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Onium Line Shape Fitting

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Abstract

This is a follow-up to CLNS 03/1837, in which I consider the effect on the observed line shape for e^+e^- resonance production due to interference with the continuum e^+e^- annihilation into hadrons or into lepton pairs. Relative to the original version of this CBX, this new version corrects for a missing factor of π in equation (4) and in the tabulated values for κ and y_{int}. I have also simplified some of the formulas.

1 Review of Onium Line Shape Fundamentals

In order to determine the e^+e^- coupling to a $c\bar{c}$ or $b\bar{b}$ bound state resonance, we scan over the resonance, measuring the cross section for hadronic final states. The dependence of the observed cross section above continuum $\bar{\sigma}(W_{CESR})$ on the nominal setting of the two-beam CESR energy is a convolution of three energy spreading effects [1]:

$\tilde{\sigma}(W_{res}, M)$, the intrinsic dependence of the resonant cross section on the total energy W_{res} of the resonance, nominally a Breit-Wigner centered at $W_{res} = M$ and having a width (FWHM) equal to Γ;

$F_{KF}(W_{ee}, W_{res})$, the spectrum of energy $W_{ee} - W_{res}$ radiated before the formation of the resonance, including the dependence on W_{ee} itself and the effect of higher order virtual radiative processes on the overall scale of the cross section (see ref. [2])

$G_{\delta W}(W_{ee} - W_{CESR})$, the spreading of the e^+e^- collision energies relative to the centroid CESR two-beam energy setting, which we take to be a Gaussan with a slowly energy dependent r.m.s. width δW.

For the $c\bar{c}$ and $b\bar{b}$ bound state ${}^1S_1$ "onium" resonances the width Γ is between 0.02 and 0.09 MeV for the Υ's and the ψ, and is 0.30 MeV for the ψ'. The r.m.s. beam energy spread δW is of the order of 1 MeV for the ψ and ψ' (depending on the number of wiggler magnets), and is about 4 MeV for the Υ's (see Table 1). Thus, except possibly for the case of the ψ', we do not expect the observed line shape to be very sensitive to the distinction between the actual Breit-Wigner and a delta function. Roughly speaking, the resonance line shape will be a delta function with a radiative tail on the high energy side, smeared by a Gaussian of width δW. However, since our goal is to extract a $\mathcal{O}(1\%)$ measurement of Γ_{ee} from the observed resonance line shape, we have to keep track of small effects.

Once the convolution integral $\mathcal{F}(W_{CESR})$ is calculated ([1] eq. 19), with the appropriate normalization, one can fit it to the observed cross section data and determine the idealized, radiationless area $\mathcal{A}$ of the resonance peak and then extract $\Gamma_{ee}\Gamma_{had}/\Gamma$ ([1] eq 5,21):

$$\bar{\sigma}(W_{CESR}) = \mathcal{A} \times \mathcal{F}(W_{CESR}), \tag{1}$$

$$\mathcal{A} = \frac{6\pi^2\Gamma_{ee}\Gamma_{had}}{M^2\Gamma}. \tag{2}$$

The fit result is roughly given in terms of the observed peak cross section and full-width at half-maximum:

$$\mathcal{A} \sim 1.69 \times \bar{\sigma}(M) \times \Delta W_{fwhm}. \tag{3}$$

2 Interference

One potentially important effect is the distortion in the shape and area of the intrinsic Breit-Wigner resonance line shape by interference between resonance decays to hadronic states and the nonresonant background annihilation of e^+e^- into the same hadronic states. The CESR beam energy resolution is not fine enough for a useful measurement of the distortion, but if ignored, it may nevertheless have an undesirable effect on the result $\mathcal{A}$ of the $\bar{\sigma}$-to-$\mathcal{F}$ fitting. It can shift the apparent M of the resonance. The effect on the area of the resonance peak is zero in the limit of a narrow resonance, but can be nonzero if the resonance width $\Gamma(W)$ is large enough to make its energy dependence significant. Ignoring the shape dependence will certainly affect the goodness of fit if the data are accurate enough.

