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Learning the weak phase γ from B decays

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Abstract. The current status of some methods to determine the weak phase γ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub}^* using B decays is discussed, and comments are made on accuracy achievable in the next few years.

1. INTRODUCTION

The observed CP violation in K and B decays can be interpreted in terms of phases of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. While $\beta = \mathrm{Arg}(V_{td}^*)$ is well-determined from the CP asymmetry in $B^0 \to J/\psi K_S$, current information on $\gamma = \mathrm{Arg}(V_{ub}^*)$ is much less precise, with $39^\circ < \gamma < 80^\circ$ at 95% c.l. [1]. In order to learn γ one must generally separate strong and weak phases from one another in two-body B decays. We describe several areas in which progress in this work has been accomplished, and what improvements lie ahead. Some additional details are noted in an earlier review [2].

In Section 2 we compare the determination of β from $B^0 \to J/\psi K_S$ with the more difficult determination of $\alpha = \pi - \beta - \gamma$ from $B^0 \to \pi^+\pi^-$. We then discuss some uses of information from various decay modes of $B \to K\pi$ in Sec. 3. One obtains useful constraints on γ with some assumptions about SU(3) flavor symmetry from the decays $B \to VP$ (Sec. 4) and $B \to PP$ (Sec. 5), where V and P denote light vector and pseudoscalar mesons. The decays $B \to D_{CP}K$, where D_{CP} denotes a CP eigenstate of a neutral charmed meson, also provide useful constraints (Sec. 6). We summarize in Sec. 7.

2. β FROM $B^0 \to J/\psi K_S$ VS. α FROM $B^0 \to \pi^+\pi^-$

The unitarity of the CKM matrix is conveniently expressed in terms of the triangle of Fig. 1. Here, for example, $1 - \bar\rho - i\bar\eta = -V_{tb}^* V_{td}/V_{cb}^* V_{cd}$. (See Ref. [2] for other definitions.)

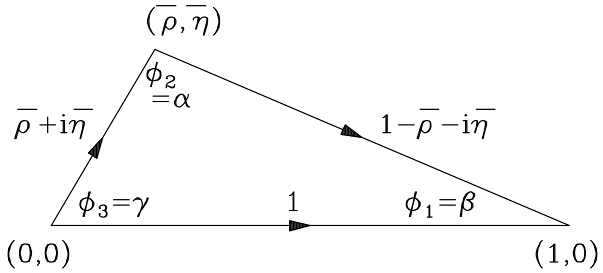

FIGURE 1. The unitarity triangle. Two conventions for its angles are shown.

2.1. $B^0 \to J/\psi K_S$

The CP asymmetry in the decay $B^0 \to J/\psi K_S$ is simple to analyze because there is only one main subprocess $\bar b \to \bar c c \bar s$. The direct decay (with zero weak phase) interferes with $B^0 \to \bar B^0$ mixing (with weak phase $e^{-2i\beta}$). The most recent BaBar and Belle measurements, when averaged, provide $\sin 2\beta = 0.736 \pm 0.049$ [3] without much ambiguity.

2.2. $B^0 \to \pi^+\pi^-$

Here there are two types of amplitude, "T" (tree) and "P" (penguin), contributing to the decay. (For a discussion of amplitudes within flavor SU(3) see Refs. [4] and [5].) Different weak and strong phases can complicate the analysis. If one had only a tree amplitude, the direct amplitude $A(B^0 \to \pi^+\pi^-) \sim e^{i\gamma}$ would interfere with $A(B^0 \to \bar B^0 \to \pi^+\pi^-) \sim e^{-2i\beta}e^{-i\gamma}$ to provide a measure of the relative phase $2(\beta+\gamma) = 2\pi - 2\alpha$. So, in the absence of the penguin contribution, one would measure α. One seeks an estimate of $|P/T|$ or observables not requiring this ratio.

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The neutral B mass eigenstates may be written

$$B_L^0 = p|B^0\rangle + q|\bar{B}^0\rangle \;;\; B_H^0 = p|B^0\rangle - q|\bar{B}^0\rangle \;, \qquad (1)$$

where

$$q/p \;=\; e^{-2i\beta} \;,\; \lambda \equiv (q/p)(\bar{A}/A) \;, \qquad (2)$$

$$A \;\equiv\; A(B \to f) \;,\; \bar{A} \equiv A(\bar{B} \to \bar{f}) \;. \qquad (3)$$

Observables in the time-dependence of $\left\{ \begin{array}{c} B^0 \\ \bar{B}^0 \end{array} \right\}_{t=0} \to$ $\pi^+\pi^-$ (or any other final state) are:

$$\Gamma(t) \sim e^{-\Gamma|t|}[1 \mp S \sin \Delta m t \mp A \cos \Delta m t] \;,\; t \equiv t_{\rm decay} - t_{\rm tag} \;, \qquad (4)$$

with $S = 2{\rm Im}\lambda/(1+|\lambda|^2)$ and

$$A = \frac{|\lambda|^2 - 1}{|\lambda|^2 + 1} = A_{CP} = \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)} \;. \qquad (5)$$

The experimental data [6, 7] on these asymmetries in $B \to \pi^+\pi^-$ are shown in Table 1.

