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CHARMLESS $B \to PP$ DECAYS USING FLAVOR SU(3) SYMMETRY *

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Abstract

The decays of B mesons to a pair of charmless pseudoscalar (P) mesons are analyzed within a framework of flavor SU(3). Symmetry breaking is taken into account in tree (T) amplitudes through ratios of decay constants; exact SU(3) is assumed elsewhere. Acceptable fits to $B \to \pi\pi$ and $B \to K\pi$ branching ratios and CP asymmetries are obtained with tree, color-suppressed (C), penguin (P), and electroweak penguin (P_{EW}) amplitudes. Crucial additional terms for describing processes involving η and η' include a large flavor-singlet penguin amplitude (S) as proposed earlier and a penguin amplitude P_{tu} associated with intermediate t and u quarks. For the $B^+ \to \pi^+\eta'$ mode a term S_{tu} associated with intermediate t and u quarks also may be needed. Values of the weak phase γ are obtained consistent with an earlier analysis of $B \to VP$ decays, where V denotes a vector meson, and with other analyses of CKM parameters.

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I. INTRODUCTION

A central objective of the study of B meson decays is to help determine the phases and magnitudes of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, through the measurement of branching ratios and CP-violating observables. It is important to have accurate and self-consistent information on CKM matrix elements if they are ever to be compared with fundamental theories predicting them. At present no such theories exist. A further objective is to learn about possible new physics at higher mass scales, affecting rare B decays by giving observables that appear to be inconsistent with others. One wishes to know whether there are any sources of CP violation other than the phases in the CKM matrix first proposed by Kobayashi and Maskawa [1].

Charmless B meson decays, many of whose branching ratios and CP asymmetries (CPA's) have been measured to good accuracy, are an interesting and useful set of modes. Following the method presented in Ref. [2] for B decays into a vector meson (V) and a pseudoscalar meson (P), we analyze observables in B decays into two pseudoscalar mesons ($B \to PP$ decays) in the present paper. From the results of fits involving a small set of invariant amplitudes, one can extract information about the parameters in theory, compare with other known constraints, and predict as-yet-unreported observables. In particular, the amplitudes contributing to two-body hadronic charmless B decays involve only one nontrivial weak phase γ within the standard model (SM). In a previous analysis of $B \to VP$ decays [2], we found good agreement between the favored range of γ from a fit to the VP modes and that from fits to CKM parameters [3] based on other measurements. It is therefore of great interest to see if the PP modes give a consistent result.

In the present analysis, we take flavor SU(3) symmetry [4–10] as a working hypothesis. Motivated by factorization in tree-level amplitudes, we take symmetry breaking due to decay constant differences into account in these amplitudes when relating strangeness-conserving and strange-changing processes. We leave the issue of SU(3) symmetry breaking in penguin-type amplitudes to experimental data. As a test, one can compare the $B^+ \to \pi^+ K^0$ mode (involving purely a strangeness-changing QCD penguin amplitude) with the $B^+ \to K^+ \overline{K}^0$ and $B^0 \to K^0 \overline{K}^0$ modes (involving purely strangeness-conserving QCD penguin amplitudes). In the limit of flavor $SU(3)$ symmetry, they should differ by a ratio of CKM factors, V_{cs}/V_{cd}. If penguin amplitudes P_{tu} associated with intermediate t and u quarks are important, the predictions for these modes will be affected.

We find acceptable fits to $B \to \pi\pi$ and $B \to \pi K$ branching ratios and CP asymmetries with a combination of tree, color-suppressed (C), penguin (P), and electroweak penguin (P_{EW}) amplitudes. In contrast to an earlier analysis of $B \to PP$ decays [11], in order to describe these decays we must introduce a rather large value of $|C/T|$ and a non-trivial relative phase between C and T. A large $|C/T|$ value could improve agreement between the QCD factorization approach and experiment [12]. Our conclusion is driven in part by the large branching ratio for $B^0 \to \pi^0\pi^0$ reported recently [13, 14].

The data on processes involving η and η' also have made some progress since our earlier analysis [11]. Crucial additional terms for describing these decays include not only a large flavor-singlet penguin amplitude (S) as proposed (e.g.) in Refs. [15], but also a penguin amplitude P_{tu} associated with intermediate t and u quarks, and (for the $B^+ \to \pi^+\eta'$ mode) a term S_{tu} associated with intermediate t and u quarks.

Values of the weak phase $\gamma \simeq 60^\circ$ are obtained consistent with our earlier analysis of $B \to VP$ decays [2]. Other robust aspects of our fit include the magnitude of the strangeness-changing penguin amplitude, the strong phase of the tree amplitude relative to the penguin ($\sim 20^\circ$–30°), the size of electroweak penguin contributions, the correlation of a large direct CP asymmetry in $B^0 \to \pi^+\pi^-$ with a small one in $B^0 \to \pi^-K^+$, the correct prediction of signs and magnitudes of all other measured direct CP asymmetries as well, and a fairly large negative value of the time-dependent CP asymmetry parameter $S_{\pi\pi}$. Some other aspects of the fit are less likely to remain unchanged in the face of further data; we shall comment on them in due course.

We review our conventions for the quark content of pseudoscalar mesons and topological amplitudes in Section II. Experimental data and topological decompositions of decay amplitudes are presented in Section III. In Section IV we enumerate the data that will be used in our χ^2 fit. Two fits to $\pi\pi$ and πK observables are presented in Section V, while modes with η or η' in the final state are included in Section VI. We comment on robust and less-stable aspects of the fits in Section VII. Based upon our fitting results, we discuss our predictions for as-yet-unreported modes in Section VIII. Comparisons with other recent approaches (e.g., Refs. [12, 16–18]) are pursued in Section IX. We summarize our findings in Section X.

II. NOTATION

Our quark content and phase conventions [7, 9] are:

- *Bottom mesons*: $B^0 = d\bar{b}$, $\overline{B}^0 = b\bar{d}$, $B^+ = u\bar{b}$, $B^- = -b\bar{u}$, $B_s = s\bar{b}$, $\overline{B}_s = b\bar{s}$;
- *Charmed mesons*: $D^0 = -c\bar{u}$, $\overline{D}^0 = u\bar{c}$, $D^+ = c\bar{d}$, $D^- = d\bar{c}$, $D_s^+ = c\bar{s}$, $D_s^- = s\bar{c}$;
- *Pseudoscalar mesons*: $\pi^+ = u\bar{d}$, $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\pi^- = -d\bar{u}$, $K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\overline{K}^0 = s\bar{d}$, $K^- = -s\bar{u}$, $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$, $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$;

The η and η' correspond to octet-singlet mixtures

$$\eta = \eta_8 \cos\theta_0 - \eta_1 \sin\theta_0 \ , \quad \eta' = \eta_8 \sin\theta_0 + \eta_1 \cos\theta_0 \ , \tag{1}$$

with $\theta_0 = \sin^{-1}(1/3) = 19.5^\circ$.

In the present approximation there are seven types of amplitudes: a "tree" contribution t; a "color-suppressed" contribution c; a "penguin" contribution p; a "singlet penguin" contribution s, in which a color-singlet $q\bar{q}$ pair produced by two or more gluons or by a Z or γ forms an $SU(3)$ singlet state; an "exchange" contribution e, an "annihilation" contribution a, and a "penguin annihilation" contribution pa. These amplitudes contain both the leading-order and electroweak penguin contributions, and appear in the independent combinations

$$\begin{aligned} t &\equiv T + P_{\rm EW}^C \ , & c &\equiv C + P_{\rm EW} \ , \\ p &\equiv P - P_{tu} - \tfrac{1}{3} P_{\rm EW}^C \ , & s &\equiv S - S_{tu} - \tfrac{1}{3} P_{\rm EW} \ , \\ a &\equiv A \ , & e + pa &\equiv E + PA \ , \end{aligned} \tag{2}$$

where the capital letters denote the leading-order contributions [7–9, 15] while $P_{\rm EW}$ and $P_{\rm EW}^C$ are respectively color-favored and color-suppressed electroweak penguin amplitudes [8]. We shall neglect smaller terms [19, 20] $PE_{\rm EW}$ and $PA_{\rm EW}$ [(γ, Z)-exchange and (γ, Z)-direct-channel electroweak penguin amplitudes]. We shall denote $\Delta S = 0$ transitions by unprimed quantities and $|\Delta S| = 1$ transitions by primed quantities. The hierarchy of these amplitudes can be found in Ref. [21]. By writing QCD and flavor-singlet penguins as $P - P_{tu}$ and $S - S_{tu}$, we adopt the so-called c-quark convention, in which the heavy top quark is integrated out from the theory. For penguin-type amplitudes, we use the unitarity relation $V_{tb}^* V_{td} + V_{cb}^* V_{cd} + V_{ub}^* V_{ud} = 0$ to remove any top quark dependence. The $V_{ub}^* V_{ud}$ term of the top quark mediated penguin is combined with the up quark mediated penguin to form P_{tu} or

S_{tu}. Similarly, the $V_{cb}^* V_{cd}$ term is united with the charm quark mediated penguin into P or S. As a consequence, the strangeness-conserving P and S and strangeness-changing P' and S' penguin amplitudes have real weak phases in our discussions. The relation between the c-quark convention and the t-quark convention, where the c quark dependence is removed instead, can be found in, e.g., Ref. [23].

