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Measurement of Hadronic Spectral Moments in τ Decays and a Determination of α_s

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Abstract

We present measurements of spectral moments extracted from the invariant mass distributions of the final states of hadronic τ decays recorded in the CLEO detector. From the fit of theoretical predictions to the measurements of spectral moments and the total hadronic decay width of τ, we determine the strong coupling constant and a set of non-perturbative QCD parameters. The strong coupling constant is measured to be $\alpha_s(M_\tau)$=0.309$\pm$0.024 which, extrapolated to the Z^0 mass, yields $\alpha_s(M_{Z^0}) = 0.114\pm0.003$; where the error includes contributions from experimental and theoretical uncertainties.

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I. INTRODUCTION

The τ, which is the only known lepton heavy enough to decay into hadrons, presents a clean environment to study the hadronic weak currents. The total τ hadronic width has been calculated [1,2] using analyticity and the Operator Product Expansion (OPE). This is one of very few processes calculated to $\mathcal{O}(\alpha_s^3)$ and the non-perturbative corrections have been shown to be largely suppressed. Thus this process has been advertized as the best method to determine the most precise value of α_s.

Of broader interest are the invariant mass distributions of the final state in the hadronic decay channels of the τ, which have not yet been calculated within the existing framework of QCD. However, the so called spectral moments, built from the weighted integrals of these distributions, have been calculated [2,3]. As opposed to the total hadronic width of the τ, the non-perturbative effects in spectral moments may not necessarily be suppressed and this may worsen the precision of the theoretical predictions. Indeed, choosing the appropriate weight functions, one can make the non-perturbative effects dominate the final results. However, this feature of spectral moments only make them more interesting and it can be used to measure the parameters characterizing the non-perturbative dynamics and hence improve our understanding of QCD at long distances [3].

It is a well known fact that in the determination of α_s by other methods, theoretical uncertainties are dominant over experimental uncertainties, mainly due to the missing higher order corrections in the theory. The ALEPH collaboration [4] has used the above procedure to determine the value of α_s from τ hadronic decays and has demonstrated that the experimental uncertainties are dominant over the theoretical uncertainties in this procedure. Thus, it becomes of great interest to use the large data sample and the excellent π^0 reconstruction capabilities of CLEO in order to improve the experimental precision on this measurement.

In this note, we report our preliminary results on the determination of $\alpha_s(M_\tau)$ from a) the total hadronic decay width alone, b) the spectral moments alone, and c) a global fit of hadronic width and spectral moments. In the global fit, we also determine the values for the non-perturbative QCD parameters. The final results are obtained from the global fit. The other fits mentioned above provide a consistency check of the procedure. The more detailed experimental tests of the theory suggested in ref. [3] are underway.

II. EVENT SELECTION AND CUTS

The data used in this analysis were collected with the CLEO-II detector, described elsewhere [5], at CESR at about 10 GeV center of mass energy. We tagged the τ events with the decay channel $\tau^- \to \mu^- \bar{\nu_\mu} \nu_\tau$ and studied the hadronic decay of the other τ (the inclusion of charge conjugate modes is implied throughout the paper). The other τ was required to have either one (ie 1-1 topology for $\tau \to \pi(n\pi^0)\nu_\tau$) or three (ie 1-3 topology in case for $\tau \to 3\pi n\pi^0\nu_\tau$) charged tracks.

The opening angle between the two charged tracks in 1-1 topology and the minimum angle between the lepton and any of the other three charged tracks in 1-3 topology were required to exceed 90^0. Only events with zero total charge and 2 (for 1-1 topology) or 4 (for 1-3 topology) "good" charged tracks were considered. A good charged track was defined as below: 1)Impact parameter does not exceed the value of 0.01 m; 2)Momentum should be

less than 85% of the beam momentum; and 3)The angle made with the beam direction must be greater than 37^0.

The total visible energy of the event was required to exceed 0.15 E_{cm} to suppress two-photon background. To reject the QED background, it was required that the net transverse momentum of the tracks be greater than 200 MeV and the ratio of the vector to scalar sum of charged tracks momenta be greater than 0.05. Also events with energy deposited in the endcap greater than 200 MeV were rejected.

