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An Improved Brane Anti-Brane Action from Boundary Superstring Field Theory and Multi-Vortex Solutions

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Abstract: We present an improved effective action for the D-brane-anti-D-brane system obtained from boundary superstring field theory. Although the action looks highly non-trivial, it has simple explicit multi-vortex (*i.e.* codimension-2 multi-BPS D-brane) multi-anti-vortex solutions. The solutions have a curious degeneracy corresponding to different "magnetic" fluxes at the core of each vortex. We also generalize the brane anti-brane effective action that is suitable for the study of the inflationary scenario and the production of defects in the early universe. We show that when a brane and anti-brane are distantly separated, although the system is classically stable it can decay via quantum tunneling through the barrier.

Keywords: D-Branes; Tachyon Condenstaion; String Theory; Brane-World Cosmology.

Contents

1. Introduction

D-branes play a crucial role in string theory [1]. To understand the D-brane anti-D-brane ($\mathrm{D}\overline{\mathrm{D}}$) system involves off-shell physics. A powerful way to study it is to write down its effective space-time action from background-independent or boundary string field theory (BSFT) [2–4]. Following the work on the non-BPS D-brane effective action in open superstring theory [5], this program was carried out by two groups (KL [6] and TTU [7]). Here we seek to improve on their effective $\mathrm{D}\overline{\mathrm{D}}$ action and study its properties.

The effective action in Refs [6, 7] has a number of interesting properties. It includes all powers of the single derivative of the tachyon field T, a feature very important for time dependent, or rolling tachyon, solutions [8, 9]. This feature is also necessary to lead to the fact that the lower dimensional branes appear as soliton solutions in tachyon condensation. In particular, KL/TTU find a codimension-two BPS brane as a solitonic solution, with the correct brane tension and the correct RR charge [10–12]. However, that vortex solution does not have "magnetic" flux inside it, contrary to our intuition from the Abelian Higgs model. As written, the KL/TTU effective action that involves all powers of the first derivative of T does not respect the $U(1)\times U(1)$ gauge symmetry of the $\mathrm{D}\overline{\mathrm{D}}$ system; the derivatives of T do not generalize to covariant derivatives, as is necessary since the complex tachyon field T is charged under the relative $U(1)$. Without the correct gauge covariant action, it is not clear whether the vortex solution, and more generally the multi-vortex solutions, should have "magnetic" flux inside them or not.

We improve the D$\overline{\rm D}$ effective action by restoring the covariance and the $U(1) \times U(1)$ gauge symmetry of the system so the tachyon field couples to one of the gauge fields as expected. This improved action is summarized in Eq.(2.13). Starting with this D$\overline{\rm D}$ action we find analytic multi-vortex multi-anti-vortex solutions (all parallel with arbitrary positions and constant velocities), summarized in Eq.(3.10). The solution with n vortices (*i.e.* n parallel codimension-2 branes) and m anti-vortices has total tension $\varepsilon_{p-2} = (n+m)\tau_{p-2}$ and Ramond-Ramond (RR) charge $\mu_{p-2} = (n-m)\tau_{p-2}g_s$ under the spacetime $(p-1)$-form potential. Here τ_{p-2} is the D$(p-2)$-brane tension and g_s is the string coupling constant. The simplicity of the solution suggests that the D$\overline{\rm D}$ effective action may be useful to study the brane dynamics. For $m = 0$ and an appropriate choice of the magnetic flux, the solution is supersymmetric and corresponds to n BPS D$(p-2)$-branes.

These solutions have a curious degeneracy. Each unit of winding (*i.e.* a vortex corresponding to a D-brane) can have up to one unit of "magnetic" flux inside it. That is, both the tension and the RR charge are independent of the presence (or absence) of the "magnetic" flux. We expect this degeneracy to be lifted by the quantum corrections to the D$\overline{\rm D}$ action and/or the corrections from the higher derivative and gauge field-strength terms. However, it is not clear exactly how the degeneracy will be lifted.

One motivation to understand the D$\overline{\rm D}$ system better is its role in cosmology. D-brane interaction in the brane world scenario provides a natural setting for an inflationary epoch in the early universe [13–16]. There, the inflaton is simply the brane-brane separation while the inflaton potential comes from their interaction. The simplest such scenario involves a brane-anti-brane pair [14, 17, 18]. Toward the end of inflation, as the brane and the anti-brane approach each other and collide, a tachyon emerges and tachyon condensation (*i.e.* the tachyon field rolling down its potential) is expected to reheat the universe and produce solitons (even codimensional branes) that appear as cosmic strings in our universe [16]. The cosmic string density is estimated to be compatible with present day observations, but will be critically tested by cosmic microwave background radiation and gravitational wave detectors in the near future [19]. To study inflation and how it ends, we also construct the $(\mathrm{D}\overline{\mathrm{D}})_p$ effective action when the brane is separated from the anti-brane. We find a separation dependent tachyon potential which predicts that the D$\overline{\rm D}$ system is classically stable when the brane and anti-brane are further than $\frac{2\pi^2\alpha'}{2\ln 2}$ apart, but can quantum mechanically decay with the tachyon tunneling through its potential. The critical separation agrees with the result known from other methods [20] aside from the factor of $2\ln 2$.

The paper is organized as follows. In §2, we briefly review the BSFT derivation of the D$\overline{\rm D}$ action. Then we use Lorentz and gauge symmetry to complete the terms in the effective action. As a check, we expand it to next to leading order and show agreement with known results. In §3, we present the general multi-vortex multi-anti-vortex solutions, with zero and non-zero gauge field strengths inside the vortices. We calculate the RR charge and the total energy of these solutions and reveal the degeneracy. We discuss how this degeneracy may be lifted. In §4, we construct the effective action when the Dp-brane and the $\overline{\rm D}p$-brane are separated. The barrier potential to tunelling is evaluated. §5 is the conclusion.

2. Brane Anti-Brane Effective Actions

2.1 Linear Tachyon Action from BSFT

We summarize the brane anti-brane effective action from BSFT calculated by KL and TTU [6,7]. We restrict attention to D9-branes in type IIB theory, and generalize using T-Duality later. BSFT essentially extends the sigma-model approach to string theory [21], in that (under certain conditions [2,4]) the disc world-sheet partition function with appropriate boundary insertions gives the classical spacetime action. This framework for the bosonic BSFT was extended to the open superstring in [5] and formally justified in [22]. In the NS sector the spacetime action is

$$S_{\text{spacetime}} = -\int \mathcal{D}X\mathcal{D}\psi\mathcal{D}\tilde{\psi}\; e^{-S_\Sigma - S_{\partial\Sigma}}. \tag{2.1}$$

where Σ is the worldsheet disc and $\partial\Sigma$ is its boundary. The worldsheet bulk disc action is the usual one

$$\begin{aligned} S_\Sigma &= \frac{1}{2\pi\alpha'}\int d^2z\; \partial X^\mu \overline{\partial} X_\mu + \frac{1}{4\pi}\int d^2z \left(\psi^\mu \overline{\partial}\psi_\mu + \tilde{\psi}^\mu \partial \tilde{\psi}_\mu\right) \\ &= \frac{1}{2}\sum_{n=1}^{\infty} n X^\mu_{-n} X_{n\ \mu} + i\sum_{r=\frac{1}{2}}^{\infty} \psi^\mu_{-r}\psi_{r\ \mu}, \end{aligned}$$

after expanding the fields in the standard modes. To reproduce the Dirac-Born-Infeld (DBI) action for a single brane, the appropriate boundary insertion is the boundary pullback of the $U(1)$ gauge superfield to which the open string ends couple; for the N brane M anti-brane system, the string ends couple to the superconnection [11,23], hence the boundary insertion should be

$$e^{-S_{\partial\Sigma}} = \text{Tr}\;\hat{P}\exp\left[\int d\tau d\theta \mathcal{M}(\mathbf{X})\right], \quad \mathcal{M}(\mathbf{X}) = \begin{pmatrix} iA^1_\mu(\mathbf{X})D\mathbf{X}^\mu & \sqrt{\alpha'}T^\dagger(\mathbf{X}) \\ \sqrt{\alpha'}T(\mathbf{X}) & iA^2_\mu(\mathbf{X})D\mathbf{X}^\mu \end{pmatrix} \tag{2.2}$$

where the insertion must be supersymmetrically path ordered to preserve supersymmetry and gauge invariance. $A^{1,2}$ are the $U(N)$ and $U(M)$ connections, and T is the tachyon matrix transforming in the $(N,\overline{M})$ of $U(N)\times U(M)$. The lowest component of $\mathcal{M}$ is proportional to the superconnection. To proceed, it is simplest to perform the path-ordered trace by introducing boundary fermion superfields [24]; we refer the reader to [6] for details. The insertion (2.2) can then be simplified to be

$$\text{Tr}\;P\exp\left[i\alpha'\int d\tau \begin{pmatrix} F^1_{\mu\nu}\psi^\mu\psi^\nu + iT^\dagger T + \frac{1}{\alpha'}A^1_\mu \dot{X}^\mu & -iD_\mu T^\dagger \psi^\mu \\ -iD_\mu T\psi^\mu & F^2_{\mu\nu}\psi^\mu\psi^\nu + iTT^\dagger + \frac{1}{\alpha'}A^2_\mu \dot{X}^\mu \end{pmatrix}\right], \tag{2.3}$$

where the tachyon covariant derivatives are

$$D_\mu T = \partial_\mu T + iA^1_\mu T - iTA^2_\mu. \tag{2.4}$$

This expression reproduces the expected results when the tachyon and its derivatives vanish: the only open string excitations will be the gauge fields on the branes and the anti-branes, for each of which the action is the standard DBI action. For instance, with $N = M = 1$, $DT = T = 0$, the partition function (2.1) with the insertion (2.3) leads to

$$S_{\mathrm{D\overline{D}}} = -\tau_9 \int d^{10}x \left[\sqrt{-\det(g+2\pi\alpha' F^1)} + \sqrt{-\det(g+2\pi\alpha' F^2)}\right]. \tag{2.5}$$