The partial decay width of two-body B decays is

$$\Gamma(B \to M_1 M_2) = \frac{p_c}{8\pi m_B^2} |\mathcal{A}(B \to M_1 M_2)|^2 \ , \tag{3}$$

where p_c is the momentum of the final state meson in the rest frame of B, m_B is the B meson mass, and M_1 and M_2 can be either pseudoscalar or vector mesons. Using Eq. (3), one can extract the invariant amplitude of each decay mode from its experimentally measured branching ratio. To relate partial widths to branching ratios, we use the world-average lifetimes $\tau^+ = (1.653 \pm 0.014)$ ps and $\tau^0 = (1.534 \pm 0.013)$ ps computed by the LEPBOSC group [22]. Unless otherwise indicated, for each branching ratio quoted we imply the average of a process and its CP-conjugate.

III. AMPLITUDE DECOMPOSITIONS AND EXPERIMENTAL RATES

The experimental branching ratios and CP asymmetries on which our analysis is based are listed in Tables I and II. Contributions from the CLEO [24–27], BaBar [13, 28–32, 35–37], and Belle [14, 36, 38–46] Collaborations are included [47]. In order to implement upper bounds in a consistent manner we have computed our own experimental averages for $B^+ \to \pi^+ \eta'$ and $B^0 \to \eta K^0$. These two modes were observed by BaBar with a significance of 3.4 and 3.3 standard deviations, respectively.

We list theoretical predictions and averaged experimental amplitudes for charmless $B \to PP$ decays involving $\Delta S = 0$ transitions in Table III and those involving $|\Delta S| = 1$ transitions in Table IV. Theoretical predictions are shown in terms of topological amplitudes t, c, p and s while e, a and pa contributions are neglected. They are expected to be suppressed by a factor of order $1/m_b$ relative to tree and penguin amplitudes [48]. A suppression factor proportional to f_B/m_b was suggested in [7, 8]. Future measurements of the $B^0 \to K^+ K^-$ decay mode which only receives contributions from exchange and penguin annihilation diagrams will test this suppression.

TABLE I: Experimental branching ratios of selected $\Delta S = 0$ decays of B mesons. CP-averaged branching ratios are quoted in units of 10^{-6}. Numbers in parentheses are upper bounds at 90 % c.l. References are given in square brackets. Additional lines, if any, give the CP asymmetry $\mathcal{A}_{CP}$ (second line) or $(\mathcal{S}, \mathcal{A})$ (second and third lines) for charged or neutral modes, respectively. The error in the average includes the scale factor S when this number is shown in parentheses.

	Mode	CLEO	BaBar	Belle	Average
$B^+ \to$	$\pi^+\pi^0$	$4.6^{+1.8+0.6}_{-1.6-0.7}$ [24]	$5.5^{+1.0}_{-0.9} \pm 0.6$ [28]	$5.0 \pm 1.2 \pm 0.5$ [14]	5.2 ± 0.8
		-	$-0.03^{+0.18}_{-0.17} \pm 0.02$ [28]	$-0.14 \pm 0.24^{+0.05}_{-0.04}$ [38]	-0.07 ± 0.14
	$K^+\overline{K}^0$	< 3.3 [24]	$1.1 \pm 0.75^{+0.14}_{-0.18}$ (< 2.5) [29]	< 3.3 [14]	< 2.5
	$\pi^+\eta$	$1.2^{+2.8}_{-1.2}$ (< 5.7) [25]	$5.3 \pm 1.0 \pm 0.3$ [30]	$5.4^{+2.0}_{-1.7} \pm 0.6$ [39]	4.9 ± 0.9
		-	$-0.44 \pm 0.18 \pm 0.01$ [30]	-	-0.44 ± 0.18
	$\pi^+\eta'$	$1.0^{+5.8}_{-1.0}$ (< 12) [25]	$2.7 \pm 1.2 \pm 0.3$ (< 4.5) [30]	< 7 [40]	2.4 ± 1.1 (< 4.5)
$B^0 \to$	$\pi^+\pi^-$	$4.5^{+1.4+0.5}_{-1.2-0.4}$ [24]	$4.7 \pm 0.6 \pm 0.2$ [31]	$4.4 \pm 0.6 \pm 0.3$ [14]	4.6 ± 0.4
		-	$\left\{\begin{array}{l} -0.40 \pm 0.22 \pm 0.03 \\ 0.19 \pm 0.19 \pm 0.05 \end{array}\right.$ [32]	$\left\{\begin{array}{l} -1.00 \pm 0.21 \pm 0.07 \\ 0.58 \pm 0.15 \pm 0.07 \end{array}\right.$ [41]	$\left\{\begin{array}{l} -0.70 \pm 0.30\,(S = 1.91) \\ 0.42 \pm 0.19\,(S = 1.52) \end{array}\right.$
	$\pi^0\pi^0$	< 4.4 [24]	$2.1 \pm 0.6 \pm 0.3$ [13]	$1.7 \pm 0.6 \pm 0.2$ [14]	1.9 ± 0.5
	K^+K^-	< 0.8 [24]	< 0.6 [31]	< 0.7 [14]	< 0.6
	$K^0\overline{K}^0$	< 3.3 [24]	$0.6^{+0.7}_{-0.5} \pm 0.1$ (< 1.8) [29]	< 1.5 [14]	< 1.5
	$\pi^0\eta$	$0.0^{+0.8}_{-0.0}$ (< 2.9) [25]	$0.7^{+1.1}_{-0.9} \pm 0.3$ (< 2.5) [33]	-	< 2.5
	$\pi^0\eta'$	$0.0^{+1.8}_{-0.0}$ (< 5.7) [25]	$1.0^{+1.4}_{-1.0} \pm 0.8$ (< 3.7) [33]	-	< 3.7
	$\eta\eta$	< 18 [27]	$-0.9^{+1.6}_{-1.4} \pm 0.7$ (< 2.8) [34]	-	< 2.8
	$\eta\eta'$	< 27 [27]	$0.6^{+2.1}_{-1.7} \pm 1.1$ (< 4.6) [34]	-	< 4.6
	$\eta'\eta'$	< 47 [27]	$1.7^{+4.8}_{-3.7} \pm 0.6$ (< 10) [34]	-	< 10

IV. χ^2 FIT AND DATA POINTS

We define for n experimental observables $X_i \pm \Delta X_i$ and the corresponding theoretical predictions $X_i^{\rm th}$,

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{X_i^{\rm th} - X_i}{\Delta X_i} \right)^2 \quad . \tag{4}$$

The data points are the branching ratios and the CP asymmetries. We write the corresponding theoretical predictions in terms of topological amplitudes and extract their magnitudes, weak phases and strong phases by minimizing χ^2.

Tables I and II contain a total of 26 data points, including 9 observables from $\Delta S = 0$ decays and 17 from $|\Delta S| = 1$ decays. The modes involving $\pi\pi$ and πK consist of the following 15 pieces of data:

- The $\pi^+\pi^0$ decay involving the t and c amplitudes provides two data points. Since both

TABLE II: Same as Table I for $|\Delta S| = 1$ decays of B mesons.

	Mode	CLEO	BaBar	Belle	Average
$B^+ \to$	$\pi^+ K^0$	$18.8^{+3.7+2.1}_{-3.3-1.8}$ [24]	$22.3 \pm 1.7 \pm 1.1$ [29]	$22.0 \pm 1.9 \pm 1.1$ [14]	21.8 ± 1.4
		$0.18 \pm 0.24 \pm 0.02$ [26]	$-0.05 \pm 0.08 \pm 0.01$ [29]	$0.07^{+0.09+0.01}_{-0.08-0.03}$ [42]	0.02 ± 0.06
	$\pi^0 K^+$	$12.9^{+2.4+1.2}_{-2.2-1.1}$ [24]	$12.8^{+1.2}_{-1.1} \pm 1.0$ [28]	$12.0 \pm 1.3^{+1.3}_{-0.9}$ [14]	12.5 ± 1.0
		$-0.29 \pm 0.23 \pm 0.02$ [26]	$-0.09 \pm 0.09 \pm 0.01$ [28]	$0.23 \pm 0.11^{+0.01}_{-0.04}$ [38]	$0.00 \pm 0.12\,(S = 1.79)$
	ηK^+	$2.2^{+2.8}_{-2.2}$ (< 6.9) [25]	$3.4 \pm 0.8 \pm 0.2$ [30]	$5.3^{+1.8}_{-1.5} \pm 0.6$ [39]	3.7 ± 0.7
		-	$-0.52 \pm 0.24 \pm 0.01$ [30]	-	-0.52 ± 0.24
	$\eta' K^+$	$80^{+10}_{-9} \pm 7$ [25]	$76.9 \pm 3.5 \pm 4.4$ [35]	$78 \pm 6 \pm 9$ [43]	77.6 ± 4.6
		$0.03 \pm 0.12 \pm 0.02$ [26]	$0.037 \pm 0.045 \pm 0.011$ [35]	$-0.015 \pm 0.070 \pm 0.009$ [44]	0.02 ± 0.04
$B^0 \to$	$\pi^- K^+$	$18.0^{+2.3+1.2}_{-2.1-0.9}$ [24]	$17.9 \pm 0.9 \pm 0.7$ [31]	$18.5 \pm 1.0 \pm 0.7$ [14]	18.2 ± 0.8
		$-0.04 \pm 0.16 \pm 0.02$ [26]	$-0.107 \pm 0.041 \pm 0.013$ [36]	$-0.088 \pm 0.035 \pm 0.018$ [36]	-0.09 ± 0.03
	$\pi^0 K^0$	$12.8^{+4.0+1.7}_{-3.3-1.4}$ [24]	$11.4 \pm 1.7 \pm 0.8$ [29]	$11.7 \pm 2.3^{+1.2}_{-1.3}$ [14]	11.7 ± 1.4
		-	$\begin{cases} 0.48^{+0.38}_{-0.47} \pm 0.06 \\ -0.40^{+0.28}_{-0.27} \pm 0.09 \end{cases}$ [37]		$\begin{cases} 0.48 \pm 0.42 \\ -0.40 \pm 0.29 \end{cases}$
	ηK^0	$0.0^{+3.2}_{-0.0}$ (< 9.3) [25]	$2.9 \pm 1.0 \pm 0.2$ (< 5.2) [30]	< 12 [45]	$2.5 \pm 1.0\,(S = 1.08)$ (< 5.2)
	$\eta' K^0$	$89^{+18}_{-16} \pm 9$ [25]	$60.6 \pm 5.6 \pm 4.6$ [35]	$68 \pm 10^{+9}_{-8}$ [43]	$65.2 \pm 6.2\,(S = 1.03)$
		-	$\begin{cases} 0.02 \pm 0.34 \pm 0.03 \\ -0.10 \pm 0.22 \pm 0.04 \end{cases}$ [35]	$\begin{cases} 0.43 \pm 0.27 \pm 0.05 \\ -0.01 \pm 0.16 \pm 0.04 \end{cases}$ [46]	$\begin{cases} 0.27 \pm 0.21 \\ -0.04 \pm 0.13 \end{cases}$

