TECHNION-PH-2004-21
CLNS-04-1879
hep-ph/yymmnnn
May 2004

IMPLICATIONS OF CP ASYMMETRIES IN $B \to \pi^+\pi^-$ [1]

Michael Gronau
Physics Department, Technion – Israel Institute of Technology
32000 Haifa, Israel

Jonathan L. Rosner[2]
Laboratory for Elementary-Particle Physics, Cornell University
Ithaca, New York 14850

CP asymmetries in $B^0(t) \to \pi^+\pi^-$ are studied by relating this process in broken flavor SU(3) with $B^+ \to K^0\pi^+$ and $B^0 \to K^+\pi^-$. Using two different scenarios for SU(3) breaking, we show that the range of values of the weak phase α permitted by the measured asymmetries overlaps with that obtained from other CKM constraints, supporting the KM origin of the asymmetries. We evaluate the potential precision of this method to improve the determination of α.

PACS codes: 12.15.Hh, 12.15.Ji, 13.25.Hw, 14.40.Nd

Measurements of CP-violating asymmetries in the decays $B^0(t) \to \pi^+\pi^-$ and their charge conjugates have reached a very interesting stage. The BaBar [1] and Belle [2] collaborations measured two asymmetries, $C_{\pi\pi} \equiv -A_{\pi\pi}$ and $S_{\pi\pi}$, defined by [3]

$$\frac{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)}{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)} = -C_{\pi\pi}\cos(\Delta m t) + S_{\pi\pi}\sin(\Delta m t) \ , \quad (1)$$

obtaining values

$$C_{\pi\pi} = \begin{cases} -0.19 \pm 0.19 \pm 0.05 \ , \\ -0.58 \pm 0.15 \pm 0.07 \ , \end{cases} S_{\pi\pi} = \begin{cases} -0.40 \pm 0.22 \pm 0.03 \ , & \text{BaBar} \ , \\ -1.00 \pm 0.21 \pm 0.07 \ , & \text{Belle} \ . \end{cases} \quad (2)$$

The Belle measurement rules out the case of CP-conservation, $C_{\pi\pi} = S_{\pi\pi} = 0$, at a level of 5.2 standard deviations. The current average values of the two asymmetries are [4]

$$C_{\pi\pi} = -0.46 \pm 0.13 \ , \quad S_{\pi\pi} = -0.74 \pm 0.16 \ . \quad (3)$$

An immediate question, motivated by a search for physics beyond the Standard Model, is whether these values are consistent with other constraints on Cabibbo-Kobayashi-Maskawa (CKM) parameters. If confirmed, the next question is whether

[1]To be submitted to Physics Letters B.

[2]On leave from Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637.

reducing experimental errors in the asymmetries may improve these constraints, thereby tightening the current range of the weak phase $\alpha \equiv \phi_2$ [5], $75^\circ < \alpha < 120^\circ$.

An extraction of α from the CP asymmetry in $B^0 \to \pi^+\pi^-$ is obstructed by the effect of a penguin amplitude [3, 6]. The theoretically cleanest way of obtaining α from these measurements is based on isospin symmetry [7]. It includes electroweak penguin effects [8], and requires in addition to the measured rate of $B^+ \to \pi^+\pi^0$ separate decay rate measurements of B^0 and $\overline{B}^0$ to $\pi^0\pi^0$. Isospin breaking effects are expected to introduce an uncertainty of only a few degrees in the determination of the weak phase. Prior to a $B^0/\overline{B}^0$ separation, the measured combined decay rate of B^0 and $\overline{B}^0$ into $\pi^0\pi^0$ provides a measure for the uncertainty in α [9]. Current branching ratios imply an uncertainty of about 50° for arbitrary asymmetry measurements [1, 10]. A higher precision may be achieved for special values of the asymmetries [11, 12]. In order to obtain more precise knowledge of α before $B^0 \to \pi^0\pi^0$ and $\overline{B}^0 \to \pi^0\pi^0$ are separately measured, further assumptions beyond isospin symmetry are required.

A powerful approach to B decays into a pair of charmless pseudoscalar mesons is based on the broader but the less precise flavor SU(3) symmetry [13, 14]. Introducing SU(3) breaking effects in a controllable and testable manner [15] improves the precision of this approach. A variety of studies along this line, focusing on $B \to \pi\pi$ and $B \to K\pi$ decays, were performed in the past ten years [16]. A crucial factor in determining α in $B^0 \to \pi^+\pi^-$ is a knowledge of the ratio of penguin and tree amplitudes contributing to this process [17].

