**CLNS 04/1863**

# The status of $|V_{ub}|$[1]

Lawrence Gibbons

*(Representing the CLEO Collaboration)*

*Cornell University, Department of Physics, Ithaca, NY 14850, U.S.A.*

**Abstract.** I survey the theoretical and experimental information available for determination of $|V_{ub}|$ with inclusive and exclusive techniques. Using recent experimental and theoretical advances, I outline a procedure in which the inclusive information can be combined to obtain an inclusive $|V_{ub}|$ that includes experimentally–derived uncertainty estimates for outstanding theoretical corrections.

## Introduction

The magnitude of the Cabibbo–Kobayashi–Maskawa (CKM)[1, 2] matrix element $V_{ub}$ remains a crucial input into tests of the unitarity of the 3-generation CKM matrix, yet a $|V_{ub}|$ averaging procedure that results in a defensible uncertainty remains elusive. Recent experimental and theoretical progress provides us with an opportunity to improve this situation. Here I review the theoretical and experimental concerns in extraction of $|V_{ub}|$, and outline a potential inclusive "averaging" procedure in which information from all regions of phase space can be combined to bound experimentally the missing theoretical contributions that plague our averaging attempts.

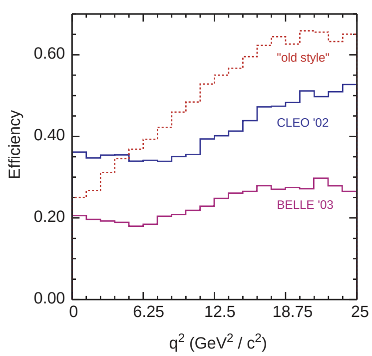



**FIGURE 1.** Efficiency as a function $q^2$ for the "traditional" lepton endpoint analysis (dotted curve) and more recent CLEO and BELLE analyses (solid curves).

Inclusive theory has progressed significantly in categorization of the important corrections in different regions of phase space, and in determination of some of the hitherto unknown corrections. Experimentally, measurements have significantly reduced model dependence in extraction of rates. For example, the reduced dependence of efficiency on dilepton mass $q^2 = M^2_{W^*}$ in new rate measurements near the $E_\ell$ endpoint[3–5] (Figure 1) has reduced model dependence by a factor of three. Recent measurements have also minimized reliance on detailed $d\Gamma(B \to X_u \ell \nu)/dE_\ell dM_{X_u} dq^2$ modeling to separate the $b \to u\ell\nu$ signal from $b \to c\ell\nu$. We thus have rate measurements with well-defined sensitivities within well-defined regions of phase space for which we can categorize the important theoretical uncertainties. Both the theoretical and experimental improvements are key to more robust evaluation of $|V_{ub}|$.

## Inclusive $b \to u\ell\overline{\nu}$

Theoretically, issues regarding the calculation of the total semileptonic partial width $\Gamma(B \to X_u \ell \nu)$ via the operator product expansion (OPE) are well-controlled[6–11]. This nonperturbative power series in $1/m_b$ and perturbative expansion in $\alpha_s$ has, at order $1/m_b^2$, two nonperturbative parameters: $\lambda_1$ or $\mu_\pi^2$, which is related to the Fermi momentum of the $b$ quark in the meson, and $\lambda_2$ or $\mu_G^2$, which parameterizes the hyperfine interaction between the heavy quark and the light degrees of freedom. The $\lambda$ and $\mu$ parameters differ in their infrared behavior. In terms of these parameters, the OPE at $1/m_B^2$ yields

$$\Gamma(B \to X_u e \nu) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} m_b^5 \times \left[1 - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \ldots - \mathscr{O}(\frac{\alpha_s}{\pi})\right]. \quad (1)$$

[1] The following article has been submitted to the Proceedings of Beauty 2003. After it is published, it will be found at `http://proceedings.aip.org`.

The perturbative corrections are known to order $\alpha_s^2$ [12]. The error induced by uncertainties in the nonperturbative parameters $\lambda_{1,2}$ is relative small, and an evaluation by the LEP Heavy Flavors [13] working group yielded

$$|V_{ub}| = 0.00445\left(\frac{\mathrm{B}(b\to u\ell\bar{\nu})}{0.002}\,\frac{1.55\mathrm{ps}}{\tau_b}\right)^{1/2} \times\left(1\pm 0.020_{\lambda}\pm 0.052_{m_b}\right) \quad (2)$$

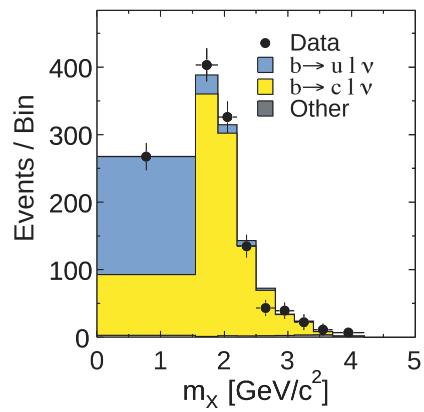
The mass $m_b^{1S}(1GeV) = 4.58\pm 0.09$ GeV, which agrees with a recent survey[14], dominates the uncertainty. Because the OPE is a quark-level calculation, the estimate rests on the assumption of global quark-hadron duality, which is well-motivated[15] for $b\to u\ell\bar{\nu}$ (particularly with its broad range of hadronic final states). We can bound $\sigma_{\mathrm{duality}}$ by comparison of the more precise inclusive and exclusive evaluations of $|V_{cb}|$. The difference[16] of $(0.8\pm 1.6)\times 10^{-3}$ implies $\sigma_{\mathrm{duality}} = 4\%$, for a total uncertainty of 6.8% on the total rate.

To overcome the 100 times larger $b\to c\ell\bar{\nu}$ background, inclusive $b\to u\ell\bar{\nu}$ measurements utilize restricted regions of phase space in which the background is suppressed. Extraction of $|V_{ub}|$ then requires the fraction of the total $b\to u\ell\bar{\nu}$ rate that lies within the given region of phase space. This complicates the theoretical issues and uncertainty considerably. We first consider measurements restricted either to the region $p_\ell \gtrsim 2.2$ GeV$/c$ or to the region of low hadronic mass ($M_X \lesssim M_D$). Within both, the parton–level OPE fails because its expansion parameter $E_X\Lambda_{QCD}/m_x^2 \sim 1$. We will return to this issue and its impact on rate estimations.

Table 1 summarizes the rate measurements by CLEO[3], BaBar[4] and Belle[5] near the $p_\ell$ endpoint. These analyses must suppress background from continuum processes, and can reach down to about 2.2 GeV$/c$ before control of $b\to c\ell\nu$ becomes problematic. The continuum suppression induces the efficiency variation with $q^2$ discussed above. The remainder of the model dependence could be eliminated by coarsely binning in $q^2$, or, preferably, through use of a tagged $B$ sample.

