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Strings in Ramond-Ramond backgrounds

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Abstract

We write the full and explicit component form of the type IIB worldsheet action in classes of bosonic curved backgrounds threaded with Ramond-Ramond fluxes. The fixing of the kappa symmetry in the light-cone gauge and the use of the Bianchi identities of the supergravity theory lead to an expression of a relatively simple form, yet rich with new physical information about how fundamental strings react to the presence of RR fields. The results are useful in particular to the study of vacuum structure and dynamics in the context of the Holographic duality; and to possibly formulate an open-closed string duality at the level of the worldsheet.

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1 Introduction and Results

In many realizations of the Holographic duality [1, 2, 3], where a perturbative string theory is found dual to strongly coupled dynamics in a field theory or in another string theory, the closed strings on the weakly coupled side of the duality are emersed in background Ramond-Ramond (RR) fluxes. Knowledge of the couplings of strings to such fluxes is then an important ingredient in exploring the principles of the duality itself and in understanding the dual theory.

The matter becomes particularly urgent with the discovery of Non-Commutative Open String (NCOS) theories [4, 5, 6]. In these settings, a perturbative definition of a new theory of open strings is known – it being inherited from the parent string theory whose particular low energy truncation the NCOS dynamics corresponds to; and a definition of this open string theory at strong coupling is provided via a perturbative closed string theory in a certain curved background with RR fluxes [7, 8]. Information about the particular forms of the couplings to these fluxes in the closed string sigma model holds the promise to help us better understand strong coupling dynamics of an open string theory, and to correspondingly formulate the open-closed string duality as a map at the level of the worldsheet.

There are three main approaches in writing down an action of closed superstrings in an arbitrary background. In the RNS formalism, powerful computational techniques are available, yet the vertex operators sourced by RR fields involve spin fields. Consequently, the resulting action is not terribly useful in practice (see also the proposal of [9]). A second approach is the GS formalism with spacetime supersymmetry, generally leading to actions that are useful in unraveling the semi-classical dynamics of the sigma model. On the down side, manifest Lorentz symmetry is lost with the choice of the light-cone gauge; and, at one loop level for example, the lost symmetry results in serious complications. The third approach was developed recently [10] and involves a hybrid picture; in this strategy, part of the spacetime symmetries remain manifest yet couplings to the RR fields take relatively simple forms. The cost is the introduction of several auxiliary fields, and certain assumptions on the form of the background.

In this work, we focus on the second GS method with spacetime supersymmetry and on determining the component form of the action. Our interest is to eventually study, semi-classically, closed string dynamics in certain backgrounds that may not be endowed with a lot of symmetries. Other attempts involve focusing on particularly symmetric cases; see, for example, [11]-[15] for realizations in AdS spaces; and [16] for the case involving the Melvin background. Most of the difficulties involved in writing down this string action in general form are due to the fact that superspace in the presence of supergravity, while still being an attractive setting, can be considerably elaborate [17]. A large amount of superfluous symmetries need to be fixed and computations are often prohibitavily lengthy.

The task is significantly simplified by the use of the method of normal coordinate expansion [18, 19] in superspace. This was developed for the Heterotic string in [20], and, along with the use of computers for analytical manipulations, makes it straightforward to determine the type IIA and IIB sigma models as well. The additional complications that arise in these cases, and that are absent in the Heterotic string case, are due entirely to the presence of the RR fields. We concentrate for now on the IIB theory. In [21], part of this action, to quadratic order in the fermions, was derived starting from the supermembrane action and using T-duality. And in [22], for certain classes of backgrounds that are subsets of the ones we consider, the action was written down through a more direct approach. Starting from superspace in IIB theory and using different techniques, we will write the full-form of this action in the light-cone gauge relevant to most backgrounds of interest.

There are two additional steps within the normal coordinate expansion technique that help to simplify matters further. One involves fixing the κ symmetry early on in the computation. We will show that this truncates the action to quartic order in the fermions. The second step involves making certain general assumptions on the form of the background fields. These assumptions are crucial; otherwise, expressions explodes in size by many orders of magnitude.