The Breit-Wigner resonance amplitude and no-interference cross section are given by

$$a_{BW}(W_{res}) = \frac{\sqrt{\kappa}}{W_{res} - M + i\Gamma/2},$$

$$\sigma_{BW}(W_{res}) = |a_{BW}|^2 = \frac{\kappa}{(W_{res} - M)^2 + \Gamma^2/4},$$

where

$$\kappa = 3\pi\Gamma_{ee}\Gamma_{had}/M^2. \tag{4}$$

Note that the amplitude a_{BW} is complex, with a phase $\phi_{res} = -\tan^{-1}[(\Gamma/2)/(W_{res} - M)]$ that starts near $0°$ at low W_{res}, passes through $90°$ at $W_{res} = M$, and approaches $180°$ for large W_{res}. Nonresonant $e^+e^- \to q\bar{q} \to hadrons$ has cross section and amplitude given by

$$\sigma_{cont}(W) = R\sigma_{\mu\mu} \approx 3\Sigma(q_i^2) \times \tfrac{4}{3}\pi\alpha^2/W^2, \tag{5}$$

$$a_{cont}(W) = \sqrt{\sigma_{cont}}e^{i\phi}.$$

The phase ϕ should be energy independent. What matters for the interference is what its value is relative to the energy dependent phase I have already defined for the resonant amplitude.

The interference term to be added to the Breit-Wigner cross section is

$$2\mathcal{R}e(a^*_{BW}a_{cont}) = 2|a_{BW}|\sqrt{\sigma_{cont}}\frac{(W_{res}-M)\cos\phi + (\Gamma/2)\sin\phi}{(W_{res}-M)^2+\Gamma^2/4}$$

$$= 2\sigma_{BW}\sqrt{f\sigma_{cont}/\kappa}[(W_{res}-M)\cos\phi + (\Gamma/2)\sin\phi]$$
$$= y_{int}\sigma_{BW}[2(W_{res}-M)/\Gamma)\cos\phi + \sin\phi],$$

where I have absorbed into y_{int} the factors determining the magnitude of the interference, including κ from eq 4, σ_{cont} from eq 5, and the fraction f of the resonant amplitude that is coherent with the continuum amplitude.

How much interference should we expect? To what extent are the hadronic final states in resonance decay coherent with the corresponding final states in continuum annihilation?

First, we can expect any cascade decays of higher resonances to lower ones (via γ, $\pi\pi$, etc.) to be completely incoherent with continuum background. The existence of a lower resonance in the cascade final state would make it observably different, at least in principle, from a continuum final state, even though the eventual resonance decay products may cause the two processes to look the same in practice.

The decay $res \to ggg \to hadrons$ and the continuum process $e^+e- \to \gamma^* \to q\overline{q} \to hadrons$ share some of the same exclusive final states. Although the final products have different distributions (in sphericity, multiplicity, and momentum, for example) one cannot experimentally separate them into ggg and $q\overline{q}$ event by event. If the ggg and/or $q\overline{q}$ intermediate states lived long enough for them to be identifiable before hadronization, *even in principle* if not in practice, then they would of course be incoherent. The theorists that I have polled [3] do not expect coherence.

There is, however, a fraction of the resonance decays that go though the same $J^P = 1^-$ intermediate state $\gamma^* \to q\overline{q}$ as in the continuum annihilation. The corresponding branching ratio is

$$B_{q\overline{q}} = \frac{res \to \gamma^* \to q\overline{q}}{res \to hadrons} = R \times \frac{B_{\mu\mu}}{1-nB_{\mu\mu}} \approx 3\Sigma(q_i^2) \times \frac{B_{\mu\mu}}{1-nB_{\mu\mu}}, \tag{6}$$

where q_i are the charges of the quarks with masses below $W/2$, $B_{\mu\mu} = \Gamma_{\mu\mu}/\Gamma$ for the resonance, and $n = 2(3)$ for ψ's(Υ's). $B_{q\overline{q}}$ varies from 1.4% (ψ') to 13% (ψ) (Table 1). These decays will be completely indistinguishable from the continuum production. I would therefore expect them to be fully coherent and interfere maximally, that is, $f = B_{q\overline{q}}$.

