

## STATUS OF $\psi''$ DECAYS TO CHARMLESS FINAL STATES <sup>1</sup>

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The non- $D\bar{D}$  decays of the  $\psi'' = \psi(3770)$  resonance are reviewed and possibilities for further measurements are noted. These decays can shed light on S-D mixing, the “missing”  $\psi' = \psi(3686)$  decays, a possible discrepancy between the total and  $D\bar{D}$  cross sections at the  $\psi''$ , and rescattering effects contributing to enhanced  $b \rightarrow s$  penguin amplitudes in  $B$  meson decays.

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### I. INTRODUCTION

The  $\psi'' \equiv \psi(3770)$  particle is the lowest-mass charmonium resonance above  $D\bar{D}$  threshold.<sup>3</sup> It is produced through virtual photons in  $e^+e^-$  collisions, and can serve as a well-defined source of charmed particle pairs [1] in the same way that the  $\Upsilon(4S)$  state, lying just above  $B\bar{B}$  threshold, is a good source of nonstrange  $B$  mesons. While discovered a number of years ago [2], the  $\psi''$  is now being exploited through high-intensity studies under way at the CLEO Detector at Cornell [3] and at the BES Detector in China [4]. The couplings of the  $\psi''$  to charmless states are of interest for a number of reasons.

(1)  $\psi''$  production and decays are sensitive to the mixing between S and D waves in its wave function. It is predominantly a  $c\bar{c}(1^3D_1)$  state,<sup>4</sup> but contains important contributions from mixing with the  $2^3S_1$  level (of which  $\psi' = \psi(3686)$  contains the major share) as well as with possible other  $^3S_1$  levels and  $D\bar{D}$  continuum states [5, 6]. This mixing can affect not only  $\psi''$  modes, but also  $\psi'$  modes, suppressing some of them while leading to contributions in  $\psi''$  decays [7] which may be hard to see in those decays as a result of interference effects [8, 9, 10, 11].

(2) New measurements of the cross section  $\sigma(e^+e^- \rightarrow \psi'' \rightarrow D\bar{D}) \equiv \sigma(D\bar{D})$  have recently been reported by the BES [4] and CLEO [12] Collaborations. The CLEO measurement employs a “double-tag” method pioneered by the Mark III group in a previous study [13]. Although the values appear higher than that of Mark III,  $\sigma(D\bar{D})$

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<sup>3</sup>Numbers in parentheses will denote masses of particles, in MeV/ $c^2$ .

<sup>4</sup>The spectroscopic shorthand is  $n^{2S+1}L_J$ , where  $n = 1, 2, 3, \dots$  is the radial quantum number;  $S = 0$  or 1 is the  $c\bar{c}$  spin;  $L = S, P, D, \dots$  ( $l = 0, 1, 2, \dots$ ) is the orbital angular momentum, and  $J = 0, 1, 2, \dots$  is the total spin.

still seems less than the total cross section  $\sigma(e^+e^- \rightarrow \psi'' \rightarrow \dots) \equiv \sigma(\psi'')$ , for which several groups have reported measurements [14, 15, 16, 17]. If  $\sigma(D\bar{D})$  is really less than  $\sigma(\psi'')$ , this would be a question of intrinsic interest and would provide an estimate for rates for channels other than  $D\bar{D}$  during the forthcoming extensive accumulations of data by CLEO and BES-III at the  $\psi''$  energy.

(3) The non-charm decays of  $\psi''$ , if appreciable, provide a possible laboratory for the study of rescattering effects relevant to  $B$  meson decays. As one example, if the  $\psi''$  decays to  $D\bar{D}$  pairs which subsequently re-annihilate into non-charmed final states, similar effects could be responsible for enhanced penguin amplitudes (particularly in  $b \rightarrow s$  transitions) in  $B$  decays. One particle whose enhanced production in both exclusive and inclusive  $B$  decays is not well understood is the  $\eta'$ . It should be looked for in inclusive  $\psi''$  decays.

The importance of a re-annihilation mechanism for possible decays of the  $\psi''$  into non-charmed final states was stressed quite early [18]. Similar mechanisms are relevant not only to heavy quarkonium decays [19] but also to the decays of the  $\phi$  meson into non- $K\bar{K}$  states [20]. Non-charmed final states of the  $\psi''$  were discussed in two doctoral theses [21, 22] based on Mark III data. However, no statistically significant signals were obtained. Whereas the total width of  $\psi''$  is quoted [23] as  $\Gamma(\psi'') = 23.6 \pm 2.7$  MeV, partial widths to such known channels as  $\gamma\chi_{cJ}$  ( $J = 1, 2$ ) and  $J/\psi\pi\pi$  are expected not to exceed a few tens of keV, with a few hundred keV expected for  $\Gamma(\psi'' \rightarrow \gamma\chi_{c0})$ . Thus any significant non- $D\bar{D}$  branching ratio in excess of a percent or two must come from as-yet-unseen hadronic channels or something more exotic.

I begin in Section II by reviewing the total and  $D\bar{D}$  cross sections at the  $\psi''$ . Information from its leptonic and radiative decays (as well as those of  $\psi'$ ) is presented in Section III, while Section IV treats the corresponding information available from  $\psi'' \rightarrow J/\psi\pi^+\pi^-$ . An important contribution to hadron production at the  $\psi''$  energy comes from the continuum process  $e^+e^- \rightarrow \gamma^* \rightarrow$  light  $q\bar{q}$  pairs, treated in Section V. The question of whether there is a significant non- $D\bar{D}$  cross section is examined in Section VI. If so, charmless decays can illuminate certain classes of  $B$  decays, as pointed out in Section VII, where the particular utility of *inclusive* measurements is stressed. Section VIII concludes. An Appendix contains details of a model for re-annihilation of  $D\bar{D}$  pairs into light quarks.

## II. CROSS SECTIONS AT THE $\psi''$ RESONANCE PEAK

One can measure the cross section for  $D\bar{D}$  production at the  $\psi''$  by comparing the rates for  $e^+e^- \rightarrow \psi'' \rightarrow f_i + \dots$  and  $e^+e^- \rightarrow \psi'' \rightarrow f_i\bar{f}_j$ , where  $f_i$  and  $f_j$  are final states in  $D$  decay. Unknown branching ratios can be determined, but one must have good knowledge of detector efficiency. This method was first used by the Mark III Collaboration [13] to determine  $\sigma(D\bar{D}) = (5.0 \pm 0.5)$  nb, based on an integrated luminosity  $\int \mathcal{L}dt = (9.56 \pm 0.48)$  pb $^{-1}$ .

The CLEO Collaboration has recently measured  $\sigma(D\bar{D})$  using this same double-tag method but with  $\int \mathcal{L}dt \simeq 56$  pb $^{-1}$  [12]. The values are compared with those from Mark III and from a single-tag measurement by the BES Collaboration [4] (with  $\int \mathcal{L}dt = 17.7$  pb $^{-1}$ ) in Table I.