**TABLE 1.** Partial branching fractions for $b\to u\ell\bar{\nu}$ near the $p_l$ endpoint. The rates are integrated up to $p_\ell^{\max} = 2.6$ GeV$/c$ ($\Upsilon(4S)$ frame). The estimated rate fraction $f_E$ is given for each range. The dagger (†) indicates where the QED radiative correction was applied.

| $p_\ell^{\min}$ (GeV$/c$) | $\Delta\mathscr{B}_u(p)$ $(10^{-4})$ | $f_E$ | |
|---|---|---|---|
| 2.0 | $4.22\pm 1.81$ | †$0.266\pm 0.048$ | CLEO $(e,\mu)$ |
| 2.1 | $3.28\pm 0.77$ | †$0.198\pm 0.040$ | CLEO $(e,\mu)$ |
| 2.2 | $2.30\pm 0.38$ | †$0.130\pm 0.028$ | CLEO $(e,\mu)$ |
| 2.3 | $1.43\pm 0.16$ | †$0.074\pm 0.017$ | CLEO $(e,\mu)$ |
| | †$1.52\pm 0.20$ | $0.078\pm 0.017$ | BaBar $(e)$ |
| | $1.19\pm 0.15$ | †$0.072\pm 0.016$ | BELLE $(e)$ |
| 2.4 | $0.64\pm 0.09$ | †$0.037\pm 0.008$ | CLEO $(e,\mu)$ |




*Source: Aubert et al. [17]*

**FIGURE 2.** Reconstructed $M_X$ distribution from the BaBar low $M_X$ analysis.

BaBar[17] and Belle[18] have new analyses of the low $M_X$ region (combined with a moderate $p_\ell > 1.0$ GeV$/c$ requirement), which was first studied by DELPHI[19]. Finite $M_X$ resolution smears the $b\to c\ell\bar{\nu}$ background below its theoretical limit of $M_D$, forcing more stringent $M_X$ requirements that approach a pole in the parton level spectrum. The preliminary Belle analysis utilizes a $D^{(*)}\ell\nu$ tag, with $M_X$ calculated directly from all particles after removal of tag and lepton. This tag still results in a significantly larger background than signal level, and the systematic estimates (very preliminary) appear quite aggressive. The BaBar analysis uses fully reconstructed hadronic $B$ tags, again with direct $M_X$ calculation. This analysis achieves a beautiful $b\to u\ell\bar{\nu}$ signal in the region $M_X < 1.55$ GeV$/c$ with signal to background ratio (see Figure 2) approaching 2:1. This ratio shows the anticipated power of the hadronic $B$ tags, which afford unsurpassed resolution on $M_X$. The efficiency versus $M_X$, while not featureless, appears promisingly uniform. BaBar has also extracted the yields with a fit to the integrated $M_X < 1.55$ GeV$/c$ interval. Both features minimize dependence of the extracted rate on detailed modeling of the $b\to u\ell\bar{\nu}$, which allows for cleaner theoretical interpretation and simplifies improved determination of $|V_{ub}|$ as theory advances.

Determination of the fraction of the $b\to u\ell\bar{\nu}$ rate in the $p_\ell$ endpoint or the low $M_X$ region requires resummation of the OPE to all orders in $E_X\Lambda_{QCD}/m_x^2$[6, 20–23]. The resummation results, at leading-twist order, in a nonperturbative shape function $f(k_+)$. The argument $k_+ = k^0 + k_\|$, where $k^\mu = p_b^\mu - m_b v^\mu$ is the $b$ quark momentum with the "mechanical" portion subtracted. The spatial components $k_\|$ and $k_\perp$ are defined relative to the $m_b v^\mu - q^\mu$ (roughly the recoiling $u$ quark) direction. At leading twist, effects like the "jiggling" of $k_\perp$ are ignored, and the differential partial width is given by the

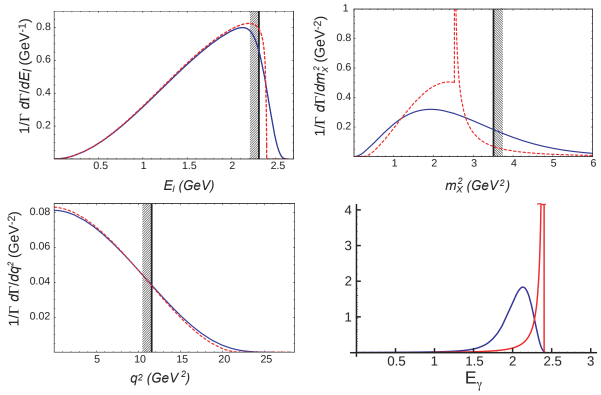



**FIGURE 3.** Parton level (blue) and model convoluted (red) spectra for $B \to X_u \ell \nu$ (top, bottom left; from ref. [24]) and for $B \to X_s \gamma$ (bottom right).

convolution of the shape function with the parton level differential distribution (see Figure 3):

$$d\Gamma = \int dk_+ \, f(k_+) d\Gamma^{(parton)}_{m_b \to m_b + k_+} . \tag{3}$$

Because the shape function depends only on parameters of the $B$ meson, this description holds for any $B$ decay to a light quark, such as $B \to s\gamma$.

It has been known for some time[6, 21] that measurement of the $E_\gamma$ spectrum in $b \to s\gamma$ can yield (at leading twist) $f(k_+)$. Ideally, $|V_{ub}|$ would be determined directly from integrated spectra[21, 25–29] without extraction of an intermediate shape function. For the lepton spectrum, for example, one takes

$$\left| \frac{V_{ub}}{V_{tb} V_{ts}^*} \right|^2 = \frac{3\alpha}{\pi} K_{\rm pert} \frac{\widehat{\Gamma}_u(E_0)}{\widehat{\Gamma}_s(E_0)} + O(\Lambda_{\rm QCD}/M_B) , \tag{4}$$

where $K_{pert}$ is a calculable perturbative kernel, and $\widehat{\Gamma}_u(E_0)$ and $\widehat{\Gamma}_s(E_0)$ are appropriately weighted integrals over, respectively, the $E_\ell$ and $E_\gamma$ spectra above the cutoff $E_0$. A similar expression exists for the $M_X$ spectrum[26]. Practical application by the endpoint analyses is currently difficult because of integration of the rate in the $\Upsilon(4S)$ frame, which introduces significant smearing. With tagged $B$ samples, endpoint measurements can be made in the $B$ frame (with relaxed continuum suppression), which would allow for direct application of the weighted integral approach. In principle, current $M_X$ analyses can already use this approach, though experimental efficiency and $p_\ell$ cutoffs must be incorporated into the integrals. All current analyses have, instead, relied on modeling an intermediate shape function.

The role of the shape function can be mitigated by restricting measurement to regions of large $q^2$, where validity of the OPE expansion is restored[30, 31]. Restriction to the region kinematically forbidden to $b \to c\ell\overline{\nu}$, $q^2 > (M_B - M_D)^2$, however, introduces a low mass scale [32, 33] into the OPE, introducing uncertainties of order $(\Lambda_{QCD}/m_c)^3$. However, the combination of an $M_X$ with a looser $q^2$ requirement can suppress $b \to c\ell\overline{\nu}$ background yet still reduce shape function contributions. Furthermore, the $q^2$ restriction moves the parton level pole away from typical $M_X$ cuts. For a practical choice of region, $f(k_+)$ contributions are suppressed but not negligible. The elimination of high energy hadronic final states by the $q^2$ restriction may exacerbate duality concerns.