The measure in (2.1) was defined to reproduce the correct tension for the D9-branes, $\tau_9 = 1/[(2\pi)^9 g_s \alpha'^5]$[1]. Unfortunately (2.3) cannot in general be simplified, but for a single brane anti-brane pair, $N = M = 1$, demanding that the gauge field to which the tachyon couples vanishes, $A^- \equiv A^1 - A^2 = 0$, the path-ordered trace can be performed using worldsheet boundary fermions. Writing $A^+ = A^1 + A^2$, we have [6]

$$S_{\partial\Sigma} = -\int d\tau \left[\alpha' T\overline{T} + \alpha'^2 (\psi^\mu \partial_\mu T)\frac{1}{\partial_\tau}(\psi^\nu \partial_\nu \overline{T}) + \frac{i}{2}\left(\dot{X}^\mu A^+_\mu + \frac{1}{2}\alpha' F^+_{\mu\nu}\psi^\mu\psi^\nu\right)\right]. \tag{2.6}$$

The operator $1/\partial_\tau$ acting on a function $f(\tau)$ is defined to be the convolution of f with $\mathrm{sgn}(\tau)$ over the worldsheet boundary. For linear tachyon profiles, gauge and spacetime rotations allow us to write $T = u_1 X^1 + i u_2 X^2$, and (2.1) can be calculated, since the functional integrals are all Gaussian. The result when $A^+ = 0$ is derived in [6, 7]:

$$S_{\mathrm{D\overline{D}}} = -2\tau_9 \int d^{10}X_0 \, \exp\left[-2\pi\alpha'[(u_1 X_0^1)^2 + (u_2 X_0^2)^2]\right] \mathcal{F}(4\pi\alpha'^2 u_1^2)\, \mathcal{F}(4\pi\alpha'^2 u_2^2). \tag{2.7}$$

where the function $\mathcal{F}(x)$ is given by [5]

$$\mathcal{F}(x) = \frac{4^x x \Gamma(x)^2}{2\Gamma(2x)} = \frac{\sqrt{\pi}\Gamma(1+x)}{\Gamma(\frac{1}{2}+x)}. \tag{2.8}$$

Note that $\mathcal{F}(x) = 0$ at $x = -1/2$, and

$$\mathcal{F}(x) = \begin{cases} 1 + (2\ln 2)x + \left[2(\ln 2)^2 - \frac{\pi^2}{6}\right]x^2 + \mathcal{O}(x^3), & 0 < x \ll 1, \\ \sqrt{\pi x}\left[1 + \frac{1}{8x} + \mathcal{O}(\frac{1}{x^2})\right], & x \gg 1, \\ -1/(1+x), & x \to -1. \end{cases} \tag{2.9}$$

This action exhibits all the intricate properties of the $\mathrm{D\overline{D}}$ system expected from Sen's conjectures: the tachyon potential at its minima $T \to \infty$ completely cancels the brane tensions; even codimension solitons can appear on the D9-brane worldvolume, with exactly the correct tension to be lower dimensional D-branes; odd codimension solitons on which tachyonic fields reside can appear, with exactly the tension of the unstable non-BPS branes of type II string theories [10].

BSFT can also give the analogue of the D-brane Chern-Simons action for the $\mathrm{D\overline{D}}$ system, defined similarly to (2.1), but with all fermions in the Ramond sector. The bulk contribution to the partition sum can be written as the wave-functional [6, 7]

$$\Psi^{RR}_{\mathrm{bulk}} \propto \exp\left[-\frac{1}{2}\sum_{n=1}^{\infty} n X^\mu_{-n} X_{n\,\mu} - i\sum_{n=1}^{\infty}\psi^\mu_{-n}\psi_{n\,\mu}\right] C,$$
$$C = \sum_{\mathrm{odd}\ p} \frac{(-i)^{\frac{9-p}{2}}}{(p+1)!} C_{\mu_0\cdots\mu_p}\psi_0^{\mu_0}\cdots\psi_0^{\mu_p}.$$

[1]Throughout this work we assume the dilaton is stabilized to give an effective string coupling $e^\phi = g_s$.

The ψ_0^μ are the zero modes of the Ramond sector fermions, and $C_{\mu_0\cdots\mu_p}$ are the even RR forms of IIB string theory. The normalization of Ψ can be set later by demanding that the correct brane charge is reproduced. The Chern-Simons action is then defined by

$$S_{\rm CS} = \int \mathcal{D}X\mathcal{D}\psi\, \Psi_{\rm bulk}^{RR}\,{\rm Tr}^*\, Pe^{-S_{\partial\Sigma}},$$

in which the trace given by

$${\rm Tr}^*\, O \equiv {\rm Tr}\, \left[\begin{pmatrix} \mathbb{1}_{N\times N} & 0 \\ 0 & -\mathbb{1}_{M\times M}\end{pmatrix} O\right]$$

results from the periodicity of the worldsheet fermion superfield which was necessary to implement to the supersymmetric path ordering. Again $e^{-S_{\partial\Sigma}}$ can be written as (2.3), with Ramond sector fermions. This expression can be viewed as a one dimensional supersymmetric partition function on $\mathbb{S}^1$, and because the Ramond sector fermions are periodic, this is equivalent to Tr $(-1)^F e^{-\beta H}$. By Witten's argument [25], only the zero modes contribute to the partition sum, giving [6, 7, 26]

$$S_{\rm CS} = \tau_9 g_s \int C\wedge {\rm Tr}^*\, e^{2\pi\alpha' i\mathcal{F}}, \tag{2.10}$$
$$\mathcal{F} = \begin{pmatrix} F^1 + iT^\dagger T & -i(DT)^\dagger \\ -iDT & F^2 + iTT^\dagger \end{pmatrix}$$

$\mathcal{F}$ is the curvature of the superconnection, and as usual, the fermion zero modes form the basis for the dual vector space and all forms above are written with $\psi_0^\mu \to dx^\mu$. This expression is exact[2] and although it was derived for 2^{m-1} brane anti-brane pairs in [6, 7] it appears to have the correct properties for the general N brane M anti-brane case.

As for the action (2.7), this result affirms Sen's conjectures in that it exhibits appropriate coupling to the RR 10-form potential, and the even codimension solitons have the correct couplings to the relevant RR forms to be identified as lower dimensional branes.

2.2 An Improved D$\overline{\rm D}$ Action

As written, the action (2.7) for a single brane anti-brane pair does not manifest the necessary gauge covariance, and this form of the action is valid only for linear tachyon profiles[3]. We now generalize the pure tachyon action of KL and TTU. Note that there are precisely two independent Lorentz and $U(1)$ invariant expressions in terms of first derivatives of the complex tachyon T [7, 27],

$$\mathcal{X} \equiv 2\pi\alpha'^2 g^{\mu\nu}\partial_\mu T\partial_\nu \overline{T}, \qquad \mathcal{Y} \equiv \left(2\pi\alpha'^2\right)^2 \left(g^{\mu\nu}\partial_\mu T\partial_\nu T\right)\left(g^{\alpha\beta}\partial_\alpha\overline{T}\partial_\beta\overline{T}\right),$$

[2] As discussed in [6], this action is exact in T and $A^\pm$ and their derivatives, but has corrections for non-constant RR forms.

[3] A covariant perturbative action was derived in [7] to order α'^2, but we seek covariance of the complete action, up to higher derivative terms.