With no penguin contributions, $S_{\pi\pi} = \sin 2\alpha < 0$ would favor $\alpha > 90°$. With a penguin-to-tree ratio $|P/T| \simeq 0.3$ estimated from $B \to K\pi$ using flavor SU(3) symmetry, one finds instead the parametric dependence of the time-dependent asymmetries on α and a relative strong phase δ [8, 9] as shown in Fig. 2 [10].

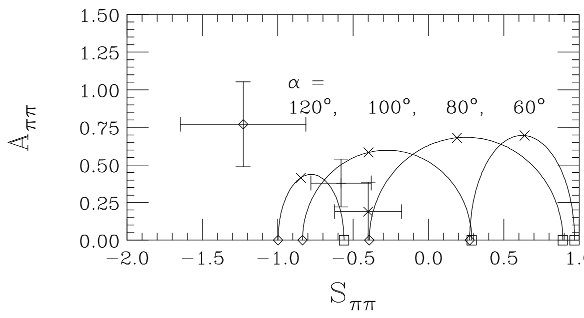

FIGURE 2. Curves describing behavior of $S_{\pi\pi}$ and $A_{\pi\pi}$ as the relative strong phase δ between penguin and tree amplitudes is varied from 0° (diamonds) through 90° (crosses) to 180° (squares). The curves are labeled by values of α. Plotted points: Babar (cross), Belle (diamond), average (no symbol).

Unless δ is near $\pi/2$, curves for different values of α intersect at the same values of $S_{\pi\pi}$ and $A_{\pi\pi}$. A quantity which is useful in resolving this discrete ambiguity is $R_{\pi\pi} = \frac{\Gamma(B^0 \to \pi^+\pi^-)}{\Gamma(B^0 \to \pi^+\pi^-)_{\rm tree}}$. In Ref. [11] it was found that $R_{\pi\pi} = 0.87^{+0.11}_{-0.28}$, which slightly favors larger strong phases and hence larger values of α for given $(S_{\pi\pi}, A_{\pi\pi})$. A related analysis has appeared recently in Ref. [12]. Information on $B_s \to K^+K^-$ may be combined with that on $B \to \pi^+\pi^-$ with the help of flavor SU(3) to separate out penguin and tree contributions [13]. The time-dependence of $B_s(t) \to K^+K^-$ provides a complementary method [14].

3. INFORMATION FROM $B \to K\pi$

A great deal of information can be obtained from $B \to K\pi$ decay rates averaged over CP, supplemented with measurements of direct CP asymmetries. One probes in this manner tree-penguin interference in various processes. Denoting amplitudes with $|\Delta S| = 1$ by primed quantities, several comparisons can be made:

- $B^0 \to K^+\pi^-$ $(T' + P')$ vs. $B^+ \to K^0\pi^+$ (P') [15–18];
- $B^+ \to K^+\pi^0$ $(T' + P' + C')$ vs. $B^+ \to K^0\pi^+$ (P') [18–21];
- $B^0 \to K^0\pi^0$ vs. other modes [18, 22–26].

The data which are used in these analyses are summarized in Table 2.

In all these comparisons it is helpful to use flavor SU(3) (often only U-spin, i.e., $s \leftrightarrow d$). We give the example of $B^0 \to K^+\pi^-$ in detail. The tree amplitude for this process is $T' \sim V_{us}V_{ub}^*$, with weak phase γ, while the penguin amplitude is $P' \sim V_{ts}V_{tb}^*$ with weak phase π. We denote the penguin-tree relative strong phase by δ and define $r \equiv |T'/P'|$. Then we may write

$$A(B^0 \to K^+\pi^-) \;=\; |P'|[1 - re^{i(\gamma+\delta)}] \;, \qquad (6)$$

$$A(\bar{B}^0 \to K^-\pi^+) \;=\; |P'|[1 - re^{i(-\gamma+\delta)}] \;, \qquad (7)$$

$$A(B^+ \to K^0\pi^+) \;=\; A(B^- \to \bar{K}^0\pi^-) = -|P'| \;, \qquad (8)$$

where the last two amplitudes are expected to be equal in the approximation that small annihilation amplitudes are neglected. A test for this assumption is the absence of a CP asymmetry in $B^+ \to K^0\pi^+$ (or in $B^+ \to \bar{K}^0K^+$, where it would be bigger [27]).

