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CORRELATED BOUNDS ON CP ASYMMETRIES IN $B^0 \to \eta' K_S$

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Flavor SU(3) is used to constrain the coefficients of $\sin \Delta m t$ and $\cos \Delta m t$ in the time-dependent CP asymmetry of $B^0 \to \eta' K_S$. Correlated bounds in the $(S_{\eta' K},\ C_{\eta' K})$ plane are derived, by using recent rate measurements of B^0 decays into K^+K^-, $\pi^0\pi^0$, $\pi^0\eta$, $\pi^0\eta'$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$. Stringent bounds are obtained when assuming a single SU(3) singlet amplitude and when neglecting annihilation-type amplitudes.

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I Introduction

Measurements of time-dependent CP asymmetries in $b \to c\bar{c}s$ decays including $B^0 \to J/\psi K_S$ [1] are interpreted in the Standard Model as $\sin 2\beta \sin \Delta m t$, where $\beta \equiv \arg(-V_{tb}V_{td}^*V_{cd}V_{cb}^*)$. These measurements have provided a crucial test [2] of the Kobayashi-Maskawa mechanism [3]. This test is theoretically clean because a single weak phase $\arg(V_{cb}^*V_{cs})$ dominates $B \to J/\psi K_S$ within a fraction of a percent [4, 5].

An interesting class of processes, susceptible to new physics effects [6], consists of $b \to s$ penguin-dominated B^0 decays into CP-eigenstates. This includes the final states ϕK_S, $(K^+K^-)_{(\text{even }\ell)}K_S$, $\pi^0 K_S$ and $\eta' K_S$. Here decay amplitudes contain two terms: a penguin amplitude, A'_P, including a dominant Cabibbo-Kobayashi-Maskawa (CKM) factor $V_{cb}^*V_{cs}$, and a color-suppressed tree amplitude, A'_C, with a smaller CKM factor $V_{ub}^*V_{us}$. The first amplitude by itself would imply a CP asymmetry of magnitude $\sin 2\beta \sin \Delta m t$. The second amplitude modifies the coefficient of this term, and introduces a $\cos \Delta m t$ term in the asymmetry [4]. The coefficients of $\sin \Delta m t$ and

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$\cos\Delta mt$ for a final state f are denoted by S_f and $-C_f$, respectively. The observables, $\Delta S_f \equiv S_f \pm \sin 2\beta$ (where the sign depends on the final state CP) and C_f, increase with $|A'_C/A'_P|$, which depends on hadron dynamics, and are functions of unknown strong interaction phases. A search for new physics effects in these processes requires a careful theoretical analysis of ΔS_f and C_f within the Standard Model.

Model-independent studies of the ratios $|A'_C/A'_P|$ in $B^0 \to \phi K_S$ [7, 8], $B^0 \to (K^+K^-)_{(\text{even } \ell)}\, K_S$ [7, 9], $B^0 \to \pi^0 K_S$ [10] and $B^0 \to \eta' K_S$ [7, 11] have been carried out using flavor SU(3). The idea is simple. Flavor SU(3) relates the hadronic amplitudes A'_P and A'_C in each of these processes to corresponding classes of hadronic amplitudes in $\Delta S = 0$ decay processes. Strangeness-changing and strangeness-conserving amplitudes include CKM factors which satisfy well-defined ratios. Consequently, rate measurements in the $\Delta S = 0$ sector may provide bounds on ratios of amplitudes $|A'_C/A'_P|$ in $\Delta S = 1$ decays.

In the present Letter we will calculate correlated bounds on $\Delta S_{\eta' K}$ and $C_{\eta' K}$ in $B^0(t) \to \eta' K_S$, using very recent branching ratio measurements of B^0 decays into a pair of neutral charmless light pseudoscalars. The two asymmetries are proportional to the ratio $|A'_C/A'_P|$ in this process. Approximate bounds on $|A'_C/A'_P|$, based on flavor SU(3), were presented in [7] and in the appendix of [11] using earlier data. These bounds neglected interference effects and $\mathcal{O}(\lambda^2)$ terms ($\lambda = 0.22$) and are expected to hold within a factor of about 1.5. The measurables $\Delta S_{\eta' K}$ and $C_{\eta' K}$ are of order $|A'_C/A'_P|$. However, they depend also on weak and strong phases. An SU(3) method for obtaining correlated bounds on $\Delta S_{\pi K}$ and $C_{\pi K}$ in $B^0(t) \to \pi^0 K_S$ was proposed in [10], taking account of this dependence and avoiding the above approximation. Here we will apply this method to the asymmetries in $B^0(t) \to \eta' K_S$.