Electromagnetic clusters which had energy $\geq$ 60 MeV, not within 20^0 of the lepton, not associated with charged tracks and contained in the barrel calorimeter were used to reconstruct π^0's. In the reconstruction of π^0, it was required that the angle between two clusters should not exceed 135^0 and that at least one cluster have a minimum energy of 80 MeV. The reconstructed π^0 was required to lie within 90^0 of the π direction (for the 1-1 topology) and the average direction of 3 pions (for the 1-3 topology). For each $n\pi^0$ case, all possible combinations of clusters forming π^0 were considered and the one with minimum overall $\chi^2 = \frac{1}{n}\sum_{i=1}^{n}(m_i - m_\pi)^2/\sigma_{m_i}^2$ was selected, where m_i is the invariant mass of the two clusters forming the i'th π^0 candidate, and σ_{m_i} is the uncertainty on m_i calculated from the measured angular and energy resolution of the calorimeter.

III. EXPERIMENTAL AND THEORETICAL DEFINITIONS OF SPECTRAL MOMENTS

Analogous to the hadronic to leptonic decay width ratio in e^+e^- collisions, one may define the ratio of total hadronic decay width of τ as:

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau hadrons)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = \frac{1 - B_e - B_\mu}{B_l} = \frac{1}{B_l} - f_e - f_\mu \tag{1}$$

where B_e and B_μ are the branching fractions of τ decays to e and μ respectively, and f_e=1 and f_μ=0.9726 are the phase space factors. Theoretically, R_τ may be written as

$$R_\tau = \frac{1}{\Gamma_l}\int_0^{s_0} ds \frac{d\Gamma_h}{ds} \tag{2}$$

where s is the invariant mass-squared of the hadronic final state and $s_0 = m_\tau^2$. Within the existing theoretical skills, it is not possible to calculate $d\Gamma_h/ds$ directly. However, it is possible to calculate the weighted integrals of $d\Gamma_h/ds$, R_τ^{kl}, written as

$$R_\tau^{kl} = \frac{1}{\Gamma_l}\int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^l \frac{d\Gamma_h}{ds}. \tag{3}$$

One should note that R_τ is the special case of R_τ^{kl} when k=0 and l=0. The normalized spectral moments, D_τ^{kl}, are defined as:

$$D_\tau^{kl} = \frac{R_\tau^{kl}}{R_\tau^{00}} \tag{4}$$

The R_τ^{kl} have been predicted using OPE, and can be written in the form

$$R_\tau^{kl}(u,d) = |V_{ud}|^2 \, S_{EW} r_{kl} \left[1 + \Delta_{m_q} + \Delta_p + \Delta_{np}\right] \quad (5)$$

where V_{ud}=0.9753 $\pm$ 0.0004 [6] is a CKM matrix element, $S_{EW} = 1.0194$ an electroweak correction [7], r_{kl} the terms predicted at parton level, Δ_{m_q} the quark mass corrections, Δ_p the perturbative terms in powers of α_s, and Δ_{np} are the non-perturbative corrections. A more detailed discussion of this expression is provided in the appendix.

The measured values of R_τ and the spectral moments, D_τ^{kl}, are used to perform a global fit in order to determine the value of $\alpha_s(M_\tau)$ and three non-perturbative parameters labelled as $< \frac{\alpha_s}{\pi} GG >$, $\mathcal{O}(6)$ and $\mathcal{O}(8)$ following the notation of ref. [3] (see the appendix).

IV. MEASUREMENT OF R_τ AND SPECTRAL MOMENTS

The value of R_τ is determined using equation 1 where B_l is determined by the following three methods: 1) $B_l = B_e/f_e = 0.1801 \pm 0.0018$, 2) $B_l = (\frac{\tau_\tau}{\tau_\mu})(\frac{m_\tau}{m_\mu})^5 = 0.1817 \pm 0.0019$, 3) $B_l = \frac{B_\mu}{f_\mu} = 0.1816 \pm 0.0024$; where the values for leptonic branching ratios, τ lifetime, and τ mass have all been taken from Ref. [6]. These three measurements yield the weighted average of $B_l = 0.1810 \pm 0.0012$ which yields $R_\tau = 3.552 \pm 0.035$.

The invariant mass distributions of the hadronic final states were computed from the selected events for the following decay channels:

1. $\tau^- \to \pi^-\pi^0\nu_\tau$
2. $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$
3. $\tau^- \to \pi^-\pi^0\pi^0\pi^0\nu_\tau$
4. $\tau^- \to \pi^-\pi^0\pi^0\pi^0\pi^0\nu_\tau$
5. $\tau^- \to \pi^+\pi^-\pi^-\nu_\tau$
6. $\tau^- \to \pi^+\pi^-\pi^-\pi^0\nu_\tau$
7. $\tau^- \to \pi^+\pi^-\pi^-\pi^0\pi^0\nu_\tau$

The branching ratios of these modes are taken from ref. [6] and listed in Table 1. The decay channel $\tau^- \to 5\pi^\pm$, with a branching ratio of about 0.06 % [6], is considered negligible and is ignored.