The form of the background fields we focus on are inspired by [20] and by the need to apply the results to settings that arise in the context of the Holographic duality. In particular, fields generated by electric and magnetic D-branes of various configurations share certain general features of interest. We list all the conditions we require on the background fields so that the results in this work are applicable:

- The fermionic fields must be zero. In particular, the gaugino and gravitino of the IIB theory have no condensates.
- We choose a certain space direction that, along with the time coordinate, we will associate with the light-cone gauge fixing later. We refer to the other eight spatial directions as being transverse to the light-cone. Then all background fields must depend only on the transverse coordinates.
- Tensorial fields can have indices in the transverse directions; and in the two light-cone directions only if the light-cone coordinates appear in pairs.
- We assume that the metric can be put into diagonal form.

If we were to consider, for example, a background consisting of a number of Dp branes, we choose the light-cone directions *parallel* to the worldvolume of the branes. All conditions listed above are then satisfied. The conditions are of course satisfied in more general cases than this particular example.

Under these assumptions, and once the κ symmetry is fixed, the IIB action takes the form

$$I = I^{(0)} + I^{(2)} + I^{(4)} \ , \tag{1}$$

where the superscripts denote the number of fermionic fields in each part. The first term is the standard bosonic part[2]

$$I^{(0)} = \int d^2\sigma \ \left[\frac{1}{2}\sqrt{-h}h^{ij}G_{mn}\partial_i x^m \partial_j x^n + \frac{1}{2}\varepsilon^{ij}\partial_i x^m \partial_j x^n b^{(1)}_{nm}\right] \ . \tag{2}$$

We represent the two spacetime spinors by a single Weyl – but otherwise complex – 16 component spinor θ. At quadratic order in θ, the action involves the following couplings to the background fields

$$\begin{aligned}
I^{(2)} \ = \ & \int d^2\sigma \ \Big[-iV_i^+ \left(\varepsilon^{ij}\theta\sigma^- \hat{D}_j\theta + \sqrt{-h}h^{ij}\bar{\theta}\sigma^- \hat{D}_j\theta\right) \\
+ \ & \omega^2 V_i^+ V_j^a \times (- \sqrt{-h}h^{ij} Z_{abcde}\bar{\theta}\sigma^- \sigma^{bcde}\theta \\
+ \ & 8\sqrt{-h}h^{ij}Z^{-+}{}_{bcd}\bar{\theta}\sigma^- \sigma_a{}^{bcd}\theta - 48\varepsilon^{ij}Z^{-+}{}_{abc}\theta\sigma^-\sigma^{bc}\theta \\
+ \ & i\varepsilon^{ij}\bar{f}^{-+}{}_b\bar{\theta}\sigma^-\sigma_a{}^b\theta + i\varepsilon^{ij}\bar{f}^{-+}{}_a\bar{\theta}\sigma^-\theta \\
+ \ & \frac{i}{4}\varepsilon^{ij}\bar{f}_{abc}\bar{\theta}\sigma^-\sigma^{bc}\theta - \frac{i}{12}\varepsilon^{ij}\bar{f}_{bcd}\bar{\theta}\sigma^-\sigma_a{}^{bcd}\theta \\
- \ & i\sqrt{-h}h^{ij}\bar{f}^{-+}{}_b\theta\sigma^-\sigma_a{}^b\theta + \frac{i}{4}\sqrt{-h}h^{ij}\bar{f}_{abc}\theta\sigma^-\sigma^{bc}\theta \\
+ \ & \sqrt{-h}h^{ij}q_a\bar{\theta}\sigma^-\theta - \varepsilon^{ij}q_b\theta\sigma^-\sigma_a{}^b\theta\Big) + \text{c.c.}\Big] \ .
\end{aligned} \tag{3}$$