A recent Belle analysis [34] in this region employs a $p_\ell > 1.2$ GeV$/c$ requirement followed by an "annealing" procedure to sort reconstructed particles into the "signal" and "other" $B$. The analysis integrates over the rate in the region $M_X < 1.7$ GeV and $q^2 > 8$ GeV$^2$ to extract $|V_{ub}|$, which again has the desired effect of minimizing dependence of the analysis on detailed modeling of the $b \to u\ell\overline{\nu}$ process. The signal to background ratio of the analysis, about 1:6, does not approach that of the $B$ tag technique. Belle finds a rate $\Delta\mathscr{B}$ in that $q^2$–$M_X$ region of

$$\frac{\Delta\mathscr{B}}{10^{-4}} = 7.37 \pm 0.89_{stat} \pm 1.12_{sys} \pm 0.55_{c\ell\nu} \pm 0.24_{u\ell\nu} . \tag{5}$$

I look forward to analysis of this region with the significantly cleaner $B$ tag technique.

Extraction of $|V_{ub}|$ from these measurements has required modeling of $f(k_+)$ for estimation of the fraction $(f)$ of the rate in each region of phase space. The endpoint analyses use fractions estimated by CLEO[3] based on an $f(k_+)$ derived from the CLEO $b \to s\gamma$ $E_\gamma$ spectrum. CLEO employed several two–parameter functional forms $f[\Lambda^{SF}, \lambda_1^{SF}](k_+)$[35, 36] that were convolved with the parton-level $E_\gamma$ calculation in fits to the measured spectrum between 1.5 and 2.8 GeV. These parameters are related to the HQET parameters of similar name, and play a similar role in evaluation of the rates. At this time, however, we do not know the relationship between the shape function parameters (or the moments of the shape function) and the HQET nonperturbative parameters $\overline{\Lambda} = m_B - m_b$ and $\lambda_1$[37, 38]. The fact that $\Lambda^{SF}$ and $\lambda_1^{SF}$ depend on the functional ansatz adopted, while the HQET parameters depend on the renormalization scheme, underscores the current ambiguity.

Figure 4 shows the best fit parameters[39] and the one standard deviation contour for the exponential form[36]. The strong correlation between the two parameters results from the interplay between the $b$ quark's effective mass (controlled by $\Lambda^{SF}$) and kinetic energy (controlled by $\lambda_1^{SF}$) in determining the *mean* of the $E_\gamma$ spectrum (and the mean energy available for the final state in $b \to u\ell\nu$). The best fit corresponds to $(\Lambda^{SF}, \lambda_1^{SF}) = (0.545, -0.342)$ and the rate fractions $f_E = 0.14$, $f_M = 0.53$ and $f_{qM} = 0.34$ for the CLEO $p_l > 2.2$ GeV$/c$, the BaBar low $M_X$, and the BELLE $M_X - q^2$ analyses, respectively. The

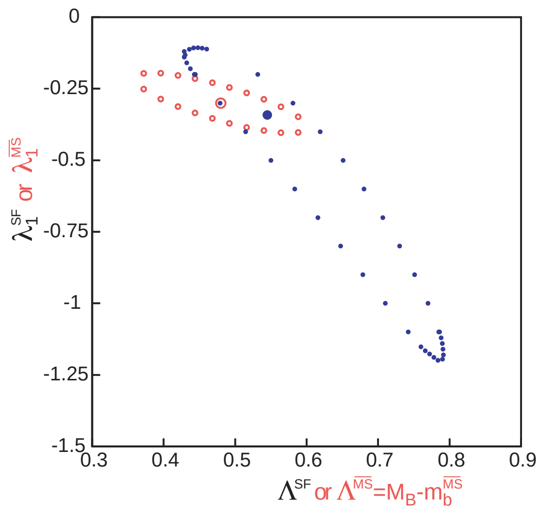



**FIGURE 4.** Shape function parameters from best fit to $b \to s\gamma$ photon energy spectrum and one standard deviation "ellipse" (solid blue circles). Also shown is the ($\overline{\Lambda}^{MS}$,$\lambda_1^{\overline{MS}}$) ellipse corresponding to the BaBar shape function parameter choice (open red circles).

**TABLE 2.** Summary of inclusive $|V_{ub}|$ results. The errors on the first group are experimental and theoretical uncertainties, respectively. The second group, adjusted to a common $f(k_+)$, lists the total experimental, $f(k_+)$–related, and $\Gamma_{tot}$ uncertainties. The two groups are not directly comparable.

| | $\lvert V_{ub}\rvert(10^{-3})$ | |
|---|---|---|
| ALEPH[43] | $4.12 \pm 0.67 \pm 0.76$ | neur. net |
| L3[44] | $5.70 \pm 1.00 \pm 1.40$ | cut, count |
| DELPHI | $4.07 \pm 0.65 \pm 0.61$ | $M_X$ |
| OPAL[45] | $4.00 \pm 0.71 \pm 0.71$ | neur. net |
| LEP Avg.[13] | $4.09 \pm 0.37 \pm 0.56$ | |
| CLEO[46] | $4.05 \pm 0.61 \pm 0.65$ | triple diff. |
| BELLE | $5.00 \pm 0.64 \pm 0.53$ | $M_X$ |
| CLEO | $4.11 \pm 0.34 \pm 0.46 \pm 0.28$ | $p_\ell > 2.2$ |
| BaBar | $4.31 \pm 0.28 \pm 0.49 \pm 0.30$ | $p_\ell > 2.3$ |
| BELLE | $3.99 \pm 0.23 \pm 0.45 \pm 0.27$ | $p_\ell > 2.3$ |
| BELLE | $4.63 \pm 0.48 \pm 0.48 \pm 0.32$ | $M_X$<1.7 $q^2$>8 |
| BaBar | $4.79 \pm 0.40 \pm 0.60 \pm 0.33$ | $M_X < 1.55$ |

statistical uncertainty derives from the extremes in the fractions found on the contour of Figure 4. The endpoint analysis was symmetrized by taking the average of those extremes, resulting in $f_E = 0.13 \pm 0.02$, $f_M = 0.52 \pm 0.12$ and $f_{qM} = 0.32 \pm 0.06$. The fractions are almost completely correlated as one moves around the contour. A small correction resulting from a re-optimization of background normalization in the $b \to s\gamma$ analysis[40] (about 1/5 the assigned systematic) is applied. After including the remaining background subtraction systematic and smaller contributions from the $\alpha_s$ uncertainty and from variation of the $f(k_+)$ ansatz[35, 36], the rate fraction results are $f_E = 0.13 \pm 0.03$ (with radiative corrections), $f_M = 0.55 \pm 0.14$ and $f_{qM} = 0.33 \pm 0.07$. A more detailed description is forthcoming[41].

