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Flavor-Singlet B-Decay Amplitudes in QCD Factorization

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Abstract

Exclusive hadronic B-meson decays into two-body final states consisting of a light pseudoscalar or vector meson along with an η or η' meson are of great phenomenological interest. Their theoretical analysis involves decay mechanisms that are unique to flavor-singlet states, such as their coupling to gluons or their "intrinsic charm" content. These issues are studied systematically in the context of QCD factorization and the heavy-quark expansion. Theory can account for the experimental data on the $B \to K^{(*)}\eta^{(\prime)}$ branching fractions, albeit within large uncertainties.

1 Introduction

The unexpectedly large branching fraction of order $6 \cdot 10^{-4}$ for the inclusive production of high-momentum η' mesons in B decays first reported by the CLEO Collaboration in 1997 [1] has triggered some theoretical activity aiming at explaining the dynamical origin of this enhancement. The large mass of the η' meson as compared to other pseudoscalar mesons is one of the clearest manifestations of the axial anomaly in QCD. It is therefore natural to ask how the anomaly affects the production process of η' mesons [2, 3, 4]. Other possible enhancement mechanisms (not mentioning those that invoke new fundamental interactions) include a large $c\bar{c}$ content of the η' [5, 6]. Given the theoretical uncertainties involved in the calculation of semi-inclusive branching fractions, the dynamical details remain unclear except that there appears to be no fundamental problem to account for the observed branching fraction.

Recent data on two-body final states containing η' (summarized in Table 1) show that the branching fraction for $B^- \to K^-\eta'$ decay is about six times larger than that for the corresponding $B^- \to K^-\pi^0$ decay, confirming the semi-inclusive enhancement. The comparison with other two-body final states allows us to extract a more detailed pattern. For instance, if η' is replaced by η the corresponding branching fraction is suppressed rather than enhanced compared to the π^0 mode. Furthermore, the enhancement and suppression appears to be absent or even reversed when the pseudoscalar kaon is replaced by the corresponding vector meson, and with the current limited data no particularly striking pattern is visible when the kaons are replaced by pions or ρ mesons.

The theoretical description of exclusive two-body decays has also improved, since the concept of factorization is now better understood [10]. The QCD factorization approach, in particular, has successfully explained the magnitude of tree and penguin amplitudes of final states with pions and kaons [11], while the calculation of strong interaction phases remains to be validated by experiments. It is therefore clearly interesting to see whether this approach can explain the pattern of branching fractions containing $\eta^{(\prime)}$ mesons as described above. The new element that has to be understood for this purpose is a flavor-singlet amplitude, defined as the amplitude for producing a quark–antiquark pair not containing the spectator quark in the coherent flavor state $(u\bar{u} + d\bar{d} + s\bar{s})$ or a pair of gluons, where the quarks or gluons have small relative transverse momentum and hadronize into an $\eta^{(\prime)}$ meson.

The analysis of this singlet amplitude, which has not been considered systematically in the QCD factorization approach so far, is the main purpose of this paper. We will indeed see below that the standard QCD factorization formula does not hold, but that a suitable modification allows us to obtain the flavor-singlet amplitude at leading order in an expansion in $1/m_b$ at the price of introducing one new non-perturbative parameter. We shall find that the QCD factorization approach can qualitatively account for the pattern of exclusive branching fractions described above, including the large rate for the $K\eta'$ final state, but the theoretical uncertainties are rather large. In particular, we find that it is the constructive or destructive interference of non-singlet penguin amplitudes rather than an enhanced singlet penguin amplitude which is responsible for

Table 1: CP-averaged experimental branching ratios (in units of 10^{-6}) on charmless B decays into two-body final states containing η, η' or π^0. Upper limits are at 90% confidence level.

Mode	CLEO [7]	BaBar [8]	Belle [9]
$B^- \to K^-\eta'$	$80\,^{+10}_{-\ 9} \pm 7$	$67 \pm 5 \pm 5$	$77.9\,^{+6.2}_{-5.9}\,^{+9.3}_{-8.7}$
$\bar{B}^0 \to \bar{K}^0\eta'$	$89\,^{+18}_{-16} \pm 9$	$46 \pm 6 \pm 4$	$68.0\,^{+10.4}_{-\ 9.6}\,^{+8.8}_{-8.2}$
$B^- \to K^-\eta$	< 6.9		$5.2\,^{+1.7}_{-1.5}$
$\bar{B}^0 \to \bar{K}^0\eta$	< 9.3		
$B^- \to K^-\pi^0$	$11.6\,^{+3.0}_{-2.7}\,^{+1.4}_{-1.3}$	$12.8 \pm 1.2 \pm 1.0$	$13.0\,^{+2.5}_{-2.4} \pm 1.3$
$\bar{B}^0 \to \bar{K}^0\pi^0$	$14.6\,^{+5.9}_{-5.1}\,^{+2.4}_{-3.3}$	$10.4 \pm 1.5 \pm 0.8$	$8.0\,^{+3.3}_{-3.1} \pm 1.6$
$B^- \to K^{*-}\eta'$	< 35		
$\bar{B}^0 \to \bar{K}^{*0}\eta'$	< 24	< 13	
$B^- \to K^{*-}\eta$	$26.4\,^{+9.6}_{-8.2} \pm 3.3$	< 33.9	$26.5\,^{+7.8}_{-7.0} \pm 3.0$
$\bar{B}^0 \to \bar{K}^{*0}\eta$	$13.8\,^{+5.5}_{-4.6} \pm 1.6$	$19.8\,^{+6.5}_{-5.6} \pm 1.7$	$16.5\,^{+4.6}_{-4.2} \pm 1.2$
$B^- \to K^{*-}\pi^0$	< 31		
$\bar{B}^0 \to \bar{K}^{*0}\pi^0$	< 3.6		

the distinctive pattern of strangeness-changing transitions to $\eta^{(\prime)}$ mesons, much along the lines envisaged qualitatively by Lipkin [12]. The improved description of data compared to the naive factorization analysis [13] comes from the radiative enhancement of the non-singlet penguin amplitude, which also underlies the sizeable branching fractions for πK final states. Our findings are in contrast to other recent analyses of the $\eta^{(\prime)}$ modes [14, 15, 16], where the large branching fractions for the $K\eta^{(\prime)}$ final states were attributed due to an enhanced flavor-singlet penguin amplitude. Similarly, the singlet amplitudes we obtain are smaller than those inferred from phenomenological analyses using SU(3) symmetry and certain dynamical assumptions about flavor topologies [17]. For completeness we note that the decays $B \to K\eta^{(\prime)}$ have also been analyzed using the perturbative QCD approach [18], however no singlet-specific mechanisms besides η–η' mixing were investigated in this study.

2 Implementation of η–η' mixing

In the calculation of weak decay amplitudes with an $\eta^{(\prime)}$ meson in the final state, we need several matrix elements of local operators evaluated between the vacuum and $\eta^{(\prime)}$. These are the matrix elements of the flavor-diagonal axial-vector and pseudoscalar current densities,

$$
\begin{aligned}
\langle P(q)|\bar{q}\gamma^\mu\gamma_5 q|0\rangle &= -\frac{i}{\sqrt{2}}\, f_P^q\, q^\mu\,, \qquad & 2m_q\langle P(q)|\bar{q}\gamma_5 q|0\rangle &= -\frac{i}{\sqrt{2}}\, h_P^q\,, \\
\langle P(q)|\bar{s}\gamma^\mu\gamma_5 s|0\rangle &= -if_P^s\, q^\mu\,, & 2m_s\langle P(q)|\bar{s}\gamma_5 s|0\rangle &= -ih_P^s\,,
\end{aligned}
\tag{1}
$$

where $q = u$ or d. We assume exact isospin symmetry and identify $m_q \equiv \frac{1}{2}(m_u + m_d)$. We also need the anomaly matrix elements

$$
\langle P(q)|\frac{\alpha_s}{4\pi}\, G_{\mu\nu}^A\, \widetilde{G}^{A,\mu\nu}|0\rangle = a_P\,,
\tag{2}
$$

where we use the convention

$$
\widetilde{G}^{A,\mu\nu} = -\frac{1}{2}\,\epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^A \qquad (\epsilon^{0123} = -1)
\tag{3}
$$

for the dual field-strength tensor. In all cases $P = \eta$ or η' denotes the physical pseudoscalar meson state. We also need the generalization of the local quark operators to the corresponding light-cone operators, which define the twist-2 and twist-3 light-cone distribution amplitudes. The treatment of mixing for these generalizations will be analogous to the case of the local operators. Finally, we need the current matrix elements $\langle P|\bar{q}\Gamma b|\bar{B}\rangle$, which we decompose into Lorentz-invariant form factors as for any other pseudoscalar meson.

The equations above define ten non-perturbative parameters f_P^i, h_P^i, and a_P, which however are not all independent. Taking the divergence of the flavor-diagonal axial-vector current,

$$
\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = 2im_q\,\bar{q}\gamma_5 q - \frac{\alpha_s}{4\pi}\, G_{\mu\nu}^A\, \widetilde{G}^{A,\mu\nu}
\tag{4}
$$

(and similarly with q replaced by s) yields four relations between the various parameters, which can be summarized as

$$
a_P = \frac{h_P^q - f_P^q\, m_P^2}{\sqrt{2}} = h_P^s - f_P^s\, m_P^2\,.
\tag{5}
$$

Without further assumptions this leaves us with six independent parameters. It is conventional to write each set of two parameters corresponding to $P = \eta, \eta'$, such as $\{f_\eta^s, f_{\eta'}^s\}$, in terms of a parameter (such as f_s) and a mixing angle (such as ϕ_s). A general treatment then implies three such parameters and three independent mixing angles.

