimeters, 2) using charged tracks only, and 3) using clusters only. The α_s values determined from analysis 1) are the quoted central values. The larger of the deviations of α_s values from procedures 2) and 3) from that from 1) is taken as a symmetric error which, added in quadrature with the statistical error from the data and the correction factors, provides the quoted experimental error.
2. **Hadronization uncertainties:** The data, which are collected at hadron level, must be corrected for hadronization before comparing to the QCD calculations which are performed at parton level. Our quoted results are extracted from the data corrected to the parton level using the QCD parton shower MC model in JETSET [14] with the string hadronization scheme. The analysis was repeated with the data corrected to parton level by using the HERWIG MC, tuned to explain our data reasonably well at hadron level, with the cluster hadronization scheme. Both the models agree better at hadron level than at parton level as they start from parton level by using the different hadronization models to reach the agreement. The difference in final results obtained by using corrections computed from these two models is quoted as a symmetric error due to hadronization.
3. **Uncertainty in parton virtuality:**
   The QCD shower models contain a parameter, Q_0, which defines the cutoff value on the virtuality of the parton where the QCD shower process is halted and the hadronization process begun. Changing the value of Q_0 would change the final parton multiplicity in the shower before hadronization starts and the hadronization corrections would also change. For our central values, we used Q_0=1. We repeated our analysis for several values of Q_0 in the range of 0.7 to 2, and took the resulting deviation from the central value of α_s as the error due to parton virtuality.
4. **Uncertainty due to renormalization scale:**

One may notice from equation 10 that the jet-rate distributions do depend upon the renormalization scale μ. Had QCD been solved to all orders, the values of α_s determined from the data would have been independent of the choice of the scale. The results from $\mathcal{O}(\alpha_s^2)$ QCD are expected to be dependent on scale. One choice would be $\mu = E_{cm}$, ie $\mathbf{f}_\mu$=1. Although QCD itself would not suggest which value of the scale the fits should be made, a number of QCD inspired theoretical procedures have been suggested to make an optimal choice of the scale [20]. Some experimental procedures have also been presented to make an optimal choice of the scale [4,21,22]. It has been shown in references [4,21] that various experimental procedures to optimize the scale give similar results to those obtained by treating $\mathbf{f}_\mu$ as a free parameter in the fit and that the values of $\mathbf{f}_\mu$ so obtained agree with the values predicted by various theoretical procedures. As there is no agreement on this issue, we regard the different choices of scale a a source of systematic error. In order to accommodate all these approaches we perform fits of the theory to the data in two different ways:1) by fixing $\mathbf{f}_\mu$=1, and 2) by treating $\mathbf{f}_\mu$ as a free parameter in the fit. The average of the two α_s values thus obtained is quoted as the central result and the difference between the central value and either extreme is taken as a symmetric error due to the renormalization scale. This method has also been applied in determining the reported LEP results with which we shall compare our results.

5. **Uncertainty from the fit range:** We have already explained our criteria for choosing the y_{cut} range in which the fits are made. We use different ranges to check the consistency of our results. We find that the variations are not larger than the statistical errors. However, we quote this variation as an error due to the y_{cut} range.

   As no renormalization scale dependence is expected if QCD is solved to all orders, it may be argued that the errors due to the renormalization scale are a measure of the uncertainty introduced by the missing higher order terms in the theory [4,21–23]. As changing the value of Q_0 changes the final parton multiplicity in the shower and the hadronization corrections, the error due to Q_0 is correlated to the errors due to the hadronization and the renormalization scale. However, taking a conservative approach, we estimate and quote these errors separately. The final values of α_s along with all the errors are listed in Table 2.

Algorithm	y_{cut} fit range	$\mathbf{f}_\mu$	$\Lambda_{\overline{MS}}$	$\alpha_s(10.53)$	χ^2
JJ	0.08-0.20	1	372± 36	0.178±0.004	1.62/5
	0.07-0.20	0.05±0.02	96 ± 7	0.132±0.002	1.64/5
JP	0.06-0.20	1	391± 25	0.180±0.003	1.60/7
	0.04-0.20	0.114±0.013	180± 9	0.150±0.002	2.59/9
DP	0.05-0.20	1	351± 31	0.175±0.004	6.55/8
	0.04-0.20	0.118±0.030	192± 15	0.152±0.003	2.27/8

Table 1: Values of $\alpha_s(10.53)$ determined from fitting $\mathcal{O}(\alpha_s^2)$ QCD to the CLEO II data, with Only the experimental errors.

Algorithm	α_s	Δ_{exp}	Δ_{had}	Δ_{Q_0}	Δ_μ	Δ_y
JJ	0.156	± 0.004	$^{+0.014}_{-0.000}$	± 0.004	± 0.023	± 0.002
JP	0.165	± 0.004	± 0.005	± 0.007	± 0.015	± 0.002
DP	0.164	± 0.004	± 0.003	± 0.007	± 0.012	± 0.002

Table 2: Final values of $\alpha_s(10.53)$ determined from fitting $\mathcal{O}(\alpha_s^2)$ QCD to the CLEO II data, with all errors. For detail see text.