Two alternate approaches to shape function modeling have been taken in experimental studies. The BaBar $M_X$ analysis[17] takes the $f(k_+)$ ansaetze just discussed, but associates $(\Lambda^{SF}, \lambda_1^{SF})$ with the HQET parameters derived from spectral moments of the $b \to s\gamma$ and $b \to c\ell\overline{\nu}$ processes (see Figure 4). Note that while the ellipse appears smaller, the $\overline{\Lambda}^{MS}$–$\lambda_1^{\overline{MS}}$ correlation does not stabilize the average final state energy like the $E_\gamma$-based correlation, resulting in $f_M$ uncertainties that are comparable to the $E_\gamma$ based uncertainties. The Belle $M_X$–$q^2$ analysis uses reference [31], which employs a simpler form based on the single parameter $\Lambda^{SF}/\lambda_1^{SF}$ estimated from the $m_b^{(1S)}$ mass and typical estimates for $\lambda_1$. These approaches rest on the assumption, now agreed to be incorrect[37, 38], that an arbitrary renormalization scheme can be used in the parameterizations employed. The associated uncertainty is difficult to assess, and has not been included in the $|V_{ub}|$ determinations

Once the renormalization behavior of $f(k_+)$ is understood, the relationship between HQET parameters, kinematic definitions of the $b$ quark mass, and moments of $f(k_+)$ will be better defined. Moments of the $b \to c\ell\nu$ process and $m_b$ constraints can then inform extraction of $f(k_+)$. Correlations from the $m_b$ dependence in the total rate and in the rate fractions must then be incorporated. Given the complete independence of the kinematic mass determination used for the total rate and the effective quark mass in the $E_\gamma$–derived $f(k_+)$, coupled with the large effective mass range sampled in the latter, a linear combination of those two uncertainties[34] seems overly conservative for the present $E_\gamma$–derived results.

Table 2, based on the Heavy Flavors Averaging Group (HFAG) summary, [42], summarizes the full set of inclusive $|V_{ub}|$ results. Given the strong correlations among the rate fractions, $|V_{ub}|$ comparisons are meaningful only for results evaluated with common theoretical input. The $E_\gamma$–derived shape function is currently the best motivated theoretically and has the most complete categorization of uncertainties. I therefore adjust recent results and $f(k_+)$–related uncertainties to use the rate fractions discussed above. The errors do not include uncertainties from potentially large theoretical corrections that have been categorized but not calculated, as discussed below.

## Combining inclusive information

Evaluation of the total uncertainty on $|V_{ub}|$ remains problematic because of three main theoretical complications ([see, *e.g.*, 47, 48]. The first arises from subleading (higher twist) contributions to the OPE resummation[49–52], which are not universal. Hence with use of $b \to s\gamma$ to constrain $f(k_+)$, there exist subleading contributions both to the use of a shape function in $b \to u\ell\bar{\nu}$ itself and to the derivation of $f(k_+)$ from $b \to s\gamma$. The subleading contributions, formally of order $\Lambda_{QCD}/m_b$, can be as large as $\sim 15\%$. Indeed, a partial estimate[51] for the endpoint region finds corrections that are approximately the same size as the other combined uncertainties.

The second contribution, from "weak annihilation" processes[53, 54], is formally of order $(\Lambda_{QCD}/m_b)^3$ but receives a $16\pi^2$ enhancement. The contribution, which requires factorization violation to be nonzero, is expected to be localized near $q^2 \sim m_b^2$. This results in further enhancement of the effect on $|V_{ub}|$ measurements. For the endpoint region, an effect on the total rate of 2-3% (for factorization violation of about 10%), corresponds to 20-30% on the endpoint rate.

Finally, while global quark hadron duality is well-motivated for spectral moments, the OPE cannot predict the detailed inclusive spectra. The extent of violation of quark–hadron duality locally depends on the size and nature of the region of phase space considered: including a large fraction of the rate is best. The associated uncertainty is difficult to assess.

The problems outlined present a considerable obstacle to a meaningful average of the inclusive results. Results with a potentially large bias will be included with neither correction nor meaningful uncertainty: $|V_{ub}|$ will be biased and have an unreliable uncertainty. As an alternative, we can choose a region of phase space that provides a reasonable compromise among the unknown contributions. The choice is inherently subjective given the different viewpoints within the theory community ([see, *e.g.*, 29, 31]). In this reviewer's opinion, the opportunity to bound experimentally the uncertainties we know of, thereby providing as complete an uncertainty estimate as possible, is more important than achieving the smallest statistical precision. Currently, I find the region restricted to low $M_X$ and higher $q^2$ the most compelling. It has reduced corrections from $f(k_+)$ and hence from subleading contributions, yet has a sufficient fraction of the spectrum to dilute weak annihilation and local quark hadron duality concerns. The low $M_X$ and $p_l$ endpoint regions, in which one or more of the corrections is more pronounced, then play critical roles in limiting the uncertainty in this region. I stress that estimating a complete inclusive uncertainty is of fundamental importance, and hence that I consider measurements in all three regions of equal importance. Indeed, I view a single coherent analysis of all three phase space regions simultaneously an important milestone for both $B$ factories, particularly with application of the powerful and clean $B$ tag samples.

For now, however, only BELLE has contributed a result for this region of phase space, and I quote that result as my "central value":

$$|V_{ub}|/10^{-3} = 4.63 \pm 0.28_{stat} \pm 0.39_{sys} \pm 0.48_{f_{qM}} \pm 0.32_{\Gamma_{thy}} \pm \sigma_{\mathrm{WA}} \pm \sigma_{SSF} \pm \sigma_{LQD}. \tag{6}$$

New measurements in this region can be easily combined when available, and will improve the experimental uncertainties. We must determine the uncertainties for weak annihilation (WA), subleading shape function corrections (SSF) and local quark hadron duality (LQD) within this region. Note that the data are not yet precise enough to draw conclusions regarding the presence or absence of these corrections; we use them to provide bounds.

Each phase space region considered should largely contain the WA contribution, which will be most (least) diluted in the low $M_X$ (endpoint) region. For a neglected WA contribution, comparison of $|V_{ub}|$ from these two regions would predict a bias in the $M_X$, $q^2$ region to be

$$[(1-f_{qM})/f_{qM}][f_e f_M/(f_M - f_e)] \approx 0.39 \tag{7}$$

of the observed difference. The quoted value, which is model dependent, is based on the fractions found for the $E_\gamma$–derived $f(k_+)$. Comparison of the CLEO endpoint and BaBar low $M_X$ values, taking into consideration the almost total correlation in the shape function and $\Gamma_{tot}$ uncertainties, yields $\Delta|V_{ub}|/10^{-3} = 0.69 \pm 0.53$. I take the larger of the central value and error and scale according to Eq. 7 to obtain $\sigma_{WA} \approx 0.27$.

To estimate $\sigma_{SSF}$, I assume that subleading corrections scale like the fractional change in the rate prediction $(\Delta\Gamma/\Gamma)$ that $f(k_+)$ induces relative to the parton-level calculation. Comparison of the low $M_X$ region ($[\Delta\Gamma/\Gamma]_M \sim 0.15$) to the combined $M_X$, $q^2$ region ($[\Delta\Gamma/\Gamma]_{qM} \sim -0.075$) with theory correlations considered, gives $\Delta|V_{ub}|/10^{-3} = 0.16 \pm 0.63$. Scaling by $|(\Delta\Gamma/\Gamma)_{qM}/(\Delta\Gamma/\Gamma)_M| = 0.49$, which is model dependent, we obtain $\sigma_{SSF} \approx 0.31$.