In Section II we write expressions for the observed asymmetries in terms of hadronic amplitudes, noting their dependence on weak and strong phases. Section III provides an SU(3) decomposition for the amplitude of $B^0 \to \eta' K^0$ and for a class of $\Delta S = 0$ related processes. In Section IV we obtain bounds on $\Delta S_{\eta' K}$ and $C_{\eta' K}$ in two ways, both in a general SU(3) framework and also by neglecting small annihilation-type amplitudes. Section V concludes by comparing these bounds with expectations in other approaches, including a recent global SU(3) fit of all B decays to pairs of charmless pseudoscalars.

II Asymmetries and amplitudes in $B^0 \to \eta' K_S$

The CP asymmetry in B^0 decays to the CP eigenstate $\eta' K_S$ has the general expression [4]:

$$A(t) \equiv \frac{\Gamma(\bar{B}^0(t) \to \eta' K_S) - \Gamma(B^0(t) \to \eta' K_S)}{\Gamma(\bar{B}^0(t) \to \eta' K_S) + \Gamma(B^0(t) \to \eta' K_S)} = -C_{\eta' K}\cos(\Delta mt) + S_{\eta' K}\sin(\Delta mt)\ . \tag{1}$$

Measurements obtained by the BaBar [12] and Belle [13] Collaborations,

$$S_{\eta' K} = \begin{cases} 0.02 \pm 0.34 \pm 0.03\ , \\ 0.43 \pm 0.27 \pm 0.05\ , \end{cases} C_{\eta' K} = \begin{cases} 0.10 \pm 0.22 \pm 0.04\ , & \text{BaBar}\ , \\ 0.01 \pm 0.16 \pm 0.04\ , & \text{Belle}\ , \end{cases} \tag{2}$$

imply averages

$$S_{\eta' K} = 0.27 \pm 0.21 \ , \qquad C_{\eta' K} = 0.04 \pm 0.13 \ . \tag{3}$$

The two measurables $S_{\eta' K}$ and $C_{\eta' K}$ can be expressed in terms of the amplitude of $B^0 \to \eta' K^0$. As mentioned in the introduction, it is convenient to decompose this amplitude into two terms, A'_P and A'_C, involving intrinsic CKM factors $V^*_{cb} V_{cs}$ and $V^*_{ub} V_{us}$, and strong and weak phases δ and γ, respectively,

$$A(B^0 \to \eta' K^0) = A'_P + A'_C = |A'_P| e^{i\delta} + |A'_C| e^{i\gamma} \ . \tag{4}$$

Expressions for $S_{\eta' K}$ and $C_{\eta' K}$ in terms of A'_P and A'_C can be obtained from definitions, taking into account the negative CP eigenvalue of $\eta' K_S$ in B^0 decays [4]:

$$S_{\eta' K} \equiv \frac{2 \mathrm{Im}(\lambda_{\eta' K})}{1 + |\lambda_{\eta' K}|^2} \ , \qquad C_{\eta' K} \equiv \frac{1 - |\lambda_{\eta' K}|^2}{1 + |\lambda_{\eta' K}|^2} \ , \tag{5}$$

where

$$\lambda_{\eta' K} \equiv -e^{-2i\beta} \frac{A(\bar{B}^0 \to \eta' \bar{K}^0)}{A(B^0 \to \eta' K^0)} \ . \tag{6}$$