The fermion-pair decays of a vector resonance, such as a $^1S_1$ $c\overline{c}$ or $b\overline{b}$, should interfere destructively with the corresponding continuum production below the resonance and constructively above [4]. This has been confirmed by SPEAR data on ψ and ψ' to $\mu^+\mu^-$ [5]. It implies that $\phi = 0°$.

Table 1: Input parameters for FUNCTION GBWKF (energies in MeV)

parameter	source	ψ	ψ'	Υ(1S)	Υ(2S)	Υ(3S)
Fixed quantities:						
Γ	[7]	0.087	0.30	0.0525	0.055	0.0263
ϕ	[5]	0.	0.	0.	0.	0.
y_{int}	eq 7	0.012	0.010	0.018	0.018	0.018
$y_{int}^{\mu\mu}$	§4	0.094	0.69	0.22	0.39	0.29
Suggested starting values for fitted quantities:						
M	[7]	3096.87	3685.96	9460.3	10023.26	10355.2
δW	[1]	0.7	0.9	3.8	4.3	4.8
Input to calculation of y_{int}:						
R	[7]	2.1	2.1	3.5	3.5	3.5
$B_{\mu\mu}$	[7]	.059	.0072	.024	.013	.018

3 The Convolution Function

The convolution of Breit-Wigner, radiative tail, and energy Gaussian must be done numerically. I have programmed it as a Fortran function [6],

$$\mathcal{F}(\Delta W) = \text{GBWKF}(M, \Gamma, \delta W, y_{int}, \phi, \Delta W),$$

where

- M is the central mass in MeV of the resonance (floated in the fit),
- Γ is the intrinsic full-width at half-maximum in MeV of the resonance (assumed fixed value),
- δW is the r.m.s. spread in MeV of the CESR total e^+e^- center-of-mass energy (floated in the fit),
- y_{int} is the ratio of the interference contribution to the no-interference Breit-Wigner cross-section, evaluated for $W_{res} - M = \Gamma/2$ and $\phi = 0°$ (assumed fixed value),
- ϕ is the phase in degrees of the non-resonant hadronic production amplitude (assumed fixed value),
- ΔW $= W_{CESR} - M$ is the relative energy in the scan in MeV (the independent variable for the fitted function).

When GBWKF is first called it goes through a lengthy series of initializations. Small changes in the fitted parameters do not trigger reinitialization, but any change in the fixed quantities

Table 2: Comparison of $\mathcal{F}$ with and without interference

	ψ	ψ'	Υ(1S)	Υ(2S)	Υ(3S)
$W_{CESR} - M$ at maximum, MeV					
no interference	.081	.097	.460	.491	.515
interference (Table 1)	.087	.101	.491	.526	.555
change	.005	.004	.031	.035	.040
maximum value of $\mathcal{F}$					
no interference	.33363	.23634	.06113	.05448	.05006
interference	.33348	.23623	.06108	.05444	.05001
% change	$-.04$	$-.05$	$-.08$	$-.08$	$-.08$
integral of $\mathcal{F}(W_{CESR} - M)$, MeV					
no interference	.908	.887	.892	.888	.879
interference	.913	.889	.896	.892	.883
% change	.4	.2	.5	.5	.5

does. One could still make a crude fit for one of the fixed quantities (say Γ for instance) by making repeated multiparameter fits, each time with a different fixed Γ, keeping track of the best-fit χ^2 for each. The numerical approximations in GBWKF should be good to better than 0.5% for the bound state $c\bar{c}$ and $b\bar{b}$ resonances (though not for the $\psi''(3770)$ or the Υ(4S)) provided $\Delta W < 50\delta W$.