Table I: Comparison of cross sections  $\sigma(D\bar{D}) \equiv \sigma(e^+e^- \rightarrow \psi'' \rightarrow D\bar{D})$ , in nb.

Collaboration	$\sigma(D^+D^-)$	$\sigma(D^0\bar{D}^0)$	$\sigma(D\bar{D})$
BES-II <sup>a</sup> [4]	$2.52 \pm 0.07 \pm 0.23$	$3.26 \pm 0.09 \pm 0.26$	$5.78 \pm 0.11 \pm 0.38$
CLEO <sup>a</sup> [12]	$2.58 \pm 0.15 \pm 0.19$	$3.90 \pm 0.42 \pm 0.28$	$6.48 \pm 0.44 \pm 0.49$
Mark III [13]	$2.1 \pm 0.3$	$2.9 \pm 0.4$	$5.0 \pm 0.5$

<sup>a</sup> Preliminary.

Table II: Comparison of total cross sections  $\sigma(\psi'') \equiv \sigma(e^+e^- \rightarrow \psi'' \rightarrow \dots)$ , in nb.

Collaboration	$\sigma(\psi'')$
Crystal Ball [14]	$6.7 \pm 0.9$
Lead-Glass Wall [15]	$10.3 \pm 1.6$
Mark II [16]	$9.3 \pm 1.4$
BES <sup>a</sup> [17]	$7.7 \pm 1.1$
Average	$7.9 \pm 0.6$

<sup>a</sup> Estimate based on fit (see Sec. VI).

The ratios  $\sigma(D^+D^-)/\sigma(D^0\bar{D}^0)$  are consistent at present with the ratio of kinematic factors  $(p_{+-}^*/p_{00}^*)^3 = 0.685$  appropriate for the P-wave decay  $\psi'' \rightarrow D\bar{D}$  (where  $p^*$  denotes the magnitude of the center-of-mass [c.m.] 3-momentum). Coulomb and other final-state-interaction effects can alter this ratio and lead to its dependence on energy [24], but these phenomena remain to be studied.

The values in Table I are to be compared with those for the total cross section  $\sigma(\psi'')$  in Table II. It is possible that  $\sigma(D\bar{D})$  falls short by one or more nb from the total cross section  $\sigma(\psi'')$ , but the difference is not statistically significant. Improved measurements of both quantities by the *same* experiment will be needed to resolve the question. In Section VII we will take an illustrative example in which this difference, taken to be 18% of  $\sigma(\psi'')$ , is ascribed to re-annihilation of  $D\bar{D}$  into light-quark states.

### III. INFORMATION FROM LEPTONIC AND RADIATIVE DECAYS

A simple model of S-D wave mixing for the  $\psi'$  and  $\psi''$  is to write

$$\psi'' = \cos \phi |1^3D_1\rangle + \sin \phi |2^3S_1\rangle \quad , \quad \psi' = -\sin \phi |1^3D_1\rangle + \cos \phi |2^3S_1\rangle \quad . \quad (1)$$

The ratio  $R_{\psi''/\psi'}$  of leptonic widths (scaled by factors of  $M^2$ ) and the partial widths  $\Gamma(\psi' \rightarrow \chi\gamma)$  and  $\Gamma(\psi'' \rightarrow \chi\gamma)$  may then be calculated as functions of  $\phi$  [7, 25]. Specifically, it was found in Ref. [7] that

$$R_{\psi''/\psi'} \equiv \frac{M_{\psi''}^2 \Gamma(\psi'' \rightarrow e^+e^-)}{M_{\psi'}^2 \Gamma(\psi' \rightarrow e^+e^-)} = \left| \frac{0.734 \sin \phi + 0.095 \cos \phi}{0.734 \cos \phi - 0.095 \sin \phi} \right|^2 = 0.128 \pm 0.023 \quad , \quad (2)$$

Table III: Partial widths in keV predicted in Ref. [6] without (a) or with (b) couplings to open channels and in Ref. [7].  $M(\psi'') = 3772 \text{ MeV}/c^2$  is taken in accord with the fit of Sec. VI; the nominal mass quoted in Ref. [23] is  $3769.9 \pm 2.5 \text{ MeV}/c^2$ .

$\psi''$ decay	$E_\gamma$ (MeV)	Ref. [6]		Ref. [7]
		(a)	(b)	( $\phi = 12 \pm 2^\circ$ )
$\gamma\chi_{c2}$	210	3.2	3.9	$24 \pm 4$
$\gamma\chi_{c1}$	252	183	59	$73 \pm 9$
$\gamma\chi_{c0}$	340	254	225	$523 \pm 12$

while

$$\Gamma(\psi'' \rightarrow \gamma\chi_{c0}) = 145 \text{ keV} \cos^2 \phi (1.73 + \tan \phi)^2, \quad (3)$$

$$\Gamma(\psi'' \rightarrow \gamma\chi_{c1}) = 176 \text{ keV} \cos^2 \phi (-0.87 + \tan \phi)^2, \quad (4)$$

$$\Gamma(\psi'' \rightarrow \gamma\chi_{c2}) = 167 \text{ keV} \cos^2 \phi (0.17 + \tan \phi)^2, \quad (5)$$

and

$$\Gamma(\psi' \rightarrow \gamma\chi_{c0}) = 67 \text{ keV} \cos^2 \phi (1 - 1.73 \tan \phi)^2, \quad (6)$$

$$\Gamma(\psi' \rightarrow \gamma\chi_{c1}) = 56 \text{ keV} \cos^2 \phi (1 + 0.87 \tan \phi)^2, \quad (7)$$

$$\Gamma(\psi' \rightarrow \gamma\chi_{c2}) = 39 \text{ keV} \cos^2 \phi (1 - 0.17 \tan \phi)^2. \quad (8)$$

These quantities are plotted as functions of  $\phi$  in Fig. 1.

The observed ratio  $R_{\psi''/\psi'}$  agrees with predictions only for  $\phi = (12 \pm 2)^\circ$  or  $(-27 \pm 2)^\circ$ , as shown by the vertical bands in Fig. 1. Only the solution with  $\phi = (12 \pm 2)^\circ$  is remotely consistent with the observed partial widths [23]  $\Gamma(\psi' \rightarrow \gamma\chi_{cJ}) = 20\text{--}30 \text{ keV}$ . This range of  $\phi$  favors the decay  $\psi'' \rightarrow \gamma\chi_{c0}$  over  $\psi'' \rightarrow \gamma\chi_{c1,2}$  by a substantial amount. The choice  $\phi = (12 \pm 2)^\circ$  also is favored by the comparison of  $\psi'$  and  $\psi''$  decays to  $J/\psi\pi^+\pi^-$ . With the choice  $\phi = (-27 \pm 2)^\circ$ , a larger rate would be predicted for  $\psi'' \rightarrow J/\psi\pi^+\pi^-$  than for  $\psi' \rightarrow J/\psi\pi^+\pi^-$ , in conflict with experiment [26]. It has recently been argued [27] that the mixing could be larger,  $|\phi| \simeq 40^\circ$ , but this conclusion depends on specific production models for the  $\psi''$  in inclusive  $e^+e^-$  annihilations and in  $B$  decays.