Using Eq. (4), the asymmetries $S_{\eta' K}$ and $C_{\eta' K}$ are then written in terms of $|A'_C / A'_P|$, δ, γ, and $\alpha \equiv \pi - \beta - \gamma$:

$$S_{\eta' K} = \frac{\sin 2\beta + 2 |A'_C / A'_P| \cos\delta \sin(2\beta + \gamma) - |A'_C / A'_P|^2 \sin(2\alpha)}{R_{\eta' K}} \ , \tag{7}$$

$$C_{\eta' K} = \frac{2 |A'_C / A'_P| \sin\delta \sin\gamma}{R_{\eta' K}} \ , \tag{8}$$

$$R_{\eta' K} \equiv 1 + 2 |A'_C / A'_P| \cos\delta \cos\gamma + |A'_C / A'_P|^2 \ . \tag{9}$$

The amplitudes A'_P and A'_C are expected to obey a hierarchy, $|A'_C| \ll |A'_P|$ [16]. In the limit of neglecting A'_C, one has the well-known result $S_{\pi K} = \sin 2\beta, C_{\pi K} = 0$. Keeping only linear terms in $|A'_C / A'_P|$, one has [4]

$$\begin{aligned} \Delta S_{\eta' K} \equiv S_{\eta' K} - \sin 2\beta &\approx 2 |A'_C / A'_P| \cos 2\beta \cos\delta \sin\gamma \ , \\ C_{\eta' K} &\approx 2 |A'_C / A'_P| \sin\delta \sin\gamma \ . \end{aligned} \tag{10}$$

Thus, within this approximation, the allowed region in the $(S_{\eta' K}, C_{\eta' K})$ plane is confined to an ellipse centered at $(\sin 2\beta, 0)$, with semi-principal axes $2[|A'_C / A'_P| \sin\gamma]_{\max} \cos 2\beta$ and $2[|A'_C / A'_P| \sin\gamma]_{\max}$. In our study below we will use the exact expressions (7)–(9).

III SU(3) decomposition of amplitudes

A convenient way of introducing flavor symmetry in charmless B decays is through graphical representations of SU(3) amplitudes [14, 15, 16, 17]. This parametrization is equivalent to a pure group-theoretical presentation [14, 18, 19, 20], having the advantage of anticipating that certain amplitudes are smaller than others. Our analysis will be carried out both in a general SU(3) framework, and also neglecting small annihilation-type amplitudes. We use quark content for mesons and phase conventions as in [16, 17]:

$$B^0 = d\bar{b},\ \pi^0 = (d\bar{d}-u\bar{u})/\sqrt{2},\ K^0 = d\bar{s}, \eta = (s\bar{s}-u\bar{u}-d\bar{d})/\sqrt{3}, \eta' = (u\bar{u}+d\bar{d}+2s\bar{s})/\sqrt{6}\,. \tag{11}$$

The η and η' correspond to octet-singlet mixtures

$$\eta = \eta_8 \cos\theta_0 - \eta_1 \sin\theta_0\ ,\quad \eta' = \eta_8 \sin\theta_0 + \eta_1 \cos\theta_0\quad , \tag{12}$$

with $\theta_0 = \sin^{-1}(1/3) = 19.5^\circ$.

The flavor flow amplitudes, which occur in B^0 decays into relevant pairs of neutral charmless pseudoscalar mesons, are the following: a "penguin" contribution p; a "singlet penguin" contribution s, in which a color-singlet $q\bar{q}$ pair produced by two or more gluons or by a Z or γ forms an $SU(3)$ singlet state; a "color-suppressed" contribution c; an "exchange" contribution e, and a "penguin annihilation" contribution pa. The three amplitudes, p, s and c contain both leading-order and electroweak penguin contributions [16].

We note that in a general SU(3) analysis decays of B mesons into pairs of pseudoscalars, consisting of a singlet and an octet of SU(3), are described by three SU(3) amplitudes [14]. Our parametrization uses the single amplitude s, neglecting two other amplitudes in which the spectator quark enters the decay Hamiltonian [17]. The other amplitudes, in which the spectator quark participates in decay processes, are e and pa. These amplitudes may be assumed to be smaller than the others [16], and will be neglected in part of our discussion. Experimental evidence for the suppression of the combination $e + pa$, already exhibited by the current upper bound on $B^0 \to K^+K^-$ [21], is expected to be strengthened in future measurements. We expect the approximation involved in neglecting small amplitudes to be comparable to that associated with assuming flavor SU(3) symmetry. For generality, we will also give exact results within SU(3) which do not neglect small contributions.