amplitudes have the same weak phase except for a small contribution from EWP, no significant CPA is expected.

- The $\pi^+\pi^-$ decay involves the t and p amplitudes with different weak phases. Time-dependent CPA's have been observed by both BaBar and Belle groups. Thus, this mode provide three data points.

- The $\pi^0\pi^0$ decay involving the c and p amplitudes only provides one data point because no CPA has been measured yet.

- The $\pi^+ K^0$ decay involving only the p' amplitude provides two data points, although no significant CPA is expected. This mode plays a dominant role in constraining the magnitude of the P' amplitude.

- The $\pi^0 K^+$ decay involving the p', t', and c' amplitudes provides two data points.

- The $\pi^- K^+$ decay involving the p' and t' amplitudes provides two data points.

- The $\pi^0 K^0$ decay involves the p' and c' amplitudes. Time-dependent CPA's have been reported by the BaBar group. Thus, this mode provides three data points.

TABLE III: Summary of predicted contributions to $\Delta S = 0$ decays of B mesons to two pseudoscalars. Amplitude magnitudes $|A_{\rm exp}|$ extracted from experiments are quoted in units of eV.

	Mode	Amplitudes	p_c (GeV)	$\|A_{\rm exp}\|^a$
$B^+ \to$	$\pi^+\pi^0$	$-\frac{1}{\sqrt{2}}(t+c)$	2.636	23.4 ± 1.7
	$K^+\overline{K}^0$	p	2.593	< 16.4
	$\pi^+\eta$	$-\frac{1}{\sqrt{3}}(t+c+2p+s)$	2.609	22.9 ± 2.0
	$\pi^+\eta'$	$\frac{1}{\sqrt{6}}(t+c+2p+4s)$	2.551	16.2 ± 3.8 (< 22.2)
$B^0 \to$	$\pi^+\pi^-$	$-(t+p)$	2.636	22.8 ± 1.1
	$\pi^0\pi^0$	$-\frac{1}{\sqrt{2}}(c-p)$	2.636	14.7 ± 1.8
	K^+K^-	$-(e+pa)$	2.593	< 8.3
	$K^0\overline{K}^0$	p	2.592	< 13.2
	$\pi^0\eta$	$-\frac{1}{\sqrt{6}}(2p+s)$	2.610	< 17.0
	$\pi^0\eta'$	$\frac{1}{\sqrt{3}}(p+2s)$	2.551	< 20.9
	$\eta\eta$	$\frac{\sqrt{2}}{3}(c+p+s)$	2.582	< 18.1
	$\eta\eta'$	$-\frac{\sqrt{2}}{3}(c+p+\frac{5}{2}s)$	2.522	< 23.4
	$\eta'\eta'$	$\frac{1}{3\sqrt{2}}(c+p+4s)$	2.460	< 35.0

[a] $|A_{\rm exp}|$ is defined by Eq. (3) as an amplitude related to a CP-averaged branching ratio quoted in Table I.

TABLE IV: Same as Table III for $|\Delta S| = 1$ decays of B mesons.

	Mode	Amplitudes	p_c (GeV)	$\|A_{\rm exp}\|$
$B^+ \to$	π^+K^0	p'	2.614	48.2 ± 1.6
	π^0K^+	$-\frac{1}{\sqrt{2}}(p'+t'+c')$	2.615	36.6 ± 1.5
	ηK^+	$-\frac{1}{\sqrt{3}}(s'+t'+c')$	2.588	19.9 ± 1.9
	$\eta' K^+$	$\frac{1}{\sqrt{6}}(3p'+4s'+t'+c')$	2.528	92.5 ± 2.7
$B^0 \to$	π^-K^+	$-(p'+t')$	2.615	45.7 ± 1.0
	π^0K^0	$\frac{1}{\sqrt{2}}(p'-c')$	2.614	36.6 ± 2.2
	ηK^0	$-\frac{1}{\sqrt{3}}(s'+c')$	2.587	17.0 ± 3.5 (< 24.6)
	$\eta' K^0$	$\frac{1}{\sqrt{6}}(3p'+4s'+c')$	2.528	88.0 ± 4.2

Successful SU(3) fits to modes with an η or η' in the final state require amplitudes beyond those mandated by the $\pi\pi$ and πK fits. A common feature of these modes, for example, is that they involve a flavor singlet amplitude s or s'. Moreover, uncertainties in η and η' wave functions and possible SU(3) breaking effects can affect such fits [12], so we list these 11 data points separately:

- The $\pi^+\eta$ mode involving the combination $t + c + 2p + s$ provides two data points.
- The $\pi^+\eta'$ mode involving the combination $t + c + 2p + 4s$ provides one data point.
- The ηK^+ mode involving the combination $s' + t' + c'$ provides two data points. Note that it does not contain p'; all three contributing amplitudes are comparable in size. One generally expects significant CPA as a result of the interference between tree-level and penguin-loop diagrams.
- The ηK^0 mode involving the combination $s' + c'$ provides one data point.
- The $\eta' K^+$ mode involving the combination $3p' + 4s' + t' + c'$ provides two data points.
- The $\eta' K^0$ mode provides three data points, including the CP-averaged branching ratio and time-dependent CPA's.

V. χ^2 FIT TO $\pi\pi$ AND πK MODES

To avoid complication from uncertainties in the flavor-singlet amplitudes, wave functions of η and η', and associated SU(3) breaking effects, we first fit the fifteen $\pi\pi$ and πK data points. A study restricted to $B \to K\pi$ decays based on similar assumptions was carried out in Refs. [49]. Guided by the relative importance of strangeness-conserving and strangeness-changing transitions, we choose T, C, P', and P_{tu} as our parameters.

We further fix the strong phase convention to be

$$T = |T|\, e^{i(\delta_T + \gamma)} \ , \tag{5}$$

$$C = |C|\, e^{i(\delta_T + \delta_C + \gamma)} \ , \tag{6}$$

$$P_{tu} = |P_{tu}|\, e^{i(\delta_{P_{tu}} + \gamma)} \ , \tag{7}$$

$$P' = -|P'| \ . \tag{8}$$

The phase convention is such that zero strong phases of T, C and P_{tu} amplitudes correspond to these amplitudes having a phase of γ with respect to the penguin-type amplitude P. Note that δ_C is defined as a relative strong phase between the C and T amplitudes. The extra minus signs for P' comes from the relative weak phase π between $P' = (V_{cs}/V_{cd})\,P$ and P amplitudes.

The expressions for the T', C', P'_{tu}, P, and P_{EW} are obtained from the above equations taking into account the following ratios

$$\frac{T'}{T} = \frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi} = \frac{\lambda}{1-\lambda^2/2}\frac{f_K}{f_\pi} \simeq 0.281\ , \tag{9}$$

$$\frac{C'}{C} = \frac{P'_{tu}}{P_{tu}} = \frac{V_{us}}{V_{ud}} = \frac{\lambda}{1-\lambda^2/2} \simeq 0.230\ , \tag{10}$$

$$\frac{P}{P'} = \frac{P_{EW}}{P'_{EW}} = \frac{V_{cd}}{V_{cs}} = -\frac{\lambda}{1-\lambda^2/2} \simeq -0.230\ , \tag{11}$$

where $\lambda = 0.224$ [50]. Therefore, a major SU(3) breaking effect from the decay constant difference is included for tree-type diagrams. No such effect is considered for penguin-type amplitudes because we do not expect factorization to work in such cases. The ratio $P_{EW}/P'_{EW} = V_{cd}/V_{cs}$ is being used for the simplicity of our analysis. We checked that using $P_{EW}/P'_{EW} = V_{td}/V_{ts}$ (to express t-quark dominance of EWP amplitudes) does not affect the results in any significant way.