The prospects for observation of  $\psi'' \rightarrow \gamma\chi_{cJ}$  have been greatly improved with the accumulation of the recent data sample of  $\int \mathcal{L} dt \simeq 56 \text{ pb}^{-1}$  in the CLEO-c detector [12]. With this sample and  $\sigma(\psi'') \geq 6 \text{ nb}$  one should see several events in the cascade  $\psi'' \rightarrow \gamma\chi_{c1} \rightarrow \gamma\gamma J/\psi \rightarrow \gamma\gamma\ell^+\ell^-$ . The inclusive signal in  $\psi'' \rightarrow \gamma\chi_{c0}$  will not be statistics-limited. All predictions of branching ratios lie in the 1–2% range.

It is important to consider coupling to open  $D\bar{D}$  channels and mixing schemes that are more general than Eq. (1) when predicting radiative decay widths [6]. Table III compares partial widths predicted in one such scheme with those depicted in Fig. 1. In Ref. [6] the  $\psi''$  is composed of only 52%  $c\bar{c}$ ; the remainder of its wave function contains additional light quark-antiquark pairs, *e.g.*, in the form of the open  $D\bar{D}$  channel. Thus the results of Fig. 1 do represent some oversimplification.

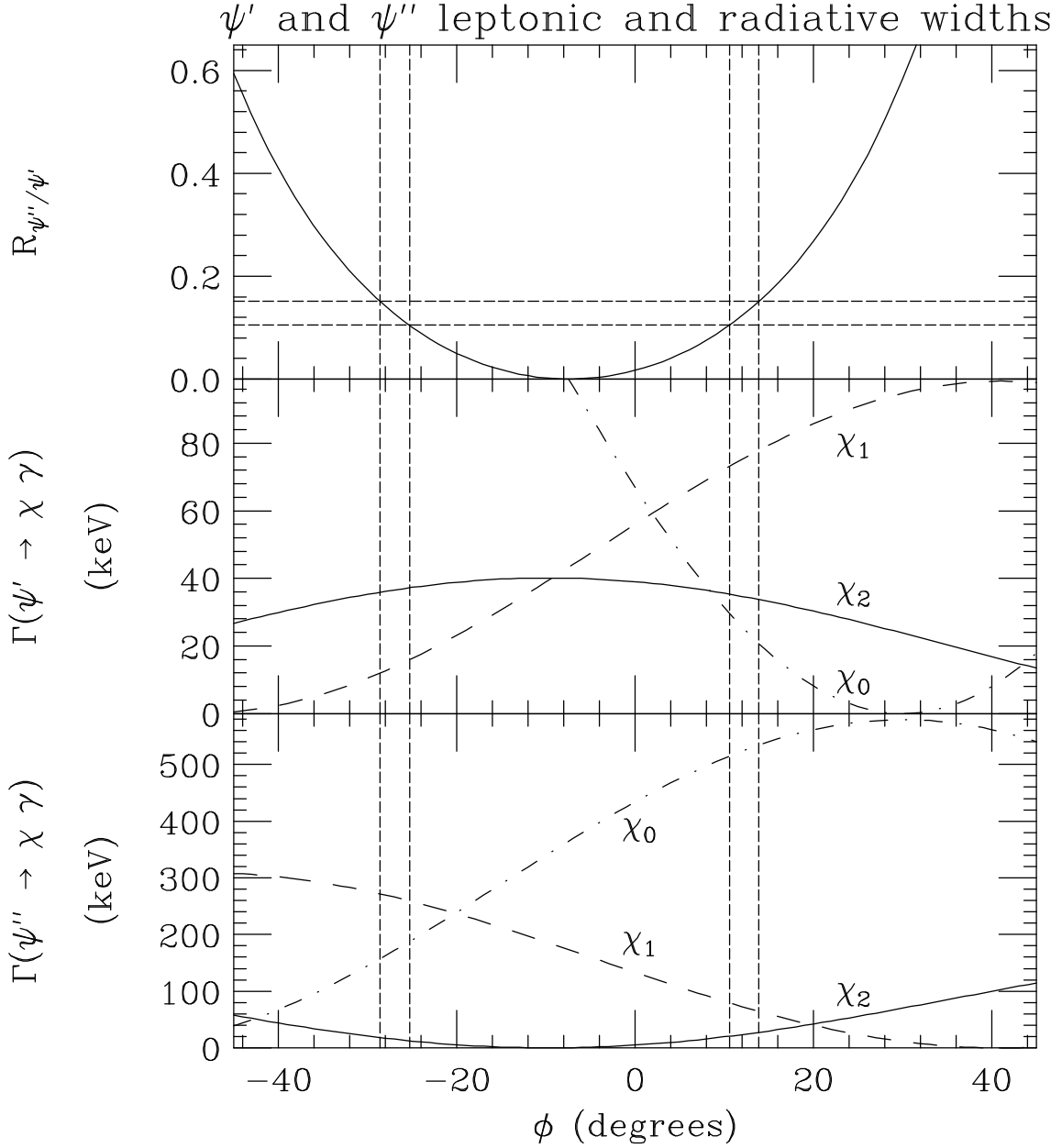


Figure 1: Sensitivity of scaled leptonic width ratio  $R_{\psi''/\psi'}$  and partial widths  $\Gamma(\psi', \psi'' \rightarrow \chi\gamma)$  to mixing angle  $\phi$ . Horizontal lines in top panel denote  $\pm 1\sigma$  limits on  $R_{\psi''/\psi'}$ , and are projected onto the  $\phi$  axis with vertical bands. In middle and bottom panels solid, dashed, and dash-dotted curves denote partial widths to  $\gamma\chi_{c2}$ ,  $\gamma\chi_{c1}$ , and  $\gamma\chi_{c0}$ , respectively.

For an exclusive decay involving  $\chi_{c1}$  suppose that  $\Gamma(\psi'' \rightarrow \gamma\chi_{c1}) = 59$  keV and use the tabulated branching ratios [23]  $\mathcal{B}(\chi_{c1} \rightarrow \gamma J/\psi) = (31.6 \pm 1.2)\%$ ,  $\mathcal{B}(J/\psi \rightarrow \ell^+\ell^-) = (5.9 \pm 0.1)\%$  ( $\ell = e$  or  $\mu$ ). With an efficiency of 1/2 for each shower or charged track<sup>5</sup> one expects to see two events.