In the present Letter we update and modify an analysis [18] of $B^0(t) \to \pi^+\pi^-$, which combines this process with $B^+ \to K^0\pi^+$. In [18] we assumed that both tree and penguin amplitudes factorize. Instead, we will now leave open the question of factorization of penguin amplitudes, comparing results obtained under different assumptions about SU(3) breaking in these amplitudes. Using current data unavailable at the time of the analysis in [18], we will argue for a ratio of penguin-to-tree amplitudes P/T larger than commonly accepted. This study will also be combined with a complementary analysis relating $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$, where similar bounds on P/T are obtained. Earlier but somewhat different studies relating these two last processes were performed in [19, 20, 21]. Finally, we use information on P/T from $K\pi$ and $\pi\pi$ rates to study the CP asymmetries in $B \to \pi^+\pi^-$ as functions of α. We will show that the current asymmetries are consistent with the allowed range of α, and will discuss the possibility of tightening this range.

We use the "c-convention" [18], in which the top-quark has been integrated out in the $b \to d$ penguin transition and unitarity of the CKM matrix has been used. Absorbing a P_{tu} term in T, one writes

$$A(B^0 \to \pi^+\pi^-) = Te^{i\gamma} + Pe^{i\delta} \ . \tag{4}$$

By convention T and P, which involve magnitudes of CKM factors, $|V^*_{ub}V_{ud}|$ and $|V^*_{cb}V_{cd}|$, are positive and the strong phase δ lies in the range $-\pi \leq \delta \leq \pi$. The amplitude for $\overline{B}^0 \to \pi^+\pi^-$ is obtained by changing the sign of γ. The asymmetries $C_{\pi\pi}$ and $S_{\pi\pi}$ are given by [3]

$$C_{\pi\pi} \equiv \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2} \ , \qquad S_{\pi\pi} \equiv \frac{2\mathrm{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2} \ , \tag{5}$$

where

$$\lambda_{\pi\pi} \equiv e^{-2i\beta} \frac{A(\overline{B}^0 \to \pi^+\pi^-)}{A(B^0 \to \pi^+\pi^-)} \quad . \tag{6}$$

Substituting (4) into these definitions, one obtains [17],

$$C_{\pi\pi} = \frac{2r\sin\delta\sin(\beta+\alpha)}{R_{\pi\pi}} , \tag{7}$$

$$S_{\pi\pi} = \frac{\sin 2\alpha + 2r\cos\delta\sin(\beta-\alpha) - r^2\sin 2\beta}{R_{\pi\pi}} , \tag{8}$$

$$R_{\pi\pi} = 1 - 2r\cos\delta\cos(\beta+\alpha) + r^2 , \tag{9}$$

where

$$r \equiv \frac{P}{T} , \tag{10}$$

is a ratio of penguin to tree amplitudes.

In the absence of a penguin amplitude ($r = 0$) one has $C_{\pi\pi} = 0$, $S_{\pi\pi} = \sin 2\alpha$. For small values of r, keeping only linear terms in this ratio, one finds

$$C_{\pi\pi} = 2r\sin\delta\sin(\beta+\alpha) + \mathcal{O}(r^2) , \tag{11}$$

$$S_{\pi\pi} = \sin 2\alpha + 2r\cos\delta\sin(\beta+\alpha)\cos 2\alpha + \mathcal{O}(r^2) . \tag{12}$$

That is, in the linear approximation the allowed region in the $(S_{\pi\pi}, C_{\pi\pi})$ plane is confined to an ellipse centered at $(\sin 2\alpha, 0)$, with semi-principal axes $2[r\sin(\beta+\alpha)\cos 2\alpha]_{\rm max}$ and $2[r\sin(\beta+\alpha)]_{\rm max}$. In our study below we will use the exact expressions (7)–(9).

Given a value of β, as already measured in $B^0(t) \to J/\psi K_S$ [22], the two measurables $C_{\pi\pi}$ and $S_{\pi\pi}$ provide two equations for the weak phase α and for the two hadronic parameters r and δ. At least one additional constraint on r and δ is needed in order to determine α. Such constraints are given (by isospin and) by flavor SU(3) symmetry considerations as described below.

Turning now to $B \to K\pi$ decays, one describes corresponding decay amplitudes in terms of primed quantities, T' and P' [14]. We introduce an SU(3) breaking factor f_K/f_π in tree amplitudes which are expected to factorize [23, 24], but assume in the first place exact SU(3) for penguin amplitudes for which factorization is not expected to hold [25],

$$T' = \frac{f_K}{f_\pi}\frac{V^*_{ub}V_{us}}{V^*_{ub}V_{ud}}\,T = \frac{f_K}{f_\pi}\bar{\lambda}\,T , \qquad P' = \frac{V^*_{cb}V_{cs}}{V^*_{cb}V_{cd}}\,P = -\bar{\lambda}^{-1}P . \tag{13}$$

Here

$$\bar{\lambda} \equiv \frac{\lambda}{1-\lambda^2/2} = 0.230 . \tag{14}$$

The assumption of SU(3) symmetry in penguin amplitudes can be tested [26] by comparing the measured rate of $B^+ \to K^0\pi^+$ with future measurements of $B^+ \to K^+\overline{K}^0$. Another test of this assumption and the effect of possible SU(3) breaking in P will be discussed below. SU(3) amplitudes represented by exchange and annihilation contributions occur in $B^0 \to \pi^+\pi^-$ and $B^+ \to K^0\pi^+$ respectively [14]. They are $1/m_b$ suppressed relative to tree and penguin amplitudes [24] and will be neglected. These approximations and the neglect of very small color-suppressed electroweak penguin contributions are testable in $B^0 \to K^+K^-$ and in other processes [27].