(with the normalizations chosen for convenience). For the linear profile $T = u_1 x_1 + i u_2 x_2$, the only translation invariant way to reexpress $u_{1,2}$ is as $u_{1,2} = \partial_{1,2} T^{1,2}$; then with $g^{\mu\nu} = \eta^{\mu\nu}$ we can calculate $\mathcal{X}$ and $\mathcal{Y}$,

$$\mathcal{X} = 2\pi\alpha'^2 (u_1^2 + u_2^2),$$
$$\mathcal{Y} = \left(2\pi\alpha'^2\right)^2 (u_1^2 - u_2^2)^2,$$

so the arguments of $\mathcal{F}$ in (2.7) can be written as

$$4\pi\alpha'^2 u_1^2 = \mathcal{X} + \sqrt{\mathcal{Y}},$$
$$4\pi\alpha'^2 u_2^2 = \mathcal{X} - \sqrt{\mathcal{Y}}.$$

This provides a unique way to covariantize (2.7) as

$$S_{\mathrm{D\overline{D}}} = -2\tau_9 \int d^{10}x \sqrt{-g}\; e^{-2\pi\alpha' T\overline{T}}\, \mathcal{F}(\mathcal{X} + \sqrt{\mathcal{Y}})\, \mathcal{F}(\mathcal{X} - \sqrt{\mathcal{Y}}), \tag{2.11}$$

which reduces to (2.7) when T is linear in two spacetime coordinates. We shall see in §3.1 that restoring the spherical and gauge symmetry in this expression allows us to construct multiple codimension-2 BPS solitons as expected from the K-theory arguments [11].

Further, we can restore the A^+ dependence of the action, since (2.6) remains quadratic when $A^+ \neq 0$ if F^+ is constant and the partition function (2.1) will be Gaussian. A similar calculation was performed for the non-BPS brane action [28], and borrowing that result gives the extended tachyon and gauge field action

$$S_{\mathrm{D\overline{D}}} = -2\tau_9 \int d^{10}x\; e^{-2\pi\alpha' T\overline{T}} \sqrt{-G}\;\; \mathcal{F}(\mathcal{X} + \sqrt{\mathcal{Y}})\, \mathcal{F}(\mathcal{X} - \sqrt{\mathcal{Y}}), \tag{2.12}$$

where now $G_{\mu\nu} = g_{\mu\nu} + \pi\alpha' F^+_{\mu\nu}$ forms the effective metric for the tachyon, as is usual for open string states in the presence of a gauge connection [29]:

$$\mathcal{X} \equiv 2\pi\alpha'^2 G^{\{\mu\nu\}} \partial_\mu T \partial_\nu \overline{T} \qquad\qquad \mathcal{Y} \equiv \left| 2\pi\alpha'^2 G^{\mu\nu} \partial_\mu T \partial_\nu T \right|^2.$$

Indices are raised and lowered with respect to G: $G^{\mu\nu} G_{\nu\alpha} = \delta^\mu_\alpha$, and $G^{\{\mu\nu\}}$ indicates the symmetric part of G; this symmetrization is necessary to obtain a real action. [4] This coupling to F^+ can be confirmed considering that the $\mathrm{D\overline{D}}$ system reduces to the non-BPS brane system under the spacetime IIA $\leftrightarrow$ IIB quotient $(-1)^{F_L}$ [10], which in this system is applied by setting $\overline{T} = T$, $F^1 = F^2$:

$$S_{\mathrm{D\overline{D}}} \xrightarrow[A^1 = A^2]{\overline{T} = T} -2\tau_9 \int d^{10}x\; e^{-2\pi\alpha' T^2} \sqrt{-G}\;\; \mathcal{F}(2\mathcal{X})\, \mathcal{F}(0) = \sqrt{2} S_{\mathrm{nBPS}}.$$

The overall normalization of the action must be divided by $\sqrt{2}$ to compensate for the extra boundary fermion in the $\mathrm{D\overline{D}}$ system which was integrated over, which is superfluous in the non-BPS brane system.

[4] It is also possible to include a term in $\mathcal{X}$ proportional to the anti-symmetric part of $G^{\mu\nu}$, which must have an imaginary coefficient for the sake of reality. The coefficient of such a term is undetermined by our arguments, and shall be unimportant in our analysis of the action.

The action (2.12) is still incomplete in that $A^- = A^1 - A^2$, the $U(1)$ connection to which the tachyon couples, was set to zero in its derivation. We can conjecture the extension to $A^- \neq 0$ based on the following information:

- Gauge covariance demands that all tachyon derivatives must be replaced by covariant derivatives. A^- cannot appear outside a covariant derivative, so (2.12) with $\partial T \to DT$ can only suffer corrections for non-constant A^- (and of course, the higher T and A^+ derivative corrections).
- (2.5) should be reproduced for $T = DT = 0$.
- We expect the gauge connections to appear in the matrix form

$$\begin{pmatrix} \frac{1}{2}F^+ & 0 \\ 0 & \frac{1}{2}F^+ \end{pmatrix} \to \begin{pmatrix} F^1 & 0 \\ 0 & F^2 \end{pmatrix},$$

  when we restore $F^- = F^1 - F^2 \neq 0$. We can insert this into the action (2.12) and trace over the $U(2)$ indices.

This leads us to the next improvement to (2.12),

$$S_{\mathrm{D}\overline{\mathrm{D}}} = -\tau_9 \int d^{10}x \; e^{-2\pi\alpha' T\overline{T}} \begin{bmatrix} \sqrt{-\det[G_1]}\,\mathcal{F}(\mathcal{X}_1 + \sqrt{\mathcal{Y}_1})\,\mathcal{F}(\mathcal{X}_1 - \sqrt{\mathcal{Y}_1}) \\ +\sqrt{-\det[G_2]}\,\mathcal{F}(\mathcal{X}_2 + \sqrt{\mathcal{Y}_2})\,\mathcal{F}(\mathcal{X}_2 - \sqrt{\mathcal{Y}_2}) \end{bmatrix}, \tag{2.13}$$

$$(G_{\mu\nu})_{1,2} \equiv \left(g_{\mu\nu} + 2\pi\alpha' F^{1,2}_{\mu\nu}\right), \qquad \begin{aligned} \mathcal{X}_{1,2} &\equiv 2\pi\alpha'^2 G^{\{\mu\nu\}}_{1,2} D_\mu T D_\nu \overline{T}, \\ \mathcal{Y}_{1,2} &\equiv \left|2\pi\alpha'^2 G^{\mu\nu}_{1,2} D_\mu T D_\nu T\right|^2, \end{aligned}$$

The tachyon is charged only under A^-:

$$D_\mu T = \partial_\mu T + i A^-_\mu T, \qquad\qquad A^\pm_\mu = A^1_\mu \pm A^2_\mu,$$

and the function $\mathcal{F}(x)$ is defined in (2.8). This is the effective action which shall be studied in this work. Corrections to this action will include higher derivative terms in T and $F^\pm$. Possible terms like $(F^-)^n T\overline{T}$ may be included in higher tachyon derivatives since $[D_\mu, D_\nu] = iF_{\mu\nu}$. Being non-supersymmetric, there will be quantum corrections to the action as well.

In the α' expansion, using (2.9), we have

$$\mathcal{F}(\mathcal{X} + \sqrt{\mathcal{Y}})\,\mathcal{F}(\mathcal{X} - \sqrt{\mathcal{Y}}) = 1 + 4(\ln 2)\mathcal{X} + \left[8(\ln 2)^2 - \frac{\pi^2}{3}\right]\mathcal{X}^2 - \frac{\pi^2}{3}\mathcal{Y} + \ldots$$

which agrees with the terms that have four powers of the single derivative of T calculated in TTU. This provides a non-trivial check on the above improved action. Note that the action is invariant under $\sqrt{\mathcal{Y}} \to -\sqrt{\mathcal{Y}}$, so in the above Taylor expansion, only integer powers of $\mathcal{Y}$ appears in the action.

For time-dependent tachyon fields, $T \to t/(\sqrt{2\pi}\alpha')$, we have $\mathcal{X} - \sqrt{\mathcal{Y}} \to 0$ and $\mathcal{X} + \sqrt{\mathcal{Y}} = -\dot{T}^2 \to -1$, so $\mathcal{F}(\mathcal{X} + \sqrt{\mathcal{Y}})\,\mathcal{F}(\mathcal{X} - \sqrt{\mathcal{Y}}) \to -1/(1 - \dot{T}^2)$. This justifies the approximation used for the rolling tachyon in Ref. [30].

3. Solitons on Brane Anti-Brane Systems

Here we shall study the solitonic solutions of the improved $D\overline{D}$ effective action (2.13). Since T is complex, the solitonic solutions will be vortices, corresponding to D7-branes with brane tension τ_7. Let $z = x^1 + ix^2$ be the coordinate in the complex plane transverse to the D7-branes. The KL/TTU solution for a single vortex is given by $T = \lim_{u\to\infty} uz$, $A^{\pm} = 0$. We shall discuss solutions for parallel vortices and anti-vortices. The n vortices are located at $\{z_i\}_{i=1,\ldots n}$ while the m anti-vortices are located at $\{z'_j\}_{j=1,\ldots m}$. Since T is uncharged under $A^+ = A^1 + A^2$ only solutions with $A^+ = 0$ are studied. We shall consider an ansatz where the energy density $\varepsilon_7 = (n+m)\tau_7$ while the total RR charge is $\mu_7 = (n-m)\tau_7 g_s$. We find that there are such solutions with and without an A^- "magnetic" flux associated with each winding number. In §3.1, we show that the RR charge is independent of the gauge field, or magnetic flux. It is a function of the winding (minus the anti-winding) number only. In §3.2, we calculate the energy density ε_7 for vortices with and without magnetic flux. The general solution (3.10) can be found at the end of this subsection. To understand better the properties of the solutions, we consider the multi-vortex case more closely in §3.3. For an appropriate choice of magnetic flux, the multi-vortex solution is supersymmetric, though the degeneracy still persists.