One should notice that although the JJ, JP and DP algorithms predict different jet rates, when applied consistently at theoretical and experimental levels they yield consistent values of the physical parameter α_s which overlap within errors. This check on the compatability of QCD calculations with the data is consistent with that from LEP experiments [24]. As we are using the JJ algorithm only as a consistency check, and the results from the JP and DP algorithms are very similar, we quote the α_s value from the DP algorithm as our final result which is

$$\alpha_s(10.53) = 0.164 \pm 0.004 \pm 0.014 \tag{14}$$

in the four flavor continuum, where the first error is the experimental error and the second error has been obtained by adding in quadrature all the other errors in Table 2. This value extrapolated to M_{Z^0} in the five flavor regime, using the formulae of Ref. [25], yields

$$\alpha_s(M_{z^0}) = 0.113 \pm 0.002 \pm 0.006 \tag{15}$$

Note that the fractional error in α_s decreases considerably in extrapolating from 10.53 to M_Z. This is due to the energy dependence of α_s in equation 8 used for extrapolation. This is one of the reasons for a growing interest among high energy physicists for determining α_s from low energy processes.

VI. SUMMARY AND DISCUSSION

In summary, we have tested the predictions of $\mathcal{O}(\alpha_s^2)$ QCD in the four flavor e^+e^- continuum by comparing them to data taken at the C.M. energy of 10.53 GeV, where only 4 flavors are produced. As the QCD predictions in the four flavor continuum are different from those in the five flavor continuum due to the different contributions from the vacuum polarization diagrams and have not been tested before, the agreement between the theory and the data constitutes a new test of the validity of QCD.

The value of α_s=0.164$\pm$0.004$\pm$0.014 obtained from fits of the theory to the data agrees well with $\alpha_s = 0.167^{+0.015}_{-0.011}$ from Υ decay [6] at about the same energy. As these are two different physical processes, the agreement between the α_s values obtained from them provides a convincing consistency check of the underlying theory. The measured α_s value at 10.53 GeV in the four flavor continuum extrapolated to 29 GeV in the five flavor continuum yields $\alpha_s(29)$=0.135 $\pm$ 0.003 which is in good agreement with $\alpha_s(29)$=0.133 $\pm$ 0.004 extracted from Mark II jet data [26] using the same procedure, where the shown errors are experimental errors only. Our α_s value extrapolated to $\alpha_s(M_{z^0}) = 0.113 \pm 0.002 \pm 0.006$ is also consistent

within errors with $\alpha_s(M_{z^0}) = 0.120 \pm 0.006$ extracted from the LEP jet data [5] using the same procedure and calculations, where all errors are included.

One should note that we have used QCD calculations of the same process from which α_s has been determined at 29 and 91 GeV by comparing the calculations to the data at those energies. The α_s value extracted from our data, following the same procedure as at higher energies, and extrapolated to those energies, agrees with the higher energy results. This is a unique test of the validity of renormalization group equation of QCD used for the extrapolation and which governs the running of α_s. Previously, all such comparisons have been made for one process on one side of the threshold to a different process on the other side of the threshold.

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VIII. LIST OF FIGURES

Fig. 1:
a) Jet rates in the JJ algorithm at parton and hadron levels as predicted by the JETSET MC.
b) Comparison of measured jet rates, corrected for detector effects and initial state radiation, to the predictions of the JETSET MC.
c) Detector correction factor (ie the ratio of jet rates measured at generator level MC and after smearing the MC with the detector effects) as a function of y_{cut}.

Fig. 2: Same as Fig. 1 but for the JP algorithm

Fig. 3: Same as Fig. 1 but for the DP algorithm

Fig. 4:
a) Measured distribution R_2 in the JJ algorithm, compared to $\mathcal{O}(\alpha_s^2)$ QCD predictions computed by using the value of α_s which was determined by comparing data and theory at one point, y_{cut}=0.06, only.
b) Measured distribution D_2 in JJ-algorithm, compared to $\mathcal{O}(\alpha_s^2)$ QCD predictions. The QCD parameters used in the predictions were determined from the fits and the fit ranges are shown in Table 1.

Fig. 5:
a) Same as 4a) but for the JP algorithm.
b) Same as 4b) but for the JP algorithm.

Fig. 6:
a) Same as 4a) but for the DP algorithm.
b) Same as 4b) but for the DP algorithm.