Under these assumptions one may write expressions for $B \to K\pi$ amplitudes in terms of amplitudes contributing to $B^0 \to \pi^+\pi^-$:

$$
\begin{aligned}
A(B^+ \to K^0\pi^+) &= -\bar{\lambda}^{-1} P e^{i\delta} \ , && (15)\\
A(B^0 \to K^+\pi^-) &= -\frac{f_K}{f_\pi}\bar{\lambda} T e^{i\gamma} + \bar{\lambda}^{-1} P e^{i\delta} \ . && (16)
\end{aligned}
$$

The CP asymmetry in the first process vanishes, while that of $B^0 \to K^+\pi^-$ is related to the asymmetry in $B^0 \to \pi^+\pi^-$ [20],

$$
\Gamma(\overline{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-) = -\frac{f_K}{f_\pi}[\Gamma(\overline{B}^0 \to \pi^+\pi^-) - \Gamma(B^0 \to \pi^+\pi^-)] \ . \quad (17)
$$

Here and below we neglect phase space factors introducing calculable corrections at a percent level. Eq. (17) may be used to test SU(3) symmetry including the SU(3) breaking factor f_K/f_π. This equality reads in units of 10^6 times branching ratios

$$
-3.5 \pm 1.0 = -5.2 \pm 1.5 \quad , \quad (18)
$$

where we use the current charge-averaged branching ratios, in units of 10^{-6} [4]:

$$
\bar{\mathcal{B}}(B \to \pi^+\pi^-) = 4.6 \pm 0.4, \ \bar{\mathcal{B}}(B \to K^0\pi^+) = 21.8 \pm 1.4, \ \bar{\mathcal{B}}(B \to K^+\pi^-) = 18.2 \pm 0.8 \ , \quad (19)
$$

and the CP asymmetry [4] $A(K^+\pi^-) = -0.095 \pm 0.028$ and the average (3) for $C_{\pi\pi}$. Although current errors are too large to provide a quantitative test of flavor SU(3), the consistency of the signs of the two asymmetries provides one test of SU(3). With future increased statistics, Eq. (17) may be used as an additional input in a determination of α as described below.

Each of the two charge averaged rates $\bar{\Gamma}(B^+ \to K^0\pi^+) \equiv [\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to \overline{K}^0\pi^-)]/2$ and $\bar{\Gamma}(B^0 \to K^+\pi^-) \equiv [\Gamma(B^0 \to K^+\pi^-) + \Gamma(\overline{B}^0 \to K^-\pi^+)]/2$ provides an additional constraint on the three parameters r, δ and α. We normalize these rates by the charge averaged rate of decays to $\pi^+\pi^-$, $\bar{\Gamma}(B^0 \to \pi^+\pi^-) \equiv [\Gamma(B^0 \to \pi^+\pi^-) + \Gamma(\overline{B}^0 \to \pi^+\pi^-)]/2$, defining the following two ratios ($\mathcal{R}_+$ corresponds to $1/b_c$ defined in [18]):

$$
\begin{aligned}
\mathcal{R}_+ &\equiv \frac{\bar{\lambda}^2\,\bar{\Gamma}(B^+ \to K^0\pi^+)}{\bar{\Gamma}(B^0 \to \pi^+\pi^-)} \ , && (20)\\
\mathcal{R}_0 &\equiv \frac{\bar{\lambda}^2\,\bar{\Gamma}(B^0 \to K^+\pi^-)}{\bar{\Gamma}(B^0 \to \pi^+\pi^-)} \ . && (21)
\end{aligned}
$$

The values (19) and the lifetime ratio [28] $\tau(B^+)/\tau(B^0) = 1.077 \pm 0.013$ imply

$$
\mathcal{R}_+ = 0.235 \pm 0.026 \ , \qquad \mathcal{R}_0 = 0.209 \pm 0.020 \ , \quad (22)
$$

in which the errors are already at a level of only 10%.