We shall denote $\Delta S = 0$ transitions by unprimed quantities and $|\Delta S| = 1$ transitions by primed quantities. The amplitudes $p^{(\prime)}$, $s^{(\prime)}$ and $pa^{(\prime)}$ contain CKM factors $V^*_{cb}V_{cd(s)}$, while $c^{(\prime)}$ and $e^{(\prime)}$ contain $V^*_{ub}V_{ud(s)}$. Flavor SU(3) symmetry implies equal $\Delta S = 0$ and $\Delta S = 1$ hadronic amplitudes multiplying these factors:

$$\begin{aligned}
\frac{p}{p'} &= \frac{s}{s'} = \frac{pa}{pa'} = \frac{V^*_{cb}V_{cd}}{V^*_{cb}V_{cs}} = -\bar{\lambda}\ ,\\
\frac{c}{c'} &= \frac{e}{e'} = \frac{V^*_{ub}V_{ud}}{V^*_{ub}V_{us}} = \bar{\lambda}^{-1}\ ,\\
\bar{\lambda} &= \frac{\lambda}{1-\lambda^2/2} = 0.230\ .
\end{aligned} \tag{13}$$

Expressions for decay amplitudes in terms of graphical contributions are obtained in a straightforward manner [16]. For $B^0 \to \eta' K^0$ one finds

$$\sqrt{6}A(B^0 \to K^0\eta') = A'_P + A'_C = 3p' + 4s' + c'\ , \tag{14}$$

implying

$$A'_P = 3p' + 4s' \ , \qquad A'_C = c' \ . \tag{15}$$

Similarly, one finds expressions for a set of SU(3) related strangeness-conserving amplitudes of which we list those that are useful for constraining the asymmetry in $B^0 \to \eta' K_S$:

$$\begin{aligned}
A(B^0 \to K^+K^-) &= -e - pa \ , \\
A(B^0 \to K^0\bar{K}^0) &= p + pa \ , \\
\sqrt{2}A(B^0 \to \pi^0\pi^0) &= p - c + e + pa \ , \\
\sqrt{6}A(B^0 \to \pi^0\eta) &= -2p - s + 2e \ , \\
\sqrt{3}A(B^0 \to \pi^0\eta') &= p + 2s - e \ , \\
(3/\sqrt{2})A(B^0 \to \eta\eta) &= p + s + c + e + (3/2)pa \ , \\
3\sqrt{2}A(B^0 \to \eta'\eta') &= p + 4s + c + e + 3pa \ , \\
3\sqrt{2}A(B^0 \to \eta\eta') &= -2p - 5s - 2c - 2e \ .
\end{aligned} \tag{16}$$

We denote the amplitudes of these processes by $A(f)$, where f stands for a given final state. Eqs. (13)–(16) provide a starting point for our analysis, in which correlated bounds on $\Delta S_{\eta' K}$ and $C_{\eta' K}$ will be obtained in terms of rate measurements of the processes occurring in (16).

IV Correlated bounds on $\Delta S_{\eta' K}$ and $C_{\eta' K}$

The bases of potential bounds on $\Delta S_{\eta' K}$ and $C_{\eta' K}$ are SU(3) relations between the amplitude of $B^0 \to \eta' K^0$ (14) and certain linear combinations of the amplitudes (16). The role of such relations in setting approximate bounds on $|A'_C/A'_P|$ was pointed out in [7]. The method for deriving correlated bounds on the asymmetries S and C has already been applied to $B^0 \to \pi^0 K_S$ [10]. In order to implement the method in $B^0 \to \eta' K_S$, one is searching for a linear superposition of the $\Delta S = 0$ amplitudes $A(f)$ in (16), with given real coefficients a_f, which acquires an expression similar to (14),

$$\sqrt{6}\Sigma_f a_f A(f) = A_P + A_C = 3p + 4s + c \ . \tag{17}$$

Since the eight physical amplitudes $A(f)$ are expressed in terms of five SU(3) contributions, one may form a whole continuum of such combinations. Constraints on the asymmetry in $B^0 \to \eta' K_S$ will be shown to follow from upper bounds on rates of processes appearing on the left-hand-side of (17). The choice of a combination leading to the strongest constraints on the asymmetry depends on experimental upper bounds available at a given time.