The number of  $\psi'' \rightarrow \gamma\chi_{c0}$  events expected in the current CLEO-c sample of  $\simeq 56$  pb<sup>-1</sup> may be estimated as follows. Suppose the cross section for  $\psi''$  production is at least 6 nb. Assume a branching ratio of at least a percent and a photon detection efficiency of at least 50%. Then one expects at least  $56000 \times 6 \times 0.01 \times 0.5 = 1680$  events containing a monochromatic photon with energy 340 MeV.

The Mark III collaboration [21] reported some marginal signals for  $\psi''$  radiative decays (quoted in Ref. [7]), whose partial widths we now adjust for the ratio of the Mark III total cross section [13]  $\sigma(\psi'') = 5.0 \pm 0.5$  nb and our average of  $\sigma(\psi'') = 7.9 \pm 0.6$  nb:

$$\Gamma(\psi'' \rightarrow \gamma\chi_{c0}) = (320 \pm 120) \text{ keV} , \quad (9)$$

$$\Gamma(\psi'' \rightarrow \gamma\chi_{c1}) = (280 \pm 100) \text{ keV} , \quad (10)$$

with an upper limit

$$\Gamma(\psi'' \rightarrow \gamma\chi_{c2}) \leq 330 \text{ keV (90\% c.l.)} . \quad (11)$$

The partial widths predicted in Table III imply that the signal for  $\psi'' \rightarrow \gamma\chi_{c0}$  could be genuine, but that for  $\psi'' \rightarrow \gamma\chi_{c1}$  is less likely to be so.

#### IV. INFORMATION FROM $\psi'' \rightarrow J/\psi\pi^+\pi^-$

The rate for the decay  $\psi'' \rightarrow J/\psi\pi^+\pi^-$  was originally estimated by Kuang and Yan [26] using a QCD multipole expansion and assuming the  $\psi''$  to be a pure  ${}^3D_1$  state. The inclusion of mixing and comparison with experimental results imply that the intrinsic  ${}^3D_1 \rightarrow J/\psi\pi^+\pi^-$  amplitude cannot be neglected but is not as large as a free-gluon approximation would predict.

An early Mark III result reported in Ref. [21] found  $\sigma(\psi'')\mathcal{B}(\psi'' \rightarrow J/\psi\pi^+\pi^-) = (1.2 \pm 0.5 \pm 0.2) \times 10^{-2}$  nb, implying  $\mathcal{B}(\psi'' \rightarrow J/\psi\pi^+\pi^-) = (0.15 \pm 0.07)\%$ . This result is compared with others in Table IV. The average (not including information from the CLEO upper limit) is  $\mathcal{B}(\psi'' \rightarrow J/\psi\pi^+\pi^-) = (0.18 \pm 0.06)\%$ , corresponding to a partial width of  $43 \pm 14$  keV. Adding another 50% for the  $\psi'' \rightarrow J/\psi\pi^0\pi^0$  mode, one finds  $\Gamma(\psi'' \rightarrow J/\psi\pi\pi) = (64 \pm 21)$  keV, or at most about 100 keV.

Kuang and Yan [26] predicted  $\Gamma(\psi'' \rightarrow J/\psi\pi^+\pi^-) = 107$  keV for a free-gluon estimate of  ${}^3D_1 \rightarrow J/\psi\pi^+\pi^-$  (based on the observed  $\psi' \rightarrow J/\psi\pi^+\pi^-$  rate) and  $\Gamma(\psi'' \rightarrow J/\psi\pi^+\pi^-) = 20$  keV if  $\Gamma({}^3D_1 \rightarrow J/\psi\pi^+\pi^-)$  were reduced by a factor of 3 from a free-gluon estimate. (This estimate is lower than 107/3 because of the interplay of S-wave and D-wave contributions to the  $\psi''$  decay.) This may have implications for the search for  $\Upsilon(1D) \rightarrow \Upsilon(1S)\pi^+\pi^-$ . A recent CLEO upper limit [30] for  $\Upsilon(1D) \rightarrow \Upsilon(1S)\pi^+\pi^-$  lies about a factor of 7 below the Kuang-Yan [26] free-gluon prediction. Mixing in the  $\Upsilon(1D)$  may be different from that in  $\psi''$ , however.

<sup>5</sup>This is a conservative estimate; the CLEO-c detector can probably do considerably better.

Table IV: Comparison of experimental branching ratios  $\mathcal{B}(\psi'' \rightarrow J/\psi\pi^+\pi^-)$ , in percent.

Collaboration	$\mathcal{B}(\psi'' \rightarrow J/\psi\pi^+\pi^-)$
Mark II [21]	$0.15 \pm 0.07$
BES [28]	$0.34 \pm 0.14 \pm 0.08$
Mark II – BES average	$0.18 \pm 0.06$
CLEO [29]	$< 0.26$ (90% c.l.)

A further complication in analysis of the  $\psi'' \rightarrow J/\psi\pi^+\pi^-$  partial width arises from the tail of the  $\psi'$ , whose contribution is non-negligible at the  $\psi''$  mass as a result of the large branching ratio for  $\psi' \rightarrow J/\psi\pi^+\pi^-$ . A thorough analysis of this effect probably requires measurement of the energy dependence of the apparent  $\psi'' \rightarrow J/\psi\pi^+\pi^-$  signal.

## V. CONTINUUM EXPECTATIONS

The total cross section for  $e^+e^- \rightarrow \psi''$ , whether it is 6, 8, or 10 nb, is by no means the only contribution to hadron production at a c.m. energy of 3770 MeV. One expects hadron production from  $e^+e^- \rightarrow q\bar{q}$  ( $q = u, d, s$ ) to account for

$$\begin{aligned} \sigma(e^+e^- \rightarrow q\bar{q}) &= N_c \left[ \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] \sigma(e^+e^- \rightarrow \mu^+\mu^-) \left[ 1 + \frac{\alpha_s}{\pi} + \dots \right] , \\ &= 2 \sigma(e^+e^- \rightarrow \mu^+\mu^-) [1 + \text{QCD correction}] , \end{aligned} \quad (12)$$

where  $N_c = 3$  is the number of quark colors and (neglecting the muon mass)

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} = 6.1 \text{ nb} \quad (13)$$

for  $s \equiv E_{\text{c.m.}}^2 = (3770 \text{ MeV})^2$ . This contribution will be referred to as *continuum*. In addition  $\tau^+\tau^-$  pair production would account for

$$\sigma(e^+e^- \rightarrow \tau^+\tau^-) = \left(1 - \frac{4m_\tau^2}{s}\right)^{1/2} \left(1 + \frac{2m_\tau^2}{s}\right) \sigma(e^+e^- \rightarrow \mu^+\mu^-) \quad (14)$$

or about 2.9 nb if initial-state-radiation effects were neglected. Such effects will change the observed cross section. The separation of  $\tau^+\tau^-$  from  $q\bar{q}$  final states requires good understanding of detector sensitivities and  $q\bar{q}$  fragmentation.