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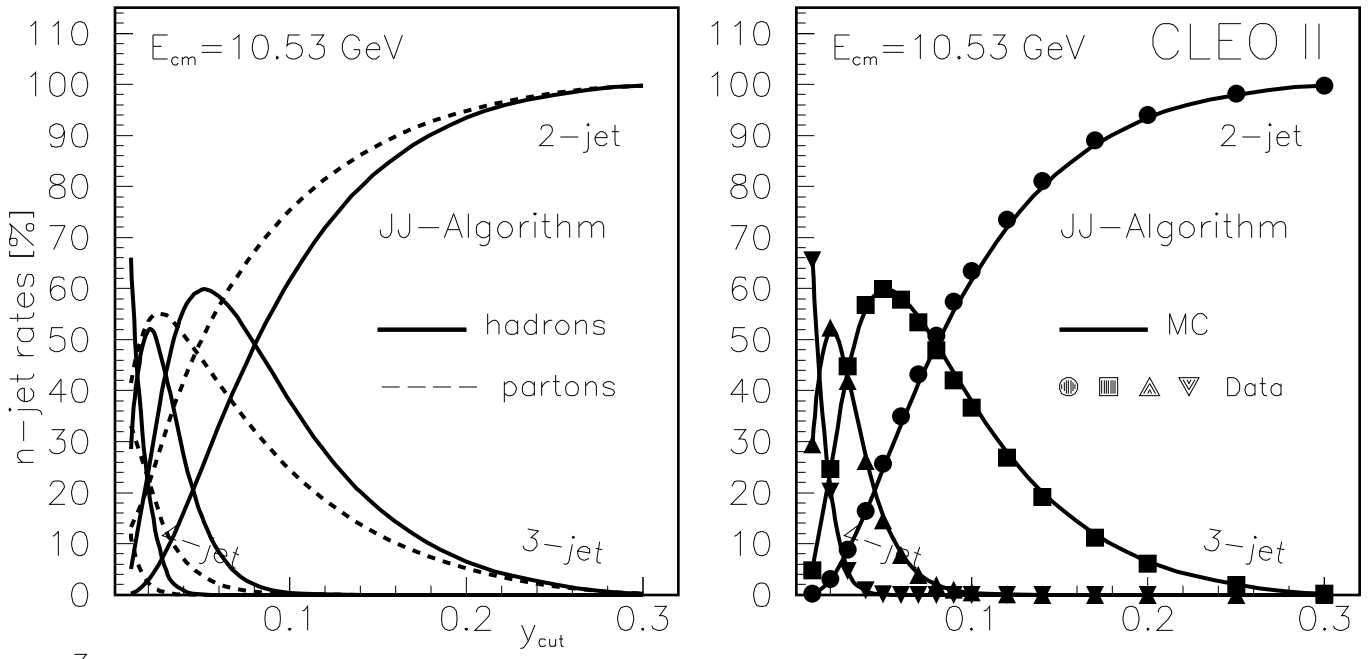

E_{cm}=10.53 GeV
2−jet
JJ−Algorithm
hadrons
partons
3−jet
4−jet
n−jet rates [%]
y_{cut}
E_{cm}=10.53 GeV
CLEO II
2−jet
JJ−Algorithm
MC
Data
3−jet
4−jet