Mode	Branching ratio (%)
$\pi\nu_\tau$	11.7$\pm$0.4
$\pi\pi^0\nu_\tau$	25.2$\pm$0.4
$\pi 2\pi^0\nu_\tau$	9.52$\pm$0.40
$\pi 3\pi^0\nu_\tau$	1.28 $\pm$ 0.24
$\pi 4\pi^0\nu_\tau$	0.20 $\pm$ 0.11
$3\pi\nu_\tau$	8.20$\pm$0.30
$3\pi\pi^0\nu_\tau$	5.14$\pm$0.30
$3\pi 2\pi^0\nu_\tau$	0.49 $\pm$ 0.05

Table 1: Branching ratios of hadronic tau decays, taken from ref. [6], used to construct the spectral moments from the invariant mass distributions.

The number of events which passed our selection criteria are listed in Table 2 for each channel.

Mode	Events	Background(%)		
		f_τ	f_{π^0}	f_h
$\pi\pi^0\nu_\tau$	38388	4.71±0.10	2.07±0.07	0.0
$\pi 2\pi^0\nu_\tau$	7892	4.99±0.24	2.59±0.17	0.0
$\pi 3\pi^0\nu_\tau$	623	8.49±1.00	7.31±0.93	0.0
$\pi 4\pi^0\nu_\tau$	32	13.51 ± 6.04	5.40±3.82	0.0
$3\pi\nu_\tau$	22810	11.20±0.22	—	0.0
$3\pi\pi^0\nu_\tau$	4906	4.52±0.26	6.95±0.32	0.0
$3\pi 2\pi^0\nu_\tau$	285	20±4	7.5±1.7	0.0

Table 2: Number of selected events and the background fractions for various decay channels. For details see text.

From these distributions, the background from the following sources was subtracted: 1) Cross channel contamination from other τ decay channels including Cabbibo suppressed channels, f_τ; 2) Fake π^0's constructed from unrelated photons, f_{π^0}; and 3) Contamination from hadronic events, f_h. To estimate the hadronic background, 582,500 multihadron events, generated by JETSET MC [8] and smeared through the detector effects, were put through our analysis chain. In any decay channel, not more than two out of 582,500 events could sneak through our analysis procedure. Background from other τ decay channels including the Cabbibo suppressed channels, f_τ, was determined from KORALB MC [9]. These background fractions are also listed in Table 2.

After subtracting the background, the distributions were corrected for detector effects as described below:

1. An MC sample using KORALB was generated and processed through the detector simulation program subject to to the same reconstruction procedure and event selection criteria as the experimental data. The invariant mass distributions computed from this sample of MC are compared to those measured from the data before corrections, in Figures 1 and 2. The distributions N_G^{MC} and N_O^{MC} were measured before and after processing respectively. The correction matrix was computed by defining its matrix elements as

$$M(n_g, n_o) = \frac{No.\ of\ events\ generated\ in\ mass\ bin\ n_g\ and\ observed\ in\ mass\ bin\ n_o}{No.\ of\ events\ observed\ in\ mass\ bin\ \ n_o}$$

2. To correct the distributions for selection efficiency, bin by bin correction factors, $C(n_g)$, were computed from N_G^{MC} described above and N_G^{MC-B} before the event selection and cuts as below:

$$C(n_g) = \frac{N_G^{MC-B}(n_g)/N_G^{MC-B}(total)}{N_G^{MC}(n_g)/N_G^{MC}(total)} \tag{6}$$

The corrected distributions, $N_{cor}^{exp}(n)$, were obtained from the observed distributions, $N_{obs}^{exp}(n_o)$, by using the following formula:

$$N_{cor}^{exp}(n) = C(n) \sum_{n_o} M(n, n_o) N_{obs}^{exp}(n_o) \tag{7}$$

The corrected distributions, normalized to unit area, weighted by the branching ratios, listed in Table 1, are combined into a single distribution presented in Fig. 1. The π^- mass from $\pi^-\nu_\tau$ decay channel enters this distribution as a delta function (not shown in the figure). It is this distribution from which the spectral moments are computed using equation 3. The differential spectral moments are presented in Fig. 2. Their integral values are listed in Table 3 where the experimental errors are obtained by adding in quadrature the statistical errors and the systematic errors. The last column in Table 3 contains the values of the spectral moments computed from the raw data before making any background subtractions or detector corrections. Moments with $k > 1$ are not used to to avoid the low mass region where the validity of the method may be questionable.