Substituting Eqs. (4), (15) and (16), we obtain

$$\mathcal{R}_+ = \frac{r^2}{R_{\pi\pi}} , \qquad (23)$$

$$\mathcal{R}_0 = \frac{r^2 + 2r\bar{\lambda}'^2 \cos\delta \cos(\beta+\alpha) + \bar{\lambda}'^4}{R_{\pi\pi}} , \qquad \bar{\lambda}' \equiv \sqrt{\frac{f_K}{f_\pi}}\,\bar{\lambda} . \qquad (24)$$

These expressions and Eq. (9) can be inverted to write r in terms of $z \equiv \cos\delta\cos(\beta + \alpha) = -\cos\delta\cos\gamma$ and one of the two measured quantities $\mathcal{R}_+$ or $\mathcal{R}_0$:

$$\mathcal{R}_+ : \qquad r = \frac{\sqrt{\mathcal{R}_+^2\, z^2 + (1-\mathcal{R}_+)\mathcal{R}_+} - \mathcal{R}_+\, z}{1-\mathcal{R}_+} , \qquad (25)$$

$$\mathcal{R}_0 : \qquad r = \frac{\sqrt{(\mathcal{R}_0 + \bar{\lambda}'^2)^2\, z^2 + (1-\mathcal{R}_0)(\mathcal{R}_0 - \bar{\lambda}'^4)} - (\mathcal{R}_0 + \bar{\lambda}'^2)\, z}{1-\mathcal{R}_0} . \qquad (26)$$

In principle, Eqs. (9), (23), and (24) may be solved for r (and z) in terms of $\mathcal{R}_+$ and $\mathcal{R}_0$,

$$r = \sqrt{\frac{\mathcal{R}_+(1+\bar{\lambda}'^2)}{1-\mathcal{R}_+ + \bar{\lambda}'^{-2}(\mathcal{R}_0 - \mathcal{R}_+)}} . \qquad (27)$$

However, in practice this provides no useful information about r because small errors in $\mathcal{R}_+$ and $\mathcal{R}_0$ are enhanced by the factor $\bar{\lambda}'^{-2}$ multiplying $\mathcal{R}_0 - \mathcal{R}_+$ in the denominator, thereby permitting very large values of r.

At this point, let us consider lower and upper bounds on r following separately from Eqs. (25) and (26), depending on branching ratio measurements of $B^+ \to K^0\pi^+$ and $B^0 \to K^+\pi^-$, respectively. For the values of $\mathcal{R}_+$ and $\mathcal{R}_0$ in (22), both expressions for r are monotonically decreasing functions of z. Using current constraints on CKM parameters [5] implying $38^\circ \le \gamma \le 80^\circ$ at 95% confidence level, the lowest and highest allowed value of z are –0.79 and 0.79, respectively. Inserting these values in (25) and (26), and using central values in (22), we find the following bounds:

$$\mathcal{R}_+ : \qquad 0.36 \le r \le 0.85 , \qquad (28)$$

$$\mathcal{R}_0 : \qquad 0.30 \le r \le 0.85 . \qquad (29)$$

Slightly wider ranges are allowed when including errors in $\mathcal{R}_+$ and $\mathcal{R}_0$.

Values of r in the lower parts of these ranges correspond to $z > 0$ or $\pi/2 < |\delta| < \pi$, while the upper parts correspond to $z < 0$ or $0 < |\delta| < \pi/2$. Assuming that δ lies in the first (positive or negative) quadrant, $|\delta| < \pi/2$, one has

$$\mathcal{R}_+ : \qquad 0.55 \le r \le 0.85 \quad (\text{assuming } |\delta| < \pi/2) , \qquad (30)$$

$$\mathcal{R}_0 : \qquad 0.51 \le r \le 0.85 \quad (\text{assuming } |\delta| < \pi/2) . \qquad (31)$$

The two lower bounds, which become slightly smaller (0.51 and 0.48 respectively) when errors are included, stand in contrast to most calculations based on QCD factorization ($r = 0.285 \pm 0.076$ [23] or $r = 0.32^{+0.16}_{-0.09}$ [29]) and perturbative QCD