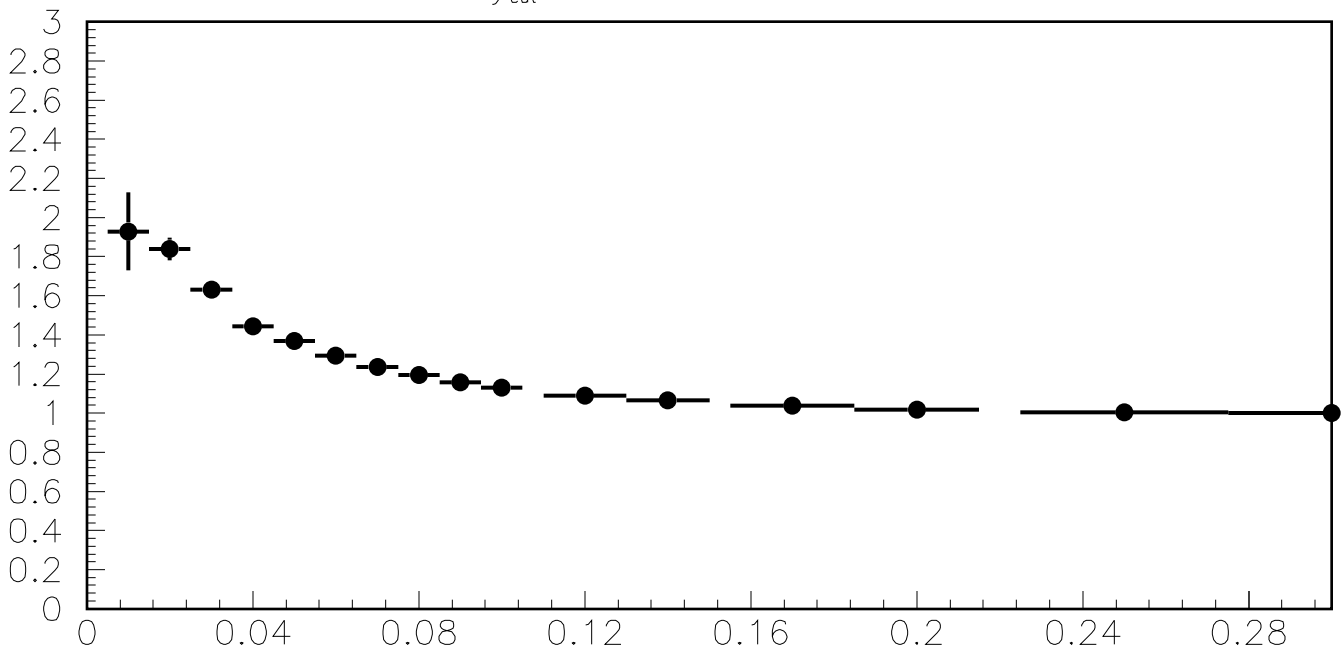
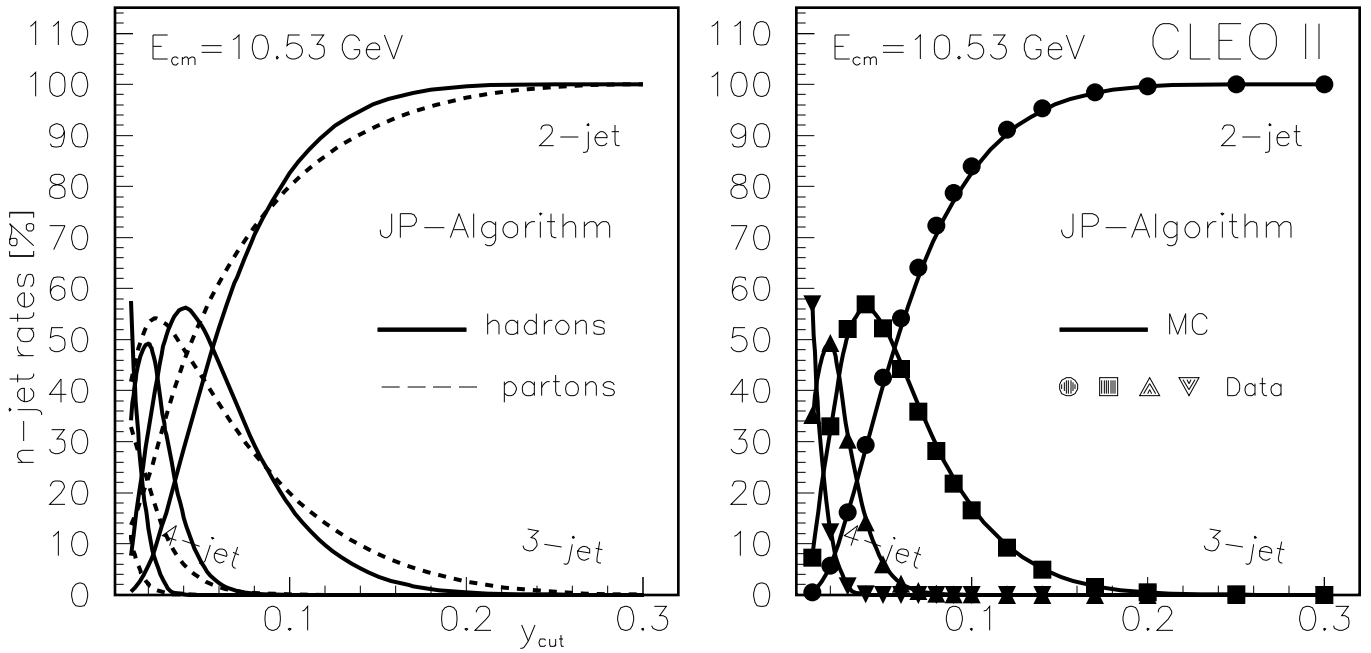


Fig. 1


E_{cm}=10.53 GeV
2−jet
JP−Algorithm
hadrons
partons
4−jet
3−jet
n−jet rates [%]
y_{cut}
E_{cm}=10.53 GeV
CLEO II
2−jet
JP−Algorithm
MC
Data
4−jet
3−jet