In the SU(3) flavor-symmetry limit, where $|\eta\rangle$ is a flavor-octet and $|\eta'\rangle$ a flavor-singlet, it follows that $f_\eta^s = -\sqrt{2}\, f_\eta^q$ and $f_{\eta'}^s = f_{\eta'}^q/\sqrt{2}$, $h_\eta^s = -\sqrt{2}\, h_\eta^q$ and $h_{\eta'}^s = h_{\eta'}^q/\sqrt{2}$, and $a_\eta = 0$. However, it is known empirically that SU(3)-breaking corrections to these relations are large. In the following we shall not rely on SU(3) flavor symmetry but instead introduce another assumption, expected to be accurate at the 10% level, to reduce the number of hadronic parameters. This assumption leads to what will be referred to as the Feldmann–Kroll–Stech (FKS) mixing scheme [19]. In the absence of the axial U(1) anomaly, the flavor states $|\eta_q\rangle = (|u\bar{u}\rangle + |d\bar{d}\rangle)/\sqrt{2}$ and $|\eta_s\rangle = |s\bar{s}\rangle$ mix only through OZI-violating effects known phenomenologically to be small. We therefore assume that the anomaly is the only effect that mixes the two flavor states. (This assumption implies, in particular, that the vector mesons ω and ϕ are pure $(u\bar{u} + d\bar{d})$ and $s\bar{s}$ states, respectively, as is indeed the case to very good approximation.) In a chiral Lagrangian treatment of the pseudoscalar mesons (including the η' meson, which can be done in a combined chiral and $1/N_c$ expansion [20]), the anomaly introduces an effective mass term for the system of $\eta^{(\prime)}$ states that is not diagonal in the flavor basis $\{|\eta_q\rangle, |\eta_s\rangle\}$, and since this is by assumption the only mixing effect, the FKS scheme amounts to a scheme with a single mixing angle in the flavor basis. If the physical states are related to the flavor states by

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \tag{6}$$

then the same mixing angle applies to the decay constants f_P^i and h_P^i with the normalization given by (1). We therefore write

$$\begin{aligned} f_\eta^q &= f_q \cos\phi\,, &\qquad f_\eta^s &= -f_s \sin\phi\,, \\ f_{\eta'}^q &= f_q \sin\phi\,, &\qquad f_{\eta'}^s &= f_s \cos\phi\,, \end{aligned} \tag{7}$$

and an analogous set of equations for the h_P^i. This defines four new parameters $f_{q,s}$ and $h_{q,s}$. Inserting these results into (5) allows us to express all ten non-perturbative parameters in terms of the decay constants f_q, f_s and the mixing angle ϕ. We obtain

$$\begin{aligned} h_q &= f_q\,(m_\eta^2\,\cos^2\phi + m_{\eta'}^2\,\sin^2\phi) - \sqrt{2} f_s\,(m_{\eta'}^2 - m_\eta^2)\,\sin\phi\cos\phi\,, \\ h_s &= f_s\,(m_{\eta'}^2\,\cos^2\phi + m_\eta^2\,\sin^2\phi) - \frac{f_q}{\sqrt{2}}\,(m_{\eta'}^2 - m_\eta^2)\,\sin\phi\cos\phi\,, \end{aligned} \tag{8}$$

and

$$\begin{aligned} a_\eta &= -\frac{1}{\sqrt{2}}\,(f_q\,m_\eta^2 - h_q)\,\cos\phi = -\frac{m_{\eta'}^2 - m_\eta^2}{\sqrt{2}}\,\sin\phi\cos\phi\,(-f_q\,\sin\phi + \sqrt{2} f_s\,\cos\phi)\,, \\ a_{\eta'} &= -\frac{1}{\sqrt{2}}\,(f_q\,m_{\eta'}^2 - h_q)\,\sin\phi = -\frac{m_{\eta'}^2 - m_\eta^2}{\sqrt{2}}\,\sin\phi\cos\phi\,(f_q\,\cos\phi + \sqrt{2} f_s\,\sin\phi)\,. \end{aligned} \tag{9}$$

The three remaining parameters of the FKS scheme have been determined from a fit to experimental data, yielding [19]

$$f_q = (1.07 \pm 0.02)\,f_\pi, \qquad f_s = (1.34 \pm 0.06)\,f_\pi, \qquad \phi = 39.3^\circ \pm 1.0^\circ, \tag{10}$$

where the errors do not include a possible systematic uncertainty from the theoretical assumptions underlying the FKS scheme. The numerical values of the other parameters

Table 2: Decay constants (in MeV), pseudoscalar densities and anomaly matrix elements (in GeV^3) of $\eta^{(\prime)}$ in the FKS mixing scheme.

h_q	0.0015 ± 0.004	f_η^q	108 ± 3	h_η^q	0.001 ± 0.003
h_s	0.087 ± 0.006	f_η^s	-111 ± 6	h_η^s	-0.055 ± 0.003
a_η	-0.022 ± 0.002	$f_{\eta'}^q$	89 ± 3	$h_{\eta'}^q$	0.001 ± 0.002
$a_{\eta'}$	-0.057 ± 0.002	$f_{\eta'}^s$	136 ± 6	$h_{\eta'}^s$	0.068 ± 0.005

derived from these inputs are given in Table 2. As can be seen from the table the expression for h_q in (8) is not useful to compute this parameter accurately in practice, since large cancellations occur between the terms proportional to f_q and f_s. In fact, from (1) it follows that h_q must vanish in the limit $m_q \to 0$ and so is expected to be very small ($h_q/h_s \approx m_q/m_s$).[1] An alternative expression is obtained with the help of the chiral and large-N_c expansion. To leading order this gives [20]

$$\begin{aligned} f_q &= f_\pi\,, & f_s &= \sqrt{2f_K^2 - f_\pi^2} = 1.41 f_\pi\,, \\ h_q &= f_q\, m_\pi^2 = 0.0025\,\text{GeV}^3\,, & h_s &= f_s\,(2m_K^2 - m_\pi^2) = 0.086\,\text{GeV}^3\,. \end{aligned} \tag{11}$$

These results are also prone to large errors, since the chiral expansion is expected to apply only if the masses of the pseudoscalar mesons (now including the η') are much smaller than the masses of the other hadrons. We conclude that h_q is poorly determined at present. Note that although h_q is small it cannot be neglected in general, since some decay amplitudes are proportional to h_q/m_q.

3 The flavor-singlet amplitude

A distinctive feature of hadronic B decays with an $\eta^{(\prime)}$ meson in the final state is that these states contain a flavor-singlet component in their wave function, which opens possibilities for novel decay mechanisms that do not occur if the final state contains only flavor non-singlet states such as pions and kaons. In this section we discuss the different contributions to the flavor-singlet amplitude in the QCD factorization approach.

Neglecting weak annihilation terms for now, we write the $\bar{B} \to \bar{K}P$ decay amplitudes (where $P = \eta^{(\prime)}$) as

$$\mathcal{A}(\bar{B} \to \bar{K}P) = i\,\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(s)}\, \mathcal{A}_p(KP)\,, \tag{12}$$

[1]Because the masses of η and η' are themselves functions of the mixing angle ϕ, it would be inconsistent to interpret (8) as equations for h_q and h_s as functions of ϕ with m_η and $m_{\eta'}$ fixed to their physical values. Rather, only those values of m_η, $m_{\eta'}$ and ϕ leading to a small value of h_q are acceptable. Setting $h_q = 0$ gives $\phi \simeq 38°$, which is close to the phenomenological value quoted above.

where $\lambda_p^{(s)} = V_{pb}V_{ps}^*$, and

$$\mathcal{A}_p(KP) = m_B^2\, F_0^{B\to P}(0)\, \frac{f_K}{\sqrt{2}} \left\{ \delta_{pu}\, \delta_{q_s u}\, \alpha_1(PK) + \alpha_4^p(PK) + \frac{3}{2}\, e_{q_s}\, \alpha_{4,\rm EW}^p(PK) \right\}$$
$$+ \, m_B^2\, F_0^{B\to K}(0) \left\{ \frac{f_P^q}{\sqrt{2}} \left[\delta_{pu}\, \alpha_2(KP_q) + 2\alpha_3^p(KP_q) + \frac{1}{2}\, \alpha_{3,\rm EW}^p(KP_q) \right] \right.$$
$$+ f_P^s \left[\alpha_3^p(KP_s) + \alpha_4^p(KP_s) - \frac{1}{2}\, \alpha_{3,\rm EW}^p(KP_s) - \frac{1}{2}\, \alpha_{4,\rm EW}^p(KP_s) \right]$$
$$\left. + f_P^c \left[\delta_{pc}\, \alpha_2(KP_c) + \alpha_3^p(KP_c) \right] \right\}. \qquad (13)$$

Here $F_0^{B\to M}(0)$ is a $B \to M$ transition form factor evaluated at $q^2 = 0$, $q_s = u$ or d denotes the flavor of the spectator antiquark in the $\bar{B}$ meson, $e_u = 2/3$ and $e_d = -1/3$ are the corresponding electric charge factors, and the symbol $\delta_{q_s u}$ (and similarly δ_{pu}) is defined such that $\delta_{q_s u} = 1$ if $q_s = u$ and zero otherwise. The notation P_i will be explained below.

The various α_i coefficients in the expression for the amplitudes correspond to different flavor topologies contributing to a given decay [21]. The arguments in parentheses are such that the first meson inherits the spectator antiquark from the B meson, while the second meson is the "emission particle" produced at the weak vertex. In naive factorization, the α_i are given in terms of combinations of Wilson coefficients of the effective weak Hamiltonian as

$$\begin{aligned} \alpha_1(PK) &= C_1 + \frac{C_2}{N_c}\,, \qquad & \alpha_4^p(PK) &= C_4 + \frac{C_3}{N_c} + r_\chi^K \left(C_6 + \frac{C_5}{N_c} \right), \\ \alpha_2(KP_i) &= C_2 + \frac{C_1}{N_c}\,, \qquad & \alpha_4^p(KP_s) &= C_4 + \frac{C_3}{N_c} + r_\chi^{P_s} \left(C_6 + \frac{C_5}{N_c} \right), \end{aligned} \qquad (14)$$

and

$$\alpha_3^p(KP_i) = C_3 + \frac{C_4}{N_c} - \left(C_5 + \frac{C_6}{N_c} \right), \qquad (15)$$

where $r_\chi^K = 2m_K^2/[m_b(m_s+m_q)]$ in the definition of $\alpha_4^p(PK)$, and $r_\chi^{P_s} = h_s^P/(f_s^P m_b m_s)$ in the case of $\alpha_4^p(KP_s)$. Analogous expressions hold for the electroweak penguin coefficients $\alpha_{3,\rm EW}^p$ and $\alpha_{4,\rm EW}^p$. Note that the singlet amplitude $\alpha_3^p(KP_i)$ is generated by penguin operators in the effective weak Hamiltonian through transitions of the type $b \to sq\bar{q}$, where the $q\bar{q}$-pair hadronizes into an $\eta^{(\prime)}$ meson.