We explore two approaches to fitting $\pi\pi$ and πK data points. One of them (Fit II) uses Eqs. (2) for the topological amplitudes t, c and p:

$$t \equiv T + P^C_{\rm EW}\ , \tag{12}$$

$$c \equiv C + P_{\rm EW}\ , \tag{13}$$

$$p \equiv P - P_{tu} - \frac{1}{3}P^C_{\rm EW}\ . \tag{14}$$

Using these three equations, we can write the amplitude for any $\pi\pi$ or πK decay mode in Tables III and IV in terms of 9 parameters: weak phase γ, topological amplitudes $|T|$, $|C|$, $|P_{tu}|$, and $|P'|$, strong phases δ_T, δ_C, and $\delta_{P_{tu}}$, and a parameter δ_{EW}. The latter relates EW penguins to tree-level diagrams and will be defined below.

In the other approach (Fit I) we use the fact that P_{tu} has the same weak factors as tree-level amplitudes T and C. This allows us to absorb the P_{tu} penguin into redefined $\tilde{T}$ and $\tilde{C}$ amplitudes:

$$\tilde{T} \equiv T - P_{tu}\ , \tag{15}$$

$$\tilde{C} \equiv C + P_{tu} \ . \tag{16}$$

By writing topological amplitudes t, c and p in terms of $\tilde{T}$, $\tilde{C}$ and P as

$$t = \tilde{T} + P^C_{\rm EW} \ , \tag{17}$$

$$c = \tilde{C} + P_{\rm EW} \ , \tag{18}$$

$$p = P - \frac{1}{3} P^C_{\rm EW} \ , \tag{19}$$

we still get the correct expressions for $B \to \pi\pi$ and $B \to \pi K$ decay amplitudes, except for $B^+ \to \pi^+ K^0$ and $B^+ \to \pi^0 K^+$. In these two cases the resulting expressions differ from the correct ones by a P'_{tu} term. Compared to the dominant QCD penguin P', this term is expected to be small. Thus, Fit I gives a good description of $B \to \pi\pi$ and $B \to \pi K$ modes in terms of redefined tree-level amplitudes. The advantage of this approach is a smaller number of fit parameters as both $|P_{tu}|$ and its strong phase $\delta_{P_{tu}}$ are absorbed into $\tilde{T}$ and $\tilde{C}$. Just 7 fit parameters are used in Fit I: weak phase γ, amplitudes $|\tilde{T}|$, $|\tilde{C}|$, and $|P'|$, strong phases $\delta_{\tilde{T}}$ and $\delta_{\tilde{C}}$, and the δ_{EW} parameter.

A relation between the EWP amplitudes and the tree-type diagrams has been found in Ref. [51] using Fierz transformation to relate EWP operators with tree-level operators. Explicitly, we have the relations

$$P'_{EW} = -\delta_{EW}\, T' e^{-i\gamma} = -\delta_{EW}\, |T'|\, e^{i\,\delta_T} \ , \tag{20}$$

$$P'^C_{EW} = -\delta_{EW}\, C' e^{-i\gamma} = -\delta_{EW}\, |C'|\, e^{i(\delta_T + \delta_C)} \ , \tag{21}$$

where both the color-allowed and color-suppressed EWP amplitudes have approximately the same proportionality constant

$$\delta_{EW} \simeq 0.65 \pm 0.15 \ . \tag{22}$$

These relations determine both the magnitudes and phases of the EW penguins. Their weak phases are equal to the weak phase of P', i.e. to $-\pi$. They appear as the minus signs in Eqs. (20) and (21). We do not use Eq. (22) as a constraint in our fit, but simply use δ_{EW} as a fit parameter and check whether it comes out within the expected bounds.

Eqs. (20) and (21) were incorporated into Fit II but Fit I only employs redefined $\tilde{T}'$ and $\tilde{C}'$ that cannot be directly related to the EW penguins. Instead, we write

$$P'_{EW} + P'^C_{EW} = -\delta_{EW}\,(T' + C')\, e^{-i\gamma} = -\delta_{EW}\,(\tilde{T}' + \tilde{C}')\, e^{-i\gamma} \tag{23}$$

and then neglect P'^{C}_{EW} which is expected to be the smaller of the two to obtain

$$P'_{EW} \simeq -\delta_{EW}\,(\tilde{T}' + \tilde{C}')\,e^{-i\gamma} = -\delta_{EW}\,(|\tilde{T}'|e^{i\,\delta_T} + |\tilde{C}'|e^{i(\delta_T+\delta_C)})~. \tag{24}$$

This relation for P'_{EW} was used in Fit I while P'^{C}_{EW} was set to zero.

The fitting parameters of both fits are shown in the columns for Fit I and Fit II in Table V. An unusually large $|\tilde{C}/\tilde{T}| \approx 1.4$ ratio predicted by Fit I is an indication of large $|P_{tu}|$ [17], destructive interference between T and P_{tu} contributions to the redefined tree amplitude $\tilde{T}$, and constructive interference between C and P_{tu} contributions to $\tilde{C}$. Indeed, Fit II which separates P_{tu} and tree-level amplitudes predicts $|P_{tu}| = 14.9$ and a much more reasonable $|C/T| = 0.46$ ratio.

Fits I and II represent a completely satisfactory description of $B \to \pi\pi$ and $B \to \pi K$ decay modes. The branching ratio for $B^0 \to \pi^0 K^0$ is predicted to be about 1.7σ below the observed value, while that for $B^+ \to \pi^0 K^+$ is predicted to be about 1.1σ below experiment. These deviations could be hints of new physics [17, 52], or simply due to underestimates of neutral-pion detection efficiencies [53]. The predictions are shown in the columns for Fits I and II in Tables VI and VII. Uncertainties for all predictions have been estimated by scanning the parameter space and studying the parameter sets that led to χ^2 values no more than 1 unit above the minimum. The spread in predictions corresponding to those parameter sets has determined the uncertainties in predictions. The same method was used in an earlier analysis of $B \to VP$ decays [2].

The confidence level of Fit II is slightly lower than in Fit I because two new parameters ($|P_{tu}|$ and its strong phase $\delta_{P_{tu}}$) have been added without a corresponding improvement in the χ^2 value. The dependence of χ^2 on the weak phase γ in Fits I and II is shown as the dotted and dash-dotted curves, respectively, in Fig. 1.

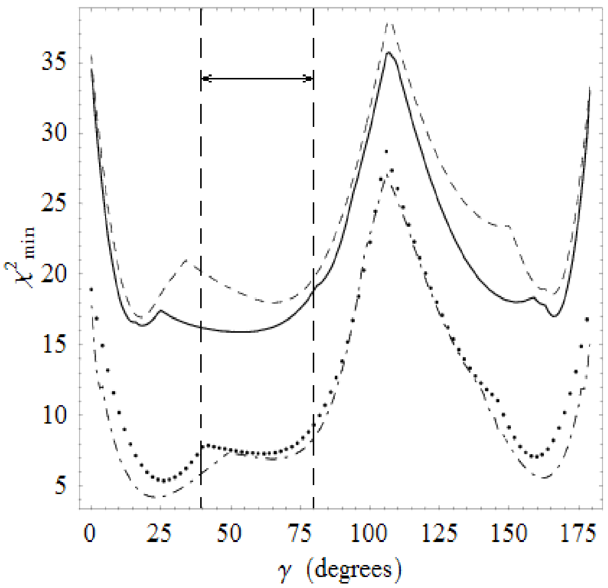

FIG. 1: $(\chi^2)_{\rm min}$, obtained by minimizing over all remaining fit parameters, as a function of the weak phase γ. Dotted curve: Fit I; dash-dotted curve: Fit II; dashed curve: Fit III; solid curve: Fit IV. Vertical dashed lines show the boundaries of the favored 95% confidence level range of γ ($39^\circ - 80^\circ$) from fits to CKM parameters [3] based on other measurements.

VI. INCLUSION OF MODES WITH η AND η'

To enlarge the fit and discussion to decays involving η or η' in the final state, we include an additional singlet amplitude. It is represented by

$$S' = -|S'|e^{i\delta_S} \ , \tag{25}$$

which gives two more fitting parameters. The relation between S and S' is the same as the one between P and P':

$$\frac{S}{S'} = \frac{V_{cd}}{V_{cs}} = -\frac{\lambda}{1-\lambda^2/2} \simeq -0.230 \ . \tag{26}$$

The importance of the S' amplitude has been discussed in Refs. [11, 15, 21] mainly to account for the large branching ratios of the $\eta' K$ modes. Moreover, we include the parameter P_{tu} and its associated strong phase $\delta_{P_{tu}}$. The penguin contribution P_{tu} is apparently required by our fits to decay modes involving η and η'. For instance, in $B^+ \to \pi^+\eta^{(\prime)}$ the P_{tu} contribution is of the same order as the other terms and cannot be neglected. The results under these assumptions are given in the column for Fit III in Table V. The χ^2 dependence on γ is shown as the dashed curve in Fig. 1.

TABLE V: Comparison of parameters extracted in fits to branching ratios and CP asymmetries under various assumptions. Values of the topological amplitudes are quoted in units of eV. The Fit I column shows values for $\tilde{T}$ and $\tilde{C}$ and their strong phases in place of T and C amplitudes and phases. Probabilities are those for χ^2 to exceed the value shown for the indicated number of degrees of freedom.