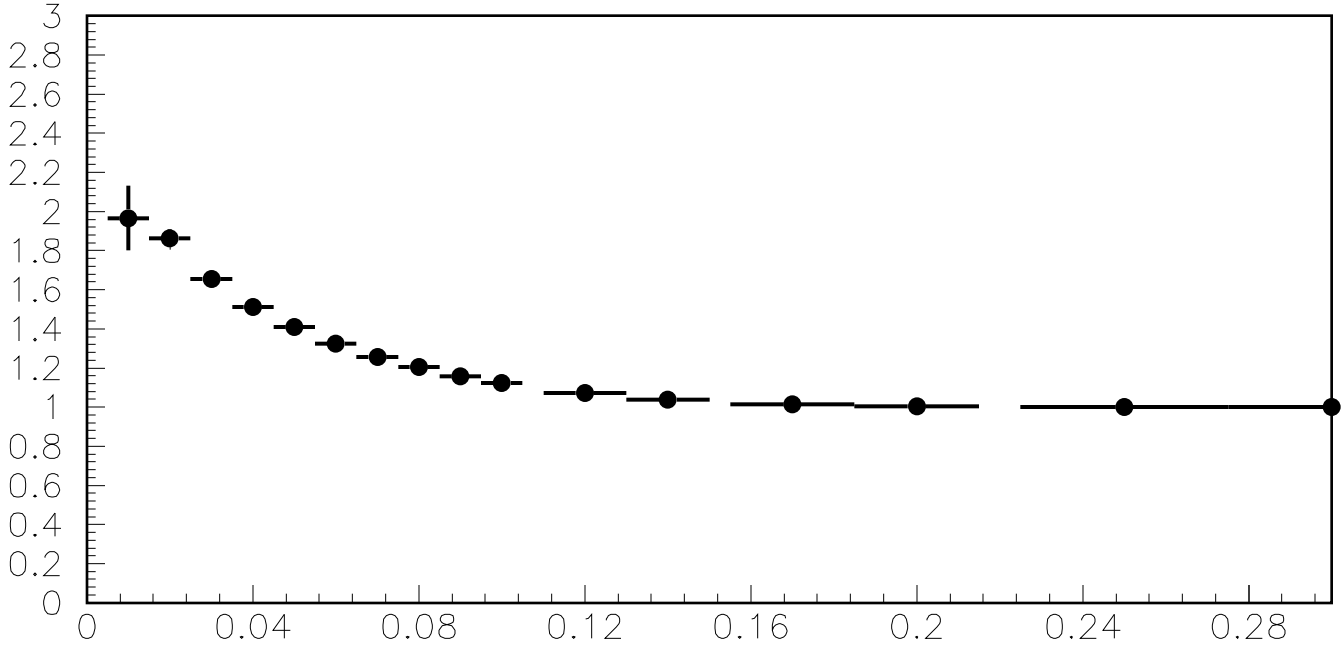
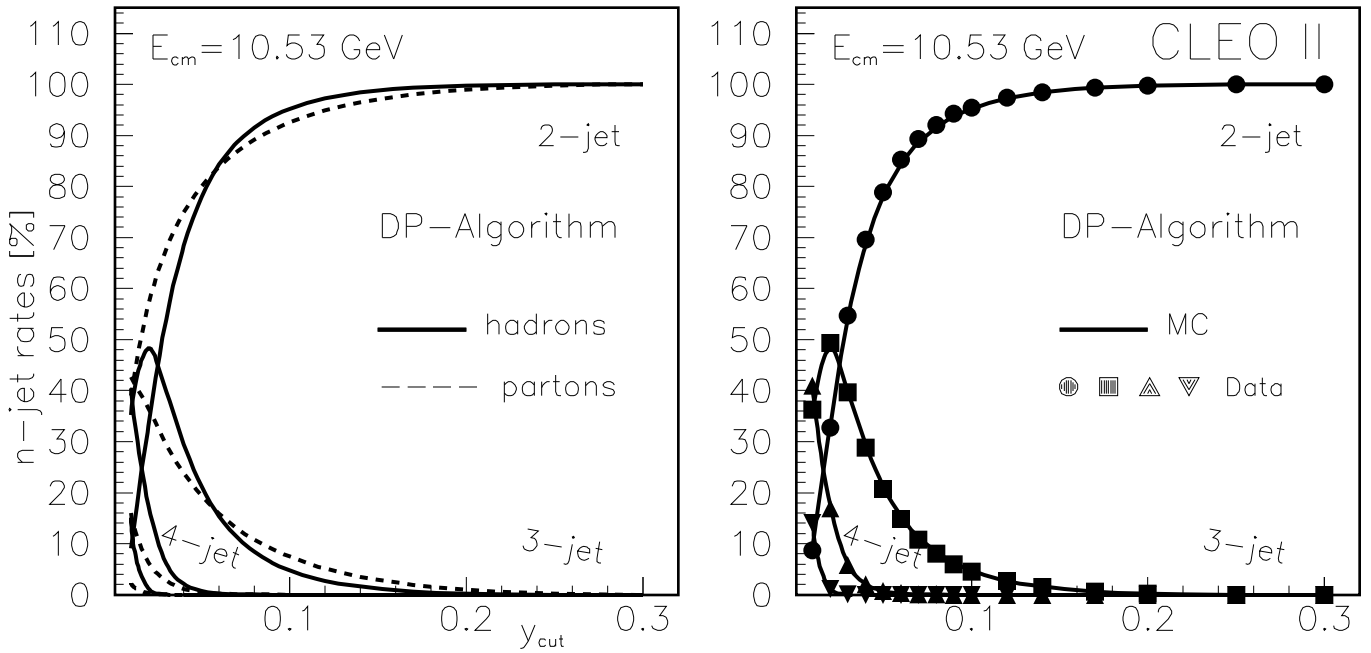