The calculation of corrections of order α_s to the coefficients $\alpha_{1,2,4}$ is analogous to the calculation for pions or kaons discussed in [11]. (In the notation of that paper, we have $\alpha_{1,2} = a_{1,2}$, $\alpha_3 = a_3 - a_5$, $\alpha_4 = a_4 + r_\chi a_6$, $\alpha_{3,\rm EW} = a_9 - a_7$, and $\alpha_{4,\rm EW} = a_{10} + r_\chi a_8$.) In the FKS scheme, the $\eta^{(\prime)}$ system is described by two leading-twist quark–antiquark light-cone distribution amplitudes, $\phi_q(x)$ and $\phi_s(x)$, defined in analogy with the decay constants f_q and f_s. The subscript on P_q and P_s in (13) means that the corresponding

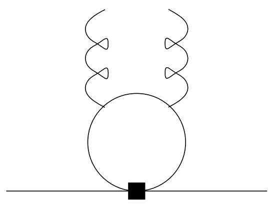

Figure 1: Two-gluon emission from a quark loop. A second diagram with the two gluons crossed is implied.

distribution amplitude must be used in the expressions for the α_i parameters. The one-loop vertex corrections to the quark–antiquark amplitude and a contribution from spectator scattering to the singlet amplitude have also already been computed in [11]. This result is not complete, however, since it ignored the possibility that the $\eta^{(\prime)}$ meson is formed from two gluons.

In the following subsections we discuss three contributions to the singlet decay amplitude related to the gluon content of the $\eta^{(\prime)}$: the $b \to sgg$ amplitude and its relation to an effective charm decay constant, spectator scattering involving two gluons, and singlet weak annihilation, the latter being suppressed by at least one power of Λ/m_b in the heavy-quark limit. Before we begin this discussion we comment on the treatment of the heavy-quark content of the $\eta^{(\prime)}$ meson. In QCD factorization the decay amplitude is factorized into short-distance kernels and light-cone distribution amplitudes. When the factorization scale is smaller than the heavy-quark mass, heavy-quark contributions are part of the short-distance kernels, and the light meson is described in terms of distribution amplitudes of light quarks and gluons. If, in the limit $m_b \to \infty$, the charm-quark mass is held fixed, one should still introduce charm light-cone distribution amplitudes to sum large logarithms of m_b/m_c, but these distribution functions (and the associated decay constants) can be computed in terms of those of light quarks. In (13) we included the possibility of such an induced charm contribution in the form of terms $f_c^P \alpha_i(KP_c)$, to which we will return below.

3.1 The $b \to sgg$ amplitude

The $b \to sgg$ amplitude for general gluon momenta has been calculated in [22] in the electroweak theory. What we need here is the special case where the two gluons have small invariant mass to form an $\eta^{(\prime)}$, and where the $b \to s$ transition is induced by an operator in the effective weak Hamiltonian. The relevant diagrams are shown in Figures 1 and 2.

The diagram of Figure 1 is only relevant when the internal quark is a charm or bottom quark. (Top quarks have already been removed from the effective weak Hamiltonian.) Diagrams with light-quark loops are governed by hadronic scales and their short-distance part consists of the O_i insertion only. This is already taken into account in the naive factorization results (14) and (15). For an operator $O = (\bar{s}\Gamma_1 b)(\bar{q}\Gamma_2 q)$, where the Γ_i

denote arbitrary spinor and color matrices, we find (assuming the two gluons to be in a color-singlet configuration)

$$\mathcal{A}(b \to sgg)|_{\text{Fig. 1}} = -(\bar{u}_s \Gamma_1 u_b)\, \langle g(q_1) g(q_2) | \text{tr}\, (\Gamma_2\, A) | 0 \rangle \,, \tag{16}$$

where u_i denote quark spinors, and the trace is performed over spin and color indices. The quantity A, which is proportional to the 3×3 identity matrix in color space, is given by

$$A = \frac{\alpha_s}{4\pi N_c} \left\{ \frac{1}{8m_q} \Big[(1-4r)\, F(r) + 4r \Big]\, G^A_{\mu\nu}\, G^{A,\mu\nu} + \frac{1}{8m_q}\, F(r)\, i\gamma_5\, G^A_{\mu\nu}\, \widetilde{G}^{A,\mu\nu} \right.$$
$$\left. + \frac{1}{q^2} \Big[1 - F(r) \Big]\, i q^\alpha \gamma_\beta \gamma_5\, G^A_{\mu\alpha}\, \widetilde{G}^{A,\mu\beta} \right\}, \tag{17}$$

where m_q is the quark mass in the loop, $r = m_q^2/q^2 - i\epsilon$, and $q^2 = (q_1 + q_2)^2$ will later be identified with the square of the pseudoscalar meson mass. The function

$$F(r) = 4r \arctan^2\left(\frac{1}{\sqrt{4r-1}} \right) = 1 + \frac{1}{12r} + O(1/r^2) \tag{18}$$

describes the dependence on the invariant mass q^2 of the two gluons and the quark mass in the loop. The last term in (17) may be simplified using the identity

$$G^A_{\mu\alpha}\, \widetilde{G}^{A,\mu}{}_\beta = \frac{g_{\alpha\beta}}{4}\, G^A_{\mu\nu}\, \widetilde{G}^{A,\mu\nu} \,, \tag{19}$$

which for local operators is implied by the antisymmetry of the field-strength tensor. In the limit where $m_q^2 \gg q^2$ (i.e., for $q = c, b$), we obtain

$$A = \frac{\alpha_s}{4\pi N_c} \left\{ \frac{1}{12m_q}\, G^A_{\mu\nu}\, G^{A,\mu\nu} - \frac{(\not{q} - 6m_q)\, i\gamma_5}{48m_q^2}\, G^A_{\mu\nu}\, \widetilde{G}^{A,\mu\nu} + O(1/m_q^3) \right\}. \tag{20}$$

For a pseudoscalar state only the matrix element of $G^A_{\mu\nu}\, \widetilde{G}^{A,\mu\nu}$ is non-zero. Evaluating the Dirac and color structures for the various operators in the effective weak Hamiltonian, we obtain for the contribution to the $\bar{B} \to \bar{K} P$ decay amplitudes

$$\mathcal{A}_p^{\text{charm}} = \frac{a_P}{12m_c^2}\, \langle \bar{K}(p') | \bar{s} \not{q} (1-\gamma_5) b | \bar{B}(p) \rangle \left\{ \left(C_2 + \frac{C_1}{N_c} \right) \delta_{pc} + \left(C_3 - C_5 + \frac{C_4 - C_6}{N_c} \right) \right\}, \tag{21}$$

where $q = p - p'$ and where we have neglected the very small contribution from bottom-quark loops. The definition (2) has been used to evaluate the matrix elements of the anomaly. It follows from (9) that, to a very good approximation (i.e., neglecting the small quantity h_q), $a_P/m_P^2 = -f_P^q/\sqrt{2}$. Comparing (21) with (13)–(15), we see that the charm-loop contribution can be incorporated into (13) if we identify

$$f_P^c = \frac{a_P}{12m_c^2} \approx -\frac{m_P^2}{12m_c^2}\, \frac{f_P^q}{\sqrt{2}} \tag{22}$$

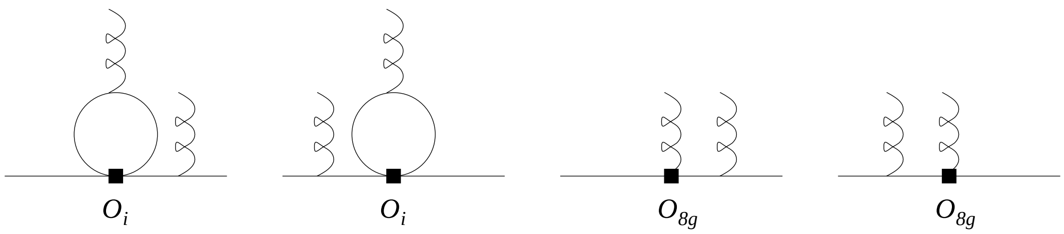

Figure 2: Contributions to the $b \to sgg$ amplitude that are power suppressed in the heavy-quark limit.

with the leading-order result (in an expansion in $1/m_c$) for a new decay constant defined as

$$\langle P(q)|\bar{c}\gamma^\mu\gamma_5 c|0\rangle = -if_P^c\, q^\mu\,, \tag{23}$$

which might be interpreted as a measure of the "intrinsic charm" component of the $\eta^{(\prime)}$ wave function. With our definitions f_P^c is negative and takes values $f_\eta^c \approx -1\,\text{MeV}$ and $f_{\eta'}^c \approx -3\,\text{MeV}$. These results for the decay constants agree with a different derivation in [23]. A similar discussion of the charm-loop diagram in the context of non-leptonic B decays has already been given in [13].

Note that the charm decay constant is formally power-suppressed for heavy charm quarks (but not power suppressed if only m_b goes to infinity), but that the factor of α_s has disappeared in the evaluation of the anomaly matrix element. The charm contributions are potentially non-negligible, since they multiply the large Wilson coefficients $C_{1,2}$. The eventual smallness of this contribution is due to the factor 1/12 in (22), and because the large Wilson coefficients enter in the color-suppressed combination $C_2 + C_1/N_c$. Since we have identified a new leading-order contribution to the factorized amplitude (13), we should in principle consider α_s corrections to $\alpha_{2,3}(KP_c)$, which would require the calculation of two-loop diagrams. We expect that these diagrams can be factored into genuine α_s^2 corrections and a contribution with the structure of a one-loop kernel (with external charm quarks) folded with a calculable charm-quark distribution amplitude. Due to the smallness of the leading-order charm contributions, however, the calculation of this correction is not required within the accuracy of our analysis.

There are other diagrams that could potentially contribute to the $b \to sgg$ amplitude, in which one or both gluons are emitted from external quark lines. Figure 2 shows the relevant graphs with one gluon emitted from a penguin loop or from the chromo-magnetic penguin operator, and the other gluon emitted from the b or s-quark lines. The loop diagrams are non-zero only for an insertion of the penguin operator O_5. In order to obtain a leading-power contribution to the decay amplitude we must pick out the large components of all momenta in the above expression. However, setting $q_1 = xq + \dots$, $q_2 = \bar{x}q + \dots$ and $p_b - p_s = q + \dots$ one finds that the leading term is proportional to q^α or q^β and thus vanishes for on-shell gluons. It follows that the diagrams in Figure 2 are power suppressed with respect to the diagram in Figure 1. A similar analysis shows that also the graphs where both gluons are emitted from the b and s-quark lines yield a power-suppressed contribution.

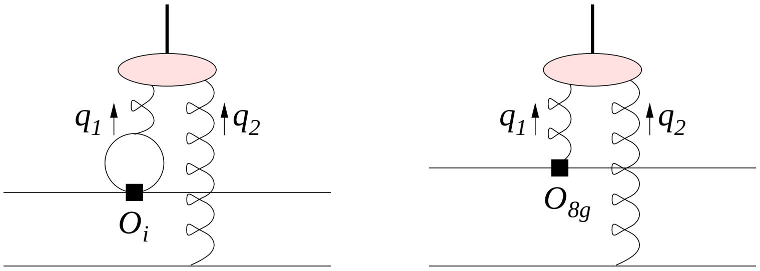

Figure 3: Spectator-scattering contributions to the $B \to K^{(*)}\eta^{(\prime)}$ decay amplitudes. The shaded blob represents the $\eta^{(\prime)}g^*g^*$ form factor.