3.1 Ramond-Ramond Charge of Multi-Soliton Solutions

We can gain more insight into the form of the solution giving multi-soliton branes by looking at the Chern-Simons action (2.10), which is known exactly. Multi-soliton solutions can be constructed with trivial gauge fields, just as in the single soliton case. In fact, we show that for soliton solutions, the RR charge is completely independent of the gauge field to which the tachyon couples.

Starting with Eq. (2.10), for D7-brane solitons on a single brane anti-brane pair, we consider only nonzero RR field C_8 , and set to zero the gauge field under which T is inert, $A^+ = 0$, $F^+ = 0$:

$$S_{\rm CS} = \tau_9 g_s \int e^{-2\pi\alpha' T\overline{T}} (-iC_8) \wedge \left[2\pi\alpha' iF^- - (2\pi\alpha')^2 DT \wedge D\overline{T}\right]. \qquad (3.1)$$

The coupling to the field strength, F^-, is the standard one, giving the unstable 9-brane system coupling to 7-branes. The second term gives the soliton coupling, and the system can decay to solitons with trivial gauge fields. For brevity, we can extract the RR charge μ_7 of the soliton under a C_8 which is constant in the plane in which T condenses

$$\mu_7 = -i\frac{\tau_7 g_s}{2\pi} \int_{\mathbb{R}^2} e^{-2\pi\alpha' T\overline{T}} \left[iF^- - (2\pi\alpha') DT \wedge D\overline{T}\right], \qquad \tau_7 = 4\pi^2\alpha'\tau_9. \qquad (3.2)$$

The single D7-brane solution [6, 7], can be written in polar coordinates on $\mathbb{R}^2$ as $A^{\pm} = 0$, $F^{1,2} = 0$, $T = uz = ure^{i\theta}$:

$$\begin{aligned}\mu_7 &= i\frac{\tau_7 g_s}{2\pi} \int e^{-2\pi\alpha' u^2 r^2} (2\pi\alpha') u^2 (-2ir) dr \wedge d\theta \\ &= \tau_7 g_s.\end{aligned}$$

Unlike in the kinetic term, here u can take any real value without altering the RR charge of the soliton.

We can construct multi-centered soliton solutions, and in general μ_7 is independent of the gauge field winding about them. To prove this, we require only that:

- $T = 0$ at the center of each soliton, and the tachyon fields winds about each of these centers.
- $T \to \infty$ far from the solitons, so that there the tachyon potential and hence the D9-brane anti-brane energy density vanishes. Away from the solitons, the D9-brane and anti-brane have annihilated and the ground state is indistinguishable from the closed string vacuum.
- A^- can wind only about the soliton centers, in analogy to vortices in the Abelian Higgs model.

We note first that the terms in (3.2) can be rewritten using

$$d\left(e^{-2\pi\alpha' T\overline{T}}A^-\right) = e^{-2\pi\alpha' T\overline{T}}\left[F^- + 2\pi\alpha' A^- \wedge (\overline{T}dT + Td\overline{T})\right],$$
$$d\left(\frac{1}{2}e^{-2\pi\alpha' T\overline{T}}\left[\frac{dT}{T} - \frac{d\overline{T}}{\overline{T}}\right]\right) = e^{-2\pi\alpha' T\overline{T}}\left(2\pi\alpha' dT \wedge d\overline{T} + \frac{1}{2}d\left[\frac{dT}{T} - \frac{d\overline{T}}{\overline{T}}\right]\right).$$

The final term is naïvely zero, but receives contributions from the poles in $dT/T - d\overline{T}/\overline{T}$ which result from the zeros or singularities of T; if T is just a polynomial in z and $\overline{z}$, then this term is just $2\pi\delta^{(2)}(T,\overline{T})$, and each zero (soliton) contributes equally to this δ-function. The integrand of (3.2) is then

$$\mu_7 = i\frac{\tau_7 g_s}{2\pi}\int\limits_{\mathbb{C}}\left\{d\left(\frac{1}{2}e^{-2\pi\alpha' T\overline{T}}\left[\frac{DT}{T} - \frac{D\overline{T}}{\overline{T}}\right]\right) - \frac{1}{2}e^{-2\pi\alpha' T\overline{T}}d\left[\frac{dT}{T} - \frac{d\overline{T}}{\overline{T}}\right]\right\},$$
$$= -i\frac{\tau_7 g_s}{2\pi}\int\limits_{\mathbb{C}}\frac{1}{2}e^{-2\pi\alpha' T\overline{T}}d\left[\frac{dT}{T} - \frac{d\overline{T}}{\overline{T}}\right]. \tag{3.3}$$

The first term in the first line is the total derivative of a one-form which vanishes at the boundary at infinity, hence its integral vanishes. The remaining term (3.3) is independent of the gauge field and essentially counts the zeros and poles of T. Although this expression might not appear to be gauge invariant, the number of zeros and singularities of T is a manifestly gauge invariant quantity. Hence μ_7 is not only gauge invariant, but completely independent of A^- and its curvature. As an example, if we construct a solution with n holomorphic and m anti-holomorphic zeros

$$T = u\prod_{i=1}^{n}(z - z_i)\prod_{j=1}^{m}(\overline{z} - \overline{z}'_j),$$

then the total RR D7-brane charge is

$$\mu_7 = \tau_7 g_s\int\limits_{\mathbb{C}} e^{-2\pi\alpha' T\overline{T}}\delta^{(2)}(T,\overline{T})dT \wedge d\overline{T} = (n - m)\tau_7 g_s. \tag{3.4}$$

Physically, every soliton contributes one topological unit to the total RR charge of the solution as we expect. More complicated solutions include tachyon fields which are multiply wound about their zeros,

$$T = u\prod_{i=1}^{N}\left(\frac{z-z_i}{\overline{z}-\overline{z}_i}\right)^{w_i/2}(z-z_i)^{l_i/2}(\overline{z}-\overline{z}_i)^{l_i/2}, \qquad (w_i\in\mathbb{Z},\ l_i\in\mathbb{R}^+)$$

$$\mu_7 = -i\frac{\tau_7 g_s}{2\pi}\int_{\mathbb{C}} e^{-2\pi\alpha' T\overline{T}}\frac{1}{2}d\left[\sum_{i=1}^{N} w_i\left(\frac{dz}{z-z_i}-\frac{d\overline{z}}{\overline{z}-\overline{z}_i}\right)\right] = \tau_7 g_s\sum_{i=1}^{N} w_i. \qquad (3.5)$$

As we shall see, for this solution to be BPS it must be accompanied by a gauge field which winds about each $\{z_i\}$; one can explicitly check that the solution for A^- does not provide any contribution to μ_7.

These calculations reveal that (3.3) behaves in an intuitive manner; holomorphic or "positively wound" zeros of T correspond to D7-branes, and contribute one topological unit to μ_7, whereas anti-holomorphic or "negatively wound" zeros of T represent anti-D7-branes, and contribute oppositely to the RR charge, the sign arising from the antisymmetry of the volume element. As for the single soliton case, it is not necessary to take $u\to\infty$ to get the exact answer; this is not so when we consider the $\mathrm{D}\overline{\mathrm{D}}$ action.

3.2 Multi-Soliton Tensions

We now turn to the tension or energy density of the solitons, beginning with (2.13) and setting F^+ to zero. To obtain the lowest energy solution it is necessary to take u, the overall multiplying constant in the ansatz for T, to ∞. On the worldsheet, this limit corresponds to the infrared conformal limit, or equivalently to on-shell physics. In the effective theory, the limit allows the tension to be calculated exactly, and since we are searching for solutions representing classical D-branes, which have zero width, the regions in the plane at which we require that $V(T\overline{T})=1$ must be points with all other regions having $V(T\overline{T})=0$. Since $V(T\overline{T})=\exp[-2\pi\alpha' T\overline{T}]$, the potential will be maximal at the zeros of T, and shall vanish elsewhere when $u\to\infty$.