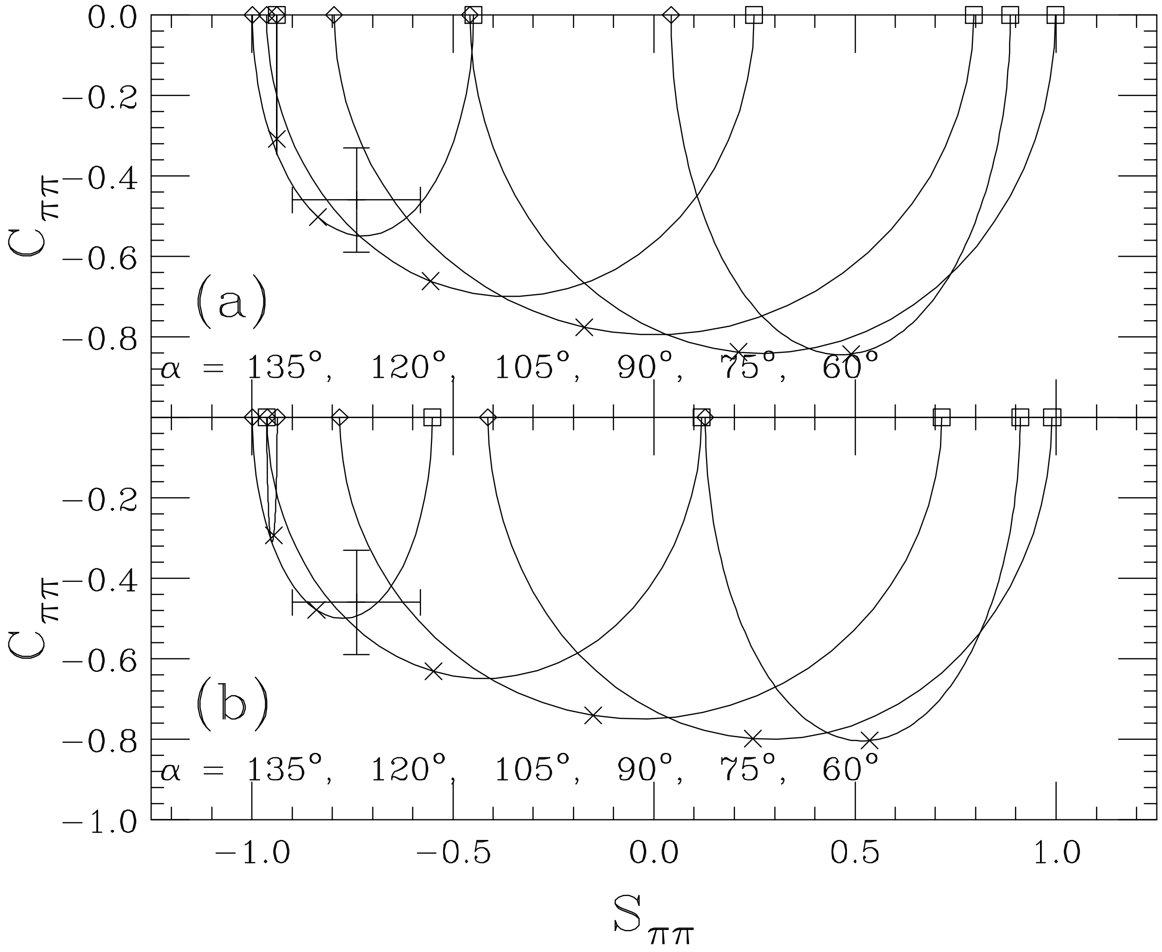

Figure 1: Parametric curves of $C_{\pi\pi}$ vs. $S_{\pi\pi}$ for the range $-\pi \leq \delta \leq 0$, based on measurements of (a) $\mathcal{R}_+$ and (b) $\mathcal{R}_0$. Points marked with diamonds, crosses and squares denote $\delta = 0$, $-\pi/2$ and $-\pi$, respectively. Plotted point denotes the experimental average (3).

($r = 0.23^{+0.07}_{-0.05}$ [30]), where values of δ were obtained in the range $|\delta| < \pi/2$. A value $r = 0.26 \pm 0.08$ was estimated [31] by applying factorization to $B \to \pi\ell\nu$, but disregarding the P_{tu} term in T. On the other hand, a recent a global SU(3) fit to all $B \to \pi\pi$ and $B \to K\pi$ decays [32], including a sizable P_{tu} contribution, obtained values $r = 0.69 \pm 0.09$ and $\delta = (-34^{+11}_{-25})^\circ$, in obvious agreement with the bounds (30) and (31). Note that these bounds do not rely on the asymmetry measurements in $B^0 \to \pi^+\pi^-$. As we will see below, where we make no assumption about δ, the measured asymmetries also seem to favor negative values of δ in the first quadrant.

Each of the two relations (25) and (26) may be used separately together with (7)–(9) to express $C_{\pi\pi}$ and $S_{\pi\pi}$ in terms of δ, α and the measured values of $\beta, \mathcal{R}_+$ or $\mathcal{R}_0$. We draw two separate plots, using in one case the measurement of $\mathcal{R}_+$ and in the other case that of $\mathcal{R}_0$. Values of $S_{\pi\pi}$ and $C_{\pi\pi}$, for $\beta = 23.7^\circ$ [5], the central value of $\mathcal{R}_+$ in (22), and for a set of four values of α in the currently allowed range [5] $75^\circ \leq \alpha \leq 120^\circ$ and two values outside this range, are plotted in Fig. 1(a). We plot only the case $\delta \leq 0$ since the experimental average of BaBar and Belle values corresponds to $C_{\pi\pi} \leq 0$, and the signs of $\sin\delta$ and $C_{\pi\pi}$ are correlated by Eq. (11).