Three cases are of particular interest because of their current implications on the $\eta' K$ asymmetry:

1. One combination, involving pairs including π^0, η and η' in the final state, was proposed in [7] by using a complete SU(3) analysis, and in [11] by applying

simple U-spin symmetry arguments:

$$\begin{aligned}\Sigma_f a_f A(f) &= \frac{1}{4\sqrt{3}}A(\pi^0\pi^0) - \frac{1}{3}A(\pi^0\eta) + \frac{5}{6\sqrt{2}}A(\pi^0\eta') \\ &+ \frac{2}{3\sqrt{3}}A(\eta\eta) - \frac{11}{12\sqrt{3}}A(\eta'\eta') - \frac{5}{3\sqrt{3}}A(\eta\eta') \ . \end{aligned} \tag{18}$$

2. Another combination, based on the assumption that a single SU(3) amplitude dominates decays into a singlet and an octet pseudoscalar, involves four decay processes including $B^0 \to K^+K^-$:

$$\Sigma_f a_f A(f) = \frac{1}{3\sqrt{3}}A(\pi^0\pi^0) + \frac{1}{3\sqrt{6}}A(K^+K^-) - \frac{2}{3}A(\pi^0\eta) - \frac{2}{\sqrt{3}}A(\eta\eta') \tag{19}$$

3. A third superposition, satisfying (17) in the limit $e = pa = 0$, involves only three strangeness-conserving amplitudes:

$$\Sigma_f a_f A(f) = -\frac{5}{6}A(\pi^0\eta) + \frac{1}{3\sqrt{2}}A(\pi^0\eta') - \frac{\sqrt{3}}{2}A(\eta\eta') \ . \tag{20}$$

The coefficients a_f in these three cases can be read off Eqs. (18), (19), and (20).

Using (13), every linear combination satisfying (17), including (18), (19), and (20), can be written as

$$\Sigma_f a_f A(f) = -\bar{\lambda}A'_P + \bar{\lambda}^{-1}A'_C \ . \tag{21}$$

One now forms the ratio of squared amplitudes, averaged over B^0 and $\bar{B}^0$ and multiplied by $\bar{\lambda}^2$,

$$\begin{aligned}\mathcal{R}^2 &\equiv \frac{\bar{\lambda}^2[|\Sigma_f a_f A(f)|^2 + |\Sigma_f a_f \bar{A}(f)|^2]}{|A(B^0 \to \eta' K^0)|^2 + |A(\bar{B}^0 \to \eta' \bar{K}^0)|^2} \\ &= \frac{|A'_C/A'_P|^2 + \bar{\lambda}^4 - 2\bar{\lambda}^2|A'_C/A'_P|\cos\delta\cos\gamma}{1 + |A'_C/A'_P|^2 + 2|A'_C/A'_P|\cos\delta\cos\gamma} \ , \end{aligned} \tag{22}$$

where $\bar{A}(f)$ are decay amplitudes for a $\bar{B}^0$. This expression may be inverted to become an expression for $|A'_C/A'_P|$:

$$\frac{|A'_C|}{|A'_P|} = \frac{\sqrt{[(\bar{\lambda}^2 + \mathcal{R}^2)\cos\delta\cos\gamma]^2 + (1-\mathcal{R}^2)(\mathcal{R}^2 - \bar{\lambda}^4)} + (\bar{\lambda}^2 + \mathcal{R}^2)\cos\delta\cos\gamma}{1 - \mathcal{R}^2} \ . \tag{23}$$

Noting that $-1 \leq \cos\delta\cos\gamma \leq 1$, one has

$$\frac{||A'_C/A'_P| - \bar{\lambda}^2|}{1 + |A'_C/A'_P|} \leq \mathcal{R} \leq \frac{|A'_C/A'_P| + \bar{\lambda}^2}{1 - |A'_C/A'_P|} \ , \tag{24}$$

and

$$\frac{|\mathcal{R} - \bar{\lambda}^2|}{1 + \mathcal{R}} \leq |A'_C/A'_P| \leq \frac{\mathcal{R} + \bar{\lambda}^2}{1 - \mathcal{R}} \ . \tag{25}$$