One now forms the ratio

$$R \;\equiv\; \frac{\Gamma(B^0 \to K^+\pi^-) + \Gamma(\bar{B}^0 \to K^-\pi^+)}{2\Gamma(B^+ \to K^0\pi^+)}$$

$$=\; 1 - 2r\cos\gamma\cos\delta + r^2 \;. \qquad (9)$$

Fleischer and Mannel [15] pointed out that $R \geq \sin^2\gamma$ for any r, δ so if $1 > R$ one can get a useful bound. However, if one uses

$$RA_{CP} = -2r\sin\gamma\sin\delta \qquad (10)$$

as well and eliminates δ one can get a more powerful constraint, illustrated in Fig. 3.

At the 1σ level, $R < 1$, leading to an upper bound $\gamma <$ 80° which happens to coincide with that of Ref. [15]. We have used $R = 0.898 \pm 0.071$ and $A_{CP} = -0.095 \pm 0.029$ based on recent averages [7] of CLEO, BaBar, and Belle data, and $r = |T'/P'| = 0.142^{+0.024}_{-0.012}$. The most conservative bound arises for the smallest $|A_{CP}|$ and largest r. The allowed region lies between the curves $A_{CP} = 0$ and $|A_{CP}| = 0.124$ (1σ). In order to estimate the tree amplitude and $r = |T'/P'|$ we have used factorization

TABLE 1. Time-dependent asymmetries in $B^0 \to \pi^+\pi^-$.

Observable	BaBar	Belle	Average
$S_{\pi\pi}$	$-0.40\pm0.22\pm0.33$	$-1.23\pm0.41^{+0.08}_{-0.07}$	-0.58 ± 0.20
$A_{\pi\pi}$	$0.19\pm0.19\pm0.05$	$0.77\pm0.27\pm0.08$	0.38 ± 0.16

TABLE 2. Branching ratios and CP asymmetries for $B \to K\pi$ decays [7].

Decay mode	Amplitude	$\mathscr{B}$ (units of 10^{-6})	A_{CP}
$B^+ \to K^0\pi^+$	P'	21.78 ± 1.40	0.016 ± 0.057
$B^+ \to K^+\pi^0$	$-(P'+C'+T')/\sqrt{2}$	12.53 ± 1.04	0.00 ± 0.12
$B^0 \to K^+\pi^-$	$-(T'+P')$	18.16 ± 0.79	-0.095 ± 0.029
$B^0 \to K^0\pi^0$	$(P'-C')/\sqrt{2}$	11.68 ± 1.42	0.03 ± 0.37

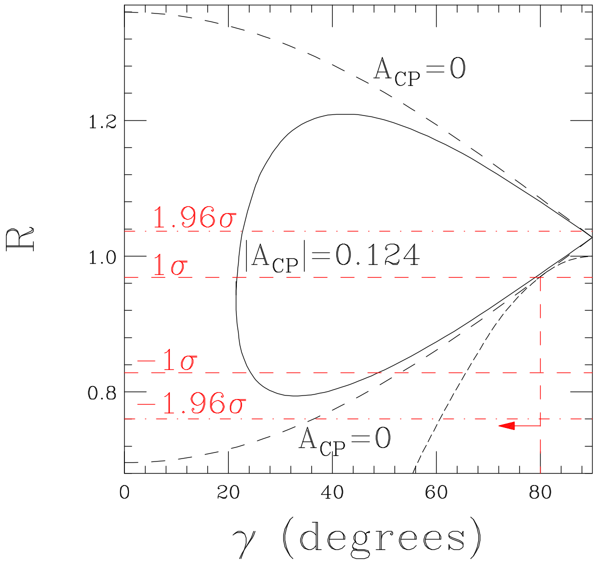

FIGURE 3. Behavior of R for $r=0.166$ and $A_{CP}=0$ (dashed curves) or $|A_{CP}|=0.124$ (solid curve) as a function of the weak phase γ. Horizontal dashed lines denote $\pm1\sigma$ experimental limits on R, while dot-dashed lines denote 95% c.l. ($\pm1.96\sigma$) limits. The short-dashed curve denotes the Fleischer-Mannel bound $\sin^2\gamma \le R$.

in $B^+ \to \pi^-\ell^+\nu_\ell$ at low q^2 [11] and $\left|\frac{T'}{T}\right| = \frac{f_K}{f_\pi}\left|\frac{V_{us}}{V_{ud}}\right| \simeq (1.22)(0.23) = 0.28$. One could use processes in which T dominates, such as $B^0 \to \pi^+\pi^-$ or $B^+ \to \pi^+\pi^0$, but these are contaminated by contributions from P and C, respectively.