As anticipated, curves of fixed α and varying δ are approximate ellipses. Points marked with diamonds, crosses and squares denote $\delta = 0$, $-\pi/2$ and $-\pi$, respectively. A consequence of the second term in the numerator of (8) is that a point $\delta = -\pi$ on each ellipse is located to the right of a point $\delta = 0$ on the same ellipse. The plotted point including errors corresponds to the present averaged asymmetries (3). Fig. 1(b) is plotted in an analogous manner, using the central value of $\mathcal{R}_0$ in (22).

We note the similar dependence in Figs. 1(a) and 1(b) of $S_{\pi\pi}$ and $C_{\pi\pi}$ as functions of δ and α. This common behavior supports our assumption of flavor SU(3), also adding to the statistical significance of the plots, which are based on central values of measurements of $B^+ \to K^0\pi^+$ and $B^0 \to K^+\pi^-$. The approximate ellipses in Fig. 1(a), for fixed values of α and varying δ, are only slightly larger than those in Fig. 1(b). This follows from the somewhat larger values of r permitted by $\mathcal{R}_+$ than those allowed by $\mathcal{R}_0$, as given in the bounds (28) and (29).

An important question is: what can be learned about δ and α from the present average asymmetries (3)? A negative value of $C_{\pi\pi}$, favored by the data, implies $-\pi < \delta < 0$. A negative $S_{\pi\pi}$, supported by both the BaBar and the Belle results (2), favors $-\pi/2 < \delta < 0$ for all plotted values of α except $\alpha = 120^\circ$ and 135°.

As for α, it is already remarkable that the two measured asymmetries lie in an area in the $(S_{\pi\pi}, C_{\pi\pi})$ plane overlapping with that corresponding to the range $75^\circ < \alpha < 120^\circ$ obtained from other constraints [5]. Larger values in this range are favored. The measured asymmetries exclude smaller values of α than in this range (e.g., $\alpha = 60^\circ$), for which corresponding ellipses lie too much to the right (e.g., corresponding to $S_{\pi\pi} \geq 0$). Larger values of α than in this range (e.g., $\alpha = 135^\circ$) are described by shorter and narrower ellipses, implying values of $|C_{\pi\pi}|$ smaller than the measured central value. The consistency between the range of α allowed by the asymmetries and by all other constraints is certainly nontrivial, indicating that the origin of the asymmetry is largely the KM phase.

Using Fig. 1(b) [slightly more restrictive than Fig. 1(a)], an error ellipse with center and principal axes specified as in Eq. (3) just touches the curve for $\alpha = 86^\circ$ at a single point, excluding lower values and implying $\alpha = (103 \pm 17)^\circ$. An actual determination of α at this precision (rather than the use of the upper bound of 120° as we have done) requires reducing the experimental errors in the two asymmetries, since as α grows one must contend with discrete ambiguities in which curves for different α intersect at a single point. Neglecting for the moment this feature, the horizontal distance between the two ellipses drawn for $\alpha = 90^\circ$ and 105° corresponds to a change in $S_{\pi\pi}$ of magnitude $\Delta S_{\pi\pi} = 0.18$. A reduction of the current experimental error, $\Delta S_{\pi\pi} = 0.16$, by a factor two will result in a comparable reduction in the error of α to $\Delta\alpha = 9^\circ$. This seems like an ultimate precision considering the approximations made in this analysis. Both Figs. 1(a) and 1(b) show that as α approaches its current upper limit of 120° a higher precision in $S_{\pi\pi}$ is required in order to achieve this precision in α, both because of the discrete ambiguity just mentioned and because the curves for a given change in α lie closer to to one another.

Before concluding, we wish to comment on the effect of SU(3) breaking in P. For illustration, let us assume $P' = -(f_K/f_\pi)\bar{\lambda}^{-1}P$ instead of (13), as would be the case if penguin amplitudes were to factorize. In this case the right-hand-side of (17) includes