We seek to calculate the tension of various solitons; the energy per 7-volume of the action (2.13) is

$$\begin{aligned}\varepsilon_7 &\equiv \int d^2x\frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{D}\overline{\mathrm{D}}}}{\delta g^{0\nu}}g^{\nu 0}\\ &= \tau_9\int d^2x\; e^{-2\pi\alpha' T\overline{T}}\sqrt{\det(\delta+\pi\alpha' F^-)}\begin{bmatrix}\mathcal{F}(\mathcal{X}_+ +\sqrt{\mathcal{Y}_+})\,\mathcal{F}(\mathcal{X}_+ -\sqrt{\mathcal{Y}_+})\\ +\mathcal{F}(\mathcal{X}_- +\sqrt{\mathcal{Y}_-})\,\mathcal{F}(\mathcal{X}_- -\sqrt{\mathcal{Y}_-})\end{bmatrix}, \end{aligned}\qquad (3.6)$$

when we assume $F^+=0$, all fields are time independent, and work with flat spacetime. $\mathcal{X}_\pm,\mathcal{Y}_\pm$ are the tachyon derivative terms containing the open string metrics $G=g\pm\pi\alpha' F^-$ respectively. When F^- has only a B-field component in the plane of tachyon condensation, (2.9) and some simple manipulations yield

$$\begin{aligned}\lim_{DT\to\infty}\mathcal{F}(\mathcal{X}+\sqrt{\mathcal{Y}})\,\mathcal{F}(\mathcal{X}-\sqrt{\mathcal{Y}}) &= \pi\sqrt{\mathcal{X}^2-\mathcal{Y}},\\ &= 2\pi^2\alpha'^2\sqrt{\left(D_zTD_{\overline{z}}\overline{T}-D_{\overline{z}}TD_z\overline{T}\right)\left((G^{\{z\overline{z}\}})^2D_zTD_{\overline{z}}\overline{T}-(G^{\{\overline{z}z\}})^2D_{\overline{z}}TD_z\overline{T}\right)}.\end{aligned}$$

Until this point, few conditions needed to be placed on the form of the solutions. Now there are two simplifying constraints we can impose; $A^- = 0$ motivated from the fact that it seems to be possible to construct sensible soliton solutions without gauge field winding, in direct contrast to the solitons of standard field theory. Secondly, the condition $D_{\overline{z}}T = 0$ was found in [31] by worldsheet methods to be the condition which must be satisfied if $\mathcal{N} = 2$ worldsheet supersymmetry and hence spacetime supersymmetry is to be preserved; configurations of multiple parallel branes are mutually BPS and must preserve some spacetime supersymmetry. We begin by considering examples that satisfy these conditions and proceed to other cases.

The first example is one satisfying both conditions; assume T is a holomorphic function with n zeros at the points $\{z_j\}$, $T = \lim_{u\to\infty} u \prod_{j=1}^{n}(z - z_j)$. Then T represents n separated D7-branes, although the result is identical when some D7-brane locations coincide. The gauge field is trivial, hence $G^{z\overline{z}} = 2$ and the tension (3.6) becomes (after taking $u \to \infty$)

$$\begin{aligned}\varepsilon_7 &= 2\tau_9 \int e^{-2\pi\alpha' T\overline{T}} 4\pi^2\alpha'^2 \; |\partial_z T \partial_{\overline{z}} \overline{T}| \; \frac{i}{2} dz \wedge d\overline{z}, \\ &= i\tau_9 \int e^{-2\pi\alpha' T\overline{T}} (2\pi\alpha')^2 dT \wedge d\overline{T}.\end{aligned}$$

Since $\partial_z T \partial_{\overline{z}} \overline{T}$ is always positive, the absolute value could be ignored. This is identical (up to the factor of g_s) to the D7-brane charge under the RR 8-form field (3.1); it was shown above that this is always equal to $n\tau_7 g_s$, hence the multi-solitons have the correct tension ($\varepsilon_7 = n\tau_7$) and exhibit BPS properties. Further, we can formulate solutions with vortices moving at constant velocities, $z_j = z_{j,0} + v_j t$. Of course, the second line of (3.6) is no longer valid when the solution has time dependence, so the first line must be used to give the general form when T is time dependent. The velocity dependence leads to the special relativistic γ-factors in the energy density of the resultant solution,

$$\varepsilon_7 = \tau_7 \sum_{j=1}^{n} \frac{1}{\sqrt{1 - |v_j|^2}}. \tag{3.7}$$

This result relies on the form of the kinetic action; for instance, in the case where $n = 1$, $\mathcal{X} \sim 2 - |v_1|^2$ and $\mathcal{Y} \sim |v_1|^4$, so only the combination $\sqrt{\mathcal{X}^2 - \mathcal{Y}} \sim \sqrt{1 - |v_1|^2}$ together with the variations of $\mathcal{F}(\mathcal{X} \pm \sqrt{\mathcal{Y}})$ with respect to g^{00} give the correct dilation factors above.

Another example is condensation to a D7-brane anti-brane pair which is obviously non-supersymmetric; in this case we still expect tension of $2\tau_7$, whereas (3.4) shows that the total RR charge is zero. Placing the soliton and anti-soliton on the real axis at x_0 and $-x_0$ respectively $T = u(z - x_0)(\overline{z} + x_0)$, after taking $u \to \infty$ we have

$$\varepsilon_7 = 2\tau_9 \int e^{-2\pi\alpha' T\overline{T}} (2\pi\alpha')^2 |\partial_z T \partial_{\overline{z}} \overline{T} - \partial_{\overline{z}} T \partial_z \overline{T}| \; \frac{i}{2} dz \wedge d\overline{z}.$$

For $x_0 = 0$, the brane and anti-brane are coincident, there is no winding of the tachyon field, and the total tension is zero (the tachyon derivative terms cancel). For any $x_0 > 0$, the absolute value gives two regions of integration with opposite signs

$$\varepsilon_7 = i\tau_9 \left(\int\limits_{\Re(z)>0} - \int\limits_{\Re(z)<0} \right) e^{-2\pi\alpha' T\overline{T}} (2\pi\alpha')^2 dT \wedge d\overline{T}.$$

By the arguments of the previous section, the first integral receives only the positive contribution from the zero of T at $z = x_0$, the second only the negative contribution from the zero at $\overline{z} = -x_0$, giving $2\tau_7$ as the total tension. Care must be taken because the arguments of the previous section relied on Stoke's theorem, and here we are introducing a new boundary along the imaginary axis, however since we work in the limit $u \to \infty$ and the boundary integrand is proportional to $\exp[-2\pi\alpha' T\overline{T}]$, the boundary term vanishes, and the result remains valid.

We can use the understanding gained from this experience to calculate the tension of the configuration $T = \lim_{u\to\infty} u \prod_{i=1}^{n}(z - z_i) \prod_{j=1}^{m}(\overline{z} - \overline{z}'_j)$, representing n D7-branes and m anti-branes (all parallel). We assume no brane and anti-brane position coincides $z_i \neq z'_j, \forall\{i, j\}$. The tension is

$$\varepsilon_7 = 2\tau_9 \int e^{-2\pi\alpha' T\overline{T}} (2\pi\alpha')^2 T\overline{T} \left| \sum_{i,k=1}^{n} \frac{1}{z - z_i} \frac{1}{\overline{z} - \overline{z}_k} - \sum_{j,l=1}^{m} \frac{1}{z - z'_j} \frac{1}{\overline{z} - \overline{z}'_l} \right| \frac{i}{2} dz \wedge d\overline{z}.$$

In regions about each z_i or z'_j (D7-brane or D7-anti-brane) the term in absolute values is positive or negative respectively. Denoting these regions by Γ_i and Γ'_j the tension is

$$\varepsilon_7 = i\tau_9 \left(\sum_{i=1}^{n} \int\limits_{\Gamma_i} - \sum_{j=1}^{m} \int\limits_{\Gamma'_j} \right) e^{-2\pi\alpha' T\overline{T}} (2\pi\alpha')^2 dT \wedge d\overline{T}.$$

Each integral precisely resembles (3.1) and so the value of the integral is proportional to $+1$ for each holomorphic zero and -1 for each anti-holomorphic zero of T in the region. The boundary terms again vanish because we take $u \to \infty$, giving the expected result

$$\varepsilon_7 = (n + m)\tau_7, \qquad\qquad \mu_7 = (n - m)\tau_7 g_s,$$

where the result of the RR charge calculated earlier has been included for comparison.