Table 1 lists the recommended values to use in GBWKF for the fixed quantities Γ, ϕ, y_{int} and for the starting values of the fitted quantities M, δW. The listed values for δW at the $c\bar{c}$ energies are estimated [1] for CESR running with one wiggler magnet installed. The actual values are uncertain. If we assume that the resonance interferes with the continuum only through the fully coherent $q\bar{q}$ final state, the dimensionless input parameter y_{int} that specifies the magnitude of the interference term is given by

$$y_{int} = \Gamma\sqrt{B_{q\bar{q}}\sigma_{cont}(M)/\kappa} =^{\frac{2}{3}} R\alpha/(1 - nB_{\mu\mu}). \tag{7}$$

For the $\psi(\Upsilon)$ resonances $y_{int} \approx 1\%(1.8\%)$ (Table 1). Note that the relative uncertainty in y_{int} is essentially the relative experimental error in R, which is of the order of a few percent [7].

With these values the effect on GBWKF relative to the no interference $y_{int} = 0$ case is slight (Table 2). The lower energy side of the peak is depressed, even subtracting from the continuum background, and the higher energy side is enhanced. This causes an upward shift of less than 0.04 MeV in the peak maximum. Although interference decreases the peak value of $\mathcal{F}(\Delta W)$ by less than 0.1%, the peak gets wider, causing the effective peak integral to increase by 0.2% to 0.5%. Ignoring this effect would cause a corresponding positive error in the fitted value of $\mathcal{A}$.

4 Muon (or Tau) Pairs

Since the initial state radiation and beam energy spreading are the same for mu pairs as for hadrons, the same GBWKF routine can also be used for fitting the line shape for $e^+e^- \to$ *onium* $\to \mu^+\mu^-$ or $\to \tau^+\tau^-$, provided one uses the appropriate value for the y_{int} input. That is, in analogy with eqs 1 and 2, one fits the observed $\bar{\sigma}_{\mu\mu}(\Delta W)$ to the product $\mathcal{A}_{\mu\mu}\mathcal{F}(\Delta W)$. where $\mathcal{A}_{\mu\mu} = 6\pi^2\Gamma_{ee}B_{\mu\mu}/M^2$ and $\mathcal{F}$ is the GBWKF function using y_{int} calculated from the first form of eq 7 with $\kappa_{\mu\mu} = 3\pi\Gamma_{ee}\Gamma_{\mu\mu}/M^2$, $\sigma^{\mu\mu}_{cont}(M) = \frac{4}{3}\pi\alpha^2/M^2$, and with $f = 1.0$ replacing $B_{q\bar{q}}$. That is,

$$y^{\mu\mu}_{int} =\frac{2}{3}\,\Gamma\alpha\frac{1}{\sqrt{B_{\mu\mu}(1-nB_{\mu\mu})}}.$$

As long as the phase space suppression of $\tau^+\tau^-$ can be neglected, the same formula holds for the tau pair case. The $y^{\mu\mu}_{int}$ is much larger (Table 1) and the interference is much more pronounced [5] than in the hadron final state.

It may be convenient to fit the energy scan data to the GBWKF peak shape using the sum of the hadronic and tau pair cross sections weighted by relative detection efficiencies, since the $\tau^+\tau^- \to$ *hadrons* final state may not be distinguishable from the direct decay of the resonance to hadrons. In that case, one has to fit to a sum of GBWKF$_h$ for hadronic decays and GBWKF$_\tau$ for tau pair hadronic decays, each with the appropriate branching fraction and efficiency coefficient and y_{int} argument. One can get the correct relative branching fraction from the observation of $e^+e^- \to$ *resonance* $\to \tau^+\tau- \to$ *leptons*, or from $e^+e^- \to$ *resonance* $\to \mu^+\mu^-$ with the appropriate phase space factor for τ/μ. One has to rename all the GBWKF routines **AND** common blocks used for GBWKF$_\tau$ so that the two peak functions do not get confused and the initializations do not get retriggered each time you call a different GBWKF.

References

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