The σ^a matrices are 16×16 gamma matrices. We denote tangent space indices by $a, b, ...$, while spacetime indices are labeled, as in (2), by $m, n, ...$ All tensors will be written with their indices in the tangent space by using the vielbein $X_{ab..} = e_a^m e_b^n X_{mn}$. The $'+'$ and $'-'$ tangent space labels refer to the light-cone directions as in $x^\pm \equiv (x^0 \pm x^a)/2$, with x^0 and x^a being respectively the time and some chosen space direction defining the light-cone. We also have

$$V_i^a \equiv \partial_i x^m e_m^a \ . \tag{4}$$

Furthermore, all Latin indices run over only directions transverse to the light-cone. Finally, the covariant derivative is given by

$$\hat{D}_j\theta^\alpha \equiv \partial_j\theta^\alpha + \frac{1}{4}\sigma^{ab\alpha}_\beta\partial_j x^m \Omega_{m,ab}\theta^\beta \ . \tag{5}$$

The various background fields appearing in (3), and in subsequent equations, are:

[2]Note that the signature of the metric we use is ‘unconventional’. See Appendix A for details.

- The IIB dilaton

$$\omega \equiv e^{\phi/2} \ . \tag{6}$$

- The field strengths for the IIB scalars

$$p_m \equiv \frac{1}{2}\left(i\hat{D}_m\chi - e^{-\phi}\hat{D}_m\phi\right) \ ; \tag{7}$$

$$q_m \equiv -\frac{1}{4}\hat{D}_m\chi \ , \tag{8}$$

with χ being the IIB axion.

- The complex field strength

$$f_{abc} \equiv \frac{1}{2}(1 + e^{-\phi} + i\chi)\mathcal{F}_{abc} + \frac{1}{2}(-1 + e^{-\phi} + i\chi)\bar{\mathcal{F}}_{abc} \ ; \tag{9}$$

$$\mathcal{F}_{abc} \equiv \frac{h^{(1)}_{abc}}{2} + i\frac{h^{(2)}_{abc}}{2} \ , \tag{10}$$

with $h^{(1)}$ and $h^{(2)}$ being, respectively, the field strengths associated with fundamental string and D-string charge.

- And the five-form self-dual field strength

$$Z_{abcde} \equiv \frac{1}{192}g_{abcde} \ . \tag{11}$$

At quartic order in the fermions, the action involves many more terms:

$$\begin{aligned}
I^{(4)} \ = \ & \int d^2\sigma \ \sqrt{-h}h^{ij}(\theta\sigma^-\sigma^{a_1a_2}\theta)(\bar{\theta}\sigma^-\sigma^{a_1a_2}\bar{\theta})V_i^+V_j^+ \ \times \\
& \left(-\frac{1}{16}R^{-+-+} - 72\omega^4 Z^{-+abc}Z^{-+}{}_{abc} - \frac{11}{256}\omega^4 f^{-+a}\bar{f}^{-+}{}_a - \frac{29}{18432}\omega^4 f^{abc}\bar{f}_{abc}\right) \\
+ \ & \sqrt{-h}h^{ij}(\theta\sigma^-\sigma^{b_1b_2}\theta)(\bar{\theta}\sigma^-\sigma_{b_1}{}^{b_3}\bar{\theta})V_i^+V_j^+ \ \times \\
& \left(-\frac{\omega^4}{8}f^{-+}{}_{b_2}p_{b_3} + \frac{5}{96}R^{-+}{}_{b_2b_3} + \frac{1}{96}R^-{}_{b_2}{}^+{}_{b_3} + \frac{23}{96}R^-{}_{b_3}{}^+{}_{b_2}\right. \\
+ \ & 768\omega^4 Z^{-+ab}{}_{b_2}Z^{-+}{}_{abb_3} \\
+ \ & \frac{19}{256}\omega^4 f_{ab_2b_3}\bar{f}^{-+a} - \frac{55}{128}\omega^4 f^{-+}{}_{b_3}\bar{f}^{-+}{}_{b_2} - \frac{67}{384}\omega^4 f^{-+}{}_{b_2}\bar{f}^{-+}{}_{b_3} \\
+ \ & \left.\frac{37}{3072}\omega^4 f_{abb_3}\bar{f}^{ab}{}_{b_2} - \frac{13}{3072}\omega^4 f_{abb_2}\bar{f}^{ab}{}_{b_3} - \frac{37}{768}\omega^4 f^{-+a}\bar{f}_{ab_2b_3}\right)
\end{aligned}$$