Moment	Value	$\sigma(stat)$	$\sigma(sys)$	$\sigma(exp)$	$\sigma(\%)$	Value (raw)
$D_\tau^{1,0}$	0.7287	0.0021	0.0052	0.0056	0.77	0.7265±0.0021
$D_\tau^{1,1}$	0.1553	0.0007	0.0007	0.0010	0.64	0.1542±0.0007
$D_\tau^{1,2}$	0.0559	0.0003	0.0007	0.0008	1.43	0.0554±0.0003
$D_\tau^{1,3}$	0.0250	0.0002	0.0005	0.0005	2.00	0.0246±0.0002

Table 3: The measured spectral moments from the corrected mass distributions.

V. DETERMINATION OF α_s

The fits of the theoretical predictions to the experimental measurements were made for various combinations of the spectral moments and the results are listed in Table 4.

Observable	$\alpha_s(M_\tau)$	$< \frac{\alpha_s}{\pi} GG >$	O(6)	O(8)	χ^2
$R_\tau, D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.309±0.024	0.024±0.010	-0.0036±0.0010	0.0035±0.0008	2.1/1
R_τ	0.327±0.025	0.025	0.0022	0	—
$D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.327±0.037	0.025	0.0022	0	10.81/3
$D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.377±0.046	0.024	-0.0036	0.0035	0.31/3
$D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.349±0.030	0.024	-0.0036	0.0035	1.98/3

Table 4: Values of $\alpha_s(M_\tau)$ determined from the fits of the theory to the measured spectral moments. In case of non-perturbative parameters, no error means the parameter was kept fixed in the fit.

The simultaneous global fit to the measured values of R_τ and $D_\tau^{1,l}$ is a four-parameter fit where the values of α_s and non-perturbative parameters $< \frac{\alpha_s}{\pi} GG >$, $\mathcal{O}(6)$ and $\mathcal{O}(8)$ are determined together.This yields

$$\alpha_s(M_\tau) = 0.309 \pm 0.018 \pm 0.016, \tag{8}$$

where the first error is experimental and the second theoretical. These results are listed in the first row of Table 4. The other rows of Table 4 contain the values of $\alpha_s(M_\tau)$ determined when the non-perturbative parameters (NPP) are kept fixed at various sets of values. Row 2 is obtained when only the measured value of R_τ is compared with the theory for fixed values of NPP taken from ref. [2]. Row 3 uses the measured values of $D_\tau^{1,l}$ from Table 3 and the NPP values from ref. [2]. Row 4 uses the same set of values for $D_\tau^{1,l}$ but the NPP values from the global fit in row 1. The last row is the same fit but with modified theoretical errors on moments according to the errors on NPP determined from the global fit. The results for $\alpha_s(M_\tau)$ from R_τ alone and $D_\tau^{1,l}$ alone are consistent with each other. This checks the consistency of the method.

In the global fit, as the NPP are determined from the fit, the uncertainties on their values do not contribute to the theoretical errors on the spectral moments. In the fits of $D_\tau^{1,l}$ alone, the uncertainties on the values of NPP become the dominant source of theoretical errors (Table 7 in the Appendix). This results in the larger errors on the values of $\alpha_s(M_\tau)$ listed in rows 3 and 4. For row 5, we recompute the theoretical erors on the moments going into the fit according to the errors on the NPP parameters determined from the global fit.

The correlation coefficients between $\alpha_s(M_\tau)$ and NPP obtained from the global fit are presented in Table 5.

Observable	$\alpha_s(M_\tau)$	$< \frac{\alpha_s}{\pi} GG >$	O(6)	O(8)
$\alpha_s(M_\tau)$	1	-0.56	0.69	-0.66
$< \frac{\alpha_s}{\pi} GG >$	-0.56	1	-0.82	0.82
O(6)	0.69	0.82	1	-0.97
O(8)	-0.66	0.82	-0.97	1

Table 5: Correlation coefficients for $\alpha_s(M_\tau)$, $< \frac{\alpha_s}{\pi} GG >$), O(6) and O(8) obtained from the global fit.