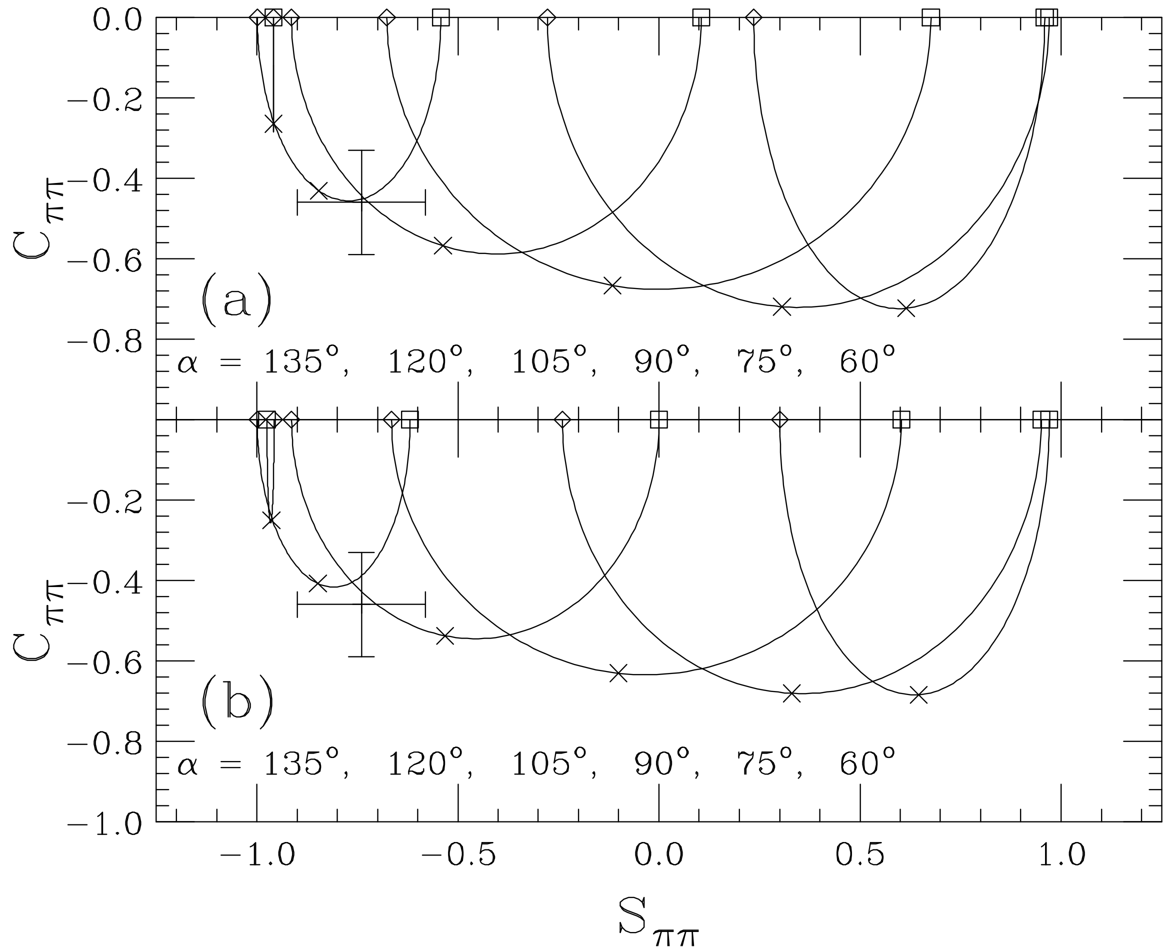

Figure 2: Same as Figs. 1 but with SU(3) breaking included in penguin terms as explained in text.

a factor $(f_K/f_\pi)^2$, implying a central value on the right-hand-side of (18), -6.3 ± 1.9, almost twice as large as the central value on the left-hand-side. As a result, one must replace $\mathcal{R}_+ \to \mathcal{R}_+/(f_K/f_\pi)^2$ in (25), and $\mathcal{R}_0 \to \mathcal{R}_0/(f_K/f_\pi)^2$, $\bar{\lambda}' \to \bar{\lambda}$ in (26). The bounds (30) and (31) are replaced by tighter ones, $0.43 \le r \le 0.61$ using $\mathcal{R}_+$, and $0.40 \le r \le 0.61$ using $\mathcal{R}_0$. The two lower bounds are still somewhat higher than most QCD calculations.

The resulting plots of $C_{\pi\pi}$ versus $S_{\pi\pi}$ are shown in Figs. 2(a) and 2(b). The constraints on α are stronger than those obtained from Figs. 1(a) and 1(b) which assumed no SU(3) breaking in P. The use of Fig. 2(b) [again, slightly more restrictive than Fig. 2(a)], allows one to exclude values of $\alpha < 94°$ by the error-ellipse method mentioned in the previous paragraph. We would then conclude that $\alpha = (107 \pm 13)°$. In any event, this example of SU(3) breaking shows that the limits obtained in the absence of SU(3) breaking in P are conservative ones.

We thank Andreas Höcker, Dan Pirjol, and Jure Zupan for helpful discussions. JLR is grateful to Maury Tigner for extending the hospitality of the Laboratory for Elementary-Particle Physics at Cornell during this research. This work was supported in part by the United States Department of Energy through Grant No. DE FG02 90ER40560 and in part by the John Simon Guggenheim Memorial Foundation.

References

[1] BaBar Collaboration, presented by H. Jawahery, Proceedings of the XXI International Symposium on Lepton and Photon Interactions, Fermilab, Batavia, USA, August 11–16 2003.

[2] Belle Collaboration, K. Abe *et al.*, Belle Report 2004-1, hep-ex/04011029.

[3] M. Gronau, Phys. Rev. Lett. **63** (1989) 1451.

[4] Updated results and references are tabulated periodically by the Heavy Flavor Averaging Group: `http://www.slac.stanford.edu/xorg/hfag`.