In such an approach one always must question the validity of SU(3) flavor symmetry. SU(3) breaking is taken into account in the ratio of tree amplitudes, but no breaking is taken in other amplitudes, since we do not assume factorization for C or P and therefore cannot account for the breaking merely via ratios of decay constants. We have assumed the same relative tree-penguin strong phases for $|\Delta S|=1$ and $\Delta S=0$ amplitudes. Tests of these assumptions will be available once one observes penguin-dominated $B \to K\bar{K}$ decays and charmless B_s decays; there are also numerous relations implied between CP-violating rate differences [28, 29].

The process $B^+ \to K^+\pi^0$ also provides constraints on γ. The deviation of the ratio

$$R_c \equiv \frac{\Gamma(B^+ \to K^+\pi^0)+\Gamma(B^- \to K^-\pi^0)}{\Gamma(B^+ \to K^0\pi^+)} = 1.15\pm0.12 \tag{11}$$

from 1, when combined with $A_{CP}=0.00\pm0.12$, $r_c = |(T'+C')/P'| = 0.195\pm0.016$ and an estimate of the electroweak penguin (EWP) $\delta_{EW} \equiv |P'_{EW}|/|T'+C'| = 0.65\pm0.15$, leads to a 1σ lower bound $\gamma > 40°$. Details may be found in Refs. [2, 18–21]. The most conservative bound arises for smallest A_{CP}, largest r_c, and largest $|P'_{EW}|$, and is shown in Fig. 4.

Another ratio

$$\begin{aligned} R_n &\equiv \frac{\bar{\Gamma}(B^0 \to K^+\pi^-)}{2\bar{\Gamma}(B^0 \to K^0\pi^0)} \\ &= \left|\frac{p'+t'}{p'-c'}\right|^2 = 0.78\pm0.10 \end{aligned} \tag{12}$$

involves the decay $B^0 \to K^0\pi^0$. Here the bar denotes CP-averaged decay widths, while small letters denote amplitudes which include EWP contributions. This ratio should be same to leading order in $|t'/p'|$ and $|c'/p'|$ as

$$R_c = \left|\frac{p'+t'+c'}{p'}\right|^2 , \tag{13}$$

but the two ratios differ by 2.4σ. Possibilities (see, e.g., Refs. [18, 30]) include (1) new physics, e.g., in the EWP amplitude, and (2) an underestimate of the π^0 detection efficiency in all experiments, leading to an overestimate of any branching ratio involving a π^0. The latter possibility can be taken into account by considering the ratio $(R_nR_c)^{1/2} = 0.96\pm0.08$, in which the π^0 efficiency cancels. As shown in Fig. 5, this ratio leads only to the conservative bound $\gamma \le 88°$.

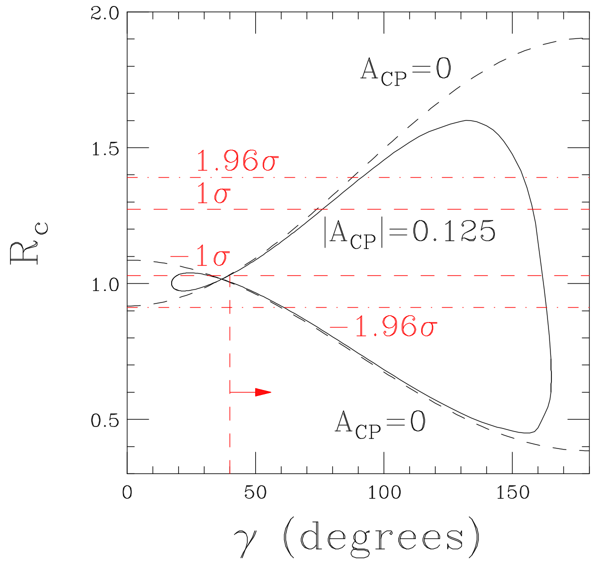

FIGURE 4. Behavior of R_c for $r_c = 0.21$ (1σ upper limit) and $A_{CP}(K^+\pi^0) = 0$ (dashed curves) or $|A_{CP}(K^+\pi^0)| = 0.125$ (solid curve) as a function of the weak phase γ. Horizontal dashed lines denote $\pm 1\sigma$ experimental limits on R_c, while dotdashed lines denote 95% c.l. ($\pm 1.96\sigma$) limits. We have taken $\delta_{EW} = 0.80$ (its 1σ upper limit), which leads to the most conservative bound on γ.