VI. SYSTEMATIC ERRORS

In this analysis, systematic errors may arise from the following sources:

1. Uncertainty in the mass scale,
2. Uncertainty on the background evaluation,
3. Uncertainty on π^0 reconstruction,
4. The limited MC statistics to correct the data, and
5. Uncertainty in the unfolding procedure.

The error due to the mass scale uncertainty of 0.25 % was estimated by multiplying the mass-squared scale (X-axis) of Fig 1. by a factor of ± 1.005 and taking the resulting shifts in the moments as the errors due to the mass scale uncertainty. The errors due to the uncertainties in the branching ratios were estimated in a rather conservative way: Spectral moments were calculated by using all the branching ratios simultaneously 1)increased by their error, and 2) decreased by their errors. The larger resulting shift in the central value of a given moment is quoted as a symmetric error on the moment due to the uncertainties in the branching fractions. The errors due to the limited MC statistics used to correct the data are also included in the systematic errors. For example, the systematic error of $\pm$ 0.0052 on $D_\tau^{1,0}$ contains contributions of ± 0.0009 from mass scale uncertainty, ± 0.0025 from MC statistics, and $\pm$ 0.0044 from branching ratio uncertainties. The errors due to other sources listed above are largely included in the errors on the branching fractions, to the extent that these sources do not affect the mass shape. However, we made the following qualitative checks on the effect of those uncertainties on our final results:

1. The analysis was repeated without making any corrections to or background subtractions from the data. The final results agree within the statistical errors .
2. We repeated our analysis with no kaon subtraction from the data and the final result on $\alpha_s(M_\tau)$ shifted by 0.4 %, ie 7% of the overall experimental error.
3. The cut on the χ^2 per degree of freedom of π^0 reconstruction was varied between 4 and 14 for the $\pi 2\pi^0$ channel. The variations in the moments were less than 0.04 % and there was a negligible effect on $\alpha_s(M_\tau)$.
4. The cut on the χ^2 of π^0 reconstruction in the $\pi\pi^0$ channel was varied between 9 and 16. The variations in the moments were less than 0.06 % and there was a negligible effect on $\alpha_s(M_\tau)$.
5. The analysis was repeated by replacing the branching ratios of the $3\pi^-$ and $3\pi^-\pi^0$ channels listed in Table 1 with 9.34% and 4.25% respectively, consistent with the current CLEO measurements presented in this conference. The resulting change in the value of $\alpha_s(M_\tau)$ was 0.001, ie about 5 % of the overall experimental error.

More accurate and detailed studies of the systematics are in progress.

VII. SUMMARY AND DISCUSSION

In summary, we have determined $\alpha_s(M_\tau)$ from hadronic decays of τ recorded by the CLEO II detector. Comparing the calculations, performed within OPE framework, to the data, we have determined $\alpha_s(M_\tau)$=0.309$\pm$0.018$\pm$ 0.016, where the first error is experimental and the second theoretical. Our result agrees with the ALEPH measurement [4] of $\alpha_s(M_\tau) =$ 0.330$\pm$ 0.046.

The agreement between the values of α_s obtained from R_τ alone and from spectral moments $D_\tau^{1,l}$ points towards the applicability of perturbative QCD calculations in this low energy regime [3]. The values of the non-perturbative parameters obtained from the fits of the theory to the data agree well with those previously obtained from the fits of largely non τ data [2].

Using the formulae of ref. [11], the value of $\alpha_s(M_\tau)$=0.309$\pm$0.018$\pm$0.016, extrapolated to 10.53 GeV, yields:

$$\alpha_s(10.53) = 0.167 \pm 0.005 \pm 0.004 \tag{9}$$

which is in good agreement with $\alpha_s(10.53) = 0.164 \pm 0.015$ determined from jet production rates at CLEO [12]. Extrapolated to M_Z, our result yields:

$$\alpha_s(M_{Z^0}) = 0.114 \pm 0.002 \pm 0.002 \tag{10}$$

Adding an error of $\pm$0.001 from the uncertainties in the extrapolation procedure arising from the crossings of c and b thresholds, this result becomes $\alpha_s(M_{Z^0}) = 0.114 \pm 0.003$, which is consistent within errors with $\alpha_s(M_{Z^0})$=0.120$\pm$0.006 determined from LEP data [13] on jet rates and event shapes.