The couplings of virtual photons to two pseudoscalar mesons  $P$  or one pseudoscalar and one vector  $V$  can be evaluated straightforwardly [31, 32]. They are proportional to  $\text{Tr}(Q[P_1, P_2])$  or  $\text{Tr}(Q\{P, V\})$ , where  $Q = \text{Diag}(2/3, -1/3, -1/3)$  and [for one  $(\eta, \eta')$  mixing scheme which fits other data well [33, 34]]

$$P \equiv \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + 2\frac{\eta'}{\sqrt{6}} \end{bmatrix}, \quad (15)$$

$$V \equiv \begin{bmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{bmatrix}. \quad (16)$$

These couplings lead to the characteristic continuum ( $\gamma^*$ ) production ratios:

$$\begin{aligned} \pi^+\pi^- : K^+K^- : K^0\bar{K}^0 &= 1 : 1 : 0 ; \\ \omega\pi^0 : \rho\eta : K^{*0}\bar{K}^0 : \rho\pi : \phi\eta : K^{*+}K^- : \omega\eta : \phi\pi^0 &= 1 : \frac{2}{3} : \frac{4}{9} : \frac{4}{27} : \frac{1}{3} : \frac{1}{9} : \frac{2}{27} : 0. \end{aligned} \quad (17)$$

Here I have neglected a small admixture of nonstrange quarks in the  $\phi$  responsible for its  $\rho\pi$  decay. The contribution of the isovector photon ( $G = +$ ) dominates:  $\sigma(2\pi + 4\pi + \dots) = 9\sigma(3\pi + 5\pi + \dots)$ . Thus one has several signatures of continuum production which can be examined at a single energy, e.g., at the  $\psi''$  peak. Of course, a better way to study continuum contributions is to change the c.m. energy to one where resonance production cannot contribute. The CLEO Collaboration has done this, studying hadron production at a c.m. energy of 3670 MeV with a sample of 21 pb $^{-1}$  [35], and results are currently being analyzed.

## VI. IS THERE A SIGNIFICANT NON- $D\bar{D}$ CROSS SECTION?

At most 600 keV of the  $\psi''$  total width of  $23.6 \pm 2.7$  MeV is due to radiative decays, and perhaps as much as another 100 keV is due to  $J/\psi\pi\pi$  decays. Along with the predominant  $D\bar{D}$  decays, are these contributions enough to account for the total  $\psi''$  width?

In Fig. 2 the BES data [17] on  $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  are displayed, along with the results of a fit to the resonance shape using conventional Blatt-Weisskopf angular momentum barrier factors [36, 37]. The fit obtains  $\sigma_{\text{pk}} = 7.7 \pm 1.1$  nb, with other central values  $M = 3772$  MeV,  $\Gamma = 23.2$  MeV, and  $R_{\text{bg}} = 2.17 + 2.36(E_{\text{c.m.}} - 3.73 \text{ GeV})\theta(E_{\text{c.m.}} - 3.73 \text{ GeV})$ , where the threshold energy of 3.73 GeV is held fixed in the fit [38].

BES measurements in the energy range near the  $\psi''$  resonance are consistent with the expectation (12). In the c.m. energy range 2–3 GeV the value  $R = 2.26 \pm 0.14$  is obtained [4]. This would imply  $\sigma(e^+e^- \rightarrow \text{hadrons}) \simeq 13.8 \pm 0.9$  nb at 3770 MeV, quite a bit more than  $\sigma(\psi'')$ . Thus inferring  $\sigma(\psi'')$  by subtracting known processes such as continuum from a cross section measured at a single energy may carry large systematic errors.

Taking as an illustrative value  $\sigma(D\bar{D}) \leq 6.5$  nb and comparing it with the overall average of  $\sigma(\psi'') = 7.9$  nb in Table II, one is invited to consider how to account for a deficit of 1.4 nb, or 18% of the total. While this quantity is not statistically significant, it is interesting to speculate on possible sources pending (1) a scan of the  $\psi''$  peak to measure  $\sigma(\psi'')$  more accurately and (2) reduction of the error on  $\sigma(D\bar{D})$ .