$$
\begin{aligned}
+ &\quad \sqrt{-h}h^{ij}(\theta\sigma^-\sigma^{c_1c_2}\theta)(\bar\theta\sigma^-\sigma^{c_3c_4}\bar\theta)V_i^+V_j^+ \times \\
&\Big(12i\omega^2\hat D_{c_3}Z^{-+}{}_{c_1c_2c_4} + 12i\omega^2\Omega_{c_3c_1c_2c_4} + \frac{1}{32}\omega^4 f_{c_1c_2c_4}p_{c_3} + \frac{1}{32}R_{c_1c_2c_3c_4} \\
+ &\quad 672\omega^4 Z^{-+a}{}_{c_1c_4}Z^{-+}{}_{ac_2c_3} - 624\omega^4 Z^{-+a}{}_{c_1c_2}Z^{-+}{}_{ac_3c_4} \\
+ &\quad 24\omega^4 Z^{-+}{}_{abc_4}Z^{ab}{}_{c_1c_2c_3} - 24\omega^4 Z^{-+}{}_{abc_2}Z^{ab}{}_{c_1c_3c_4} \\
+ &\quad \frac{31}{512}\omega^4 f_{c_2c_3c_4}\bar f^{-+}{}_{c_1} + \frac{61}{1536}\omega^4 f_{c_1c_2c_4}\bar f^{-+}{}_{c_3} \\
- &\quad \frac{13}{2048}\omega^4 f_{ac_3c_4}\bar f^{a}{}_{c_1c_2} + \frac{13}{1536}\omega^4 f_{ac_2c_4}\bar f^{a}{}_{c_1c_3} \\
+ &\quad \frac{35}{2048}\omega^4 f_{ac_1c_2}\bar f^{a}{}_{c_3c_4} - \frac{179}{1536}\omega^4 f^{-+}{}_{c_3}\bar f_{c_1c_2c_4} + \frac{143}{1536}\omega^4 f^{-+}{}_{c_2}\bar f_{c_1c_3c_4}\Big) \\
+ &\quad \text{c.c.}\ , \qquad (12)
\end{aligned}
$$

with the same conventions as before, and

$$R^{ab}{}_{cd} \equiv r^{ab}{}_{cd} + \delta^{[a}_{[c}\Omega^{b]}_{d]}\ , \qquad (13)$$

$$\Omega^a_b \equiv 2(\hat D^a\hat D_b \ln\omega) + (\hat D_b\ln\omega)(\hat D^a\ln\omega) - \frac{1}{2}(\hat D^c\ln\omega)(\hat D_c\ln\omega)\delta^a_b\ , \qquad (14)$$

$$\Omega_{c_3c_1c_2c_4} \equiv 5(\hat D_{c_3}\ln\omega)Z^{-+}{}_{c_1c_2c_4} - \frac{3}{2}\eta_{c_3[c_1}Z^{-+}{}_{c_2c_4]b}(\hat D^b\ln\omega)\ . \qquad (15)$$

r_{abcd} is the Riemann tensor associated with the metric G_{mn}, and the additional pieces in (14) and (15) come from rescaling the metric from the Einstein frame to the string frame. More details on the notation used can be found in the main text and in Appendix A.

Equations (3) and (12) can be, in practice, quite simple. Note that some terms may drop out at the expense of others. For example, for backgrounds which are electric or magnetic, but not dyonic, a fraction of the terms are left: say either involving forms like f_{-+a} or f_{abc}, but not both simultaneously. We will comment on some of the physical implications of this action in the Discussion section. For now, let us present some of the details on how these couplings were derived.