Finally, to bound the local duality uncertainty, I assume that the error scales with the rate fraction $f$ as $(1-f)/f$ (*ad hoc*, but goes to zero for full phase space and diverges for use of the detailed spectra). To obtain an estimate, I compare the CLEO $p_\ell > 2.2$ GeV/$c$ region to the average of BaBar and BELLE in the $p_\ell > 2.3$ GeV/$c$ region and apply the subleading correction estimates[51] $(+0.27 \times 10^{-3})$ to minimize potential cancelation between duality violation and subleading corrections. This yields $(|V_{ub}|^{2.3} - |V_{ub}|^{2.2} + 0.27)/10^{-3} = 0.29 \pm 0.38$. Scaling the uncertainty by

$$\frac{(1-f_{qM})/f_{qM}}{(1-f_{2.3})/f_{2.3} - (1-f_{2.2})/f_{2.2}} \approx 0.29 \tag{8}$$

based on the fractions in Table 1 gives $\sigma_{LQD} \sim 0.11$.

From this combination of information, we thus find

$$|V_{ub}|/10^{-3} = 4.63 \pm 0.28_{stat} \pm 0.39_{sys} \pm 0.48_{f_{qM}} \pm 0.32_{\Gamma_{thy}} \pm 0.27_{\mathrm{WA}} \pm 0.31_{SSF} \pm 0.11_{LQD} \tag{9}$$

for a total theory error of 15%. Since experimental uncertainties dominate, quadrature addition seems reasonable. The limits presented here can be improved in robustness ( *e.g.* considering potential cancellations among effects), through more sophisticated scaling estimates, and through improved and additional measurements. Improvement of the $b \to s\gamma$ photon energy spectrum is key. Measurements of $D^0$ versus $D_s$ semileptonic widths and comparison of neutral and charged $B$ decay can help limit WA contributions[54]. Finally, improved theory for the scaling of the effects over phase space could allow development of a procedure for simultaneous extraction of $|V_{ub}|$ and the corrections, with all experimental information contributing directly to $|V_{ub}|$, or could shift the choice of "preferred" region.

## Exclusive measurements of $b \to u\ell\overline{\nu}$

I devote considerably less time to discussion of exclusive determination of $|V_{ub}|$ – not because it is less important but because the story is simpler. Theoretical issues center on determination of the form factors (FF) involved in the decays. For $B \to \pi\ell\nu$, for example, one has[55]

$$\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2\, d\cos\theta_\ell} = |V_{ub}|^2 \frac{G_F^2 p_\pi^3}{32\pi^3} \sin^2\theta_\ell |f_+(q^2)|^2 \qquad (10)$$

with only the single form factor $f_+(q^2)$ for massless leptons. Final states with a vector meson depend on three form factors. The measured rates depend on the variation of the form factors with $q^2$ ("shape") and their relative normalizations[56], while extraction of $|V_{ub}|$ depends both on their shape and absolute normalization.

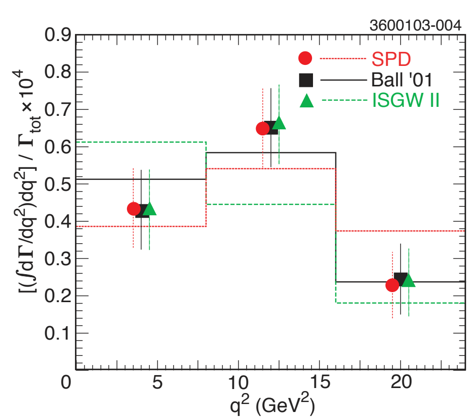
A large variety of calculations exist. For extraction of $|V_{ub}|$, the focus has sharpened onto the QCD–based calculations of lattice QCD (LQCD)[57–69] and light cone sum rules (LCSR)[70–78]. Only quenched LQCD FF calculations are available for $b \to u\ell\nu$, and these have sensitivity only in the range $q^2 \gtrsim 16$ GeV$^2$. Approximations made in the LCSR calculations are only valid for $q^2 \lesssim 16$ GeV$^2$. A recent summary[79] shows reasonable agreement where comparison is possible. The FF's have also been evaluated using many quark– or parton–model based techniques[80–94]. Uncertainties in these models are difficult to assess, and they exhibit a broad variation in shape. Finally, various studies of FF's based on constraints from dispersion relations, unitarity or Heavy Quark Symmetry have been made[95–100].

Recent work has helped assessment of the reliability of the FF's based on the available LCSR and quenched (no light quark loops in the propagators) LQCD calculations. Analysis[101–104] of the LCSR approach within the framework of soft collinear effective theory (SCET) has sparked debate regarding potential contributions missing from LCSR. The $B \to \rho\ell\nu$ FF's, in particular, may be overestimated in LCSR, biasing $|V_{ub}|$ low. Unquenched LQCD calculations have begun to appear, and comparison to experiment shows much better agreement with data (few percent) than for quenched results[105]. While work has begun on unquenched FF's, initial results are limited to valence quark masses $\sim m_s$. Initial results[106, 107] are compatible with the $\sim 15\%$ uncertainties assigned for the quenching approximation.

To date, all exclusive measurements employ detector hermeticity to estimate $p_\nu$, which allows full reconstruction of the decays. Two general strategies have been taken. The CLEO[108] and BaBar[109] $B \to \rho\ell\nu$ analyses emphasize higher efficiency and employ relatively loose event cleanliness and $\nu$-consistency criteria. With the resulting background levels, these analyses are primarily sensitive in the region $p_\ell > 2.3$ GeV$/c$. To extract total branching fractions and $|V_{ub}|$, these analyses survey a variety of FF models, including LQCD and LCSR calculations extrapolated over the full $q^2$ range.

The simultaneous $B \to \pi\ell\nu$ and $B \to \rho\ell\nu$ measurement by CLEO [110], on the other hand, applies strict criteria to achieve acceptable background levels over a broad $p_\ell$ range. With sensitivity down to 1.0 GeV$/c$ (1.5 GeV$/c$) for $\pi\ell\nu$ ($\rho\ell\nu$), this analysis was able to extract independent rates in three $q^2$ bins. This eliminated model dependence of the measured $\pi\ell\nu$ rates (Figure 5) and halved that for $\rho\ell\nu$ rates (see Ref. [56] for a discussion of model dependence). Furthermore, the analysis permits extraction of $|V_{ub}|$ from the LQCD and LCSR FF's within their valid $q^2$ ranges and so without additional modeling.