3.3 Vortex Tension with Magnetic Flux

More complicated solutions include gauge field winding about zeros of the tachyon field. For such solutions the RR 8-form charge is given by (3.5), irrespective of the behavior of the gauge field. The tachyon fields wind more than once around each zero when T is not entirely holomorphic. This represents multiple D7-branes which preserve $\mathcal{N} = 1$ spacetime supersymmetries; the necessary conditions to obtain such BPS configurations are $D_{\overline{z}} T = 0$ and $F_{zz} = F_{\overline{z}\overline{z}} = 0$ [31]. The later condition is trivial in this system, but imposing the former determines the form of A^- and its curvature

$$T = \lim_{u\to\infty} u \prod_{i=1}^{N} \left(\frac{z-z_i}{\overline{z}-\overline{z}_i}\right)^{w_i/2} (z-z_i)^{l_i/2}(\overline{z}-\overline{z}_i)^{l_i/2}, \qquad (w_i \in \mathbb{Z}^+)$$

$$A^-_{\overline{z}} = -\frac{i}{2}\sum_{i=1}^{N} \frac{a_i}{\overline{z}-\overline{z}_i}, \qquad F^-_{z\overline{z}} = -2\pi \sum_{i=1}^{N} a_i \delta^{(2)}(z-z_i, \overline{z}-\overline{z}_i), \tag{3.8}$$

For this configuration, since

$$D_{\overline{z}} T = \frac{T}{2}\sum_{i=1}^{N} \frac{l_i-(w_i-a_i)}{\overline{z}-\overline{z}_i},$$

we must have $l_i = w_i - a_i, \forall i$ to be a BPS configuration. In order to obtain a solution with just one unit of magnetic flux for each winding number, $w_i = a_i$, l_i must be zero; this can be achieved by taking the limit $l_i \to 0$ in such a way that $\lim_{l_i\to 0, u\to\infty} u l_i \to \infty$.

To calculate the tension of this ansatz, we must apply a regularization of the δ-functions, and the tension is regularization dependent; we choose that which gives the tension to be independent of the gauge field winding, to show that such a solution is possible. Formally this requires that we write the δ-function in F^- as a Gaussian of width ϵ; requiring that $\frac{1}{\epsilon} \sim u^{n>2}$ implies taking $\epsilon \to 0$ before $u \to \infty$, and the tension of the solution will be equal to the RR charge. In this regularization, before taking $u \to \infty$ we split the integral into regions about each zero of T as before, and split each region Γ_i into one about the pole at z_i ($\Gamma_{i,\leq\epsilon}$) and one over the rest of the region ($\Gamma_{i,>\epsilon}$)

$$\varepsilon_7 = \lim_{u\to\infty}\lim_{\epsilon\to 0}\sum_{i=1}^{N}\left(\int\limits_{\Gamma_i \leq \epsilon} + \int\limits_{\Gamma_i > \epsilon}\right)\mathcal{L}.$$

The first integral can be evaluated using

$$\begin{aligned} \sqrt{\det(\delta+\pi\alpha' F^-)} &= \sqrt{1+\frac{1}{2}(\pi\alpha' F^-)^2} &&\longrightarrow && 4\pi^2\alpha' \,|a_i|\, \delta^{(2)}(z-z_i,\overline{z}-\overline{z}_i), \\ (G^{-1})^{\mu\nu} &= \frac{g^{\mu\nu}-\pi\alpha' F^{-\mu\nu}}{1+\frac{1}{2}(\pi\alpha' F^-)^2} &&\longrightarrow && 0, \end{aligned}$$

about $z = z_i$, which gives for the contribution to the tension from $\Gamma_{i,\leq\epsilon}$

$$2\tau_9 \int\limits_{\Gamma_{i,\leq\epsilon}} 4\pi^2\alpha' \,|a_i|\, \delta^{(2)}(z-z_i,\overline{z}-\overline{z}_i)\frac{i}{2} dz\wedge d\overline{z} = |a_i|\,\tau_7.$$

The remaining integral is over that part of Γ_i further than ϵ from z_i. Naïvely this should be zero because we have argued that only the zeros of T contribute to the soliton tension and charge, but the specific regularization of the solution using $\frac{1}{\epsilon} \sim u^{n>2}$ gives us $\tau_7|w_i - a_i|$, it being necessary to take $u \to \infty$ after taking $\epsilon \to 0$. The total energy density of the soliton is the sum over the integrals in both regions for all Γ_i and is

$$\varepsilon_7 = \tau_7 \sum_{i=1}^{N} \left(|w_i - a_i| + |a_i|\right). \tag{3.9}$$

Since w_i are positive (allowing them to take negative values would change some branes to anti-branes), when all $w_i \geq a_i \geq 0$ the solution has minimal energy and the tension is equivalent to the RR 8-form charge (3.5), $\varepsilon_7 = \tau_7 \sum_{i=1}^N w_i = \mu_7/g_s$. Therefore we appear to have multiple solutions representing certain brane systems, with different degrees of gauge field winding but with identical energy and RR charge. At this level in the effective theory, it is a curious degeneracy of the soliton solutions, which is likely lifted by higher derivative and/or quantum corrections to the effective action.

To summarize, we give the general ansatz for a tachyon field representing a set of parallel n D7-branes and m anti-branes. The energy per 7-volume of this solution is $\varepsilon_7 = (n+m)\tau_7$ and its RR charge under the spacetime 8-form potential is $\mu_7 = (n-m)\tau_7 g_s$.

$$\begin{aligned} T &= \lim_{u\to\infty} u \prod_{i=1}^N \left(\frac{z-z_i}{\overline{z}-\overline{z}_i}\right)^{w_i/2} |z-z_i|^{l_i} \prod_{j=1}^M \left(\frac{\overline{z}-\overline{z}'_j}{z-z'_j}\right)^{w'_j/2} |z-z'_j|^{l'_j}, \\ A_{\overline{z}} &= -\frac{i}{2}\sum_{i=1}^N \frac{a_i}{\overline{z}-\overline{z}_i} - \frac{i}{2}\sum_{j=1}^M \frac{a'_j}{\overline{z}-\overline{z}'_j}, \end{aligned} \tag{3.10}$$

where z_i (z'_j) are the constant positions of the (anti-)branes. Single valuedness of T requires that $\{w_i, w'_j\}$ is some set of positive integers, and we have defined $\sum_{i=1}^N w_i = n, \sum_{j=1}^M w'_j = m$. $\{a_i, a'_j\}$ must satisfy $0 \leq a_i \leq w_i$, $0 \leq a'_j \leq w'_j$ in order to obtain the minimal energy solution as in (3.9). If the brane or anti-brane were to move at constant velocities, the tensions would pick up the relativistic γ-factors as in (3.7).

3.4 Discussion

The richness of vortex solutions examined in this section is vindication of the gauge covariant form of the tachyon kinetic terms used; the tensions of all solitons are as expected, the results can be calculated in any coordinate system, and by persuading the solitons to move at constant velocities the necessary special relativistic factors arise. All this evidence depends crucially on the $\mathcal{X} \pm \sqrt{\mathcal{Y}}$ structure of the action.

The usual topological arguments suggest the vortices of the action (2.13) are stable and we verify this by perturbing a characteristic solution representing n coincident D7-branes,

$$T = \lim_{u\to\infty} u \left(\frac{z}{\overline{z}}\right)^{n/2} (z\overline{z})^{l/2} + t(z,\overline{z}), \qquad\qquad A_{\overline{z}} = -\frac{ia}{2\overline{z}}.$$

The first order perturbations vanish for all values of $[l, n, a]$, so these are solutions of the equations of motion. When the condition for $\mathcal{N}=2$ worldsheet supersymmetry is satisfied,

$$D_{\overline{z}}T = 0, \qquad\qquad \text{or} \qquad\qquad l = n - a, \tag{3.11}$$

the second order perturbations are

$$\begin{aligned} \frac{\delta^2 S}{\delta T^2}t^2 + \frac{\delta^2 S}{\delta \overline{T}^2}\overline{t}^2 + 2\frac{\delta^2 S}{\delta T \delta \overline{T}} t\overline{t} &\propto -l e^{-2\pi\alpha' u^2 |z|^{2l}} \left(\frac{1}{2}\eta^{ij}\partial_i t\, \partial_j \overline{t} + a\delta^{(2)}(z,\overline{z}) t\overline{t}\right) \\ &\quad \underbrace{- l e^{-2\pi\alpha' u^2 |z|^{2l}} \Big[\overbrace{\left(\partial_z t \partial_{\overline{z}} \overline{t} + \partial_{\overline{z}} t \partial_z \overline{t}\right)}^{\text{from } \mathcal{X}^2} - \overbrace{2\partial_{\overline{z}} t \partial_z \overline{t}}^{\text{from } \mathcal{Y}} + \ldots \Big]}_{\text{total derivative}} \end{aligned} \tag{3.12}$$

The terms other than the kinetic terms in the off-brane directions conspire to form a total derivative (again because of the form of the tachyon kinetic terms, $\mathcal{X} \pm \sqrt{\mathcal{Y}}$) leaving just the modes in the directions along the D7-branes. When $a = 0$, they represent the two massless fluctuations of the set of branes in the two transverse dimensions. When $a \neq 0$ the gauge field must be perturbed similarly otherwise the fluctuations are massive; checks reveal that the gauge field perturbations are likewise stable.