Quantity	Fit to $\pi\pi$,πK		Global fit	
	Fit I	Fit II	Fit III	Fit IV
γ	$(61^{+14}_{-27})^\circ$	$(65^{+13}_{-35})^\circ$	$(66^{+12}_{-16})^\circ$	$(54^{+18}_{-24})^\circ$
$\lvert T\rvert$	$16.1^{+2.0}_{-1.9}$	$30.4^{+15.1}_{-8.2}$	27.5 ± 3.2	$27.4^{+7.9}_{-4.6}$
δ_T	$(34^{+25}_{-11})^\circ$	$(17^{+23}_{-12})^\circ$	$(25 \pm 9)^\circ$	$(34^{+17}_{-12})^\circ$
$\lvert C\rvert$	$22.9^{+4.3}_{-3.4}$	$13.9^{+9.0}_{-8.5}$	$19.2^{+3.1}_{-3.4}$	$24.3^{+6.9}_{-5.1}$
δ_C	$(-69^{+19}_{-22})^\circ$	$(-94^{+43}_{-52})^\circ$	$(-94^{+12}_{-11})^\circ$	$(-103^{+17}_{-21})^\circ$
$\lvert P'\rvert$	$48.2^{+0.9}_{-1.0}$	$47.7^{+0.8}_{-0.9}$	47.7 ± 0.9	$47.8^{+0.9}_{-1.1}$
$\lvert P_{tu}\rvert$	0 (input)	$14.9^{+14.0}_{-7.7}$	11.2 ± 3.4	$12.3^{+7.7}_{-5.2}$
$\delta_{P_{tu}}$	0 (input)	$(3^{+28}_{-27})^\circ$	$(21 \pm 16)^\circ$	$(37^{+17}_{-18})^\circ$
$\lvert S'\rvert$	0 (input)	0 (input)	$32.1^{+3.0}_{-3.3}$	$32.4^{+2.9}_{-3.2}$
δ_S	0 (input)	0 (input)	$(-69^{+11}_{-8})^\circ$	$(-70^{+10}_{-8})^\circ$
$\lvert S_{tu}\rvert$	0 (input)	0 (input)	0 (input)	$5.7^{+5.5}_{-4.1}$
$\delta_{S_{tu}}$	0 (input)	0 (input)	0 (input)	$(-61^{+56}_{-42})^\circ$
δ_{EW}	$0.55^{+0.44}_{-0.33}$	$0.42^{+0.50}_{-0.29}$	$0.47^{+0.32}_{-0.30}$	$0.62^{+0.39}_{-0.36}$
Fit properties:				
χ^2/d.o.f.	7.34/8	6.97/6	18.06/15	15.95/13
CL (%)	50	32	26	25
Derived quantities:				
$\lvert P'_{EW}\rvert$	$4.5^{+3.2}_{-2.6}$	$3.6^{+3.6}_{-2.3}$	$3.6^{+2.5}_{-2.3}$	$4.8^{+4.3}_{-2.9}$
$\lvert P'^C_{EW}\rvert$	0 (input)	$1.3^{+3.1}_{-1.0}$	$2.1^{+1.6}_{-1.4}$	$3.4^{+3.2}_{-2.2}$
$\lvert C/T\rvert$	$1.43^{+0.40}_{-0.31}$	$0.46^{+0.43}_{-0.30}$	0.70 ± 0.16	0.89 ± 0.21

TABLE VI: Comparison of predicted and experimental branching ratios in units of 10^{-6} and CP asymmetries for $\Delta S = 0$ $B \to PP$ decays. The predictions of Fits I and II for η and η' modes are not reliable and are given for comparison purposes only. CP asymmetries, when predicted, are displayed on second line for a decay mode, while asymmetries in curly brackets (when shown) correspond to S (second line) and A (third line).

	Mode	Fit to $\pi\pi, \pi K$		Global fit		Experimental
		Fit I	Fit II	Fit III	Fit IV	average
$B^+ \to$	$\pi^+\pi^0$	$5.12^{+0.38}_{-0.23}$	$5.11^{+0.22}_{-0.14}$	$5.11^{+0.33}_{-0.37}$	$5.13^{+0.23}_{-0.22}$	5.2 ± 0.8
		-0.00 ± 0.00	-0.00 ± 0.00	-0.00 ± 0.00	-0.01 ± 0.00	-0.07 ± 0.14
	$K^+\bar{K}^0$	1.14 ± 0.04	$1.92^{+5.45}_{-1.35}$	$1.39^{+0.45}_{-0.35}$	$1.31^{+0.99}_{-0.36}$	< 2.5
	$\pi^+\eta$	$7.10^{+1.45}_{-1.05}$	$1.84^{+1.89}_{-0.39}$	$4.09^{+0.47}_{-0.41}$	$4.58^{+0.39}_{-0.51}$	4.9 ± 0.9
		$-0.07^{+0.08}_{-0.06}$	$-0.40^{+0.90}_{-0.21}$	$-0.39^{+0.12}_{-0.11}$	$-0.40^{+0.09}_{-0.03}$	-0.44 ± 0.18
	$\pi^+\eta'$	$3.35^{+0.60}_{-0.46}$	$0.84^{+0.92}_{-0.19}$	$4.22^{+0.34}_{-0.31}$	$2.95^{+0.89}_{-0.55}$	2.4 ± 1.1 (< 4.5)
		$-0.07^{+0.08}_{-0.06}$	$-0.41^{+0.93}_{-0.21}$	-0.10 ± 0.10	$-0.03^{+0.51}_{-0.34}$	–
$B^0 \to$	$\pi^+\pi^-$	$4.58^{+0.23}_{-0.28}$	$4.55^{+0.07}_{-0.06}$	$4.58^{+0.10}_{-0.12}$	$4.58^{+0.08}_{-0.11}$	4.6 ± 0.4
		$\{-0.79^{+0.25}_{-0.16}$	$\{-0.74^{+0.26}_{-0.21}$	$\{-0.74^{+0.22}_{-0.16}$	$\{-0.89^{+0.24}_{-0.06}$	$\{-0.70 \pm 0.30$
		$0.34^{+0.02}_{-0.07}$	0.33 ± 0.02	0.31 ± 0.06	$0.30^{+0.02}_{-0.04}$	0.42 ± 0.19
	$\pi^0\pi^0$	$1.95^{+0.17}_{-0.30}$	$1.94^{+0.10}_{-0.18}$	$1.97^{+0.25}_{-0.27}$	$1.97^{+0.14}_{-0.19}$	1.9 ± 0.5
		$\{0.44^{+0.35}_{-1.02}$	$\{0.57^{+0.25}_{-1.30}$	$\{0.54^{+0.22}_{-0.55}$	$\{0.12^{+0.53}_{-0.83}$	$\{$ –
		$0.52^{+0.07}_{-0.20}$	$0.53^{+0.03}_{-0.30}$	$0.56^{+0.08}_{-0.10}$	$0.52^{+0.09}_{-0.24}$	–
	$K^0\bar{K}^0$	1.06 ± 0.04	$1.78^{+5.06}_{-1.25}$	$1.29^{+0.42}_{-0.32}$	$1.21^{+0.92}_{-0.33}$	< 1.5
	$\pi^0\eta$	0.69 ± 0.02	$1.19^{+3.40}_{-0.83}$	$1.10^{+0.30}_{-0.33}$	$0.95^{+0.39}_{-0.16}$	< 2.5
	$\pi^0\eta'$	$0.31^{+0.02}_{-0.03}$	$0.57^{+1.67}_{-0.39}$	1.34 ± 0.18	$1.00^{+0.49}_{-0.41}$	< 3.7
	$\eta\eta$	$1.67^{+0.86}_{-0.44}$	$0.68^{+2.66}_{-0.47}$	$1.54^{+0.40}_{-0.29}$	$1.92^{+1.29}_{-0.48}$	< 2.8
	$\eta\eta'$	$1.59^{+0.80}_{-0.42}$	$0.66^{+2.61}_{-0.47}$	$2.51^{+0.51}_{-0.36}$	$2.16^{+0.87}_{-0.60}$	< 4.6
	$\eta'\eta'$	$0.38^{+0.18}_{-0.10}$	$0.16^{+0.64}_{-0.12}$	$0.97^{+0.16}_{-0.11}$	0.68 ± 0.32	< 10

Finally, since there is no reason to exclude such a term, we include a contribution from a singlet-penguin amplitude S_{tu} associated with intermediate t and u quarks, consisting of

TABLE VII: Comparison of predicted and experimental branching ratios in units of 10^{-6} and CP asymmetries for $|\Delta S| = 1$ $B \to PP$ decays. The predictions of Fits I and II for η and η' modes are not reliable and are given for comparison purposes only. CP asymmetries, when predicted, are displayed on second line for a decay mode, while asymmetries in curly brackets (when shown) correspond to S (second line) and A (third line).