3.2 Spectator mechanism

The mechanisms described so far in this section provide the leading contributions to the flavor-singlet amplitude in the heavy-quark limit, which start at zeroth order in α_s (naive factorization). In practice, these contributions are numerically rather small, because they are suppressed by the small Wilson coefficients $C_{3\dots6}$ or the mass ratio $m_P^2/(12m_c^2)$. We will now identify another leading-power contribution to the flavor-singlet amplitude, which starts at order α_s but is free of such suppression factors. This contribution is related to the spectator-scattering diagrams shown in Figure 3.

The flavor-singlet spectator mechanism was considered first in [14, 15, 16] using the perturbative QCD approach. In fact, the kinematics of the spectator graph is such that at leading power the two gluons cannot be collinear with the outgoing $\eta^{(\prime)}$ meson, and at least one of them must be a (semi-) hard gluon. Specifically, assigning momentum $p_{\bar{q}} = \bar{y}p_K + k_\perp + \dots$ to the antiquark in the kaon (with $\bar{y} = 1 - y$) and a soft momentum l to the spectator quark in the B meson, it follows that for generic values of y not close to 0 or 1 the virtualities of the two gluons are $q_2^2 \approx -2\bar{y}\,p_K \cdot l = O(m_b\Lambda)$ and $q_1^2 \approx \bar{y}m_B^2 = O(m_b^2)$. The coupling of two hard gluons to the $\eta^{(\prime)}$ meson can be described by a perturbative form factor $F(q_1^2, q_2^2) = O(\alpha_s)$ defined in terms of the $\eta^{(\prime)}g^*g^*$ vertex as [24, 25]

$$iF_P(q_1^2, q_2^2)\, \epsilon_{\mu\nu\alpha\beta}\, q_1^\mu\, q_2^\nu\, \delta_{AB}\, \varepsilon_A^{\alpha*}(q_1)\, \varepsilon_B^{\beta*}(q_2)\,, \tag{24}$$

where $P = \eta'$ or η. It follows that for generic values of y the hard spectator-scattering contribution is an effect of order α_s^2 and thus is beyond the accuracy of a next-to-leading order analysis.

Nevertheless, it is instructive to work out the result for this contribution. In the kinematic region where $|q_1^2| \gg |q_2^2|$ the leading contribution to the form factor can be written as

$$F_P(q_1^2, q_2^2) = \frac{3g_s^2}{N_c\, q_1^2}\, C_P\,, \qquad (|q_1^2| \gg |q_2^2|) \tag{25}$$

where

$$C_P = \sqrt{2} f_P^q \int_0^1 dx\, \frac{\phi_q(x)}{6x\bar{x}} + f_P^s \int_0^1 dx\, \frac{\phi_s(x)}{6x\bar{x}} + \dots \tag{26}$$

contains convolution integrals involving the leading-twist quark–antiquark distribution amplitudes of the $\eta^{(\prime)}$ meson mentioned earlier (twist-3 quark–antiquark distribution amplitudes do not contribute due to chirality conservation), and the dots represent a contribution involving the two-gluon distribution amplitude, which we omit for simplicity. In the limit of asymptotic distribution amplitudes this latter contribution vanishes, and $C_P \to \sqrt{2} f_P^q + f_P^s$. At leading order in $1/m_b$, we find for the hard spectator-scattering contribution to the $B \to K\eta^{(\prime)}$ decay amplitude

$$\mathcal{A}_p^{\rm spec} = \frac{3C_F\alpha_s^2}{4N_c^2}\, C_P\, f_B f_K\, \frac{m_B}{\lambda_B} \int_0^1 \frac{dy}{\bar y} \left[P_2^p(y)\,\phi_K(y) + r_\chi^K\, P_3^p(y)\,\phi_p^K(y) \right], \qquad (27)$$

where $\phi_K(y) \approx 6y\bar y$ is the leading-twist distribution amplitude of the kaon, $\phi_p^K(y) \approx 1$ is one of the twist-3 amplitudes, and the hadronic parameter λ_B is defined as the first inverse moment of one of the two leading B-meson distribution amplitudes [10, 11]. The quantities $P_{2,3}^p(y)$ are the penguin kernels introduced in (49) and (54) of [11]. Explicitly, we have

$$\begin{aligned} P_2^p(y) &= C_1 \left[\frac{4}{3} \ln\frac{m_b}{\mu} + \frac{2}{3} - G(s_p,\bar y) \right] + C_3 \left[\frac{8}{3} \ln\frac{m_b}{\mu} + \frac{4}{3} - G(0,\bar y) - G(1,\bar y) \right] \\ &\quad + (C_4 + C_6) \left[\frac{4n_f}{3} \ln\frac{m_b}{\mu} - (n_f - 2)\, G(0,\bar y) - G(s_c,\bar y) - G(1,\bar y) \right] - \frac{2C_{8g}^{\rm eff}}{\bar y}, \qquad (28) \end{aligned}$$

with $s_q = (m_q/m_b)^2$, $n_f = 5$, and the penguin function $G(s,x)$ as defined in the same reference. The expression for $P_3^p(y)$ is identical to that for $P_2^p(y)$, except that the term $-2C_{8g}^{\rm eff}/\bar y$ must be replaced with $-2C_{8g}^{\rm eff}$. Inspection of the above expressions shows that the perturbative hard-scattering contribution has a logarithmic endpoint singularity as $y \to 1$, corresponding to the region where the gluon exchanged with the spectator quark becomes a soft gluon. For the contribution proportional to $C_{8g}^{\rm eff}$ both the leading and subleading-twist terms are divergent, whereas for the penguin loop contributions only the twist-3 term diverges. In [15, 16] these singularities were regularized by keeping subleading terms in the quark propagators, which effectively corresponds to adopting a model for the non-perturbative endpoint region. While this may be legitimate for obtaining a rough numerical estimate of the spectator-scattering effect, we do not subscribe to the conclusion reached by these authors, that the diagrams in Figure 3 are free of endpoint singularities and thus short-distance dominated. On the contrary, the logarithmic sensitivity to the endpoint region indicates that the soft spectator-scattering mechanism has one less power of α_s associated with it, and so it is formally leading with respect to the hard spectator-scattering contribution.

The appearance of a leading-power soft spectator-scattering contribution is a novel feature of our analysis, which is specific to the case of a light flavor-singlet final-state meson. The QCD factorization formula for B decays into two non-singlet mesons must be extended to account for this effect. We will now argue that factorization still holds in a generalized sense: the soft spectator-scattering contribution can be parameterized in terms of a convolution of the light-cone distribution amplitudes of the singlet meson with a perturbative kernel (the expression C_P in (26)), multiplied with a $B \to K$ "form factor" defined as the matrix element of a non-local operator. For the purpose of illustration we focus on the leading-power twist-2 contribution, corresponding to the term proportional to $C_{8g}^{\rm eff}$ in (28). Momentum conservation implies that in the endpoint region (where the momentum q_2 in Figure 3 is soft) the gluon emitted from the weak vertex is still semi-hard, since $q_1^2 = (p_P - q_2)^2 \approx -2p_P \cdot q_2 = O(m_b\Lambda)$. In the limit where $|q_1^2| \gg |q_2^2| = O(\Lambda^2)$, the leading contribution to the $\eta^{(\prime)} g^* g^*$ form factor is still given by (25), however now one of the two factors g_s is a non-perturbative coupling. Effectively, the soft gluon couples to a compact $(q\bar{q})$ pair in a color-octet state, whose internal features it cannot resolve. The dependence of the soft spectator-scattering mechanism on the soft gluon momentum q_2 can be obtained from a weighted integral over a dual gluon field-strength tensor along the light-like trajectory of the $\eta^{(\prime)}$, using the formalism introduced in Section 4.3.2 of the second paper in [10]. By dimensional analysis, the resulting non-local matrix element scales like a heavy-to-light form factor at large recoil, and we thus parameterize it as (we suppress the Wilson lines required to render the non-local matrix element gauge invariant)

$$\int_{-\infty}^{0} ds\, s\, \langle \bar{K}|\bar{s}(0)\,[\not{n}, \gamma^\mu](1+\gamma_5)\, g_s n^\alpha \widetilde{G}_{\mu\alpha}(sn)\, b(0)|\bar{B}\rangle = \frac{m_B^2}{m_b}\, \mathcal{F}_g^{B\to K}(0)\,, \tag{29}$$

where n^μ is a null-vector in the direction of the $\eta^{(\prime)}$ meson, and the "form factor" $\mathcal{F}_g^{B\to K}(0)$ scales like $(\Lambda/m_b)^{3/2}$ in the heavy-quark limit. With these definitions, the result for the soft spectator-scattering diagram can be represented as a contribution to the coefficients $\alpha_3^p(P_{q,s})$ in (13),

$$\alpha_3^p(KP_{q,s})|_{\rm soft\ spec} = -\frac{3\alpha_s(\mu_h)}{8\pi N_c}\, C_{8g}^{\rm eff}(\mu_h) \left(\int_0^1 dx\, \frac{\phi_{q,s}(x)}{6x\bar{x}} + \dots \right) \frac{\mathcal{F}_g^{B\to K}(0)}{F^{B\to K}(0)}\,, \tag{30}$$

where $\mu_h = \sqrt{m_b \Lambda_h}$ with $\Lambda_h = 0.5\,\text{GeV}$ serves as a typical scale for the semi-hard gluon propagator, and as above we have neglected a contribution from the leading-twist two-gluon distribution amplitude of the meson P. This result shows explicitly how the factorization formula must be modified to account for the effect of soft spectator scattering.

A rough estimate for the form factor $\mathcal{F}_g^{B\to K}(0)$ can be obtained by regularizing the leading-power logarithmic endpoint divergence in (27) using a cutoff such that $\bar{y} > \Lambda_h/m_B$. This yields

$$\mathcal{F}_g^{B\to K}(0) \approx \frac{24\pi C_F \alpha_s}{N_c}\, \frac{f_B f_K}{m_B \lambda_B} \left(\ln\frac{m_B}{\Lambda_h} - 1 \right) \approx 0.3\,, \tag{31}$$

which is of about the same magnitude as the conventional form factor $F^{B\to K}(0)$. The resulting contribution to $\alpha_3^p(P_{q,s})$ is then about $2\cdot 10^{-3}$, roughly half as big as the naive factorization contribution in (15).