Fig. 2


E_{cm}=10.53 GeV
2−jet
DP−Algorithm
hadrons
partons
4−jet
3−jet
n−jet rates [%]
y_{cut}
E_{cm}=10.53 GeV
CLEO II
2−jet
DP−Algorithm
MC
Data
4−jet
3−jet

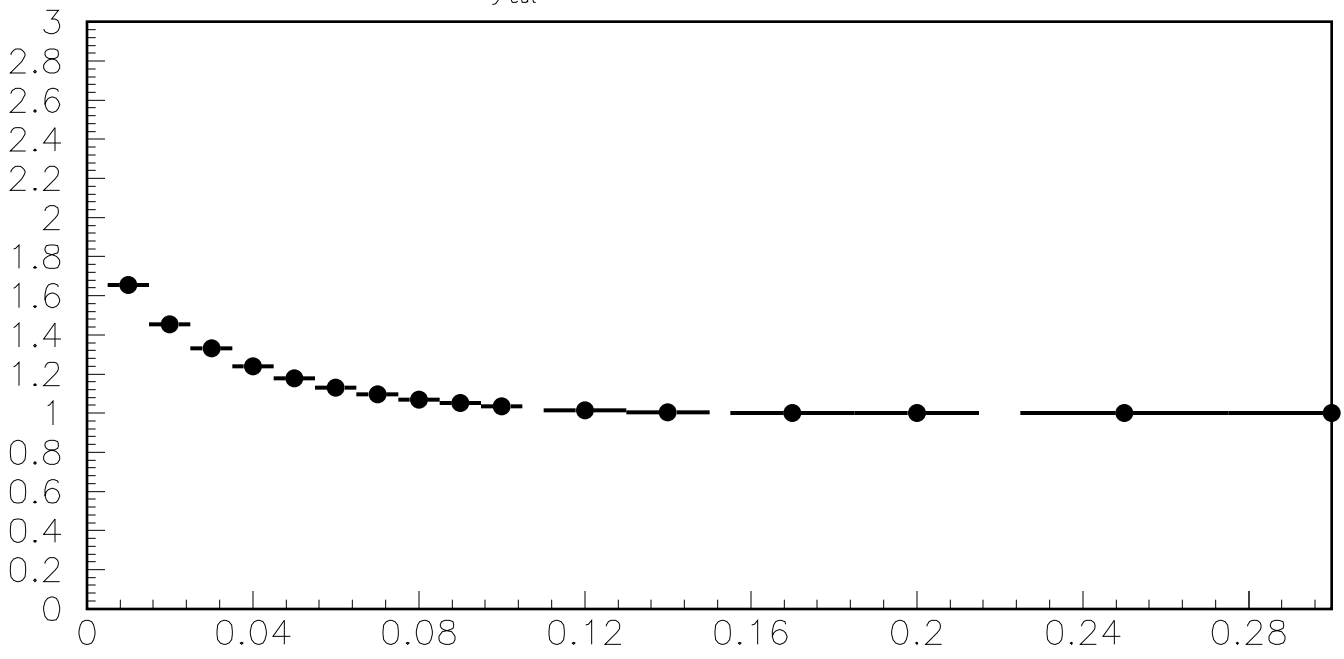


Fig. 3

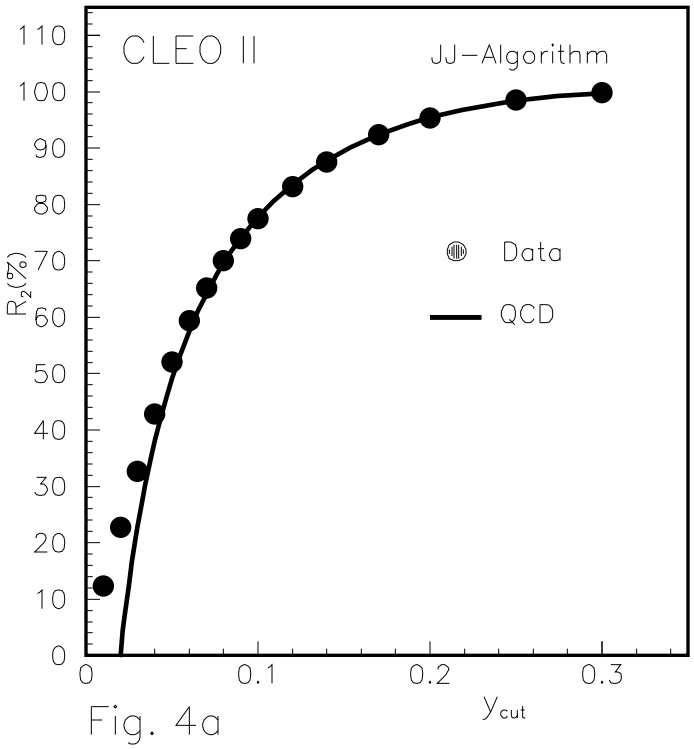

CLEO II
JJ–Algorithm
Data
QCD
R_2(%)
y_{cut}
0
10
20
30
40
50
60
70
80
90
100
110
0.1
0.2
0.3