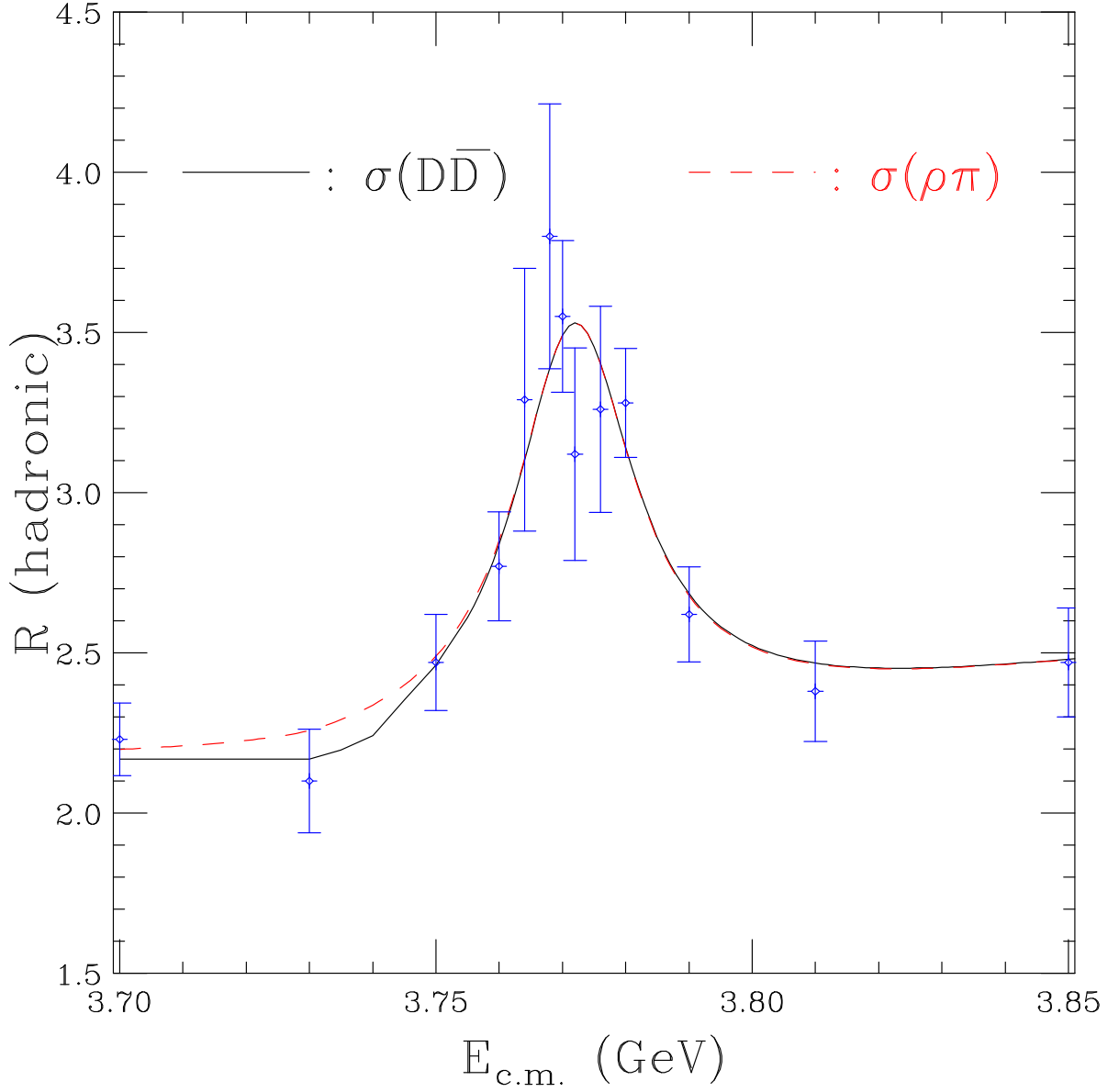


Figure 2: Fit to the  $\psi''$  peak in BES data [17]. Solid line denotes expected line shape for a  $D\bar{D}$  final state, incorporating appropriate centrifugal barrier terms, while dashed line denotes expected line shape for  $\rho\pi$  final state.

## VII. POTENTIAL INFORMATION FROM CHARMLESS MODES

The possibilities for detecting *individual* charmless decay modes of the  $\psi''$  were raised, for example, in Refs. [7] and [19]. Here I stress that more inclusive measurements at the  $\psi''$  also may be of use.

Consider a model in which the re-annihilation of charmed quarks in  $D^0\bar{D}^0$  and  $D^+D^-$  into states containing  $u$ ,  $d$ ,  $s$  accounts for the difference between  $\sigma(D\bar{D})$  and  $\sigma(\psi'')$ . The possibility of such re-annihilation was considered some time ago [18] both as a source of non- $D\bar{D}$  decays of the  $\psi''$  and as a possible source of non- $B\bar{B}$  decays of the  $\Upsilon(4S)$ . The latter do not appear to occur at any level above a few percent [39]. As an illustration, we present in Fig. 3 the case in which such re-annihilation accounts for 18% of the peak  $R$  value at  $M(\psi'') = 3772 \text{ MeV}/c^2$ . A relative phase  $\delta$  between the reannihilation amplitude and the continuum was defined in such a way that  $\delta = 0$  corresponds to constructive interference at the resonance peak. Details of this model are given in the Appendix.

Several features of this model are worth noting.

- The re-annihilation of  $D^+D^-$  and  $D^0\bar{D}^0$  pairs into light quarks will favor leading  $d\bar{d}$  and  $u\bar{u}$  pairs, with amplitudes in the ratio  $d\bar{d} : u\bar{u} \simeq 2 : 3$  in line with the cross section ratio  $\sigma(D^+D^-) : \sigma(D^0\bar{D}^0)$  (see the Appendix). The fragmentation of these quarks will populate hadronic final states in somewhat different proportions than the usual continuum process in which quark pairs are produced by the virtual photon with amplitudes proportional to their charges.
- The re-annihilation largely favors isoscalar ( $I = 0$ ) odd-G-parity final states, so one should see more effects of interference between re-annihilation and continuum in odd G ( $3\pi, 5\pi, \eta 3\pi, \eta' 3\pi, \dots$ ) states than in even-G ones ( $2\pi, 4\pi, \eta 2\pi, \dots$ ). This interference is particularly pronounced because the larger odd-G reannihilation amplitude is interfering with a smaller odd-G continuum amplitude.
- The effects of re-annihilation on the continuum contributions are quite subtle if  $\delta = 0$ , especially in the dominant  $I = 1$  (even-G-parity) channel. They are proportionately greater in the  $I = 0$  (odd-G-parity) non-strange channel (consisting, for example, of odd numbers of pions).
- The re-annihilation may be similar to that which accounts for enhanced penguin contributions in  $B$  decays, particularly in the  $b \rightarrow s$  subprocess through the chain  $b \rightarrow c\bar{c}s \rightarrow q\bar{q}s$ , where  $q = (u, d, s)$  (see also [7]). If this is so, one should look for an enhancement of  $\eta'$  production as appears to occur in both inclusive and exclusive  $B$  decays.
- As especially evident for the examples of non-zero  $\delta$ , measurement of the cross section in semi-inclusive channels with definite G-parity and especially odd G (such as final states with an odd number of pions) may show interesting interference patterns.