Table 3 summarized the results for $|V_{ub}|$ from exclusive measurements. Averages of the CLEO results are given with and without the low $q^2$ region for $\rho\ell\nu$ (for which LCSR validity is under debate). Note that FF-




*Source: Athar et al. [110]*

**FIGURE 5.** The CLEO $B^0 \to \pi^-\ell^+\nu$ partial branching fractions based on three disparate FF models.

**TABLE 3.** Summary of exclusive $|V_{ub}|$ measurements. The errors listed are the statistical and experimental systematic uncertaines combined in quadrature, and all form factor uncertainties, respectively. In the CLEO '03 averages, the LQCD and LCSR uncertainties have been treated as correlated.

| | $\lvert V_{ub}\rvert(10^{-3})$ | $q^2$ range (GeV$^2$) | FF |
|---|---|---|---|
| CLEO '00 $\rho$ | $3.23^{+0.33}_{-0.35} \pm 0.58$ | all | survey |
| BaBar '01 $\rho$ | $3.64 \pm 0.33^{+0.39}_{-0.56}$ | all | survey |
| CLEO '03 $\pi$ | $3.33 \pm 0.28^{+0.57}_{-0.40}$ | $< 16$ | LCSR |
| CLEO '03 $\pi$ | $2.88 \pm 0.63^{+0.48}_{-0.39}$ | $> 16$ | LQCD |
| CLEO '03 $\pi$ | $3.24 \pm 0.26^{+0.56}_{-0.40}$ | average | |
| CLEO '03 $\rho$ | $2.67^{+0.47}_{-0.50}{}^{+0.50}_{-0.39}$ | $< 16$ | LCSR |
| CLEO '03 $\rho$ | $3.34^{+0.42}_{-0.48}{}^{+0.69}_{-0.62}$ | $> 16$ | LQCD |
| CLEO '03 $\rho$ | $3.00^{+0.36}_{-0.41}{}^{+0.56}_{-0.47}$ | average | |
| CLEO '03 $\pi$,$\rho$ | $3.17^{+0.23}_{-0.24}{}^{+0.53}_{-0.39}$ | average | |
| CLEO '03 $\pi$,$\rho$ | $3.26 \pm 0.24^{+0.54}_{-0.39}$ | average, no $\rho$ LCSR | |

related uncertainties have been treated as completely correlated in the LCSR and LQCD averages to remain conservative. The LQCD uncertainties for the $\pi$ ($\rho$) modes include 15% (20%) quenching uncertainties, which are called out separately in the LQCD references. Ref. [56] presents a more complete review of recent $B \to X_u \ell \nu$ branching fractions, including measurements from which $|V_{ub}|$ has not yet been extracted. Of note is evidence for $B^+ \to \omega \ell^+ \nu$[111] and $B^+ \to \eta \ell^+ \nu$[110].

Potential exists for significant correlation among the dominant experimental systematics[56], so the results have been averaged assuming full correlation. The earlier results[108, 109], which depend more heavily on modeling, are deweighted by 5%. The average yields

$$|V_{ub}| = (3.27 \pm 0.13 \pm 0.19^{+0.51}_{-0.45}) \times 10^{-3} \qquad (11)$$

where the errors arise from statistical, experimental systematic and form factor uncertainties, respectively. Should the LCSR form factors prove to be overestimated, I also average without including information using $q^2 < 16\,\mathrm{GeV}^2$ in $\rho \ell \nu$, yielding $|V_{ub}| = (3.26 \pm 0.19 \pm 0.15 \pm 0.04^{+0.54}_{-0.39}) \times 10^{-3}$, where the errors are statistical, experimental systematic $\rho \ell \nu$ form factor uncertainties, and LQCD and LCSR uncertainties.

The future for exclusive determinations of $|V_{ub}|$ appears promising. The large $B$ tag samples being collected should allow significant improvement in $\nu$ resolution and reduction of backgrounds and experimental systematics. Unquenched LQCD calculations are underway and will eliminate the primary, poorly controlled, source of uncertainty. Recent advances may also allow use of the full $q^2$ range for extraction of $|V_{ub}|$ with LQCD[112, 113]. For both LQCD and experiment, $\pi \ell \nu$ appears to be the golden mode – even the $B^*$ pole now appears manageable[68]. $B \to \eta \ell \nu$ will provide a valuable cross-check. The $\rho \ell \nu$ mode will be more problematic for high precision. The broad $\rho$ width leaves experiments open to larger backgrounds, including poorly-understood nonresonant $\pi\pi$ contributions. In unquenched LQCD calculations the $\rho$ is unstable, and methods for accommodating the high energy $\pi\pi$ final state have yet to be developed. The $\omega \ell \nu$ mode may prove more tractable. Agreement between accurate $|V_{ub}|$ determinations from $\pi \ell \nu$, $\eta \ell \nu$ and $\omega \ell \nu$ will add confidence overall.

## Inclusive, exclusive averaging

Until recently, I have argued against averaging of inclusive and exclusive results because of outstanding uncertainties in the former and no checks on the latter's theory. With the experimental bounds on uncertainties determined above, and some clarification of the reliability of LCSR and of the quenching uncertainties in LQCD, my major concerns are being addressed. I therefore combine the inclusive and exclusive results and find

$$|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3}. \qquad (12)$$

Excluding results based on data below $q^2 < 16$ GeV$^2$ in the $\rho \ell \nu$ yields a similar result: $|V_{ub}| = (3.70 \pm 0.49) \times 10^{-3}$. Inclusive and exclusive averaging will likely remain controversial in the short term. However, with the progress expected both inclusive and exclusive measurements and theory, I anticipate a noncontroversial value of $|V_{ub}|$, with an uncertainty bettering the 13% presented here, in the next few years.

## ACKNOWLEDGMENTS

I would like to thank A. Kronfeld, B. Lange, G.P. Lepage, Z. Ligeti, M. Luke and M. Neubert for their input regarding the theory of $|V_{ub}|$ determinations. Special thanks to D. Cronin-Hennessy and E. Thorndike for their analysis of the rate fractions derived from the $E_\gamma$ spectrum.

## REFERENCES

1. Cabibbo, N., *Phys. Rev. Lett.*, **10**, 531–532 (1963).
2. Kobayashi, M., and Maskawa, T., *Prog. Theor. Phys.*, **49**, 652–657 (1973).
3. Bornheim, A., et al., CLEO Collaboration, *Phys. Rev. Lett.*, **88**, 231803 (2002), [arXiv:hep-ex/0202019].
4. Aubert, B., et al., BABAR Collaboration, arXiv:hep-ex/0207081.
5. Abe, K., et al., BELLE Collaboration, BELLE-CONF-0325.