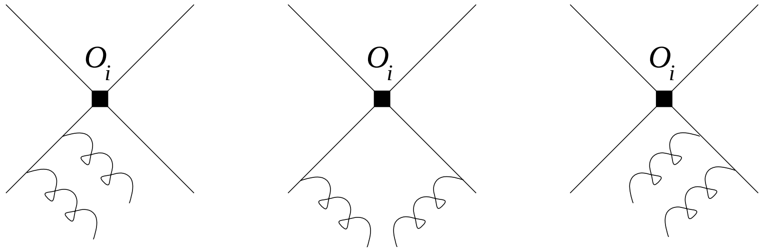

Figure 4: Representative singlet annihilation diagrams.

We remark here that we are unable to reproduce the large enhancement of the spectator-scattering effect for K mesons relative to K^*, which according to [16] is responsible for the large $K\eta'$ branching fractions. In this work the difference between pseudoscalar and vector mesons arises when twist-3 distribution amplitudes are included. We find that including the term proportional to $C_{8g}^{\rm eff}$ in $P_3^p(y)$ into our estimate, the coefficient of the logarithm in (31) changes from 1 to $1 + r_\chi^K/6 \approx 1.15$ for kaons and remains unaltered for K^*. We suspect that the huge effect observed in [16] is related to an error in the kernel for the twist-3 contribution, which leads to an artificial $1/\bar{y} \sim m_B/\Lambda$ enhancement of this contribution.

3.3 Singlet weak annihilation

The discussion of weak annihilation effects in flavor non-singlet non-leptonic decays has received considerable attention recently [11, 26, 27]. Annihilation effects belong to the class of power-suppressed corrections to the decay amplitude, which cannot be calculated in the factorization approach because they receive leading contributions from soft partons. The analysis of the flavor non-singlet penguin amplitude $\alpha_4^p(M_1M_2)$ has shown that annihilation effects are probably not large, yet they constitute the largest uncertainty in the calculation of this amplitude [11]. We now consider the corresponding effect in the flavor-singlet case.

Formally, the leading singlet annihilation amplitude is generated by the emission of two gluons, which form the $\eta^{(\prime)}$ meson. Figure 4 shows three representatives of the twenty relevant Feynman diagrams. The six diagrams where both gluons couple to the constituents of the $\bar{B}$ meson are part of the two-gluon contribution to the $B \to \eta^{(\prime)}$ form factor used as an input in the factorization formula, and need not be considered further. Heavy-quark power counting of these diagrams shows that the leading term comes from the diagrams with two gluons emitted from the spectator quark, and confirms that this contribution is of the same order in the $1/m_b$ expansion as the quark contribution to the form factor. To compute the remaining terms in the hard-scattering approach, we define the leading-twist two-gluon light-cone distribution amplitude by (omitting again a path-ordered exponential)

$$\langle P(q)|G_\mu{}^{\rho,A}(w)\,\tilde{G}^B_{\rho\nu}(z)|0\rangle = \frac{-\delta^{AB}}{N_c^2-1}\,(\sqrt{2}f_P^q + f_P^s)\,q_\mu q_\nu \int_0^1 dx\, e^{i(xq\cdot w+\bar{x}q\cdot z)}\,\phi_g^P(x)\,, \qquad (32)$$

where $(z-w)^2 = 0$, and the particular combination $\sqrt{2} f_P^q + f_P^s$ is proportional to the flavor-singlet decay constant. The corresponding momentum-space projection onto the on-shell two-gluon scattering amplitude (with the gluon polarization vectors ε^* removed) reads

$$\frac{1}{2}\,\frac{\delta^{AB}}{N_c^2-1}\,(\sqrt{2} f_P^q + f_P^s)\,\epsilon_{\alpha\beta\gamma\delta}\,\frac{q^\gamma\,\bar{n}^\delta}{q\cdot\bar{n}}\,\frac{\phi_g^P(x)}{x\bar{x}}\,, \tag{33}$$

where q is the momentum of the $\eta^{(\prime)}$ meson, $\bar{n}$ is a light-like vector in the direction of the kaon, and the index α (β) belongs to the gluon with momentum xq ($\bar{x}q$). In the FKS scheme we set $\phi_g^{\eta'}(x) = \phi_g^{\eta}(x) \equiv \phi_g(x)$. The first term in the Gegenbauer expansion of $\phi_g(x)$ reads [28, 29, 30, 31, 32]

$$\phi_g(x) = 5B_2^g(\mu)\,x^2\bar{x}^2\,(x-\bar{x}) + \dots\,. \tag{34}$$

The coefficient $B_2^g(\mu)$ as well as all higher Gegenbauer moments vanish for $\mu\to\infty$. The process $\gamma\gamma^*\to\eta^{(\prime)}$ can in principle constrain $B_2^g(\mu)$. In our normalization convention[2] the analysis performed in [32] gives $B_2^g(1\,\text{GeV}) = 2\pm 3$. Note that the second Gegenbauer moment of the singlet quark-antiquark amplitude is much smaller, of order -0.1, which underlines the weak sensitivity of the process to the gluon contribution.

The result of the computation shows that we obtain a Λ/m_b correction to the singlet decay amplitude from the diagrams where one gluon couples to the spectator quark and the other to the constituents of the kaon. The remaining diagrams are further suppressed in $1/m_b$. However, for the graphs where both gluons are emitted from the constituents of the kaon,[3] this additional suppression is compensated by the "chiral enhancement factor" r_χ^K. For this configuration we also find that the result suffers from endpoint singularities. As in the flavor non-singlet case we can then only give an estimate of annihilation effects, based on cutting off the convolution integrals at parton momenta of order Λ. To simplify this estimate, we neglect electroweak penguin annihilation and note that annihilation through the tree operators $O_{1,2}$ competes with large tree amplitudes and is CKM-suppressed for $b\to s$ transitions. We therefore concentrate on annihilation effects that contribute to the singlet penguin amplitude. In this approximation, these are accounted for by substituting

$$\alpha_3^p(KP_{q,s}) \to \alpha_3^p(KP_{q,s}) + \beta_{S3}\,, \tag{35}$$

where

$$\beta_{S3} = \left(C_6 + \frac{C_5}{N_c}\right)\frac{f_B f_K r_\chi^K}{m_B^2\,F_0^{B\to K}(0)}\,\frac{2\pi\alpha_s}{N_c}\int_0^1 dy\,\frac{\phi_p^K(y)}{y\bar{y}}\int_0^1 dx\,\phi_g(x)\,\frac{x-\bar{x}}{x^2\bar{x}^2}\,, \tag{36}$$

[2]The definition (32) implies that our distribution amplitude is $\frac{1}{2}\sqrt{C_F/n_f}$ times the distribution amplitude defined in Appendix A of [32], and $C_F/(2n_f) = 2/9$ times the distribution amplitude assumed in Sections 3 and 4 of the same paper, to which the determination of $B_2^g(\mu)$ refers.

[3]The singlet annihilation contribution where both gluons are emitted from the kaon can be interpreted in terms of a time-like $K\eta^{(\prime)}$ form factor, evaluated at $q^2 = m_B^2$.

and a small contribution proportional to C_3/N_c has been neglected.[4] Note that β_{S3} is formally power-suppressed by a factor $r_\chi^K/m_b \sim 1/m_b^2$ relative to α_3^p, similar to the suppression of the (numerically) dominant penguin annihilation amplitude for the flavor non-singlet case. Within the approximations described above, our result for β_{S3} refers to penguin annihilation into a pseudoscalar meson ($B \to K$) with both gluons radiated from the constituents of the kaon, as indicated in the last diagram in Figure 4. The corresponding diagrams for annihilation into a vector meson ($B \to K^*$) vanish for asymptotic distribution amplitudes.

To obtain a numerical estimate of β_{S3} we insert (34) for the two-gluon distribution function, the asymptotic twist-3 distribution amplitude $\phi_p^K(y) = 1$ for the kaon, and parameterize the endpoint-divergent integrals by $X_A = \int_0^1 dy/y$ as in [11]. This gives

$$\int_0^1 dy \, \frac{\phi_p^K(y)}{y\bar{y}} \int_0^1 dx \, \phi_g(x) \, \frac{x-\bar{x}}{x^2\bar{x}^2} = \frac{10}{3} \, X_A \, B_2^g \, . \tag{37}$$

Evaluating all quantities at the scale $\mu_h = \sqrt{m_b \Lambda_h}$ with $\Lambda_h = 0.5\,\text{GeV}$, and choosing $X_A = \ln(m_B/\Lambda_h)$, we find

$$\beta_{S3} \approx -9 \cdot 10^{-4} \, B_2^g \, . \tag{38}$$

The value of β_{S3} is very uncertain. Using $B_2^g = 2 \pm 3$, and allowing X_A to vary from 0 to twice the estimate given above, we obtain $-9 \cdot 10^{-3} < \beta_{S3} < 2 \cdot 10^{-3}$, but the actual size may be much smaller, since the central value for the gluon Gegenbauer moment may not reflect the true magnitude of B_2^g.

4 Phenomenological implications

We now discuss the phenomenological consequences of our results, present numerical results for the various branching fractions, and investigate to which extent the striking patterns seen in the experimental data in Table 1 can be understood in the context of QCD factorization.

The leading contributions for the $B \to K\eta^{(\prime)}$ decay amplitudes have already been given in (12) and (13). The corresponding result for the case where $P = \pi^0$ reads

$$\begin{aligned}\mathcal{A}_p(K\pi^0) &= m_B^2 \, F_0^{B\to\pi}(0) \, \frac{f_K}{\sqrt{2}} \left\{ \delta_{pu} \, \delta_{q_s u} \, \alpha_1(\pi K) + \sigma_{q_s} \left[\alpha_4^p(\pi K) + \frac{3}{2} \, e_{q_s} \alpha_{4,\text{EW}}^p(\pi K) \right] \right\} \\ &\quad + m_B^2 \, F_0^{B\to K}(0) \, \frac{f_\pi}{\sqrt{2}} \left\{ \delta_{pu} \, \alpha_2(K\pi) + \frac{3}{2} \, \alpha_{3,\text{EW}}^p(K\pi) \right\} ,\end{aligned} \tag{39}$$

where $q_s = u$ or d denotes the flavor of the B-meson spectator quark, and $\sigma_u = 1$ and $\sigma_d = -1$ are sign factors. At subleading order in the heavy-quark expansion the

[4]By dropping the contribution proportional to C_3 we eliminate the diagrams with a gluon coupling to the B-meson spectator quark, which we cannot estimate in the conventional way, since the hard scattering amplitude depends on both the plus and minus components of the spectator-quark momentum, rendering the light-cone projection onto the B meson as defined in [33, 34] invalid.

decay amplitudes receive corrections that violate factorization. Phenomenologically the most important example of such effects are weak annihilation contributions. Flavor non-singlet annihilation effects in $B \to KP$ decays can be parameterized in terms of two parameters $\beta_2(M_1M_2)$ and $\beta_3^p(M_1M_2)$ [11, 21]. Their effects can be incorporated by replacing $\alpha_1 \to \alpha_1 + \beta_2$ and $\alpha_4^p \to \hat{\alpha}_4^p \equiv \alpha_4^p + \beta_3^p$ in the expressions for the decay amplitudes. In addition, the term $\delta_{pu}\,\delta_{q_s u}\,\beta_2(KP_s)$ must be added to the parenthesis multiplying f_P^s in (13).