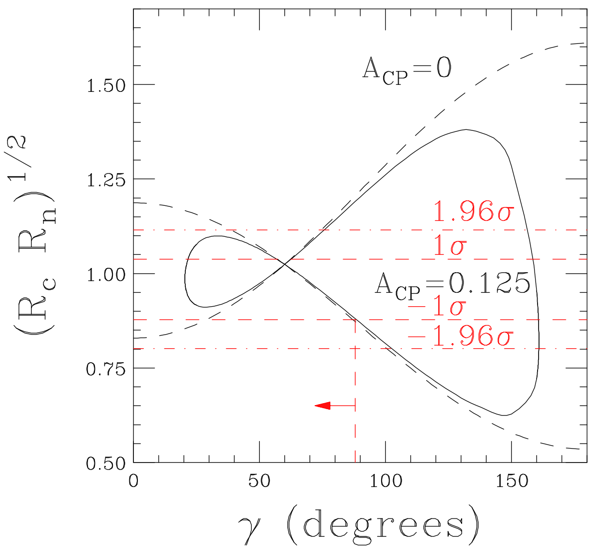

FIGURE 5. Behavior of $(R_c R_n)^{1/2}$ for $r_c = 0.18$ (1σ lower limit) and $A_{CP}(K^+\pi^0) = 0$ (dashed curves) or $|A_{CP}(K^+\pi^0)| = 0.125$ (solid curve) as a function of the weak phase γ. Horizontal dashed lines denote $\pm 1\sigma$ experimental limits on $(R_c R_n)^{1/2}$, while dotdashed lines denote 95% c.l. ($\pm 1.96\sigma$) limits. Upper branches of curves correspond to $\cos\delta_c(\cos\gamma - \delta_{EW}) < 0$, while lower branches correspond to $\cos\delta_c(\cos\gamma - \delta_{EW}) > 0$. Here we have taken $\delta_{EW} = 0.50$ (its 1σ lower limit), which leads to the most conservative bound on γ.

4. INFORMATION FROM $B \to VP$

Although the decays $B \to VP$ are characterized by more amplitudes than $B \to PP$ (since the final particles do not belong to the same flavor-symmetry multiplet), data have become so abundant that useful global fits can be performed [31, 32]. We label amplitudes by the meson (pseudoscalar P or vector V) containing the spectator quark. Some features of the fit of Ref. [32] are:

- $|t_P/t| \simeq f_\rho/f_\pi$, where t is the tree amplitude in $B \to PP$ decays, as would be expected for a weak current producing a charged meson.
- Penguin amplitudes satisfy $p'_V \simeq -p'_P$, as proposed long ago by Lipkin [33–35].
- Small CP asymmetries in many processes imply small strong phases.
- The time-dependent asymmetries in $B \to \rho\pi$ are crucial in resolving discrete ambiguities, as has also been found in the QCD factorization approach of Refs. [24–26].

Three local χ^2 minima are found: $\gamma = (26 \pm 5)^\circ$, $(63 \pm 6)^\circ$ (a range compatible with fits [1] to CKM parameters), and $(162^{+5}_{-6})^\circ$ (incompatible with $\beta \simeq 24^\circ$ since $\alpha + \beta + \gamma = \pi$). At 95% c.l. the solution compatible with CKM fits implies $51^\circ \le \gamma \le 73^\circ$ and small strong phases in accord with the expectations of QCD factorization [26]. Some predictions for as-yet-unseen decay modes are shown in Table 3.

In this fit the free parameters were:

- $p'_{P,V}$ (penguin amplitudes); their relative phase ϕ;
- $t_{P,V}$ (tree amplitudes); their strong phases $\delta_{P,V}$ with respect to $p_{P,V}$;
- Color-suppressed $c_{P,V}$ taken real with respect to $t_{P,V}$;
- Electroweak penguins $P'_{EW(P,V)}$ taken real with respect to $p'_{P,V}$;
- The weak phase γ.

One thus has 12 parameters (11 if p'_V/p'_P is assumed to be real, and 10 if we assume $p'_V/p'_P = -1$) to fit 34 data points. The resulting dependence of $\chi^2_{\rm min}$ on γ is shown in Fig. 6.

The relative phases of amplitudes are specified by CP-averaged decay rates as well as by CP asymmetries. They are illustrated in Fig. 7.