VIII. ACKNOWLEDGEMENTS

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IX. LIST OF FIGURES

Fig. 1: Fully corrected measured mass distributions for $\pi n\pi^0$. The delta function of π^- mass is not shown in the figure.

Fig. 2: Measured differential spectral moments.

Appendix: Theoretical Predictions and Uncertainties

In this section, we present some theoretical formulae, taken from ref. [3], and a discussion of theoretical uncertainties used in this analysis. The R_τ^{kl} have been predicted using OPE in which it can be written as

$$R_\tau^{kl}(u,d) = |V_{ud}|^2 \, S_{EW} r_{kl} \left[1 + \Delta_{m_q} + P^{kl}(s_0) + \sum_{D=2,4,\ldots} \Delta^{kl}(s_0, D) \right]; \tag{11}$$

where V_{ud}=0.9753 $\pm$ 0.0004 [6] is a CKM matrix element, $S_{EW} = 1.0194$ an electroweak correction [7], r_{kl} the terms predicted at parton level, and Δ_{m_q} are the quark mass corrections. The zero'th dimension (ie D=0) term, $P^{kl}(s_0)$, is the perturbative contribution and is known to third power in α_s. The higher dimension terms, $\Delta^{kl}(s_0, D)$, are the non-perturbative contributions which may be expressed as:

$$\Delta^{kl}(s_0, D) = \frac{c_D(k,l)}{m_\tau^D} \mathcal{O}(D). \tag{12}$$

The terms $\frac{c_D(k,l)}{m_\tau^D}$, computed from Ref [3], are listed in Table 6. The terms $\mathcal{O}$(D) are known as condensates and can, in principle, be obtained from the matrix element in the vacuum of QCD operators [10]. The term $\mathcal{O}(2)$ is considered absent because one cannot build an operator in this dimension from the QCD Lagrangian. The term $\mathcal{O}(4)$ is expressed in terms of quark masses and gluon condensate, $< \frac{\alpha_s}{\pi} GG >$.

D	$\frac{c_D(k,l)}{m_\tau^D}$				
	R_τ	R_τ^{10}	R_τ^{11}	R_τ^{12}	R_τ^{13}
4	0	5.65	-23.67	0	0
6	-3.75	-5.36	-7.49	20.16	0
8	-0.79	-2.83	7.11	6.38	-13.64

Table 6: Numerical values of $\frac{c_D(k,l)}{m_\tau^D}$ computed from Reference [3].

The theoretical predictions were computed using a FORTRAN program supplied by one of the authors of Ref. [3], F. Le Diberder. The spectral moments are highly correlated quantities. The correlations were taken into account and the χ^2 of the fit was defined as:

$$\chi^2 = \sum_{ij} \left[D_\tau^i(exp.) - D_\tau^i(th.) \right] V_{ij}^{-1} \left[D_\tau^j(exp.) - D_\tau^j(th.) \right]; \tag{13}$$

where D_τ^i are R_τ and D_τ^{kl}; and V_{ij}^{-1} are the elements of the inverse of the covariance matrix. The following sources of theoretical uncertainties were considered:

1. The uncertainties on the constant quantities, non-perturbative electroweak and mass corrections, used in the equation 11.
2. The uncertainties due to the missing higher order corrections in the perturbative part of the moments. This uncertainty is evaluated by taking the unknown fourth order coefficients in terms of known second and third order coefficients, namely $K_4 = 2\,|K_3(K_3/K_2)|$ and $\beta_4 = 2\,|\beta_3(\beta_3/\beta_2)|$, and computing the variation in the predicted values of the moments.

3. The contribution of renormalization uncertainty to the moments was evaluated by varying the renormalization scale μ from 1. to 2.5 GeV and also by varying the scheme dependent coefficient β_3 from 0. to twice its value in $\overline{MS}$ scheme used in these calculations.

The theoretical errors on the moments and the correlation coefficients thus computed using Ref. [2] are listed in Table 7 and Table 8. These error estimates are very similar to those in ref. [4] estimated by the similar method.

Error type	Error				
	R_τ	D_τ^{10}	D_τ^{11}	D_τ^{12}	D_τ^{13}
Σ_{th}	0.043	0.0133	0.0134	0.0032	0.0019
σ_{th}	0.033	0.0035	0.0028	0.0004	0.0002

Table 7: The theoretical errors on spectral moments computed from Reference [3].