For the case where the kaon is replaced by a vector meson K^*, one makes the following replacements in the expressions for the amplitudes: $f_K \to f_{K^*}$, $F_0^{B\to K}(0) \to A_0^{B\to K^*}(0)$, $\phi_K(y) \to \phi_{K^*}(y)$, and $r_\chi^K \to r_\chi^{K^*} = (2m_{K^*}f_{K^*}^\perp)/(m_b f_{K^*})$, where $f_{K^*}^\perp$ is the (scale-dependent) transverse decay constant of the vector meson. In addition, the relation of the parameter α_4^p to the quantities $a_{4,6}^p$ introduced in [11] changes. We have $\alpha_4^p(PK^*) = a_4^p(PK^*) + r_\chi^{K^*} a_6^p(PK^*)$ in analogy with the relation for $\alpha_4^p(PK)$, but $\alpha_4^p(K^*P_s) = a_4^p(K^*P_s) - r_\chi^{P_s} a_6^p(K^*P_s)$ with a minus sign between the two terms in the case of $\alpha_4^p(K^*P_s)$. No such sign was present for $\alpha_4^p(KP_s)$. Additional changes regarding convolutions with twist-3 vector-meson distribution amplitudes, and weak annihilation effects, will be detailed in [21].

The predictions obtained using the QCD factorization approach depend on many input parameters. First there are Standard Model parameters, such as the elements of the CKM matrix, quark masses, and the strong coupling constant. Of those, our results are by far most sensitive to the strange-quark mass. Specifically, we use $|V_{cb}| = 0.041 \pm 0.002$, $|V_{ub}/V_{cb}| = 0.09 \pm 0.02$, $\gamma = (70 \pm 20)^\circ$, and show results for $m_s = 100$ and $80\,\mathrm{MeV}$. Next there are hadronic parameters that can, in principle, be determined from experiment, such as meson decay constants and transition form factors. In practice, information about these quantities often comes from theoretical calculations. The corresponding model uncertainties in the form factors have a large impact on our results. We use (at $q^2 = 0$) $F_0^{B\to\pi} = 0.28 \pm 0.05$, $F_0^{B\to K} = 0.34 \pm 0.05$, $A_0^{B\to K^*} = 0.45 \pm 0.10$, and $\mathcal{F}_g^{B\to K^{(*)}} = 0.3 \pm 0.5$ for the "form factors" related to the soft spectator interactions discussed in Section 3.2. Finally, we need predictions for a variety of light-cone distribution amplitudes, which we parameterize by the first two Gegenbauer coefficients in their moment expansion. Experimental information can at best provide indirect constraints on these parameters. Fortunately, it turns out that the sensitivity of our predictions to the Gegenbauer coefficients is small, the only exception being the first inverse moment of the B-meson distribution amplitude, for which we take $\lambda_B = (0.35 \pm 0.15)\,\mathrm{GeV}$. (A complete list of all input parameters can be found in [11, 21].)

An important input to our analysis are the form factors $F_0^{B\to\eta^{(\prime)}}(0)$ defined in terms of the $B \to \eta^{(\prime)}$ matrix elements of the vector current $\bar{q}\gamma^\mu b$. Unfortunately, little is known about these quantities. It is not justified to assume that these form factors are related in a simple way (modulo SU(3) violations) to the $B \to \pi$ or $B \to K$ transition form factors. The reason is that there is a new mechanism available for the case of a flavor-singlet meson, where the B meson is annihilated by the current and the singlet meson is produced via the emission of two gluons. This possibility was already mentioned in Section 3.3, where it was argued that the two-gluon emission from the light spectator

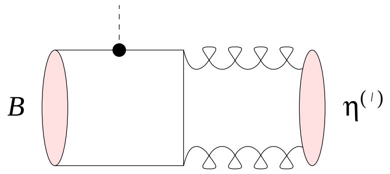

Figure 5: Leading two-gluon contribution to the $B \to \eta^{(\prime)}$ form factor. The dot represents the insertion of the current.

quark in the B meson gives a leading-power contribution to the $B \to \eta^{(\prime)}$ form factor. The corresponding diagram, shown in Figure 5, is proportional to the flavor-singlet decay constant $(\sqrt{2}f_P^q + f_P^s)/\sqrt{3}$. We thus adopt the following parameterization for the form factors ($P = \eta$ or η'):

$$F_0^{B\to P}(0) = F_1\,\frac{f_P^q}{f_\pi} + F_2\,\frac{\sqrt{2}f_P^q + f_P^s}{\sqrt{3}f_\pi}\,, \tag{40}$$

where F_1 and F_2 both scale like $(\Lambda/m_b)^{3/2}$ in the heavy-quark limit, and in the FKS scheme we expect that $F_1 \approx F_0^{B\to\pi}(0)$. Given that the ratio $(\sqrt{2}f_P^q + f_P^s)/(\sqrt{3}f_\pi)$ is about 1.2 for $P = \eta'$ and 0.2 for $P = \eta$, we expect that the extra contribution might be significant for the $B \to \eta'$ form factor, whereas its effect on the $B \to \eta$ form factor should be small. In our numerical analysis we set $F_1 = F_0^{B\to\pi}(0)$ and take $F_2 = 0$ or (somewhat arbitrarily) 0.1.

Before presenting detailed numerical results for the decay amplitudes it is useful to consider the magnitudes of certain flavor topologies in the various decays. Keeping only the leading (color-allowed and not CKM-suppressed) electroweak penguin contributions for simplicity, we write

$$\begin{aligned}
-i\mathcal{A}(B^- \to K^-\eta^{(\prime)}) &= \lambda_u^{(s)}\,(T + C + P_u + S) + \lambda_c^{(s)}\,(C_c + P_c + P_{\rm EW} + S)\,,\\
-i\mathcal{A}(\bar B^0 \to \bar K^0\eta^{(\prime)}) &= \lambda_u^{(s)}\,(C + P_u + S) + \lambda_c^{(s)}\,(C_c + P_c + P_{\rm EW} + S)\,,\\
-i\mathcal{A}(B^- \to K^-\pi^0) &= \lambda_u^{(s)}\,(T + C + P_u) + \lambda_c^{(s)}\,(P_c + P_{\rm EW})\,,\\
-i\mathcal{A}(\bar B^0 \to \bar K^0\pi^0) &= \lambda_u^{(s)}\,(C - P_u) + \lambda_c^{(s)}\,(-P_c + P_{\rm EW})\,.
\end{aligned} \tag{41}$$

The tree amplitude T contains terms proportional to α_1, the color-suppressed tree amplitude C contains terms proportional to α_2, and the penguin amplitudes P_p contain terms proportional to α_4^p and also include the dominant non-singlet penguin annihilation contributions. The singlet amplitude S contains terms proportional to α_3^p, which within our approximations does not depend on the flavor label p. The soft spectator-scattering contribution as well as flavor-singlet weak annihilation are part of S. The electroweak penguin amplitude $P_{\rm EW}$ contains the terms proportional to $\alpha_{3,\rm EW}^c$, and the small contribution proportional to $\alpha_2(KP_c)$ is denoted by C_c. The various topological amplitudes depend on the nature of the final-state particles and should thus be labeled $T(KP)$ with

Table 3: Predictions for the dominant flavor topologies (in units of 10^{-9} GeV). For a given final state the first line refers to $F_2 = 0$, the second to $F_2 = 0.1$. Errors are explained in the text.

Mode		$e^{i\gamma}\,\lambda_u^{(s)}\,T$	$\lambda_c^{(s)}\,P_c$	$\lambda_c^{(s)}\,S$
$\bar{K}\eta'$	$F_2 = 0$	$4.0 + 0.1i$	$(-65\,^{+12\,+\,9}_{-17\,-17}) + (-9\,^{+4\,+15}_{-4\,-15})i$	$(-3\,^{+\,8\,+2}_{-13\,-2}) + (4\,^{+5\,+2}_{-5\,-2})i$
	$F_2 = 0.1$	$6.6 + 0.2i$	$(-76\,^{+13\,+\,9}_{-19\,-17}) + (-10\,^{+5\,+15}_{-5\,-15})i$	$(-3\,^{+\,8\,+2}_{-13\,-2}) + (4\,^{+5\,+2}_{-5\,-2})i$
$\bar{K}\eta$	$F_2 = 0$	$4.9 + 0.1i$	$(13\,^{+8\,+5}_{-8\,-2}) + (2\,^{+1\,+4}_{-1\,-4})i$	$(-0.5\,^{+2.8\,+0.3}_{-3.2\,-0.3}) + (0.6\,^{+0.9\,+0.3}_{-0.7\,-0.3})i$
	$F_2 = 0.1$	$5.3 + 0.1i$	$(12\,^{+8\,+5}_{-8\,-2}) + (2\,^{+1\,+4}_{-1\,-4})i$	$(-0.5\,^{+2.8\,+0.3}_{-3.2\,-0.3}) + (0.6\,^{+0.9\,+0.3}_{-0.7\,-0.3})i$
$\bar{K}\pi^0$	$F_2 = 0$	$5.8 + 0.2i$	$(-29\,^{+7\,+4}_{-8\,-8}) + (-3\,^{+2\,+7}_{-2\,-7})i$	0
$\bar{K}^*\eta'$	$F_2 = 0$	$5.4 + 0.1i$	$(11\,^{+14\,+11}_{-10\,-\,5}) + (0\,^{+2\,+9}_{-2\,-9})i$	$(4\,^{+10\,+0}_{-13\,-0}) + (5\,^{+9\,+0}_{-8\,-0})i$
	$F_2 = 0.1$	$8.9 + 0.2i$	$(6\,^{+15\,+11}_{-10\,-\,5}) + (0\,^{+3\,+9}_{-3\,-9})i$	$(4\,^{+10\,+0}_{-13\,-0}) + (5\,^{+9\,+0}_{-8\,-0})i$
$\bar{K}^*\eta$	$F_2 = 0$	$6.7 + 0.2i$	$(-28\,^{+\,8\,+\,9}_{-12\,-19}) + (-1\,^{+2\,+16}_{-2\,-16})i$	$(0.6\,^{+2.6\,+0}_{-2.9\,-0}) + (0.8\,^{+1.4\,+0}_{-1.3\,-0})i$
	$F_2 = 0.1$	$7.2 + 0.2i$	$(-29\,^{+\,8\,+\,9}_{-12\,-19}) + (-1\,^{+2\,+16}_{-2\,-16})i$	$(0.6\,^{+2.6\,+0}_{-2.9\,-0}) + (0.8\,^{+1.4\,+0}_{-1.3\,-0})i$
$\bar{K}^*\pi^0$	$F_2 = 0$	$7.9 + 0.2i$	$(-14\,^{+4\,+\,5}_{-3\,-10}) + (-1\,^{+1\,+8}_{-2\,-8})i$	0

$P = \eta, \eta', \pi^0$, etc.; however, in the isospin limit the amplitudes are the same for K^- and $\bar{K}^0$ in the final state. A decomposition analogous to (41) holds for the decays $B \to K^* P$.