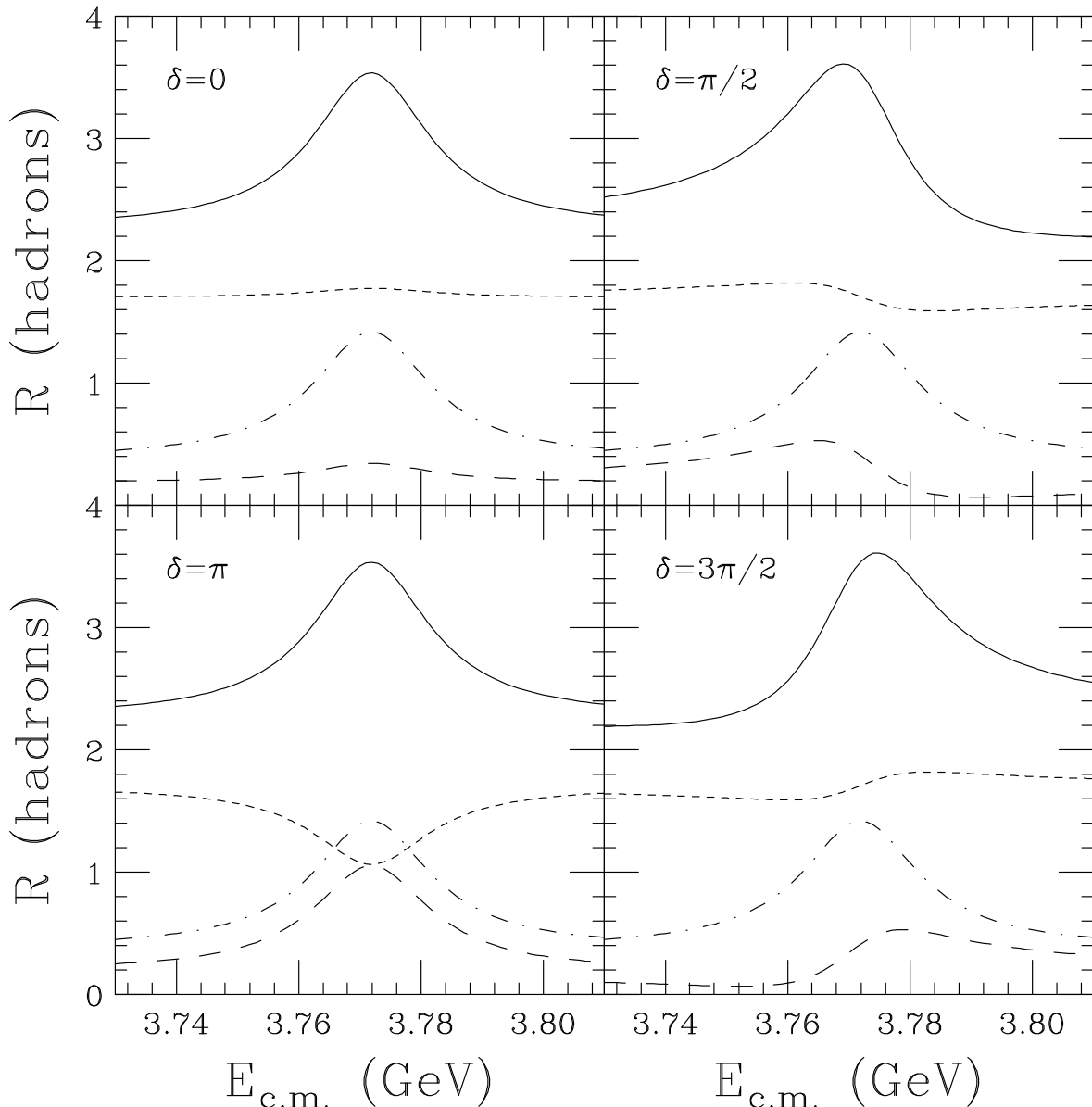


Figure 3: Contributions to  $R$  in the vicinity of the  $\psi''$  resonance energy. Solid curves: total, constrained to have a value of 3.53 at  $M(\psi'') = 3.772 \text{ GeV}/c^2$ . Short-dashed curves:  $I = 1$  continuum interfering with  $I = 1$  contribution from  $D\bar{D}$  reannihilation. Long-dashed curves:  $I = 0$  non-strange continuum interfering with  $I = 0$  nonstrange contribution from  $D\bar{D}$  reannihilation. Dot-dashed curves:  $D\bar{D}$  resonance contribution, taken to contribute 82% of resonance peak cross section, plus  $s\bar{s}$  continuum.

A Breit-Wigner amplitude is normally taken to be purely imaginary at its peak. I incorporate this phase into the definition of  $\delta$ . Thus  $\delta = \pi/2$  would correspond to no additional phase associated with the re-annihilation process. It was speculated in Refs. [7] and [40] (see also Refs. [41, 42, 43, 44]) that such an additional phase could be present and, if related to a similar phase in  $B$  decays, might account for a strong phase in penguin  $b \rightarrow s$  amplitudes. A recent fit to  $B \rightarrow PP$  decays, where  $P$  denotes a charmless pseudoscalar meson [45], finds such a phase to be in the range of roughly  $-20^\circ$  to  $-50^\circ$ . This would correspond to taking  $\delta$  in the range of  $40^\circ$  to  $70^\circ$ .

I now return briefly to a discussion of *specific* exclusive charmless decay modes of the  $\psi''$ . It was suggested in Ref. [7] that some  $\psi'$  decay modes might be suppressed via S–D mixing. In that case, they should show up in  $\psi''$  decays. Foremost among these was the  $\psi' \rightarrow \rho\pi$  decay. A prediction was made for  $\phi = (12 \pm 2)^\circ$  that  $\Gamma(\psi'' \rightarrow \rho\pi) = (9.8 \pm 3.0)$  keV, corresponding to  $\mathcal{B}(\psi'' \rightarrow \rho\pi) = (4.1 \pm 1.4) \times 10^{-4}$ . It was then pointed out [10] that because of possible interference with continuum, decays such as  $\psi'' \rightarrow \rho\pi$  might manifest themselves in various ways depending on relative strong phases, even as a *dip* in  $\sigma(e^+e^- \rightarrow \rho\pi)$  at  $M(\psi'')$ .

An estimate of suppression of a  $\psi'$  decay rate may be constructed using the quantity

$$Q(f) \equiv \frac{\mathcal{B}(J/\psi \rightarrow e^+e^-) \mathcal{B}(\psi' \rightarrow f)}{\mathcal{B}(\psi' \rightarrow e^+e^-) \mathcal{B}(J/\psi \rightarrow f)} \quad (18)$$

for any final state  $f$ . If  $Q(f) < 1$ , the decay  $\psi' \rightarrow f$  is suppressed relative to  $J/\psi \rightarrow f$ , where the ratio of leptonic widths is an attempt to correct for differing probabilities for  $c\bar{c}$  annihilation. Foremost among the  $\psi'$  modes which are candidates for some suppression is  $\psi' \rightarrow \rho\pi$ ; this and several other modes have been tabulated, for example, in Ref. [46], based on a compilation of BES results.

The suppression mechanism is ascribed in Ref. [7] to a cancellation of the S- and D-wave contributions to  $\psi' \rightarrow f$ :

$$\langle f|\psi' \rangle = \langle f|2^3S_1 \rangle \cos \phi - \langle f|1^3D_1 \rangle \sin \phi = 0, \quad (19)$$

with a corresponding enhancement of the  $\psi'' \rightarrow f$  decay:

$$\langle f|\psi'' \rangle = \langle f|2^3S_1 \rangle \sin \phi + \langle f|1^3D_1 \rangle \cos \phi = \langle f|2^3S_1 \rangle / \sin \phi. \quad (20)$$

One can then use the predicted  $\langle f|2^3S_1 \rangle$  matrix element and the measured  $\psi' \rightarrow f$  rate (whether an upper bound or observed) to predict  $\langle f|\psi'' \rangle$  and hence the  $\psi'' \rightarrow f$  rate.

All of the suppressed  $\psi''$  modes discussed in Refs. [7] and [46] are prime candidates for detection in  $\psi''$  decays. However, the interference proposed in Ref. [10] can actually lead to a *suppression* of some modes relative to the rate expected from continuum. A firm conclusion will have to await more data both at the resonance and as a function of c.m. energy in the neighboring continuum. It was anticipated in Ref. [7] that if one were to account for any “missing”  $\psi'$  decay modes by mixing with the  $\psi''$ , such an effect need not contribute more than a percent or two to the total  $\psi''$  width.