Let us recall Derrick's theorem (see [32] for instance); in a field theory of a set of scalar fields, suppose there is a time-independent solitonic solution $T(x)$ with codimension d_c. Consider the one-parameter family of field configurations defined by $T(x;\lambda) \equiv T(\lambda x)$ where λ is positive and real. In general the energy is $E(\lambda) = \lambda^{-d_c}P + \lambda^{-d_c+2}K_2 + ...$, where P is the potential energy contribution (defined so $P \geq 0$) and K_2 is the two derivative kinetic energy term, and extra terms may be present if there are more than two derivative terms in the theory. By Hamilton's principle, this must be stable at $\lambda = 1$, that is, ignoring possible extra terms, $(d_c - 2)K_2 + d_c P = 0$. Since both P and K_2 are positive in an ordinary field theory, only codimenion $d_c = 1$ soliton is possible. Vortices (codimension two) in Abelian Higgs model are possible due to the presence of magnetic flux. Now we look at vortices in the D$\overline{\text{D}}$ system. The energy of a time-independent soliton $T(x)$ with codimension d_c, in the limit $DT \to \infty$ is $E = 2\pi\tau_9 \int d^{(d_c)}x V(T\overline{T})\sqrt{\mathcal{X}^2 - \mathcal{Y}}$. That is $P = 0$ which implies E scales like a two derivative term and

$$(d_c - 2)E = 0$$

Therefore only codimension two solitons are possible. In contrast to the Abelian Higgs model, a magnetic flux is not necessary for the existence of the vortices in the D$\overline{\text{D}}$ system, as we have seen.

Returning to the issue of the apparent degeneracy between vortices with and without A^- flux, we repeat that this degeneracy is expected to be lifted by corrections to the effective action. The effective action has at least two sources of corrections:

i. Classically there are higher derivative terms.
ii. Since the D$\overline{\text{D}}$ system is non-supersymmetric, there will be quantum corrections.

Such corrections should lift the degeneracy. For any solution that involves both vortices and anti-vortices, supersymmetry is clearly broken; a more careful calculation would reveal tachyonic modes in such systems. Considering only vortices which satisfy the BPS condition $l = n - a$, the solution with no gauge field $(a = 0, l = n)$ can only receive corrections from multiple (anti-)holomorphic derivatives of T $(\overline{T})$. Since $T \sim z^n$, only the first n holomorphic derivatives of T are non-zero. Importantly the degeneracy is already present in the single vortex solution : the $n = 1$ solution may have flux $a = 0$ or up to $a = 1$, with the same energy and RR charge. Note that the $n = 1$ solution without flux [6,7] is classically exact: there are no gauge field derivative corrections and all second and higher derivatives of the tachyon field vanish. If the degeneracy is lifted, one may naïvely conclude that this zero-flux solution will be the stable one. If this is the exact BPS solution, then putting n of them together should be an exact solution too. However, higher derivative terms are non-zero for the n-vortex solution ($n > 1$, still with $a = 0$). Barring a miraculous cancellation, the n-vortex solution will have classical corrections from the higher derivative terms.

Another solution to consider is that with $n = a$, or $l = 0$ from the BPS condition (3.11). Then noting that $D_z T = \frac{(l+(n-a))}{2z} T = \frac{l}{z} T$, for $l = 0$ both $D_{\bar{z}} T = D_z T = 0$, naïvely implying that all higher derivative terms vanish and $n = a$ is an exact solution. Recall, however, that the solution with a wound gauge field requires some care in taking the limit, and we can only take $l \to 0$ with $u \to \infty$ and $lu \to \infty$, implying $\lim_{l \to 0} D_z T \neq 0$. Therefore corrections due to higher derivative terms cannot be ignored in the case where there is a flux associated with each winding, *i.e.* $n = a$. In conclusion, we do not know how the degeneracy will eventually be lifted.

4. Lower Dimensional Brane Anti-Brane Systems

Lower dimensional brane anti-brane pairs can be constructed in a straightforward manner by applying T-duality to the action (2.13). In these systems, the brane and anti-brane can be separated because under T-duality components of A^- transform into the relative separation of the pair. We follow closely the procedure of [33]. Because under T-duality, both the dilaton transforms and there is mixing between the metric and the Kalb-Ramond B-field, it is necessary to include these in our action (2.13). The T-duality properties of the various fields in the action are well known; the gauge fields in the T-dual directions transform into the adjoint scalars, the metric and Kalb-Ramond field mix, the string coupling scales. Being an open string scalar state, the tachyon is inert under T-duality. Under T-duality in directions labeled by uppercase Latin indices, (lowercase Latin indices labeling unaffected directions on the brane), the fields transform as [33]

$$\begin{array}{lll} T \to T, & A_a \to A_a, & A_I \to \dfrac{\Phi^I}{2\pi\alpha'}, \\ E_{\mu\nu} \equiv g_{\mu\nu} + B_{\mu\nu}, & e^{2\phi} \to e^{2\phi} \det E^{IJ}, & E_{IJ} \to E^{IJ} \\ E_{ab} \to E_{ab} - E_{aI} E^{IJ} E_{Jb}, & E_{aI} \to E_{aK} E^{KI}, & E_{Jb} \to -E^{JK} E_{Kb}, \end{array}$$

where E^{IJ} is the matrix inverse to E_{IJ}. The result of T-dualing $9-p$ dimensions can be written most simply by defining the pull-back in normal coordinates as:

$$P[E_{ab}]^{1,2} \equiv E_{ab} + E_{I\{a}\partial_{b\}}\Phi^{I\,1,2} + E_{IJ}\left(\partial_a \Phi^I \partial_b \Phi^J\right)^{1,2}, \quad P[E_{aI}]^{1,2} \equiv E_{aI} + E_{JI}\partial_a \Phi^{J\,1,2}.$$

Care must be taken because there are two sets of scalars which describe the position of each brane; they are denoted herein by $\Phi^{I\,1,2}$ and their difference as $\varphi^I \equiv \Phi^{I\,1} - \Phi^{I\,2}$ which is the scalar representing the $(\mathrm{D}\overline{\mathrm{D}})_p$ separation. In calculating the pull-back of any quantity only the indices corresponding to directions along the brane are affected. After T-dualing the fields in (2.13) as above and performing manipulations similar to those in [33], we obtain the improved action for a Dp brane anti-brane pair:

$$S_{(\mathrm{D}\overline{\mathrm{D}})_p} = -\tau_p \int d^{p+1}x\; e^{-2\pi\alpha' T\overline{T}} \Big[\sqrt{-\det[G_1]}\, \mathcal{F}(\mathcal{X}_1 + \sqrt{\mathcal{Y}_1})\, \mathcal{F}(\mathcal{X}_1 - \sqrt{\mathcal{Y}_1}) \\ + \sqrt{-\det[G_2]}\, \mathcal{F}(\mathcal{X}_2 + \sqrt{\mathcal{Y}_2})\, \mathcal{F}(\mathcal{X}_2 - \sqrt{\mathcal{Y}_2}) \Big] \qquad (4.1)$$

where now the effective metric contains the spacetime metric pulled-back to the brane worldvolume (and includes any non-zero NS-NS B field)

$$G^{1,2}_{ab} \equiv P[E_{ab}]^{1,2} + 2\pi\alpha' F^{1,2}_{ab},$$

and the covariant derivative dependence of $\mathcal{X}$ and $\mathcal{Y}$ in (2.13) leads to Φ dependence in the T-dual action. The complete expressions for $\mathcal{X}$ and $\mathcal{Y}$ are

$$\mathcal{X}_{1,2} = 2\pi\alpha'^2 \left[\begin{array}{c} G^{\{ab\}}_{1,2} D_a T D_b \overline{T} + \frac{1}{(2\pi\alpha')^2}\varphi^I\varphi^J T\overline{T}\left(E_{\{IJ\}} - G^{ab}_{1,2} P[E_{\{Ia}E_{bJ\}}]^{1,2}\right) \\ + \frac{i}{2\pi\alpha'}\left(G^{ab}_{1,2} P[E_{bI}]^{1,2} - G^{ba}_{1,2} P[E_{Ib}]^{1,2}\right)\left(T D_a \overline{T} - \overline{T} D_a T\right)\varphi^I \end{array} \right]$$

$$\mathcal{Y}_{1,2} = \left(2\pi\alpha'^2\right)^2 \left| \begin{array}{r} G^{ab}_{1,2} D_a T D_b T + \frac{i}{2\pi\alpha'} G^{ab}_{1,2}\left(P[E_{bI}]^{1,2} D_a T - P[E_{Ia}]^{1,2} D_b T\right) T\varphi^I \\ - \frac{1}{(2\pi\alpha')^2}\varphi^I\varphi^J T^2 \left(E_{IJ} - G^{ab}_{1,2} P[E_{Ia}E_{bJ}]^{1,2}\right) \end{array} \right|^2 .$$

These expressions simplify considerably in Minkowski spacetime when $B = 0$ and $A^{1,2} = 0$:

$$\begin{aligned} G^{1,2}_{ab} &= \eta_{ab} + \delta_{IJ}(\partial_a\Phi^I\partial_b\Phi^J)^{1,2}, \\ \mathcal{X}_{1,2} &\xrightarrow[B=0,\ A^{1,2}=0]{g=\eta} 2\pi\alpha'^2 \left[G^{ab}_{1,2}\partial_a T\partial_b\overline{T} + \frac{1}{(2\pi\alpha')^2}\varphi^I\varphi^J T\overline{T}\left(\delta_{IJ} - G^{ab}_{1,2}\partial_a\Phi^{1,2}_I\partial_b\Phi^{1,2}_J\right)\right], \\ \mathcal{Y}_{1,2} &\xrightarrow[B=0,\ A^{1,2}=0]{g=\eta} \left(2\pi\alpha'^2\right)^2 \left| G^{ab}_{1,2}\partial_a T\partial_b T - \frac{1}{(2\pi\alpha')^2}\varphi^I\varphi^J T^2\left(\delta_{IJ} - G^{ab}_{1,2}\partial_a\Phi^{1,2}_I\partial_b\Phi^{1,2}_J\right)\right|^2 . \end{aligned}$$

It is clear that the action contains relative velocity dependent terms. It would be interesting to study the implications of the velocity dependence of this action to density perturbations in brane inflationary models.