The p'_P amplitude in this diagram is taken to be real and positive. The weak phases of $t_{V,P}$ and $\bar{t}_{V,P}$ are included. There is a small relative phase between t_V and t_P, as expected in QCD factorization [26]. The relative phases of p'_V and p'_P are such that they contribute constructively to $B \to K^*\eta$, as anticipated by Lipkin [33–35]. From these phases one expects constructive tree-

TABLE 3. Predictions of the favored fit of Ref. [32] for some as-yet-unseen $B \to VP$ decays.

As yet unseen decay mode	Predicted $\mathscr{B}$ Units of 10^{-6}	Present limit	Comments
$B^+ \to \bar{K}^{*0} K^+$	0.50 ± 0.05	< 5.3	Pure p_P
$B^+ \to K^{*+} \pi^0$	$15.0^{+3.3}_{-2.8}$	< 31	EWP enhancement
$B^+ \to \rho^+ K^0$	12.6 ± 1.6	< 48	Pure p'_V
$B^0 \to \rho^0 K^0$	$7.2^{+2.1}_{-1.9}$	< 12.4	EWP enhancement

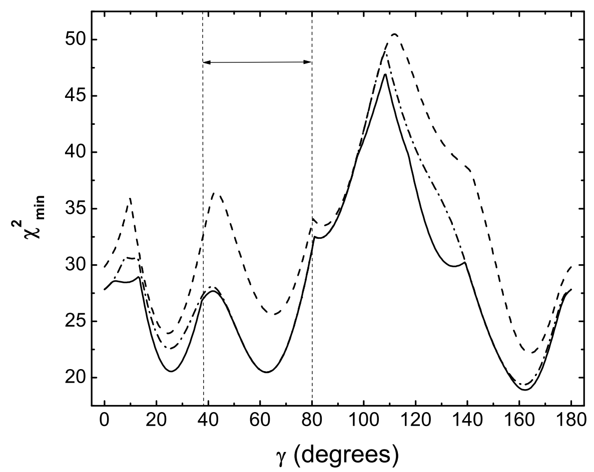

FIGURE 6. $(\chi^2)_{\min}$, obtained by minimizing over all remaining fit parameters, as a function of the weak phase γ. Dashed curve: $p'_V/p'_P = -1$ (24 d.o.f.); dash-dotted curve: p'_V/p'_P real (23 d.o.f.); solid curve: p'_V/p'_P complex (22 d.o.f.). Vertical dashed lines show the limits $39° \leq \gamma \leq 80°$ from fits [1] to CKM parameters.

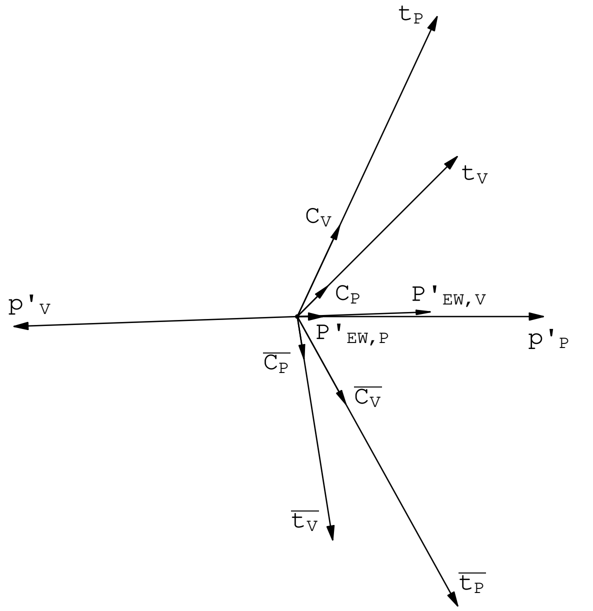

FIGURE 7. Magnitudes and phases of dominant invariant amplitudes in solution with $\gamma \simeq 63°$ and complex p'_V/p'_P [32].

penguin interference in the CP-averaged rate for $B^0 \to K^{*+}\pi^-$ and destructive interference in $B^0 \to K^+\rho^-$.

5. $B \to PP$ DECAYS WITH η AND η'