	Mode	Fit to $\pi\pi, \pi K$		Global fit		Experimental
		Fit I	Fit II	Fit III	Fit IV	average
$B^+ \to$	$\pi^+ K^0$	$21.78^{+0.81}_{-0.82}$	$22.64^{+0.83}_{-0.93}$	$22.05^{+0.89}_{-0.95}$	$22.30^{+0.84}_{-0.78}$	21.8 ± 1.4
		0	0.00 ± 0.04	$0.03^{+0.02}_{-0.03}$	$0.05^{+0.02}_{-0.03}$	0.02 ± 0.06
	$\pi^0 K^+$	$11.40^{+0.45}_{-0.70}$	$11.40^{+0.27}_{-0.72}$	$11.40^{+0.70}_{-0.77}$	$11.35^{+0.61}_{-0.68}$	12.5 ± 1.0
		$0.02^{+0.03}_{-0.04}$	$0.03^{+0.05}_{-0.07}$	0.07 ± 0.02	$0.09^{+0.01}_{-0.03}$	0.00 ± 0.12
	ηK^+	$0.16^{+0.04}_{-0.09}$	$0.21^{+0.07}_{-0.13}$	$3.44^{+0.60}_{-0.50}$	3.63 ± 0.59	3.7 ± 0.7
		0	$-0.10^{+0.08}_{-0.34}$	$-0.41^{+0.06}_{-0.04}$	$-0.34^{+0.11}_{-0.07}$	-0.52 ± 0.24
	$\eta' K^+$	$29.38^{+0.58}_{-1.21}$	$30.72^{+1.06}_{-1.11}$	$74.56^{+1.51}_{-1.92}$	$75.21^{+1.44}_{-1.73}$	77.6 ± 4.6
		0.01 ± 0.01	$0.01^{+0.04}_{-0.05}$	0.02 ± 0.01	0.01 ± 0.02	0.02 ± 0.04
$B^0 \to$	$\pi^- K^+$	$18.90^{+0.46}_{-0.41}$	$18.60^{+0.50}_{-0.47}$	$18.89^{+0.45}_{-0.44}$	$18.78^{+0.48}_{-0.39}$	18.2 ± 0.8
		$-0.10^{+0.02}_{-0.01}$	-0.10 ± 0.01	-0.10 ± 0.02	$-0.10^{+0.02}_{-0.01}$	-0.09 ± 0.03
	$\pi^0 K^0$	$9.23^{+0.67}_{-0.47}$	$9.29^{+0.67}_{-0.50}$	$9.23^{+0.76}_{-0.65}$	$9.32^{+0.64}_{-0.63}$	11.7 ± 1.4
		0.83 ± 0.01	$0.83^{+0.01}_{-0.02}$	0.83 ± 0.01	0.83 ± 0.01	0.48 ± 0.42
		$-0.11^{+0.04}_{-0.01}$	$-0.11^{+0.06}_{-0.01}$	$-0.12^{+0.03}_{-0.02}$	$-0.11^{+0.05}_{-0.02}$	-0.40 ± 0.29
	ηK^0	$0.07^{+0.00}_{-0.01}$	$0.05^{+0.10}_{-0.03}$	$2.66^{+0.46}_{-0.37}$	$2.49^{+0.43}_{-0.61}$	2.5 ± 1.0 (< 5.2)
		$-0.59^{+0.70}_{-0.19}$	$0.34^{+0.55}_{-0.53}$	$0.53^{+0.04}_{-0.03}$	$0.56^{+0.03}_{-0.02}$	–
		$0.60^{+0.35}_{-0.33}$	$0.85^{+0.15}_{-0.48}$	$0.02^{+0.05}_{-0.04}$	$0.03^{+0.05}_{-0.03}$	–
	$\eta' K^0$	$27.93^{+0.66}_{-1.09}$	$29.88^{+1.58}_{-1.47}$	$69.29^{+1.45}_{-1.84}$	$69.27^{+1.49}_{-1.72}$	65.2 ± 6.2
		$0.70^{+0.01}_{-0.00}$	$0.81^{+0.09}_{-0.05}$	0.74 ± 0.01	$0.75^{+0.00}_{-0.01}$	0.27 ± 0.21
		$0.04^{+0.00}_{-0.02}$	0.04 ± 0.06	0.07 ± 0.02	$0.06^{+0.01}_{-0.02}$	-0.04 ± 0.13

a parameter $|S_{tu}|$ and its associated strong phase $\delta_{S_{tu}}$:

$$S_{tu} = |S_{tu}| e^{i(\delta_{S_{tu}} + \gamma)} \ , \tag{27}$$

$$\frac{S'_{tu}}{S_{tu}} = \frac{V_{us}}{V_{ud}} = \frac{\lambda}{1 - \lambda^2/2} \simeq 0.230 \ . \tag{28}$$

This exercise is denoted by Fit IV. The sole improvement with respect to Fit III is a better fit to the $B^+ \to \pi^+\eta'$ branching ratio, as shown in Table VI. The tree amplitude $|T|$ extracted from both Fit III and Fit IV is in agreement with the estimate obtained from a recent application of factorization [54] to the spectrum in $B \to \pi l \nu$ [55], which yields 24.4 ± 3.8 eV.

Both Fit III and Fit IV represent a good description of $B \to PP$ decay modes, including those with η or η' in the final state. The only problematic data points are the branching ratio for $B^0 \to \pi^0 K^0$ which is predicted to be about 1.7σ below the observed value and the mixing-induced asymmetry $S(\eta' K^0)$ with the prediction ($\simeq \sin 2\beta$) at about 2.2σ above the experimental value. The predictions for all other observed η and η' modes reproduce experimental values within their uncertainties. The predictions for as-yet-unseen modes are consistent with the current experimental upper limits on their branching ratios. The predictions are shown in the columns for Fits III and IV in Tables VI and VII. The dependence of χ^2 on γ in Fit IV is shown as the solid curve in Fig. 1.

VII. STABLE AND LESS-STABLE ASPECTS OF FIT

A. Robust aspects

The value of the weak phase γ obtained in $B \to PP$ data is consistent with other determinations. All versions of the fits have a local χ^2 minimum in the range $48^\circ \leq \gamma \leq 73^\circ$ (68% c.l.) allowed by global fits to phases of the CKM matrix [3] and near the range $(63\pm6)^\circ$ obtained in a fit to $B \to VP$ data [2]. The variation of γ from fit to fit is at most about 12 degrees, providing some idea of the systematic error associated with this approach.

All fits are comfortable with a relatively large negative value of $S_{\pi\pi}$ which is the average of the Babar [32] and Belle [41] values. Large negative $S_{\pi\pi}$ is associated with larger α and smaller γ (see, e.g., the plots in Refs. [56]).

The magnitude $|P'|$ of the strangeness-changing penguin amplitude changes very little from fit to fit. It is specified by the decay $B^+ \to \pi^+ K^0$, which is expected to receive no other significant contributions. The presence of any direct CP violation in this decay would call that assumption into question, but no such asymmetry has yet been detected.

All fits obtain a much larger value of $|C/T|$ than the range of 0.08 to 0.37 assumed in Ref. [11]. Moreover, all fits (including those to $\pi\pi$ and πK modes alone) entail a large strong relative phase δ_C between the C and T amplitudes. The presence of a large color-suppressed amplitude is somewhat of a surprise from the standpoint of *a priori* calculations such as those in the QCD factorization approach [12], and probably indicates a greater-than-anticipated role for final-state rescattering, which can generate such an effective amplitude (see also [57]). Such rescattering may be the reason why the decay $B^0 \to \pi^0\pi^0$ is more prominent than had been expected. All our fits now entail a branching ratio for this mode of about 2×10^{-6}. Although the favored values of some topological amplitudes (e.g., C, P_{tu}) show noticeable variations from fit to fit, they change together in a correlated way so that the predictions for almost all of the modes that involve them remain very stable.

The strong phase δ_T of the tree amplitude T with respect to the penguin amplitude P is found to be non-zero and of the order of 20° to 30°. It is most likely driven by the need to simultaneously describe a large direct CP asymmetry (the parameter $A_{\pi\pi}$) in $B^0 \to \pi^+\pi^-$ and a small but significant direct asymmetry in $B^0 \to \pi^- K^+$. These quantities are well-fitted and their predicted values do not differ much among the four fits.

While the electroweak penguin parameter δ_{EW} was initially constrained to lie within the range (22), we found that leaving it as a free parameter led to results consistent with that range except in the cases of Fit II and Fit III. Thus, our fits do not favor a large phenomenological EWP amplitude. This should be contrasted with Refs. [17, 58] where a different assignment of weak and strong phases is given in expectation of new physics contributions. Our fits also do not favor much deviation of the predicted $S_{\pi^0 K^0}$ time-dependent asymmetry parameter from its predicted standard-model value of $\sin(2\beta) \simeq 0.74$ [59].

Once one admits enough parameters into the fits to correctly describe modes involving η and η', the negative direct CP asymmetry in $B^+ \to \pi^+\eta$ observed by BaBar [30] is correctly reproduced. The possibility that this asymmetry could be large was first noted in Ref. [60] and pursued in Refs. [15]. We predict a similarly large negative CP asymmetry in $B^+ \to \eta K^+$, as observed [30]. These asymmetries can be large because no single weak amplitude dominates the decays. As sensitivities of asymmetric e^+e^- collider experiments improve through the accumulation of larger data samples, we expect more such decay modes to emerge.

The mixing-induced and direct asymmetries $S(\eta' K^0)$ and $A(\eta' K^0)$ are predicted to be close to $\sin(2\beta)$ and 0, respectively. These two values would be expected if the $B^0 \to \eta' K^0$ decay amplitude had consisted of just QCD penguin P' and singlet penguin S'. The interference of these terms with the much smaller C', P'_{tu}, and S'_{tu} amplitudes leads to small deviations from the expected values. These deviations are to a large extent determined by the ratio $|A'_C/A'_P|$ of the terms with the weak factor $V^*_{ub}V_{us}$ (C', P'_{tu}, and S'_{tu}) and the terms with the weak factor $V^*_{cb}V_{cs}$ (P' and S'). $|A'_C/A'_P|$ is typically predicted by QCD factorization and PQCD to be smaller than 0.02 [12, 61, 62]. Our best conservative estimate of $|A'_C/A'_P|$ is based on Fit IV. We find that the SU(3) fit prefers somewhat larger values: $|A'_C/A'_P| = 0.042^{+0.017}_{-0.006}$. Fit III (somewhat more stable than Fit IV) predicts $|A'_C/A'_P| = 0.040^{+0.011}_{-0.009}$. More conservative bounds on $|A'_C/A'_P|$ and on the asymmetries $S(\eta' K^0)$ and $A(\eta' K^0)$ were obtained recently in a model-independent way using flavor SU(3) [63].