The two most important amplitudes, the penguin amplitude P_c and the tree amplitude T, are given in Table 3 together with the flavor-singlet amplitude S. For the penguin and singlet amplitudes the second error refers to the uncertainty in the calculation of weak annihilation effects (more precisely, the uncertainty parameterized by the quantity X_A), and the first error contains all other parameter uncertainties added in quadrature, excluding an overall normalization uncertainty from the CKM factor V_{cb}. In the case of the penguin amplitude the parameter uncertainties are dominated by the strange-quark mass. In contrast, the largest uncertainty for the flavor-singlet amplitude comes from the quantities λ_B and X_H (defined in [11]), which enter the calculation of the spectator-scattering effect. In particular, neither of the three effects related to the gluon content of singlet mesons, which we identified and calculated in this paper, constitutes a dominant uncertainty if the corresponding parameters lie within our estimates. The error on the tree amplitude is not given explicitly in the table. It is typically of order 20% on the real part and of order 100% on the (small) absorptive part, excluding an overall normalization uncertainty from the CKM factor V_{ub}.

The table shows that with the exception of the final state $\bar{K}^*\eta'$ the flavor-singlet amplitude S is not an important contributor to the magnitude of the decay amplitudes. In a first approximation the pattern of branching fractions is therefore controlled by the penguin amplitude P_c, which is indeed quite different for the various final states. We first note that the penguin amplitude for the $\bar{K}^*\pi^0$ final state is smaller than for $\bar{K}\pi^0$, because the $\bar{K}^*$ meson can be produced from a $(\bar{s}q)_{\mathrm{S+P}}$ current only through rescattering, which occurs at next-to-leading order in QCD factorization.[5] The new element for the $\eta^{(\prime)}$ final states is a contribution to the penguin amplitude in which the kaon (rather than the pion or $\eta^{(\prime)}$) picks up the spectator quark, while the $\eta^{(\prime)}$ is produced through the strangeness content of its wave function. It follows from (13) that

$$P_c(KP) \propto F_0^{B\to P}(0)\,\frac{f_K}{\sqrt{2}}\,\hat{\alpha}_4^c(PK) + F_0^{B\to K}(0)\,f_P^s\,\hat{\alpha}_4^c(KP_s)\,, \qquad (42)$$

and the interference of the two terms determines the magnitude of the penguin amplitude. In the case of $\bar{K}\eta'$ the two terms add constructively to yield $-20-45=-65$ (referring to the case $F_2=0$ and the real parts in Table 3). Since f_η^s is negative (see Table 2), the second amplitude changes sign for the $\bar{K}\eta$ final state, resulting in a partial cancellation $-24+37=13$ of the penguin amplitude. Our calculation therefore confirms the penguin interference mechanism suggested in [12] as the origin of the large $\bar{K}\eta'$ branching fractions. For vector mesons $\hat{\alpha}_4^c(PK^*)$ becomes smaller because the $(\bar{s}q)_{\rm S+P}$ contribution is suppressed, while $\hat{\alpha}_4^c(K^*P_s)$ becomes smaller and changes sign because the sign of the $(\bar{s}q)_{\rm S+P}$ contribution is reversed, as discussed at the beginning of this section. The two terms in (42) therefore give $-9+20=11$ for $\bar{K}^*\eta'$ and $-11-17=-28$ for $\bar{K}^*\eta$. In cases where the penguin amplitude is suppressed by cancellations there is a sizeable interfering tree amplitude, which leads to the possibility of large direct CP asymmetries.

Our predictions for the various CP-averaged branching fractions and direct CP asymmetries[6] are collected in Tables 4 and 5. Since their variations under changes of the CKM parameters $|V_{cb}|$, $|V_{ub}|$ and γ within their respective error ranges are small compared to other uncertainties, we have fixed these parameters to the central values specified above rather than including them into the error estimates. As in Table 3, the second error comes from weak annihilation and the first error from other hadronic parameters. In contrast to Table 3, however, we have made the dependence on the strange quark mass explicit by showing results for $m_s=100\,\mathrm{MeV}$ (default) and $m_s=80\,\mathrm{MeV}$, and do not include m_s into the hadronic parameter error. The following observations can be made:

i) There is a strong enhancement of some branching fractions in QCD factorization at next-to-leading order compared to naive factorization (equivalent to QCD factorization at leading order). For instance, we find $\mathrm{Br}(K^-\eta')_{\rm LO}=12^{+4}_{-5}$ compared to $\mathrm{Br}(K^-\eta')_{\rm NLO}=42\,^{+16}_{-12}\,^{+27}_{-11}$ (for default parameters) and $\mathrm{Br}(K^{*-}\eta)_{\rm LO}=4^{+1}_{-1}$ compared to $\mathrm{Br}(K^{*-}\eta)_{\rm NLO}=8.6\,^{+3.0}_{-2.6}\,^{+14.0}_{-\ 4.4}$ (all in units of 10^{-6}).

[5] The smallness of the penguin amplitude for final states containing a vector meson may appear problematic in view of the sizeable branching fraction observed for the decay $B\to\pi^+K^{*-}$. The complete set of pseudoscalar–vector final states will be discussed in [21].

[6] The sign convention for the CP asymmetry is, contrary to [11],

$$A_{\rm CP}(f)=\frac{\mathrm{Br}(\bar{B}\to f)-\mathrm{Br}(B\to\bar{f})}{\mathrm{Br}(\bar{B}\to f)+\mathrm{Br}(B\to\bar{f})}\,.$$

Table 4: Predictions for the CP-averaged branching ratios (in units of 10^{-6}), assuming $\gamma = 70°$, $|V_{cb}| = 0.041$ and $|V_{ub}/V_{cb}| = 0.09$. The first error is due to parameter variations, while the second one shows our estimate of the uncertainty due to weak annihilation (see text for explanation). The column labeled "default" refers to $m_s = 100\,\mathrm{MeV}$ and $F_2 = 0$.

Mode	Default	$m_s = 80\,\mathrm{MeV}$	$F_2 = 0.1$	Experiment
$B^- \to K^-\eta'$	$42\,{}^{+16}_{-12}{}^{+27}_{-11}$	$59\,{}^{+22}_{-16}{}^{+41}_{-17}$	$56\,{}^{+19}_{-14}{}^{+31}_{-13}$	72.2 ± 5.3
$\bar{B}^0 \to \bar{K}^0\eta'$	$41\,{}^{+15}_{-11}{}^{+26}_{-11}$	$57\,{}^{+21}_{-15}{}^{+39}_{-16}$	$56\,{}^{+18}_{-13}{}^{+30}_{-13}$	54.8 ± 10.1
$B^- \to K^-\eta$	$1.7\,{}^{+2.0}_{-1.5}{}^{+1.3}_{-0.5}$	$2.2\,{}^{+2.7}_{-2.0}{}^{+1.9}_{-0.8}$	$1.4\,{}^{+1.8}_{-1.2}{}^{+1.1}_{-0.5}$	< 6.9
$\bar{B}^0 \to \bar{K}^0\eta$	$1.0\,{}^{+1.7}_{-1.2}{}^{+1.1}_{-0.4}$	$1.4\,{}^{+2.4}_{-1.7}{}^{+1.6}_{-0.6}$	$0.7\,{}^{+1.5}_{-0.9}{}^{+0.9}_{-0.4}$	< 9.3
$B^- \to K^-\pi^0$	$9.4\,{}^{+3.2}_{-2.9}{}^{+5.6}_{-2.4}$	$12.6\,{}^{+4.3}_{-3.8}{}^{+8.2}_{-3.5}$	$9.4\,{}^{+3.2}_{-2.9}{}^{+5.6}_{-2.4}$	12.7 ± 1.2
$\bar{B}^0 \to \bar{K}^0\pi^0$	$5.9\,{}^{+2.7}_{-2.3}{}^{+4.5}_{-1.9}$	$8.5\,{}^{+3.7}_{-3.1}{}^{+6.8}_{-2.8}$	$5.9\,{}^{+2.7}_{-2.3}{}^{+4.5}_{-1.9}$	10.2 ± 1.5
$B^- \to K^{*-}\eta'$	$3.5\,{}^{+4.4}_{-3.7}{}^{+4.7}_{-1.7}$	$7.7\,{}^{+7.6}_{-6.7}{}^{+8.0}_{-3.2}$	$2.7\,{}^{+3.5}_{-2.6}{}^{+3.9}_{-1.3}$	< 35
$\bar{B}^0 \to \bar{K}^{*0}\eta'$	$2.5\,{}^{+3.8}_{-3.1}{}^{+4.3}_{-1.5}$	$6.3\,{}^{+6.8}_{-5.8}{}^{+7.4}_{-2.9}$	$1.2\,{}^{+2.7}_{-1.8}{}^{+3.2}_{-0.9}$	< 13
$B^- \to K^{*-}\eta$	$8.6\,{}^{+3.0}_{-2.6}{}^{+14.0}_{-4.4}$	$13.8\,{}^{+4.8}_{-4.2}{}^{+19.8}_{-6.7}$	$9.1\,{}^{+3.1}_{-2.7}{}^{+14.3}_{-4.6}$	26.5 ± 6.1
$\bar{B}^0 \to \bar{K}^{*0}\eta$	$8.7\,{}^{+2.9}_{-2.6}{}^{+14.0}_{-4.5}$	$13.9\,{}^{+4.6}_{-4.1}{}^{+19.5}_{-6.7}$	$9.2\,{}^{+3.0}_{-2.7}{}^{+14.2}_{-4.7}$	16.4 ± 3.0
$B^- \to K^{*-}\pi^0$	$3.2\,{}^{+1.2}_{-1.1}{}^{+4.0}_{-1.3}$	$3.3\,{}^{+1.3}_{-1.2}{}^{+4.8}_{-1.5}$	$3.2\,{}^{+1.2}_{-1.1}{}^{+4.0}_{-1.3}$	< 31
$\bar{B}^0 \to \bar{K}^{*0}\pi^0$	$0.7\,{}^{+0.6}_{-0.5}{}^{+2.4}_{-0.6}$	$0.7\,{}^{+0.6}_{-0.5}{}^{+3.0}_{-0.6}$	$0.7\,{}^{+0.6}_{-0.5}{}^{+2.4}_{-0.6}$	< 3.6

ii) While the calculation reproduces well the qualitative pattern of the known branching fractions, we find large uncertainties, principally from weak annihilation and the error on the strange quark mass, but also due to the unknown gluon contribution to the $B \to \eta^{(\prime)}$ form factors, parameterized by F_2. A smaller value of m_s helps to bring the calculation into better agreement with the data. Where cancellations occur in the penguin amplitude, the uncertainty from weak annihilation is amplified. In view of this the significance of the $B^- \to K^{*-}\eta$ branching fraction, which comes out too small with default parameters, remains unclear.