6. Bigi, I. I. Y., Shifman, M. A., Uraltsev, N. G., and Vainshtein, A. I., *Int. J. Mod. Phys.*, **A9**, 2467–2504 (1994), [arXiv:hep-ph/9312359].
7. Neubert, M., *Int. J. Mod. Phys.*, **A11**, 4173–4240 (1996), [arXiv:hep-ph/9604412].
8. Bigi, I. I. Y., Shifman, M. A., and Uraltsev, N., *Ann. Rev. Nucl. Part. Sci.*, **47**, 591–661 (1997), [arXiv:hep-ph/9703290].
9. Hoang, A. H., Ligeti, Z., and Manohar, A. V., *Phys. Rev. Lett.*, **82**, 277–280 (1999), [arXiv:hep-ph/9809423].
10. Bigi, I. I. Y., arXiv:hep-ph/9907270.
11. Ligeti, Z., arXiv:hep-ph/9908432.
12. van Ritbergen, T., *Phys. Lett.*, **B454**, 353–358 (1999), [arXiv:hep-ph/9903226].
13. Abbaneo, D., Battaglia, M., Gagnon, P., Henrard, P., Lu, J., Mele, S., Piotto, E., and Rosnet, P., The LEP VUB Working Group, lEPVUB-01/01.
14. El-Khadra, A. X., and Luke, M., *Ann. Rev. Nucl. Part. Sci.*, **52**, 201–251 (2002), [arXiv:hep-ph/0208114].
15. Bigi, I. I. Y., and Uraltsev, N., *Int. J. Mod. Phys.*, **A16**, 5201–5248 (2001), [arXiv:hep-ph/0106346].
16. Artuso, M., and Barberio, E., "Determination of $|V_{cb}|$," 2004, to appear in The Review of Particle Properties.
17. Aubert, B., et al., BABAR Collaboration, arXiv:hep-ex/0307062.
18. Schwanda, C., "$V_{cb}$ , $V_{ub}$ , HQET at Belle," in *Proceedings of the International Europhysics Conference onăHighăEnergyăPhysics*, *EPS2003*, Eur. Phys. J. C direct, 2003, digital Object Identifier (DOI) 10.1140/epjcd/s2003-03-109-2.
19. Abreu, P., et al., DELPHI Collaboration, *Phys. Lett.*, **B478**, 14–30 (2000), [arXiv:hep-ex/0105054].
20. Neubert, M., *Phys. Rev.*, **D49**, 3392–3398 (1994), [arXiv:hep-ph/9311325].
21. Neubert, M., *Phys. Rev.*, **D49**, 4623–4633 (1994), [arXiv:hep-ph/9312311].
22. Dikeman, R. D., Shifman, M. A., and Uraltsev, N. G., *Int. J. Mod. Phys.*, **A11**, 571–612 (1996), [arXiv:hep-ph/9505397].
23. Aglietti, U., and Ricciardi, G., *Nucl. Phys.*, **B587**, 363–399 (2000), [arXiv:hep-ph/0003146].
24. Bauer, C. W., Ligeti, Z., and Luke, M. E., arXiv:hep-ph/0007054.
25. Leibovich, A. K., Low, I., and Rothstein, I. Z., *Phys. Rev.*, **D61**, 053006 (2000), [arXiv:hep-ph/9909404].
26. Leibovich, A. K., Low, I., and Rothstein, I. Z., *Phys. Lett.*, **B486**, 86–91 (2000), [arXiv:hep-ph/0005124].
27. Neubert, M., *Phys. Lett.*, **B513**, 88–92 (2001), [arXiv:hep-ph/0104280].
28. Leibovich, A. K., Low, I., and Rothstein, I. Z., *Phys. Lett.*, **B513**, 83–87 (2001), [arXiv:hep-ph/0105066].
29. Bigi, I., and Uraltsev, N., *Int. J. Mod. Phys.*, **A17**, 4709–4732 (2002), [arXiv:hep-ph/0202175].
30. Bauer, C. W., Ligeti, Z., and Luke, M. E., *Phys. Lett.*, **B479**, 395–401 (2000), [arXiv:hep-ph/0002161].
31. Bauer, C. W., Ligeti, Z., and Luke, M. E., *Phys. Rev.*, **D64**, 113004 (2001), [arXiv:hep-ph/0107074].
32. Neubert, M., *JHEP*, **07**, 022 (2000), [arXiv:hep-ph/0006068].
33. Neubert, M., and Becher, T., *Phys. Lett.*, **B535**, 127–137 (2002), [arXiv:hep-ph/0105217].
34. Kakuno, H., et al., BELLE Collaboration, arXiv:hep-ex/0311048.
35. Bigi, I. I. Y., Shifman, M. A., Uraltsev, N., and Vainshtein, A. I., *Phys. Lett.*, **B328**, 431–440 (1994), [arXiv:hep-ph/9402225].
36. Kagan, A. L., and Neubert, M., *Eur. Phys. J.*, **C7**, 5–27 (1999), [arXiv:hep-ph/9805303].
37. Neubert, M. (2003), private communications, discussed in CLNS-04/1858 (in preparation).
38. Bauer, C. W., and Manohar, A. V., arXiv:hep-ph/0312109.
39. Cronin-Hennessy, D. (2003), private communication.
40. Chen, S., et al., CLEO Collaboration, *Phys. Rev. Lett.*, **87**, 251807 (2001), [arXiv:hep-ex/0108032].
41. Gibbons, L. K., Hennessy, D. C., and Thorndike, E. H. (2004), in preparation.
42. Heavy Flavor Averaging Group (HFAG) (2003), URL `http://www.slac.stanford.edu/xorg/hfag/semi/summer03-lp/summer03.shtml`.
43. Barate, R., et al., ALEPH Collaboration, *Eur. Phys. J.*, **C6**, 555–574 (1999).
44. Acciarri, M., et al., L3 Collaboration, *Phys. Lett.*, **B436**, 174–186 (1998).
45. Abbiendi, G., et al., OPAL Collaboration, *Eur. Phys. J.*, **C21**, 399–410 (2001), [arXiv:hep-ex/0107016].
46. Bornheim, A., et al., CLEO Collaboration, cLEO-CONF-02-08.
47. Luke, M., *ECONF*, **C0304052**, WG107 (2003), [arXiv:hep-ph/0307378].
48. Ligeti, Z., arXiv:hep-ph/0309219.
49. Leibovich, A. K., Ligeti, Z., and Wise, M. B., *Phys. Lett.*, **B539**, 242–248 (2002), [arXiv:hep-ph/0205148].
50. Bauer, C. W., Luke, M., and Mannel, T., *Phys. Lett.*, **B543**, 261–268 (2002), [arXiv:hep-ph/0205150].
51. Neubert, M., *Phys. Lett.*, **B543**, 269–275 (2002), [arXiv:hep-ph/0207002].
52. Bauer, C. W., Luke, M. E., and Mannel, T., *Phys. Rev.*, **D68**, 094001 (2003), [arXiv:hep-ph/0102089].
53. Bigi, I. I. Y., and Uraltsev, N. G., *Nucl. Phys.*, **B423**, 33–55 (1994), [arXiv:hep-ph/9310285].
54. Voloshin, M. B., *Phys. Lett.*, **B515**, 74–80 (2001), [arXiv:hep-ph/0106040].
55. Gilman, F. J., and Singleton, R. L., *Phys. Rev.*, **D41**, 142 (1990).
56. Gibbons, L., *ECONF*, **C0304052**, WG105 (2003), [arXiv:hep-ex/0307065].
57. Abada, A., et al., *Nucl. Phys.*, **B416**, 675–698 (1994), [arXiv:hep-lat/9308007].
58. Allton, C. R., et al., APE Collaboration, *Phys. Lett.*, **B345**, 513–523 (1995), [arXiv:hep-lat/9411011].
59. Del Debbio, L., Flynn, J. M., Lellouch, L., and Nieves, J., UKQCD Collaboration, *Phys. Lett.*, **B416**, 392–401 (1998), [arXiv:hep-lat/9708008].
60. Hashimoto, S., Ishikawa, K.-I., Matsufuru, H., Onogi, T., and Yamada, N., *Phys. Rev.*, **D58**, 014502 (1998), [arXiv:hep-lat/9711031].
61. Ryan, S., et al., *Nucl. Phys. Proc. Suppl.*, **73**, 390–392 (1999), [arXiv:hep-lat/9810041].
62. Ryan, S. M., El-Khadra, A. X., Kronfeld, A. S., Mackenzie, P. B., and Simone, J. N., *Nucl. Phys. Proc. Suppl.*, **83**, 328–330 (2000), [arXiv:hep-lat/9910010].
63. Lellouch, L., arXiv:hep-ph/9912353.
64. Bowler, K. C., et al., UKQCD Collaboration, *Phys. Lett.*, **B486**, 111–117 (2000), [arXiv:hep-lat/9911011].