We have explored the effects of changing the η–η' octet-singlet mixing angle from its nominal value $\theta \simeq 19.5^\circ$ defined in Sec. II. The angle θ assumed a value of 22.0° in Fit III with a free mixing angle while χ^2 of the fit improved by just 1.12. With one additional parameter in the fit, this did not result in a better fit quality. Fit IV with a free mixing angle preferred $\theta = 20.4^\circ$, with the fit quality dropping by 5%. Thus, leaving the η–η' mixing angle as a free parameter, we found variations of only a few degrees and negligible improvements in fits.

B. Aspects sensitive to assumptions

The possibility of a large P_{tu} term in Fit II leads to a wide range of predicted branching ratios for $B^+ \to K^+ \bar{K}^0$ and $B^0 \to K^0 \bar{K}^0$. This range is considerably reduced in other fits.

The magnitude and phase of the singlet penguin amplitude S' are probably not well-determined. The two quantities are correlated, as first pointed out in Ref. [15] and noted further in Ref. [11]. For example, a much smaller magnitude of S' is required to fit the charged and neutral $B \to \eta' K$ decay modes if S' and P' (the gluonic penguin amplitude) interfere constructively with one another. The QCD factorization approach [12] finds negligible S' contribution to these decays, explaining their enhancement by means of nonet-symmetry breaking effects as proposed, for example, in Ref. [64], and making use of the constructive interference of non-strange and strange components of the η' in the gluonic penguin amplitude [65]. One should also point out that many other explanations have been proposed for the enhancement of $B \to \eta' K$ modes [62, 66]. One also finds the magnitude of S' to be sensitive to small changes in the octet-singlet mixing in η and η'.

Predictions for the branching ratio and CP asymmetry in $B^+ \to \eta K^+$ depend crucially on the introduction of the S' amplitude. Since this amplitude is uncertain in magnitude and phase, those predictions (although apparently satisfied) should be viewed with caution. The same warning applies to the mode $B^+ \to \eta\pi^+$.

As already noted, the predicted branching ratio for $B^+ \to \pi^+\eta'$ is quite sensitive to assumptions, and was the sole quantity which could be compared to experiment that led to the introduction of the S_{tu} term in Fit IV. In Ref. [11] we noted a tight correlation between predicted branching ratios and CP asymmetries for $B^+ \to \pi^+\eta$ and $B^+ \to \pi^+\eta'$. With the added possibility of nonzero P_{tu} and S_{tu} contributions, this correlation no longer holds.

The only other prediction whose values are significantly different in Fits III and IV is the mixing-induced asymmetry $S(\pi^0\pi^0)$. One should trust the larger values of this quantity predicted by Fits I and II. These fits to $\pi\pi$ and πK data points are not affected by the uncertainties associated with η and η'. Their predictions for the asymmetries in $B^0 \to \pi^0\pi^0$ modes thus are expected to be more reliable.

The introduction of the S_{tu} term changes the favored value of γ by a noticeable amount, though still within limits from CKM global fits [3]. As noted, this provides one estimate of systematic errors associated with analyses of the present form.

VIII. MODES TO BE SEEN

Several decay modes are predicted to occur at levels just below present upper bounds, and can provide useful constraints on the residual uncertainties in our fits. For example, the decays $B^+ \to K^+\overline{K}^0$ and $B^0 \to K^0\overline{K}^0$ are predicted to have branching ratios exceeding 10^{-6} (somewhat larger than in Ref. [11]), with the exact value depending on the fit. The decay modes $B^0 \to \pi^0\eta$ and $B^0 \to \pi^0\eta'$ also should be visible at this level. The modes $B^0 \to (\eta\eta, \eta\eta', \eta'\eta')$ will probably require more work. We also make predictions for the direct and mixing-induced asymmetries in $B^0 \to \pi^0\pi^0$ and $B^0 \to \eta K^0$, with $A(\pi^0\pi^0)$ exceeding 0.5. A prediction for the branching ratio of K^+K^- cannot be made in our approach. The amplitude of this decay mode receives contributions from exchange and penguin annihilation diagrams that are neglected in this paper. It is very desirable that a more strict experimental upper limit be set for this mode to justify the assumption of negligibility of similar contributions to other neutral $\Delta S = 0$ decay modes.

TABLE VIII: Comparison of observed and predicted direct CP asymmetries for some $B \to \pi\pi$ and $B \to \pi K$ decay modes.

Decay Mode	Exptl. average	Present work (a)	QCDF [12] Full range	QCDF [12] Favored (b)	PQCD [16]	Ref. [17]
$B^0 \to \pi^+\pi^-$	0.42 ± 0.19	$0.30^{+0.02}_{-0.04}$	$-0.07^{+0.14}_{-0.13}$	0.10	0.16 to 0.30	Input
$B^+ \to \pi^- K^+$	-0.09 ± 0.03	$-0.10^{+0.02}_{-0.01}$	$0.04^{+0.09}_{-0.10}$	-0.04	$-(0.13$ to $0.22)$	$-0.14^{+0.09}_{-0.14}$
$B^0 \to \pi^0 K^0$	-0.40 ± 0.29	$-0.11^{+0.05}_{-0.02}$	-0.03 ± 0.04	0.01	–	$-0.05^{+0.29}_{-0.24}$

(a) Fit IV; (b) Scenario S4

TABLE IX: Comparison of observed and predicted direct CP asymmetries for some B decay modes involving η and η'.

Decay Mode	Exptl. average	Present work (a)	QCDF [12] Full range	QCDF [12] Favored (b)
$B^+ \to \pi^+\eta$	-0.44 ± 0.18	$-0.40^{+0.09}_{-0.03}$	-0.15 ± 0.20	0.06
$B^+ \to \eta K^+$	-0.52 ± 0.24	$-0.34^{+0.11}_{-0.07}$	$-0.19^{+0.29}_{-0.30}$	0.10

(a) Fit IV; (b) Scenario S4

IX. COMPARISON WITH OTHER APPROACHES

The signs of our predicted direct CP asymmetries agree with those measured experimentally for the five processes in which non-zero asymmetries are reported at greater than the 2σ level. We summarize these and our predictions for them in Tables VIII and IX. (For others, as shown in Tables VI and VII, negligible asymmetries are predicted, in accord with observation.)

The fact that we agree with all five signs and magnitudes is due in part to the flexibility of our SU(3) fit, but still represents a non-trivial consistency in our description of strong phases. We were not able to achieve this consistency in Ref. [11]. The same correlation between predicted signs of direct asymmetries in $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^- K^+$ occurs in all the methods compared in Table VIII. A definite prediction of the absolute signs, in accord with experiment, is made in Ref. [16].

Fits to $B \to PP$ branching ratios in the various approaches which we compare with ours

[12, 16–18] are generally acceptable, especially when allowance is made for possible large penguin amplitudes and color-suppressed contributions. These fits now are converging on a preference for γ in the range preferred by fits [3] to other observables constraining CKM parameters.

X. SUMMARY

We have analyzed the decays of B mesons to a pair of charmless pseudoscalar mesons within a framework of flavor SU(3). Acceptable fits to $B \to \pi\pi$ and $B \to K\pi$ branching ratios and CP asymmetries were obtained with tree, color-suppressed (C), penguin (P), and electroweak penguin (P_{EW}) amplitudes, but in order to describe processes involving η and η' we needed to include a large flavor-singlet penguin amplitude (S) and a penguin amplitude P_{tu} associated with intermediate t and u quarks. For the $B^+ \to \pi^+\eta'$ mode a term S_{tu} associated with intermediate t and u quarks also was employed.

We were able to achieve a good fit to the five most significant direct CP asymmetries, as noted in Tables VIII and IX. We found values of the weak phase γ roughly consistent with those obtained earlier in an analysis of $B \to VP$ decays [$(\gamma = 63 \pm 6)^\circ$], and with other analyses [3] of CKM parameters, for which the 68% confidence level limit is $48^\circ \leq \gamma \leq 73^\circ$. A global fit without S_{tu} gave $\gamma = (66^{+12}_{-16})^\circ$, while adding S_{tu} yielded $\gamma = (54^{+18}_{-24})^\circ$. The difference between these two serves as an estimate of systematic error.

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[1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] C. W. Chiang, M. Gronau, Z. Luo, J. L. Rosner and D. A. Suprun, Phys. Rev. D **69**, 034001 (2004) [hep-ph/0307395].

[3] A. Hocker, H. Lacker, S. Laplace and F. Le Diberder, Eur. Phys. J. C **21**, 225 (2001) [hep-ph/0104062]. Updated results may be found on the web site `http://ckmfitter.in2p3.fr/`. We have used those from XXI International Symposium on Lepton and Photon Interactions at High Energies (Lepton-Photon 2003).

[4] D. Zeppenfeld, Zeit. Phys. **8**, 77 (1981).

[5] M. Savage and M. Wise, Phys. Rev. D **39**, 3346 (1989); *ibid.* **40**, 3127(E) (1989).

[6] L. L. Chau *et al.*, Phys. Rev. D **43**, 2176 (1991).

[7] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994) [hep-ph/9404283].

[8] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D **52**, 6374 (1995) [hep-ph/9504327].

[9] M. Gronau and J. L. Rosner, Phys. Rev. D **53**, 2516 (1996) [hep-ph/9509325].

[10] B. Grinstein and R. F. Lebed, Phys. Rev. D **53**, 6344 (1996) [hep-ph/9602218].