	R_τ	D_τ^{10}	D_τ^{11}	D_τ^{12}	D_τ^{13}
R_τ	1	0.12	-0.22	-0.04	-0.05
D_τ^{10}	0.12	1	-0.76	-0.91	-0.93
D_τ^{11}	-0.22	-0.76	1	0.59	0.55
D_τ^{12}	-0.04	-0.91	0.59	1	0.97
D_τ^{13}	-0.05	-0.93	0.55	0.97	1

Table 8: The matrix of correlation coefficients of spectral moments.

The eros on the non-perturbative constants were conservatively taken twice the values quoted in the ref. [2]. The errors, Σ_{th}, in the first row of Table 7 contain the contribution from uncertainties in non-perturbative parameters and are used when the fixed values of non-perturbative parameters are used in the fits. The errors, σ_{th}, in the second row of Table 7 do not contain the contribution from uncertainties in non-perturbative parameters and are used in the global fit when the non-perturbative parameters are kept free.

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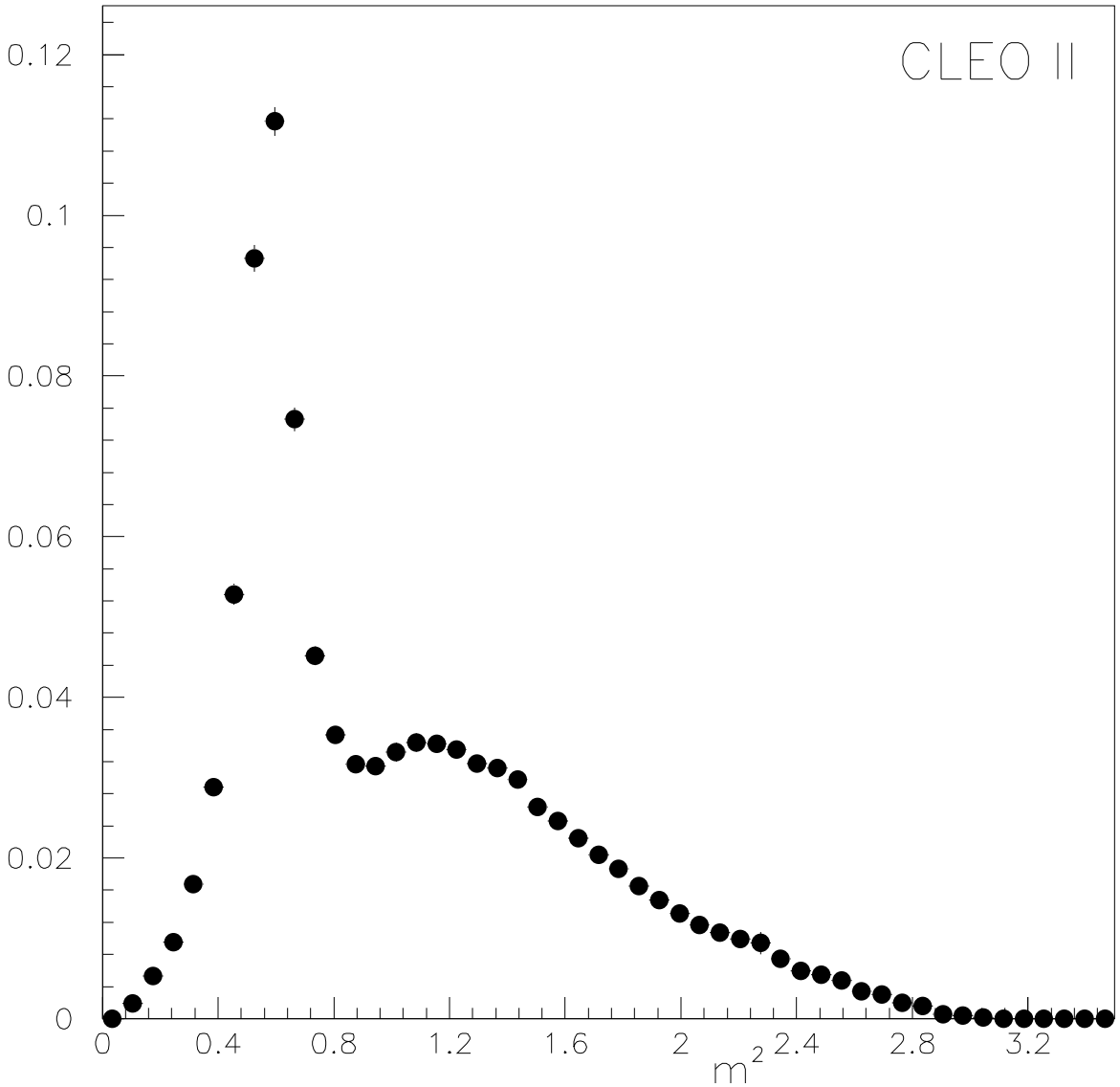

Fig. 1 Combined distribution of invariant mass squared.