iii) Given that the $K^-\eta'$ and $\bar{K}^0\eta'$ final states differ only by a CKM-suppressed tree amplitude, we are unable to explain significantly different branching fractions for these two modes, if $\gamma = (70 \pm 20)°$. We therefore suspect that the apparently different experimental results should converge. Similar remarks apply to the $K^{*-}\eta$ and $\bar{K}^{*0}\eta$ final states.

iv) Even though direct CP asymmetries are generically predicted to be small in QCD factorization (since the strong-interaction phases of the amplitudes vanish in the heavy-quark limit), appreciable asymmetries can arise in cases where there is destructive interference between the dominant amplitudes. We indeed find potentially

Table 5: Predictions for the direct CP asymmetries (in percent), assuming $\gamma = 70^\circ$ and $|V_{ub}/V_{cb}| = 0.09$, with error estimates as in Table 4. Experimental results are from [8, 35].

Mode	Default	$m_s = 80\,\mathrm{MeV}$	$F_2 = 0.1$	Experiment
$B^- \to K^-\eta'$	$2^{+1\,+4}_{-1\,-4}$	$2^{+1\,+3}_{-1\,-3}$	$3^{+1\,+4}_{-1\,-4}$	-4 ± 6
$\bar{B}^0 \to \bar{K}^0\eta'$	$2^{+1\,+1}_{-1\,-1}$	$2^{+1\,+1}_{-1\,-1}$	$2^{+1\,+0}_{-1\,-1}$	
$B^- \to K^-\eta$	$-22^{+16\,+24}_{-23\,-20}$	$-18^{+15\,+23}_{-22\,-19}$	$-26^{+21\,+32}_{-28\,-25}$	
$\bar{B}^0 \to \bar{K}^0\eta$	$-10^{+\;9\,+6}_{-17\,-5}$	$-8^{+\;8\,+6}_{-14\,-5}$	$-13^{+12\,+9}_{-31\,-8}$	
$B^- \to K^-\pi^0$	$7^{+3\,+\;9}_{-3\,-10}$	$7^{+3\,+9}_{-3\,-9}$	$7^{+3\,+\;9}_{-3\,-10}$	-10 ± 8
$\bar{B}^0 \to \bar{K}^0\pi^0$	$-4^{+3\,+2}_{-3\,-2}$	$-3^{+3\,+2}_{-3\,-2}$	$-4^{+3\,+2}_{-3\,-2}$	3 ± 37
$B^- \to K^{*-}\eta'$	$-18^{+25\,+30}_{-36\,-25}$	$-10^{+12\,+16}_{-15\,-14}$	$-34^{+48\,+60}_{-57\,-41}$	
$\bar{B}^0 \to \bar{K}^{*0}\eta'$	$-7^{+\;8\,+4}_{-15\,-4}$	$-4^{+4\,+2}_{-6\,-2}$	$-12^{+15\,+\;9}_{-39\,-12}$	
$B^- \to K^{*-}\eta$	$3^{+3\,+25}_{-3\,-24}$	$4^{+2\,+17}_{-2\,-17}$	$3^{+3\,+25}_{-4\,-24}$	-5 ± 28
$\bar{B}^0 \to \bar{K}^{*0}\eta$	$4^{+3\,+3}_{-3\,-3}$	$3^{+2\,+2}_{-2\,-2}$	$4^{+3\,+3}_{-3\,-3}$	17 ± 26
$B^- \to K^{*-}\pi^0$	$9^{+10\,+38}_{-10\,-41}$	$8^{+9\,+42}_{-9\,-44}$	$9^{+10\,+38}_{-10\,-41}$	
$\bar{B}^0 \to \bar{K}^{*0}\pi^0$	$-13^{+15\,+24}_{-16\,-28}$	$-12^{+14\,+28}_{-15\,-31}$	$-13^{+15\,+24}_{-16\,-28}$	

large direct CP asymmetries for all those modes in Table 4 whose branching fraction is of order few times 10^{-6} or less. The uncertainties in these predictions are very large, typically at least a factor of 2.

Some of the dominant theoretical uncertainties affecting our predictions cancel in ratios of branching fractions. The most interesting ratios are collected in Table 6 and compared with the experimental data. For these predictions we perform a complete error analysis, quoting (in this order) the uncertainties due to input parameter variations (other than m_s), weak annihilation, the strange-quark mass, and CKM parameters. We find that the ratios are much less sensitive to weak annihilation and the uncertainty in the strange-quark mass than the individual branching fractions, facilitating the comparison with experiment. In some cases there are, however, still sizeable uncertainties due to other input parameter variations, especially those of the heavy-to-light form factors.

A more detailed verification of the underlying mechanism of penguin interference will become possible once the branching fractions for which currently only upper limits exist have been measured. We note that we have also computed the corresponding branching fractions for $\Delta S = 0$ decays, in which the final state contains a pion instead of a kaon and a ρ meson instead of K^*. We find that these decays are tree-dominated. As a consequence the cancellations among penguin amplitudes are of lesser importance for the overall branching fractions, and no drastic signatures as in the case of strangeness-changing decays emerge.

Table 6: Predictions for some ratios of CP-averaged branching fraction, including error estimates for the variation of all input parameters (see text for explanation).

Ratio	$F_2 = 0$	$F_2 = 0.1$	Experiment
$K^-\eta'/K^-\pi^0$	$4.4\,^{+2.2\,+0.2\,+0.3\,+0.5}_{-1.4\,-0.2\,-0.3\,-0.4}$	$6.0\,^{+2.7\,+0.2\,+0.4\,+0.5}_{-1.8\,-0.2\,-0.3\,-0.5}$	5.7 ± 0.7
$K^-\pi^0/K^-\eta$	$5.6\,^{+10.0\,+0.7\,+0.1\,+2.8}_{-4.7\,-0.6\,-0.1\,-1.7}$	$7.0\,^{+12.7\,+1.0\,+0.4\,+3.7}_{-6.1\,-1.0\,-0.3\,-2.2}$	> 1.5
$\bar{K}^0\eta'/K^-\eta'$	$0.98\,^{+0.01\,+0.01\,+0.00\,+0.04}_{-0.02\,-0.01\,-0.01\,-0.04}$	$1.00\,^{+0.01\,+0.01\,+0.01\,+0.05}_{-0.02\,-0.01\,-0.01\,-0.06}$	0.76 ± 0.15
$K^{*-}\eta'/K^{*-}\pi^0$	$1.1\,^{+1.3\,+0.3\,+1.8\,+0.7}_{-1.1\,-0.2\,-0.6\,-0.4}$	$0.8\,^{+1.0\,+0.3\,+1.5\,+0.6}_{-0.8\,-0.1\,-0.4\,-0.4}$	
$K^{*-}\eta/K^{*-}\pi^0$	$2.7\,^{+1.1\,+0.5\,+2.0\,+0.6}_{-0.8\,-0.5\,-1.0\,-0.4}$	$2.8\,^{+1.1\,+0.4\,+2.1\,+0.6}_{-0.8\,-0.5\,-1.0\,-0.4}$	> 0.5
$\bar{K}^{*0}\eta/K^{*-}\eta$	$1.01\,^{+0.04\,+0.00\,+0.00\,+0.17}_{-0.05\,-0.03\,-0.01\,-0.14}$	$1.02\,^{+0.04\,+0.00\,+0.00\,+0.18}_{-0.05\,-0.03\,-0.01\,-0.15}$	0.62 ± 0.18

5 Summary

Motivated by the observation of the large branching fractions for the decays $\bar{B} \to \bar{K}\eta'$ and the distinctive pattern of other decay modes with η or η' mesons in the final state, we have computed the flavor-singlet decay amplitude in rare hadronic B decays using the framework of QCD factorization. We have considered three effects that are specific to singlet mesons due to their gluon content, and which occur at leading power in the heavy-quark expansion: the $b \to sgg$ amplitude, equivalent at leading order to an "intrinsic charm" decay constant; spectator scattering involving two gluons; and weak annihilation, where the leading-power contribution can be interpreted as a gluon contribution to the $B \to \eta^{(\prime)}$ form factors, and the remainder can be estimated in terms of the leading-twist two-gluon light-cone distribution amplitude of the $\eta^{(\prime)}$. A conceptually new result is that the spectator scattering is soft at leading order in the heavy-quark expansion, and hence appears to break factorization. Nevertheless, the long-distance contributions can be parameterized by a non-local $B \to K^{(*)}$ form factor, which is insensitive to the details of the singlet-meson wave function. In this sense factorization still holds; however, the factorization formula has to be amended for singlet-meson final states by an additional term involving this non-local form factor.

Our numerical results indicate that the flavor-singlet decay amplitude is not a key factor in explaining the pattern of the observed branching fractions, at least if the parameters that enter this amplitude (such as the non-local form factor mentioned above) do not exceed substantially our estimates. Rather we find (as has been suggested qualitatively in [12]) that the constructive or destructive interference of penguin amplitudes, where either the $K^{(*)}$ meson or the $\eta^{(\prime)}$ meson picks up the spectator quark, is responsible for this pattern. The structure of these cancellations is visible already in the naive factorization approach [13]. The important improvement from QCD factorization comes from the possibility to compute radiative corrections to the penguin amplitude, which turn out to be large enough to bring the predicted branching fractions into reasonable agreement with the data, at least for certain choices of the input parameters. The hadronic parameter uncertainties remain however large. We may therefore conclude that while it appears unlikely that one can obtain an accurate description of final states with singlet mesons from first principles, the results of our analysis clearly support the relevance of factorization to this class of charmless hadronic decays.

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