[11] C. W. Chiang, M. Gronau and J. L. Rosner, Phys. Rev. D **68**, 074012 (2003) [hep-ph/0306021].

[12] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **606**, 245 (2001) [hep-ph/0104110]; M. Beneke and M. Neubert, Nucl. Phys. B **651**, 225 (2003) [hep-ph/0210085]; Nucl. Phys. B **675**, 333 (2003) [hep-ph/0308039].

[13] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **91**, 241801 (2003) [hep-ex/0308012].

[14] Belle Collaboration, Y. Chao *et al.*, hep-ex/0311061.

[15] A. S. Dighe, M. Gronau and J. L. Rosner, Phys. Lett. B **367**, 357 (1996) [hep-ph/9509428]; *Erratum-ibid.* B **377**, 325 (1996); Phys. Rev. Lett. **79**, 4333 (1997) [hep-ph/9707521].

[16] Y. Y. Keum and A. I. Sanda, eConf **C0304052**, WG420 (2003) [hep-ph/0306004].

[17] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, hep-ph/0402112; Phys. Rev. Lett. **92**, 101804 (2004) [hep-ph/0312259].

[18] P. Żenczykowski, hep-ph/0402290.

[19] M. Gronau, Phys. Rev. D **62**, 014031 (2000).

[20] M. Gronau and J. L. Rosner, Phys. Rev. D **65**, 013004 (2002) [hep-ph/0109238].

[21] C. W. Chiang and J. L. Rosner, Phys. Rev. D **65**, 074035 (2002) [hep-ph/0112285].

[22] LEP B Oscillation Working Group, results for the summer 2003 conferences, `http://lepbosc.web.cern.ch/LEPBOSC/`.

[23] Relations between amplitudes in the two conventions are given in M. Gronau and J. L. Rosner, Phys. Rev. D **66**, 053003 (2002); *ibid.* D **66**, 119901(E) (2002) [hep-ph/0205323].

[24] CLEO Collaboration, A. Bornheim *et al.*, Phys. Rev. D **68**, 052002 (2003) [hep-ex/0302026].

[25] CLEO Collaboration, S. J. Richichi *et al.*, Phys. Rev. Lett. **85**, 520 (2000) [hep-ex/9912059].

[26] CLEO Collaboration, S. Chen *et al.*, Phys. Rev. Lett. **85**, 525 (2000) [hep-ex/0001009].

[27] CLEO Collaboration, B. H. Behrens *et al.*, Phys. Rev. Lett. **80**, 3710 (1998) [hep-ex/9801012].

[28] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **91**, 021801 (2003) [hep-ex/0303028].

[29] BABAR Collaboration, B. Aubert *et al.*, hep-ex/0312055.

[30] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **92**, 061801 (2004) [hep-ex/0311016].

[31] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **89**, 281802 (2002) [hep-ex/0207055].

[32] BABAR Collaboration, J. Olsen *et al.*, PLOT-0054, `https://oraweb.slac.stanford.edu:8080/pls/slacquery/BABAR_DOCUMENTS.DetailedIndex?P_BP_ID=3592`.

[33] BABAR Collaboration, B. Aubert *et al.*, SLAC report SLAC-PUB-10381, hep-ex/0403025.

[34] BABAR Collaboration, B. Aubert *et al.*, hep-ex/0403046.

[35] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **91**, 161801 (2003) [hep-ex/0303046].

[36] J. Fry, talk presented at Lepton Photon 2003, Fermilab, 11–16 August 2003.

[37] BABAR Collaboration, B. Aubert *et al.*, hep-ex/0403001.

[38] Belle Collaboration, K. Abe *et al.*, "Improved Measurements of Properties of $B \to K\pi$, $B \to \pi\pi$ and $B \to K\overline{K}$ Decays," Belle-CONF-0311, submitted to EPS 2003 (International Europhysics Conference on High Energy Physics), Aachen, July 17–23, 2003.

[39] Belle Collaboration, presented by H. C. Huang, XXXVII Rencontres de Moriond, ElectroWeak Interactions and Unified Theories, Les Arcs, France, March 9–16, 2002, hep-ex/0205062.

[40] Belle Collaboration, K. Abe *et al.*, Phys. Lett. B **517**, 309 (2001) [hep-ex/0108010].

[41] Belle Collaboration, K. Abe *et al.*, hep-ex/0401029.

[42] Belle Collaboration, Y. Unno *et al.*, Phys. Rev. D **68**, 011103 (2003) [hep-ex/0304035].

[43] H. Aihara, talk presented at FPCP2003 (Flavor Physics and CP Violation), Paris, June 2003.

[44] Belle Collaboration, K. F. Chen *et al.*, Phys. Lett. B **546**, 196 (2002) [hep-ex/0207033].

[45] T. Tomura, hep-ex/0305036, presented at XXXVIII Rencontres de Moriond on Electroweak interactions and Unified Theories, March 22–29, 2003, Les Arcs, France.

[46] Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **91**, 261602 (2003) [hep-ex/0308035].

[47] Updated results and references are tabulated periodically by the Heavy Flavor Averaging Group:

`http://www.slac.stanford.edu/xorg/hfag/rare`.

[48] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, hep-ph/0401188.

[49] M. Imbeault, A. L. Lemerle, V. Page and D. London, Phys. Rev. Lett. **92**, 081801 (2004) [hep-ph/0309061].

[50] M. Battaglia *et al.*, hep-ph/0304132.

[51] M. Neubert and J. L. Rosner, Phys. Lett. B **441**, 403 (1998) [hep-ph/9808493]; Phys. Rev. Lett. **81**, 5076 (1998) [hep-ph/9809311]; M. Neubert, JHEP **9902**, 014 (1999) [hep-ph/9812396]; M. Gronau, D. Pirjol and T. M. Yan, Phys. Rev. D **60**, 034021 (1999) [hep-ph/9810482].

[52] Y. Grossman, hep-ph/0310229.

[53] M. Gronau and J. L. Rosner, Phys. Lett. B **572**, 43 (2003) [hep-ph/0307095].

[54] Z. Luo and J. L. Rosner, Phys. Rev. D **68**, 074010 (2003) [hep-ph/0305262].

[55] CLEO Collaboration, S. B. Athar *et al.*, Phys. Rev. D **68**, 072003 (2003) [hep-ex/0304019].

[56] M. Gronau and J. L. Rosner, Phys. Rev. D **65**, 093012 (2002) [hep-ph/0202170]; M. Gronau and J. L. Rosner, Phys. Rev. D **66**, 053003 (2002); *ibid.* D **66**, 119901(E) (2002) [hep-ph/0205323].

[57] S. Barshay, L. M. Sehgal and J. van Leusen, hep-ph/0403049. In this work rescattering is ascribed to the coupling between the $D\bar{D}$ and $\pi\pi$ channels, which would correspond in our approach to a contribution to the $b \to d$ penguin amplitude from intermediate charmed quarks.

[58] T. Yoshikawa, Phys. Rev. D **68**, 054023 (2003) [hep-ph/0306147].

[59] M. Gronau, Y. Grossman and J. L. Rosner, Phys. Lett. B **579**, 331 (2004) [hep-ph/0310020].

[60] S. Barshay, D. Rein and L. M. Sehgal, Phys. Lett. B **259**, 475 (1991).

[61] D. London and A. Soni, Phys. Lett. B **407**, 61 (1997) [hep-ph/9704277].

[62] E. Kou and A. I. Sanda, Phys. Lett. B **525**, 240 (2002) [hep-ph/0106159].

[63] M. Gronau, J. L. Rosner and J. Zupan, hep-ph/0403287.

[64] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D **58**, 114006 (1998) [hep-ph/9802409]; T. Feldmann, Int. J. Mod. Phys. A **15**, 159 (2000) [hep-ph/9907491].

[65] H. J. Lipkin, Phys. Rev. Lett. **46**, 1307 (1981); Phys. Lett. B **254**, 247 (1991); *ibid.* B **415**, 186 (1997) [hep-ph/9710342]; *ibid.* B **433**, 117 (1998).

[66] K. Berkelman, 1997 CLEO note, unpublished; D. Atwood and A. Soni, Phys. Lett. B **405**, 150 (1997) [hep-ph/9704357]; Phys. Rev. Lett. **79**, 5206 (1997) [hep-ph/9706512]; I. Halperin and A. Zhitnitsky, Phys. Rev. D **56**, 7247 (1997) [hep-ph/9704412]; Phys. Rev. Lett. **80**, 438 (1998) [hep-ph/9705251]; F. Yuan and K. T. Chao, Phys. Rev. D **56**, R2495 (1997) [hep-ph/9706294]; H. Fritzsch, Phys. Lett. B **415**, 83 (1997) [hep-ph/9708348]; H. Y. Cheng and B. Tseng, Phys. Lett. B **415**, 263 (1997) [hep-ph/9707316]; A. Ali and C. Greub, Phys. Rev. D **57**, 2996 (1998) [hep-ph/9707251]; A. Ali, J. Chay, C. Greub and P. Ko, Phys. Lett. B **424**, 161 (1998) [hep-ph/9712372]; W. S. Hou and B. Tseng, Phys. Rev. Lett. **80**, 434 (1998) [hep-ph/9705304]; A. Datta, X. G. He and S. Pakvasa, Phys. Lett. B **419**, 369 (1998) [hep-ph/9707259]; D. Du, Y. Yang and G. Zhu, Phys. Rev. D **59**, 014007 (1999) [hep-ph/9805451].