Some important properties of the separation dependent tachyon potential can be verified. The potential is equal to that part of the Lagrangian which is independent of gauge fields and derivatives,

$$V(T,\varphi) = 2\tau_p e^{-2\pi\alpha' T\overline{T}}\,\mathcal{F}\left(\frac{1}{\pi}|\varphi|^2 T\overline{T}\right),$$
$$|\varphi|^2 \equiv E_{IJ}\varphi^I\varphi^J,$$

which gives as the position dependent mass of the tachyon

$$m_T^2 = \frac{1}{2\alpha'}\left(\frac{|\varphi|^2}{2\pi^2\alpha'} - \frac{1}{2\ln 2}\right).$$

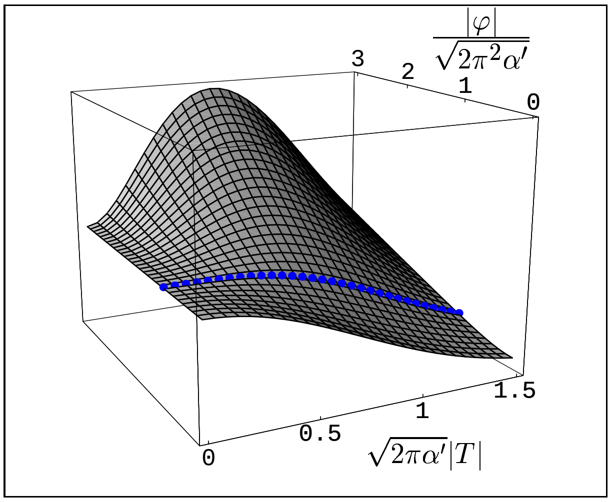

Figure 1: Separation dependent tachyon potential. The blue dotted line marks the critical separation.

Apart from the discrepancy by $2\ln 2$ which appears in BSFT calculations of the tachyon mass, this is consistent with the familiar result that as a parallel p-brane and p-anti-brane are moved toward each other, the lowest open string scalar mode becomes tachyonic at separations $|\varphi|^2 < 2\pi^2\alpha'$ [20]. We see that the separated $(\mathrm{D}\overline{\mathrm{D}})_p$ system, although classically stable, is quantum mechanically unstable for $|\varphi|^2 > \frac{2\pi^2\alpha'}{2\ln 2}$, with a tunneling barrier which increases with their separation, as in Figure 1. The system remains unstable at the critical separation $|\varphi_c|^2 \equiv \frac{2\pi^2\alpha'}{2\ln 2}$ (the dotted blue line in Figure 1), since the $|T|^4$ term in the potential has negative coefficient there. The probability of tunnelling can be calculated by instanton methods to behave like

$$\mathcal{P} \sim e^{-S_E(\varphi)}, \qquad S_E(\varphi) \simeq \frac{c(p)}{\alpha' g_s}(|\varphi|^2 - |\varphi_c|^2), \qquad |\varphi| > |\varphi_c|,$$

$S_E(\varphi)$ being the Euclidean action of the instanton, and $c(p)$ is some constant of order 1 which depends on p.

Classically, for large enough separation, when $m_T^2 > 0$, the ground state is $T = 0$, and $V(0, \varphi) = 1$. This implies that there is no force in the $(\mathrm{D}\overline{\mathrm{D}})_p$ system. However, since the system is non-supersymmetric, quantum corrections are clearly present. It is known that the one-loop open string contribution is dual to the closed string exchange. For large separation, this is dominated by the exchanges of the graviton, dilaton and RR field C_{p+1} between the Dp and the anti Dp-brane and has been calculated. These one-loop open string corrections can be included by inserting the classical closed string background produced by a Dp and a $\overline{\mathrm{D}}p$ brane into the $(\mathrm{D}\overline{\mathrm{D}})_p$ action. The supergravity solution is well known [34].

$$\begin{aligned}
ds^2 &= h(r)^{\frac{1}{2}}\left(-dt^2 + \sum_{i=1}^{p} dx^i dx^i\right) + h(r)^{-\frac{3}{2}-\frac{5-p}{7-p}}dr^2 + r^2 h(r)^{\frac{1}{2}-\frac{5-p}{7-p}}d\Omega_{8-p}^2,\\
e^{-2\phi} &= g_s^{-2}h(r)^{-\frac{p-3}{2}}, \qquad h(r) = 1 - \frac{g_s\beta}{r^{7-p}}\\
(C_{p+1})_{a_1\ldots a_{p+1}} &= \frac{\beta}{r^{7-p}}\epsilon_{a_1\ldots a_{p+1}}, \qquad \beta \equiv (4\pi)^{\frac{5-p}{2}}\alpha'^{\frac{7-p}{2}}\Gamma\left(\frac{7-p}{2}\right).
\end{aligned}$$

This classical closed string background of a brane shall be "felt" by the anti-brane, so we insert this into that part of the action corresponding to an anti-brane and the similar background into the brane action. On the brane worldvolumes the separation, r, is represented by the scalar field $|\varphi|$. The result of performing these steps is that when $|\varphi|^2 \gg \alpha'$, the total tension of the system is renormalized:

$$\begin{aligned}
S &= S_{\mathrm{D}\overline{\mathrm{D}}}(\varphi) + S_{\mathrm{CS}}(\varphi),\\
\tau_p \to \tau_p(\varphi) &= \tau_p\left(\overbrace{1 - \frac{g_s\beta}{|\varphi|^{7-p}}}^{\text{NS-NS}} \underbrace{- \frac{g_s\beta}{|\varphi|^{7-p}}}_{\text{RR}}\right).
\end{aligned}$$

Clearly for a brane-brane system, the sign of the RR contribution is reversed and the tension is unrenormalized. The renormalized tension then gives a potential for the scalar representing the separation, and we see there is an attractive force between the brane and anti-brane. When the brane separation decreases, massive closed string modes start to contribute to $\tau_p(\varphi)$. Their contributions are easy to include, except when the brane separation becomes so small that m_T^2 becomes negative. When the tachyon appears, $\tau_p(\varphi)$ becomes complex. Fortunately, $\tau_p(\varphi)$ is expected to remain finite [15] and tachyon rolling happens so fast that the precise form of $\tau_p(\varphi)$ at short distance becomes phenomenologically unimportant [30].

5. Conclusion

In this paper, we present a fully covariant D$\overline{\rm D}$ action (Eq.(2.13)) based on boundary superstring field theory. The kinetic term has some rather novel features. Its form is almost completely dictated by BSFT and symmetry properties of the system. It is quite amazing that exact multi-vortex multi-anti-vortex solutions (with arbitrary positions and arbitrary constant velocities) can be easily written down (Eq.(3.10)). The simplicity of these analytic solutions may be very useful in the study of the production of vortices. In the early universe in brane world scenarios, the production of such vortices corresponds to the production of cosmic strings towards the end of the inflationary epoch.

The solitonic solution has a large peculiar degeneracy: the energy and the RR charge of the solutions depend only on the vorticities and not on the "magnetic" flux that may or may not be present inside the vortices. Further improvement on the D$\overline{\rm D}$ action should lift this degeneracy. However we are unable to answer the question that we endeavor to address: as a soliton in the exact theory, whether the Dp-brane has a "magnetic" flux in its core.

The inclusion of the coupling of the gauge field to the tachyon in the $({\rm D}\overline{\rm D})_p$ action allows us, via T-duality, to consider the situation when the brane and the anti-brane are separated. This action is suitable for the study of the inflationary scenario in the brane world. During the inflationary epoch in early universe, the branes move slowly towards each other, since the probability of their annihilation through tunneling is exponentially small. Toward the end of inflation, the annihilation process described by tachyon condensation should be accompanied by the reheating of the universe, the defect production and tachyon matter production. The $({\rm D}\overline{\rm D})_p$ action and its generalization should provide a firm framework to study these phenomena.

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