A global fit to $B \to PP$ decays under the same assumptions as the fit to $B \to VP$ decays is still in progress [7]. However, $B \to PP$ decays with η and η' have been analyzed using flavor symmetry in Ref. [36]. It is found that the large CP asymmetry in $B^+ \to \pi^+\eta$ reported by the BaBar Collaboration [37] implies a comparable A_{CP} in $B^+ \to \pi^+\eta'$. We have

$$A_{CP}(\pi^+\eta) = -\frac{0.91 \sin\alpha \sin\delta}{1 - 0.91 \cos\alpha \cos\delta} = -0.51 \pm 0.19 \ , \tag{14}$$

$$\begin{aligned} \bar{\mathscr{B}}(\pi^+\eta) &= 4.95 \times 10^{-6}(1 - 0.91 \cos\alpha \cos\delta) \\ &= (4.12 \pm 0.85) \times 10^{-6} \ , \end{aligned} \tag{15}$$

leading to the predictions

$$A_{CP}(\pi^+\eta') = -\frac{\sin\alpha \sin\delta}{1 - \cos\alpha \cos\delta} \simeq -0.57 \ , \tag{16}$$

$$\begin{aligned} \bar{\mathscr{B}}(\pi^+\eta') &= 3.35 \times 10^{-6}(1 - \cos\alpha \cos\delta) \\ &\simeq 2.7 \times 10^{-6} \ . \end{aligned} \tag{17}$$

Tree and penguin amplitudes are of comparable magnitude in both these processes, leading to the possibility of large CP asymmetries which appears to be realized in the data. The scatter of predictions for $B^+ \to \pi^+\eta'$ is shown in Fig. 8. The central values are based on $(\alpha, \delta) \simeq (78°, 28°)$ with the discrete ambiguities $(\alpha \leftrightarrow \delta)$ or $\alpha \to \pi - \alpha$, $\delta \to \pi - \delta$.

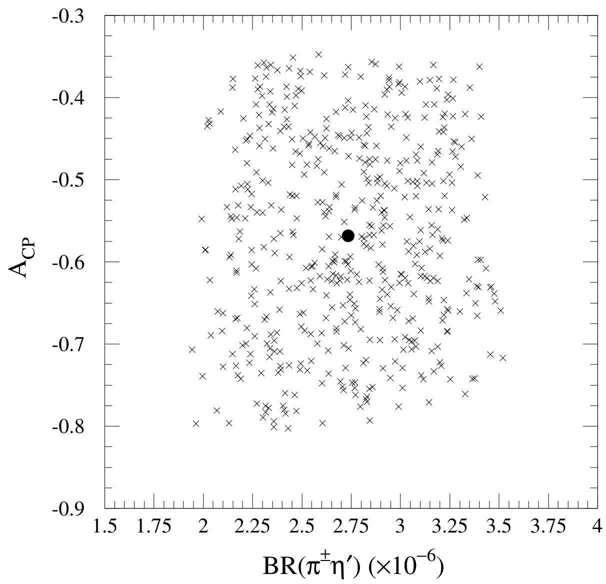

FIGURE 8. Predicted values of the averaged branching ratio and direct *CP* asymmetry for the decays $B^{\pm} \to \pi^{\pm}\eta'$.

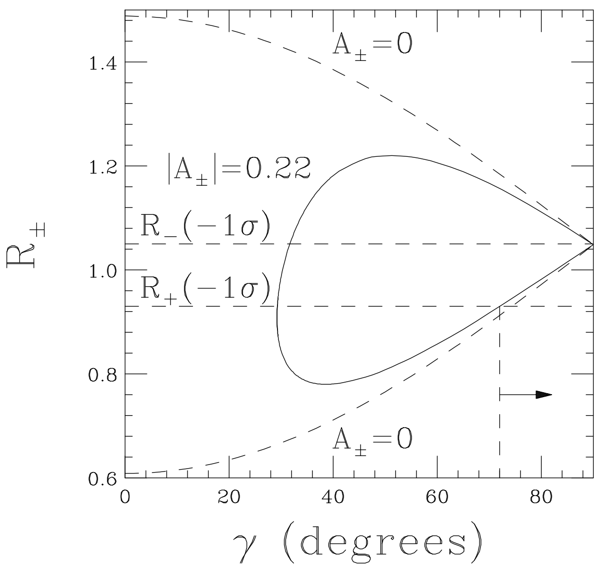

FIGURE 9. Behavior of $R_{\pm}$ for $A_{\pm} = 0$ (dashed curves) or $|A_{\pm}| = 0.22$ (solid curve) as a function of the weak phase γ. Horizontal dashed lines denote -1σ experimental limits on $R_{\pm}$. We have taken parameters (including $r = 0.22$) which lead to the most conservative bound on γ.