65. Becirevic, D., and Kaidalov, A. B., *Phys. Lett.*, **B478**, 417–423 (2000), [arXiv:hep-ph/9904490].
66. Aoki, S., et al., JLQCD Collaboration, *Nucl. Phys. Proc. Suppl.*, **94**, 329–332 (2001), [arXiv:hep-lat/0011008].
67. Abada, A., et al., *Nucl. Phys.*, **B619**, 565–587 (2001), [arXiv:hep-lat/0011065].
68. El-Khadra, A. X., Kronfeld, A. S., Mackenzie, P. B., Ryan, S. M., and Simone, J. N., *Phys. Rev.*, **D64**, 014502 (2001), [arXiv:hep-ph/0101023].
69. Aoki, S., et al., JLQCD Collaboration, *Phys. Rev.*, **D64**, 114505 (2001), [arXiv:hep-lat/0106024].
70. Ball, P., and Braun, V. M., *Phys. Rev.*, **D55**, 5561–5576 (1997), [arXiv:hep-ph/9701238].
71. Ball, P., and Braun, V. M., *Phys. Rev.*, **D58**, 094016 (1998), [arXiv:hep-ph/9805422].
72. Khodjamirian, A., Ruckl, R., Weinzierl, S., and Yakovlev, O. I., *Phys. Lett.*, **B410**, 275–284 (1997), [arXiv:hep-ph/9706303].
73. Khodjamirian, A., Ruckl, R., Weinzierl, S., Winhart, C. W., and Yakovlev, O. I., *Phys. Rev.*, **D62**, 114002 (2000), [arXiv:hep-ph/0001297].
74. Bakulev, A. P., Mikhailov, S. V., and Ruskov, R., arXiv:hep-ph/0006216.
75. Huang, T., Li, Z., and Wu, X., arXiv:hep-ph/0011161.
76. Wang, W. Y., and Wu, Y. L., *Phys. Lett.*, **B515**, 57–64 (2001), [arXiv:hep-ph/0105154].
77. Wang, W.-Y., and Wu, Y.-L., *Phys. Lett.*, **B519**, 219–228 (2001), [arXiv:hep-ph/0106208].
78. Ball, P., and Zwicky, R., *JHEP*, **10**, 019 (2001), [arXiv:hep-ph/0110115].
79. Battaglia, M., et al., arXiv:hep-ph/0304132.
80. Wirbel, M., Stech, B., and Bauer, M., *Z. Phys.*, **C29**, 637 (1985).
81. Korner, J. G., and Schuler, G. A., *Z. Phys.*, **C38**, 511 (1988).
82. Isgur, N., Scora, D., Grinstein, B., and Wise, M. B., *Phys. Rev.*, **D39**, 799 (1989).
83. Scora, D., and Isgur, N., *Phys. Rev.*, **D52**, 2783–2812 (1995), [arXiv:hep-ph/9503486].
84. Melikhov, D., *Phys. Rev.*, **D53**, 2460–2479 (1996), [arXiv:hep-ph/9509268].
85. Beyer, M., and Melikhov, D., *Phys. Lett.*, **B436**, 344–350 (1998), [arXiv:hep-ph/9807223].
86. Faustov, R. N., Galkin, V. O., and Mishurov, A. Y., *Phys. Rev.*, **D53**, 6302–6315 (1996), [arXiv:hep-ph/9508262].
87. Demchuk, N. B., Kulikov, P. Y., Narodetsky, I. M., and O'Donnell, P. J., *Phys. Atom. Nucl.*, **60**, 1292–1304 (1997), [arXiv:hep-ph/9701388].
88. Grach, I. L., Narodetsky, I. M., and Simula, S., *Phys. Lett.*, **B385**, 317–323 (1996), [arXiv:hep-ph/9605349].
89. Riazuddin, Al-Aithan, T. A., and Gilani, A. H. S., *Int. J. Mod. Phys.*, **A17**, 4927–4938 (2002), [arXiv:hep-ph/0007164].
90. Melikhov, D., and Stech, B., *Phys. Rev.*, **D62**, 014006 (2000), [arXiv:hep-ph/0001113].
91. Feldmann, T., and Kroll, P., *Eur. Phys. J.*, **C12**, 99–108 (2000), [arXiv:hep-ph/9905343].
92. Flynn, J. M., and Nieves, J., *Phys. Lett.*, **B505**, 82–88 (2001), [arXiv:hep-ph/0007263].
93. Beneke, M., and Feldmann, T., *Nucl. Phys.*, **B592**, 3–34 (2001), [arXiv:hep-ph/0008255].
94. Choi, H.-M., and Ji, C.-R., *Phys. Lett.*, **B460**, 461–466 (1999), [arXiv:hep-ph/9903496].
95. Kurimoto, T., Li, H.-n., and Sanda, A. I., *Phys. Rev.*, **D65**, 014007 (2002), [arXiv:hep-ph/0105003].
96. Ligeti, Z., and Wise, M. B., *Phys. Rev.*, **D53**, 4937–4945 (1996), [arXiv:hep-ph/9512225].
97. Aitala, E. M., et al., E791 Collaboration, *Phys. Rev. Lett.*, **80**, 1393–1397 (1998), [arXiv:hep-ph/9710216].
98. Burdman, G., and Kambor, J., *Phys. Rev.*, **D55**, 2817–2826 (1997), [arXiv:hep-ph/9602353].
99. Lellouch, L., *Nucl. Phys.*, **B479**, 353–391 (1996), [arXiv:hep-ph/9509358].
100. Mannel, T., and Postler, B., *Nucl. Phys.*, **B535**, 372–386 (1998), [arXiv:hep-ph/9805425].
101. Lange, B. O., arXiv:hep-ph/0310139.
102. Ball, P., arXiv:hep-ph/0308249.
103. Beneke, M., and Feldmann, T., arXiv:hep-ph/0311335.
104. Lange, B. O., and Neubert, M., arXiv:hep-ph/0311345.
105. Davies, C. T. H., et al., HPQCD Collaboration, arXiv:hep-lat/0304004.
106. Bernard, C., et al., MILC Collaboration, arXiv:hep-lat/0309055.
107. Okamoto, M., et al., arXiv:hep-lat/0309107.
108. Behrens, B. H., et al., CLEO Collaboration, *Phys. Rev.*, **D61**, 052001 (2000), [arXiv:hep-ex/9905056].
109. Aubert, B., et al., BABAR Collaboration, *Phys. Rev. Lett.*, **90**, 181801 (2003), [arXiv:hep-ex/0301001].
110. Athar, S. B., et al., CLEO Collaboration, *Phys. Rev.*, **D68**, 072003 (2003), [arXiv:hep-ex/0304019].
111. Abe, K., et al., Belle Collaboration, arXiv:hep-ex/0307075.
112. Foley, K. M., and Lepage, G. P., *Nucl. Phys. Proc. Suppl.*, **119**, 635–637 (2003), [arXiv:hep-lat/0209135].
113. Boyle, P. A., arXiv:hep-lat/0309100.