Fig. 4a

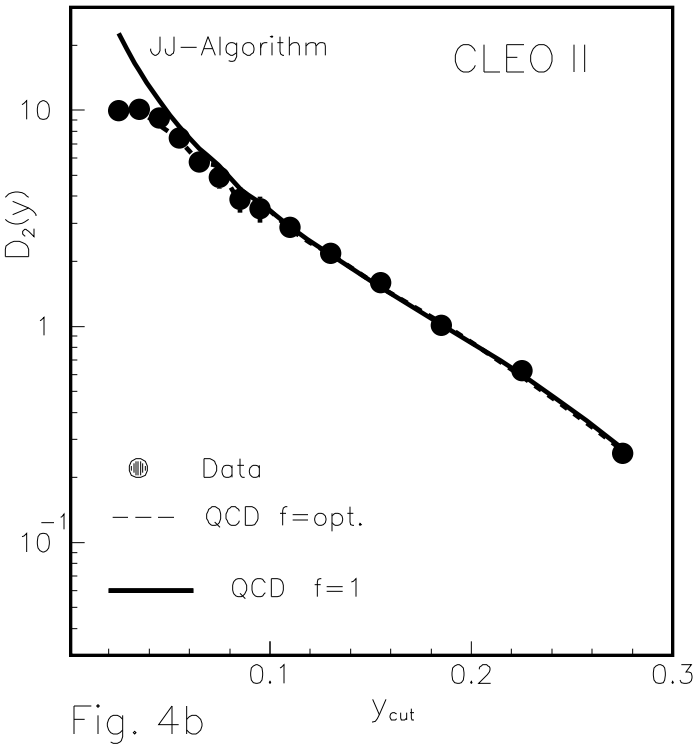

JJ–Algorithm
CLEO II
Data
QCD f=opt.
QCD f=1
$D_2(y)$
y_{cut}
10
1
10^{-1}
0.1
0.2
0.3