Table I: Branching ratios in 10^{-6} and 90% C.L. upper limits on branching ratios

Mode	$\eta' K^0$	$\pi^0\pi^0$	K^+K^-	$K^0\bar{K}^0$	$\pi^0\eta$	$\pi^0\eta'$	$\eta\eta$	$\eta'\eta'$	$\eta\eta'$
$\bar{\mathcal{B}}$	$65.2^{+6.0}_{-5.9}$	1.9 ± 0.5	< 0.6	< 1.5	< 2.5	< 3.7	< 2.8	< 10	< 4.6
CLEO					< 2.9	< 5.7	< 18	< 47	< 27

Upper bounds on $\mathcal{R}$ may be obtained from experiments using the general algebraic inequality

$$|\Sigma_f a_f A(f)|^2 + |\Sigma_f a_f \bar{A}(f)|^2 \leq \left(\Sigma_f |a_f| \sqrt{|A(f)|^2 + |\bar{A}(f)|^2}\right)^2 . \qquad (26)$$

Denoting by $\bar{\mathcal{B}}_f$ branching ratios of $\Delta S = 0$ decays, averaged over B^0 and $\bar{B}^0$, and neglecting phase space differences [in the spirit of assuming SU(3)], which can be included if desired, one has

$$\mathcal{R} \leq \bar{\lambda} \Sigma_f |a_f| \sqrt{\frac{\bar{\mathcal{B}}_f}{\bar{\mathcal{B}}(\eta' K^0)}} . \qquad (27)$$

For a given set of coefficients a_f, nonzero branching ratio measurements and upper limits on $\bar{\mathcal{B}}_f$ provide an upper bound on $\mathcal{R}$, for which the right-hand-side of (25) gives an upper bound on $|A'_C/A'_P|$. As mentioned, the coefficients a_f will be taken to have values as in (18), (19), and (20).

Nonzero branching ratios, averaged over B^0 and $\bar{B}^0$, and 90% confidence level upper limits on branching ratios [21] are listed in Table I for $B^0 \to \eta' K^0$, and for the eight strangeness-conserving processes occurring in Eqs. (16). The last five measurements involving η and η' were reported very recently by the BaBar collaboration [22]. The second line in the Table lists earlier bounds by the CLEO collaboration [23]. The new bounds for the two processes involving a π^0 and η or η' are only slightly stronger than the earlier ones. However, bounds on the three processes involving pairs with η and η' have improved considerably.

We will now consider bounds on $\mathcal{R}$ obtained in the above three cases, starting with a general SU(3) bound and continuing with bounds which neglect small amplitudes:

1. Assuming exact SU(3) and applying (18) we find, using the central value for $\bar{\mathcal{B}}(\eta' K^0)$,

$$\mathcal{R} < 0.18 . \qquad (28)$$

   This strongest bound within pure SU(3) should be compared with a bound [7, 11] $R < 0.36$ based on the earlier CLEO data, and on an earlier upper limit, $\bar{\mathcal{B}}(\pi^0\pi^0) < 5.7 \times 10^{-6}$ [21]. In the exact SU(3) limit we also find that present data imply several almost degenerate minima for upper limits on $\mathcal{R}$, beside the point in parameter space describing the combination (18).

2. Applying (19) one obtains
$$\mathcal{R} < 0.11 \ . \tag{29}$$
This bound assumes a single SU(3) amplitude (s) in decays into two pseudoscalars belonging to an SU(3) singlet and an SU(3) octet. It should be compared with $\mathcal{R} < 0.22$ obtained from the above-mentioned earlier data using the same combination of amplitudes.

3. Neglecting e and pa and using (20), which contains three processes, one finds
$$\mathcal{R} < 0.10 \ . \tag{30}$$
This bound, which improves (29) only slightly, should be compared with $R < 0.18$ based on the earlier CLEO data.

As mentioned above, the approximation involved in deriving (30), where SU(3) breaking and small amplitudes were neglected, is comparable to that associated with (28) which only neglects SU(3) breaking effects.