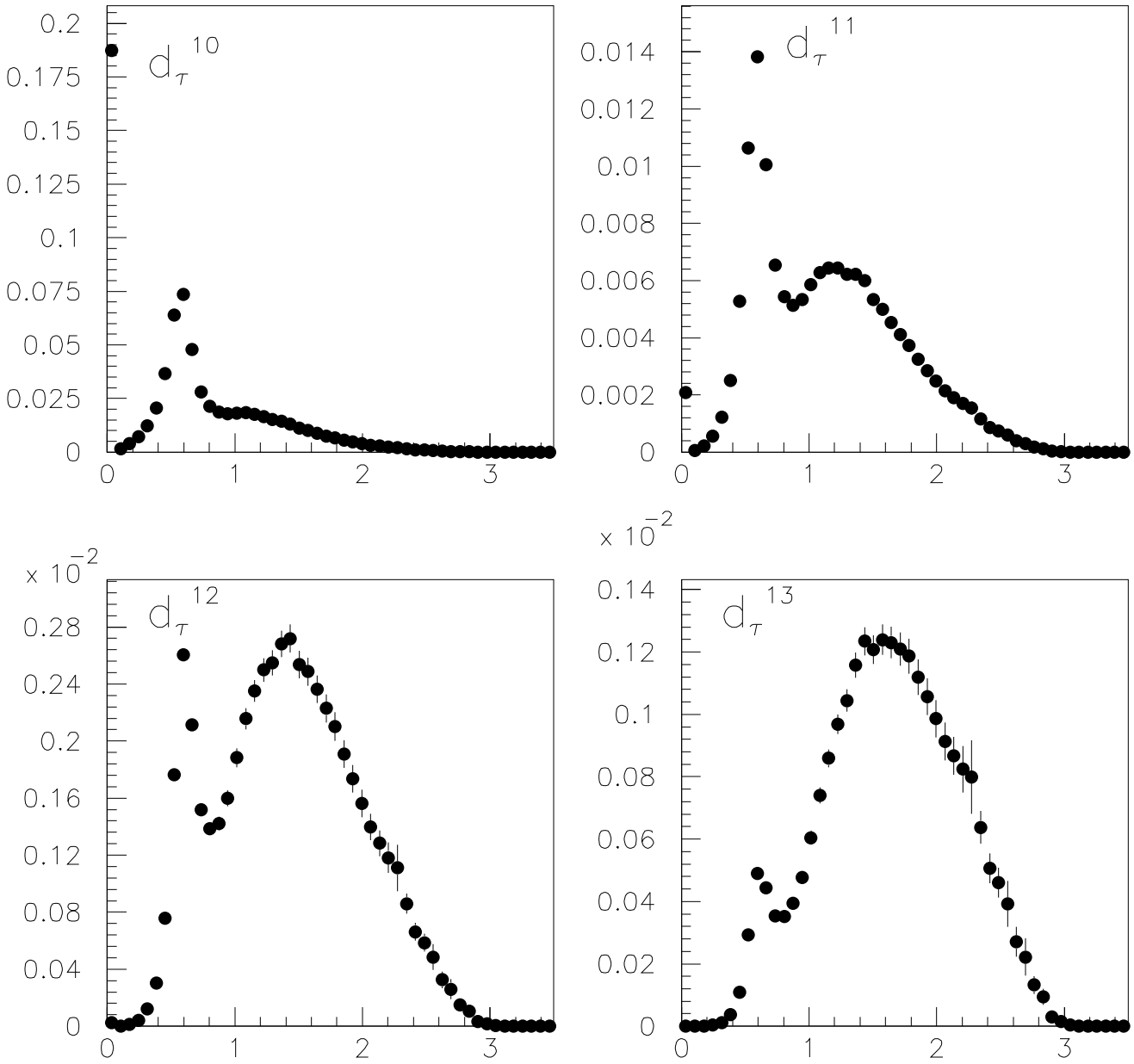

Fig. 2 Measured differential spectral moments.