Fig. 4b

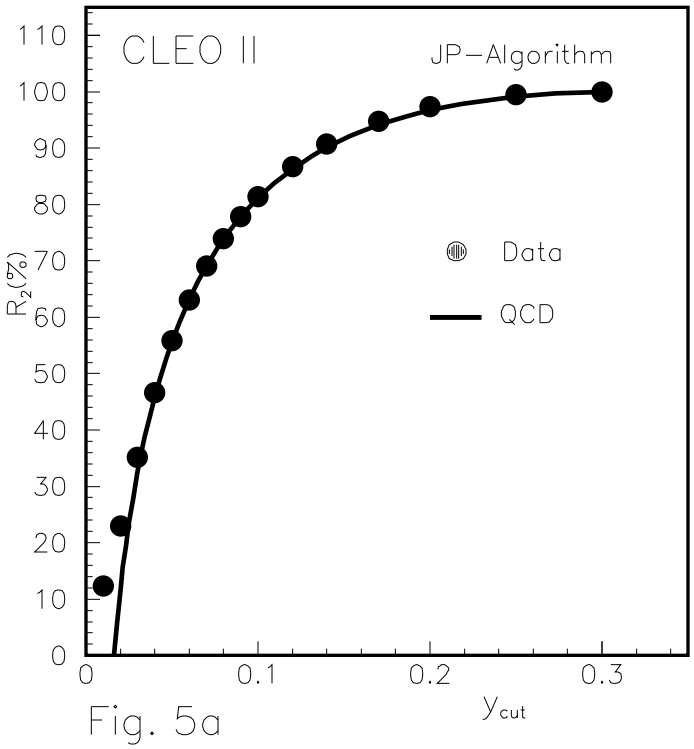

CLEO II
JP−Algorithm
Data
QCD
$R_2(\%)$
y_{cut}

Fig. 5a

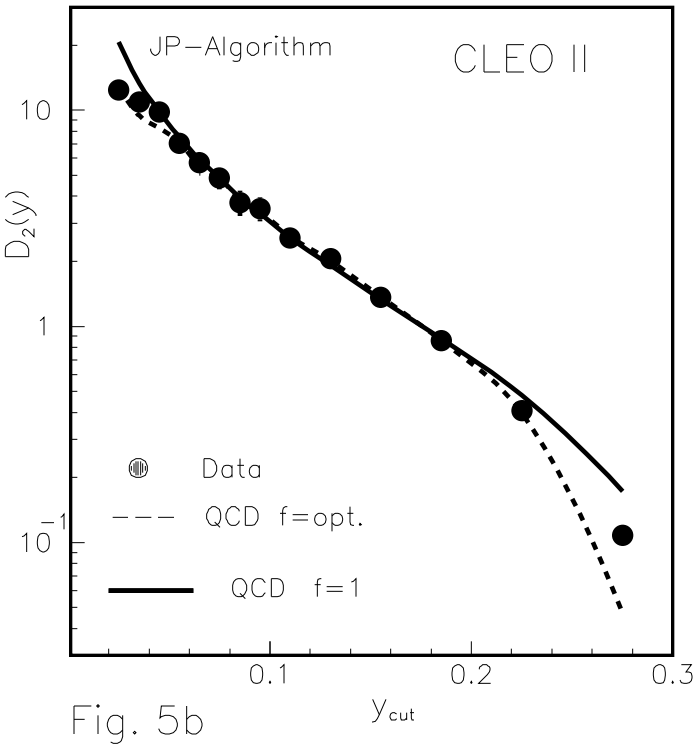

JP−Algorithm
CLEO II
Data
QCD f=opt.
QCD f=1
$D_2(y)$
y_{cut}

Fig. 5b

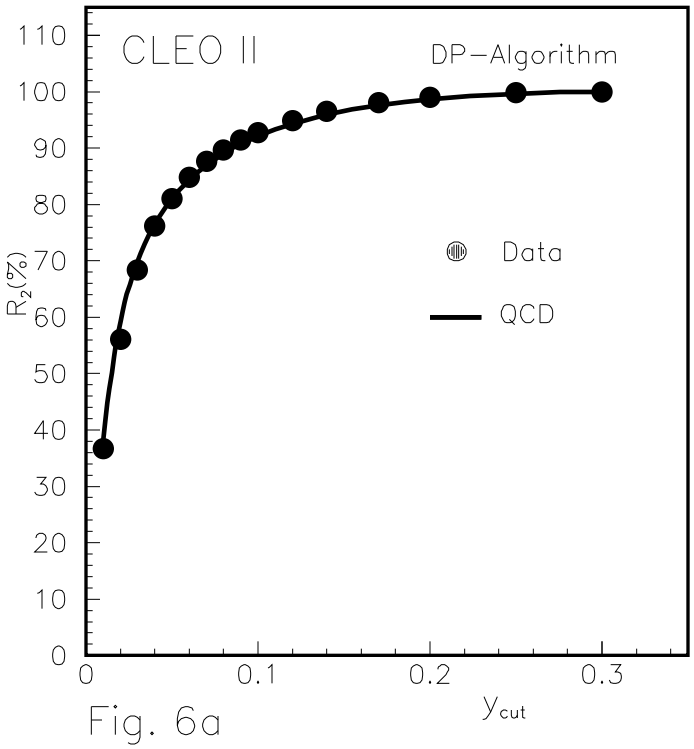

CLEO II
DP−Algorithm
Data
QCD
$R_2(\%)$
y_{cut}

Fig. 6a

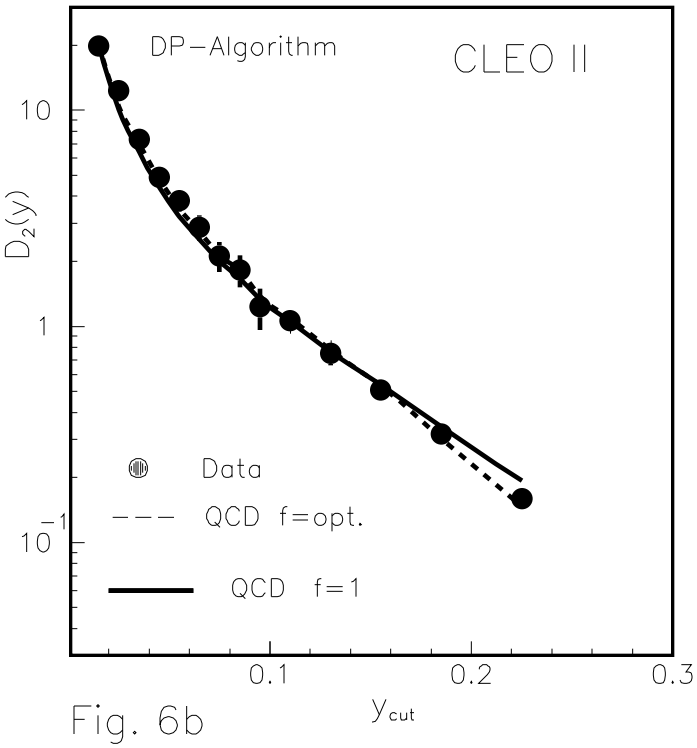

DP−Algorithm
CLEO II
Data
QCD f=opt.
QCD f=1
$D_2(y)$
y_{cut}

Fig. 6b