When comparing upper limits on $\mathcal{R}$ implied by the recent BaBar measurements with those obtained from the earlier CLEO measurements we observe in all three approximations an improvement by a factor of about two. In all three cases the present upper limit on $\bar{\mathcal{B}}(B^0 \to \eta\eta')$ contributes the largest term. Since (28)–(30) were obtained by adding linearly experimental upper limits on $|a_f|\bar{\mathcal{B}}_f^{1/2}$ at 90% confidence level, and taking a central value for $\bar{\mathcal{B}}(\eta' K^0)$ where the current error is less than 10%, statistically these bounds involve a confidence level higher than 90%. However, their systematic uncertainties caused by SU(3) breaking and e and pa corrections are expected to be at a level of 20–30%.

In order to study constraints in the $(S_{\eta' K}, C_{\eta' K})$ plane, we now apply the upper bounds (28) and (30). The exact expressions (7)–(9) imply correlated bounds on these two quantities associated with fixed values of $\mathcal{R}$. We scan over $-\pi \le \delta \le \pi$, taking a central value $\beta = 23.7^\circ$, values of γ satisfying $38^\circ \le \gamma \le 80^\circ$ [24], and values of $|A'_C/A'_P|$ in the range (25), where $\mathcal{R}$ satisfies the bound (28) or (30). The ratio $|A'_C/A'_P|$ does not saturate the bound (25) at the boundary of the allowed region, but is limited to smaller values, because $\Delta S_{\eta' K}$ and $C_{\eta' K}$ in (7)–(10) are approximately proportional to $\sin\gamma$ whereas $|A'_C/A'_P|$ in (23) increases with $\cos\gamma$. The bounds on $(S_{\eta' K}, C_{\eta' K})$ are shown in Fig. 1. Also shown are bounds based on the earlier value $\mathcal{R} < 0.36$ mentioned above, and two points corresponding to $(S_{\eta' K}, C_{\eta' K}) = (\sin 2\beta, 0)$ and $(0.75, -0.06)$ (see below).

V Conclusion

Our above discussion and Fig. 1 show that SU(3) bounds on the CP asymmetry of $B^0 \to \eta' K_S$ have improved considerably by incorporating the very recent BaBar upper bounds and by neglecting small amplitudes, narrowing drastically the region around $(S_{\eta' K} = \sin 2\beta, C_{\eta' K} = 0)$ consistent with the Standard Model. A critical and model-independent test of the Standard Model requires further improvements, both