## VIII. CONCLUSIONS

Some non- $D\bar{D}$  decay modes of the  $\psi''$  exist and are interesting in their own right, such as  $\ell^+\ell^-$  pairs,  $\gamma\chi_{cJ}$  and  $J/\psi\pi\pi$ . They tell us about mixing between S-waves, D-waves, and open  $D\bar{D}$  channels.

Most final states at the  $\psi''$  are from continuum production. Their yields will not vary much with beam energy unless their continuum production amplitudes are interfering with a genuine Breit-Wigner contribution from the  $\psi''$ . This interference is most likely to show up in odd-G-parity final states.

The suggestion that the “missing”  $\psi'$  decays, like  $\rho\pi$ , should show up instead at the  $\psi''$ , is being realized, if at all, in a more subtle manner, and does not illuminate the question of whether a substantial fraction (at least several percent) of the  $\psi''$  cross section is non- $D\bar{D}$ . I predict a substantial enhancement of  $\eta'$  production in charmless  $\psi''$  final states if the re-annihilation of  $D\bar{D}$  into light quarks is related to the generation of a  $b \rightarrow s$  penguin amplitude in  $B$  decays.

The measurement of the continuum cross section at 3670 MeV is expected to yield  $R = 2(1 + \alpha_S/\pi + \dots)$ . Its value, when extrapolated to 3770 MeV, is relevant to whether there is a cross section deficit at the  $\psi''$ .

Resolution of these questions is likely to require a measurement of the  $\psi''$  resonance shape, with an eye to possibly different behavior in different channels.

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## APPENDIX: MODEL FOR $D\bar{D}$ RE-ANNIHILATION

The BES Collaboration’s continuum value  $R = 2.26 \pm 0.14$  [4] averaged over  $2 \text{ GeV} \leq E_{\text{c.m.}} \leq 3 \text{ GeV}$  is consistent with the expected value of 2 times a QCD correction (and also is consistent with the background level obtained in the fit of Fig. 2 to the  $\psi''$  cross section). I take  $R = 2.26$  for illustration. Of this, one expects  $R(s\bar{s}) = (1/6)(2.26) = 0.377$ . The non-strange contributions may be decomposed into a 9:1 ratio of  $I = 1$  and  $I = 0$  contributions denoted by  $R_1$  and  $R_0$ , since  $(Q_u - Q_d)^2 = 9(Q_u + Q_d)^2$ . Thus  $R_1 = (5/6)(2.26)(9/10) = 1.695$  and  $R_0 = (5/6)(2.26)(1/10) = 0.188$ . The  $I = 1$  continuum corresponds to an isovector photon and even-G-parity states, while the  $s\bar{s}$  and  $I = 0$  nonstrange continua correspond to an isoscalar photon and odd-G-parity states. The  $s\bar{s}$  continuum is unlikely to lead to final states consisting exclusively of pions; one expects at least one  $K\bar{K}$  pair in its hadronic products.

A model of re-annihilation is to assume that the amplitude for  $\psi'' \rightarrow D\bar{D} \rightarrow$  (non-charmed final states) proceeds via a  $D\bar{D}$  loop diagram characterized by an amplitude

proportional to  $(p^*)^3$ , where  $p^*$  is the magnitude of the c.m. 3-momentum of either  $D$ . For  $\psi'' \rightarrow D^+ D^-$ ,  $p_{+-}^* = 250.0 \text{ MeV}/c$ , while for  $\psi'' \rightarrow D^0 \bar{D}^0$ ,  $p_{00}^* = 283.6 \text{ MeV}/c$ . The re-annihilation amplitude  $A_d^R$  into  $d\bar{d}$  pairs and the amplitude  $A_u^R$  into  $u\bar{u}$  pairs are then expected to be in the ratio  $A_d^R/A_u^R = (p_{+-}^*/p_{00}^*)^3 = 0.685$ , and the corresponding ratio for isovector and nonstrange isoscalar contributions  $A_1^R$  and  $A_0^R$  is

$$\frac{A_1^R}{A_0^R} = \frac{A_u^R - A_d^R}{A_u^R + A_d^R} = \frac{1 - 0.685}{1 + 0.685} = 0.187 \quad . \quad (21)$$

One may assume for simplicity that the re-annihilation amplitudes into  $I = 0$  and  $I = 1$  final states have the same strong phase  $\delta$  relative to the continuum, modulated by a Breit-Wigner amplitude  $f_B$  defined to be unity at the resonance peak. In the vicinity of the  $\psi''$  mass  $M_0$  one may then write the amplitudes  $A_1$  and  $A_0$  for the isovector and nonstrange isoscalar contributions to  $R$  as functions of c.m. energy  $E$ :

$$A_1 = 0.187b_0 e^{i\delta} f_B(E) + \sqrt{R_1} \quad , \quad A_0 = b_0 e^{i\delta} f_B(E) + \sqrt{R_0} \quad , \quad (22)$$

where the amplitudes have been defined such that their squares yield their contributions to  $R$ , and

$$f_B(E) = [d_B(E)]^{-1} \quad , \quad d_B(E) \equiv 1 + \frac{2i(M_0 - E)}{\Gamma} \quad . \quad (23)$$

The values  $M_0 = 3772 \text{ MeV}/c^2$  and  $\Gamma = 23.2 \text{ MeV}$  are taken from the fit of Sec. VI. This same fit implies a peak value  $R(M_0) = 3.53$  which will be taken as a constraint when choosing the arbitrary constant  $b_0$ .

The continuum away from the peak accounts for  $R = 2.26$ , so one must provide a total resonant contribution of  $\Delta R_{\text{pk}} = 3.53 - 2.26 = 1.27$ . For illustration, consider  $D\bar{D}$  pairs to provide 82% of this value, or  $\Delta R_{\text{pk}}^{D\bar{D}} = 1.04$ . This contribution will be modulated by  $|f_B(E)|^2$ . There will be a constant  $s\bar{s}$  continuum contribution of  $\Delta R^{s\bar{s}} = 0.38$ , and contributions from the isovector and non-strange isoscalar amplitudes  $A_I$  above, leading to a total of

$$R(E) = |A_1|^2 + |A_0|^2 + \Delta R_{\text{pk}}^{D\bar{D}} |f_B(E)|^2 + \Delta R^{s\bar{s}} \quad . \quad (24)$$

For  $\delta = 0$ , a relatively modest value of  $b_0 = 0.15$  provides the additional contribution needed to account for the missing 18% of the  $\psi''$  peak cross section. The corresponding values for  $\delta = \pi/2, \pi, 3\pi/2$  are 0.47, 1.46, and 0.47, respectively. It is interesting that the choice  $\delta = \pi$ , while implying large individual effects in the  $I = 1$  and nonstrange  $I = 0$  channels, leads to an identical total cross section shape when we demand  $R(M_0) = 3.53$ . This result may be demonstrated analytically with the help of the identity  $\text{Re } f_B = |f_B|^2$ .

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