[5] A. Höcker *et al.*, Eur. Phys. J. C **21** (2001) 225. Updated results may be found on the web site `http://ckmfitter.in2p3.fr`.

[6] D. London and R. D. Peccei, Phys. Lett. B **223** (1989) 257; B. Grinstein, Phys. Lett. B **229** (1989) 280.

[7] M. Gronau and D. London, Phys. Rev. Lett. **65** (1990) 3381.

[8] M. Gronau, D. Pirjol and T. M. Yan, Phys. Rev. D **60** (1999) 034021; A. Buras and R. Fleischer, Eur. Phys. J. C **11** (1999) 93.

[9] Y. Grossman and H. R. Quinn, Phys. Rev. D **58** (1998) 017504; J. Charles, Phys. Rev. D **59** (1999) 054007; M. Gronau, D. London, N. Sinha and R. Sinha, Phys. Lett. B **514** (2001) 315.

[10] M. Gronau, hep-ph/0306308, Proceedings of Flavor Physics and CP Violation (FPCP 2003), Paris, France, 3-6 Jun 2003.

[11] G. Buchalla and A. S. Safir, hep-ph/0310218; F. J. Botella and J. P. Silva, hep-ph/0312337.

[12] A. Ali, E. Lunghi and A. Y. Parkhomenko, hep-ph/0403275.

[13] D. Zeppenfeld, Zeit. Phys. **8** (1981) 77; M. Savage and M. Wise, Phys. Rev. D **39** (1989) 3346; *ibid.* **40** (1989) 3127(E); L. L. Chau *et al.*, Phys. Rev. D **43** (1991) 2176; B. Grinstein and R. F. Lebed, Phys. Rev. D **53** (1996) 6344.

[14] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D **50** (1994) 4529; **52** (1995) 6374.

[15] M. Gronau, O. F. Hernández, D. London, and J. L. Rosner, Phys. Rev. D **52** (1995) 6356.

[16] For a large list of references, see Ref. [10] and R. Fleischer, Phys. Rep. **370** (2002) 537.

[17] M. Gronau and J. L. Rosner, Phys. Rev. D **65** (2002) 093012.

[18] M. Gronau and J. L. Rosner, Phys. Rev. D **66** (2002) 053003.

[19] J. P. Silva and L. Wolfenstein, Phys. Rev. D **49** (1994) 1151.

[20] N. G. Deshpande and X. G. He, Phys. Rev. Lett. **75** (1995) 1703; M. Gronau and J. L. Rosner, Phys. Rev. Lett. **76** (1996) 1200; A. S. Dighe, M. Gronau and J. L. Rosner, Phys. Rev. D **54** (1996) 3309.

[21] J. Charles, Phys. Rev. D **59** (1999) 054007; J. Fleischer, Eur. Phys. J. C **16** (2000) 87; R. Fleischer and J. Matias, Phys. Rev. D **66** (2002) 054009; C. D. Lü and Z. Xiao, Phys. Rev. D **66** (2002) 074011.

[22] BaBar Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **89** (2002) 201802; Belle Collaboration, K. Abe *et al.*, Phys. Rev. D **66** (2002) 071102.

[23] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. B **606** (2001) 245.

[24] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, hep-ph/0401188. See also J. Chay and C. Kim, Phys. Rev. D **68** (2003) 071502; Nucl. Phys. B **680** (2004) 302.

[25] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, Phys. Lett. B **515** (2001) 33.

[26] A. F. Falk, A. L. Kagan, Y. Nir and A. A. Petrov, Phys. Rev. D **57** (1998) 4290.

[27] B. Blok, M. Gronau and J. L. Rosner, Phys. Rev. Lett. **78** (1997) 3999; **79** (1997) 1167; M. Neubert, Phys. Lett. B **424** (1998) 152; A. J. Buras, R. Fleischer and T. Mannel, Nucl. Phys. B **533** (1998) 3; M. Gronau and J. L. Rosner, Phys. Rev. D **57** (1998) 6843; D **58** (1998) 113005; A. Soni and D. Atwood, Phys. Rev. D **58** (1998) 036005; R. Fleischer, Eur. Phys. J. C **6** (1999) 451.

[28] LEP B Oscillation Working Group, results for the summer 2003 conferences, `http://lepbosc.web.cern.ch/LEPBOSC/`.

[29] M. Beneke and M. Neubert, Nucl. Phys. B **675** (2003) 333.

[30] Y. Y. Keum and A. A. Sanda, Phys. Rev. D **67** (2003) 054009.

[31] Z. Luo and J. L. Rosner, Phys. Rev. D **68** (2003) 074010.

[32] C. W. Chiang, M. Gronau, J. L. Rosner and D. A. Suprun, hep-ph/0404073. See Fit I in Table V. Inclusion of modes with η and η' lowers r somewhat, but introduces additional uncertainties associated with SU(3) breaking.