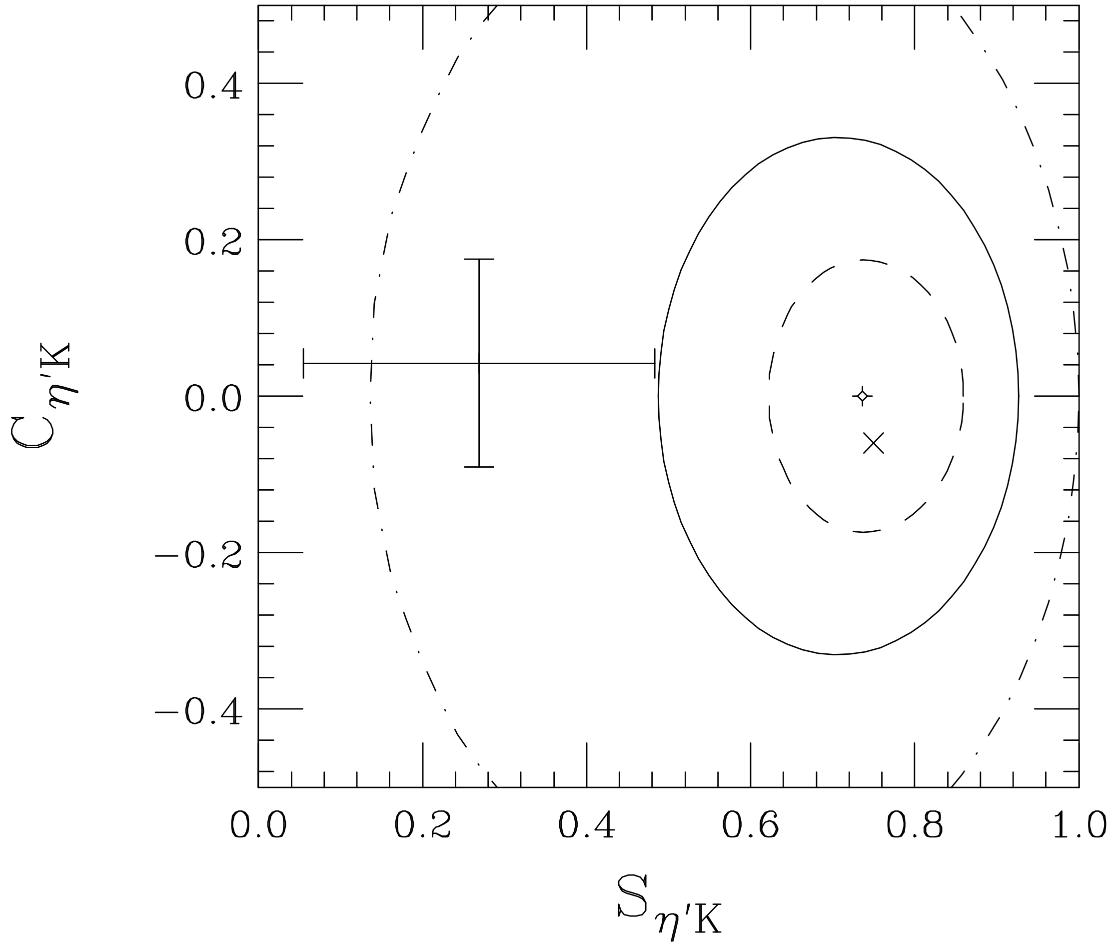

Figure 1: Regions in the $(S_{\eta' K},\ C_{\eta' K})$ plane satisfying limits (25) on the ratio $|A'_C/A'_P|$ and bounds (28) (region enclosed by the solid curve) or (30) (region enclosed by the dashed curve). The dot-dashed curve encloses the region satisfying an earlier bound $R < 0.36$. The plotted open point denotes $(S_{\eta' K},\ C_{\eta' K}) = (\sin 2\beta,\ 0)$, while the point labeled × denotes the central value of a prediction in Ref. [25].

in the asymmetry measurement and in branching ratio measurements of B^0 decays into $K\bar{K}$ and into pairs involving π^0, η and η'. The theoretical bounds plotted in Fig. 1 are based on branching ratio measurements of strangeness-conserving processes and on flavor SU(3) considerations. These considerations, including neglecting small e and pa amplitudes, may introduce theoretical errors in the bounds on $\Delta S_{\eta' K}$ and $C_{\eta' K}$ of order 20 or 30 percent.

These model-independent bounds are more conservative than other bounds which involve further assumptions. A global SU(3) fit to all B decays into pairs of charmless pseudoscalar mesons ("Fit IV" of Ref. [25]) obtains $|A'_C/A'_P| = 0.042^{+0.017}_{-0.006}$, corresponding to $0.034 < |A'_C/A'_P| < 0.064$ at 90% confidence level. [In this case the bounds (28)–(30) are satisfied automatically. A related fit ("III"), omitting one amplitude which improves a fit to a single branching ratio, obtains $|A'_C/A'_P| = 0.040^{+0.011}_{-0.009}$.] This would imply that the allowed area in the $(S_{\eta' K},\ C_{\eta' K})$ plane becomes smaller than the region corresponding to the bound $R < 0.10$. In Fig. 1 we show the value $(S_{\eta' K},\ C_{\eta' K}) = (0.75^{+0.00}_{-0.01},\ \ -0.06^{+0.02}_{-0.01})$ predicted in the favored Fit IV of Ref. [25]. [Fit III predicts $(S_{\eta' K},\ C_{\eta' K}) = (0.74 \pm 0.01,\ \ -0.07 \pm 0.02)$.] Smaller values of $|A'_C/A'_P|$ of order 0.01 have been calculated in [11, 26], implying only tiny deviations at this level from $S_{\eta' K} = \sin 2\beta,\ C_{\eta' K} = 0$.

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