6. CONSTRAINTS FROM $B \to D_{CP}K$

We present the discussion of M. Gronau [10]. One wishes to compare $B^{\pm} \to D_{CP}K^{\pm}$ with the CKM-favored process $B^{-} \to D^{0}K^{-}$, thereby probing the quark subprocesses

$$\frac{A(\bar{b} \to \bar{u}c\bar{s})}{A(\bar{b} \to \bar{c}u\bar{s})} = re^{i(\gamma+\delta)} \ , \ \frac{A(b \to u\bar{c}s)}{A(b \to c\bar{u}s)} = re^{i(-\gamma+\delta)} \ . \tag{18}$$

One then considers the ratios

$$\begin{aligned} R_{\pm} &\equiv \frac{\Gamma(D^0_{CP=\pm}K^-) + \Gamma(D^0_{CP=\pm}K^+)}{\Gamma(D^0K^-)} \\ &= 1 + r^2 \pm 2r\cos\gamma\cos\delta \end{aligned} \tag{19}$$

and the CP asymmetries

$$\begin{aligned} A_{\pm} &\equiv \frac{\Gamma(D^0_{CP=\pm}K^-) - \Gamma(D^0_{CP=\pm}K^+)}{\Gamma(D^0_{CP=\pm}K^-) + \Gamma(D^0_{CP=\pm}K^+)} \\ &= \pm 2r\sin\gamma\sin\delta / R_{\pm} \ . \end{aligned} \tag{20}$$

The relevant data are summarized in Table 4. The ratio $\Gamma(B^- \to D^0K^-)/\Gamma(B^- \to D^0\pi^-)$ was evaluated [10] by taking the average of CLEO [38], Belle [39], and BaBar [40] values.

We take $(R_+ + R_-)/2 = 1 + r^2$ so $r \geq 0.22$ at 1σ. The average $|A_{\pm}|$ is $0.11 \pm 0.11 \leq 0.22$ at 1σ; the most conservative bound is obtained for smallest r and largest $|A_{\pm}|$ and at 1σ is $\gamma \geq 72°$, as shown in Fig. 9. Note that one R must be below $1 + r^2$ while the other must be above it.

7. SUMMARY

A number of promising bounds on γ stemming from various B decays have been mentioned. So far all are statistics-limited. At 1σ we have found

- R ($K^+\pi^-$ vs. $K^0\pi^+$) gives $\gamma \leq 80°$;
- R_c ($K^+\pi^0$ vs. $K^0\pi^+$) gives $\gamma \geq 40°$;
- R_n ($K^+\pi^-$ vs. $K^0\pi^0$) should equal R_c; $(R_cR_n)^{1/2}$ gives $\gamma \leq 88°$;
- $B \to D_{CP}K$ decays give $\gamma \geq 72°$.

A flavor-SU(3) analysis of $B \to VP$ decays favors $\gamma = (63 \pm 6)°$, or $51° \leq \gamma \leq 73°$ at 95% c.l. Several as-yet-unseen decay modes are predicted, such as $B^+ \to \rho^+K^0$ and $B^+ \to K^{*+}\pi^0$. SU(3) relations among rate differences remain to be tested. We predict $2.0 \leq \bar{\mathscr{B}}(\pi^+\eta')/10^{-6} \leq 3.5$, $-0.34 \geq A_{CP}(\pi^+\eta') \geq -0.80$. A global $B \to PP$ analysis, still in progress, is complicated by possible $B \to K\pi$ inconsistencies or new physics in (e.g.) $B^0 \to K^0\pi^0$.

The future of most such γ determinations remains for now in experimentalists' hands, as one can see from Figs. 3–5 and 9. Uncertainties in SU(3) breaking are probably already the limiting factor on the error in γ from Fig. 6, and better estimates will require flavor SU(3) tests at levels of $\mathscr{B} \simeq 1/2 \times 10^{-6}$. We have noted (see, e.g., [16]) that measurements of rate ratios in $B \to K\pi$ can ultimately pinpoint γ to within about 10°. The required accuracies in R, R_c, and R_n to achieve this goal can be es-

TABLE 4. Ratios $R_{\pm}$ and CP asymmetries $A_{\pm}$ for $B \to D_{CP}K$ decays.

	R_+	A_+	R_-	A_-
Belle [39]	1.12 ± 0.24	0.06 ± 0.19	1.30 ± 0.25	-0.19 ± 0.18
BaBar [40]	1.06 ± 0.21	0.07 ± 0.18		
Average	1.09 ± 0.16	0.065 ± 0.132	1.30 ± 0.25	-0.19 ± 0.18

timated from Figs. 3–5. For example, knowing $(R_c R_n)^{1/2}$ to within 0.05 would pin down γ to within 10° if this ratio lies in the most sensitive range of Fig. 5.

A complementary approach to the flavor-SU(3) method is the QCD factorization formalism of Refs. [24–26]. It predicts small strong phases (as found in our analysis) and deals directly with flavor-SU(3) breaking; however, it involves some unknown form factors and meson wave functions and appears to underestimate the magnitude of $B \to VP$ penguin amplitudes.

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