The outline of the paper is as follows. In Section 1, we present brief reviews of the techniques we employ; these are the superspace formalism for type IIB supergravity and for the IIB string sigma model, and the normal coordinate expansion method. In Section 2, we apply these techniques to the case of interest; first, we present a series of arguments that simplify the discussion by making use of the light-cone gauge and conditions imposed on the background fields; we then outline in some detail how to obtain the terms quadratic in the fermions; then, more briefly, we sketch how to determine the terms quartic in the fermions. We end in the Discussion section with comments on the form of the action, and preliminary remarks on how the results may be applied in certain examples such as NCOS theories.

2 Preliminaries

2.1 IIB supergravity in superspace

The fields of IIB supergravity are

$$\left\{\hat{e}^a_m,\ \tau = e^{-\phi} + i\chi,\ b^{(1)}_{mn} + i\, b^{(2)}_{mn},\ b_{mnrs};\ \psi_m,\ \lambda\right\}\ ; \tag{16}$$

these are respectively the vielbein, a complex scalar comprised of the dilaton and the axion, two two-form gauge fields, a four-form real gauge field, a complex left-handed gravitino, and a complex right-handed spinor. The gauge fields have the associated field strengths defined as

$$h^{(1)} = db^{(1)}\ ,\quad h^{(2)} = db^{(2)}\ ,\quad g = db\ . \tag{17}$$

An elaborate superspace formalism can be developed for this theory. It involves the standard supergravity superfields [23]

$$\left(E^A_M, \Omega^B_{MA}\right) \to \left(T^A_{BC}, R^D_{ABC}\right)\ . \tag{18}$$

In addition, one needs five other tensor superfields

$$\left\{P_A, Q_A, \hat{\mathcal{F}}_{ABC}, G_{ABCDE}, \Lambda_A\right\} \tag{19}$$

Throughout, we accord to the standard convention of denoting tangent space superspace indices by capital letters from the beginning of the alphabet. In this setting of $\mathcal{N}=2$ chiral supersymmetry, an index such as A represents a tangent space vector index a, and two spinor indices α and $\bar{\alpha}$. Hence, superspace is parameterized by coordinates

$$z^A \in \{x^a, \theta^\alpha, \theta^{\bar{\alpha}}\}\ . \tag{20}$$

Here, θ^α and $\theta^{\bar{\alpha}} \equiv \bar{\theta}^\alpha$ have same chirality and are related to each other by complex conjugation. In this manner, unbarred and barred Greek letters from the beginning of the alphabet will be used to denote spinor indices. More details about the conventions we adopt can be found in Appendix A.

The two superfields P_A and Q_A are the field strengths of a matrix of scalar superfields

$$\mathcal{V} = \begin{pmatrix} u & v \\ \overline{v} & \overline{u} \end{pmatrix}\ , \tag{21}$$

with

$$u\overline{u} - v\overline{v} = 1\ . \tag{22}$$

This matrix describes the group $SU(1,1) \sim SL(2,R)$, which later gets identified with the S-duality group of the IIB theory. The scalars parameterize the coset space $SU(1,1)/U(1)$, with the additional $U(1)$ being a space-time dependent symmetry with an associated gauge field. We then define

$$\mathcal{V}^{-1} d\mathcal{V} \equiv \left(\begin{array}{cc} 2iQ & P \\ \overline{P} & -2iQ \end{array} \right) , \tag{23}$$

with

$$Q = \overline{Q} \tag{24}$$

being the $U(1)$ gauge field mentioned above. All fields in the theory carry accordingly various charge assignments under this $U(1)$. This is a powerful symmetry that can be used to severely restrict the superspace formalism. We also introduce the superfield strength $\hat{F}$

$$\left(\bar{\hat{\mathcal{F}}}, \hat{\mathcal{F}}\right) = \left(\bar{\hat{F}}, \hat{F}\right) \mathcal{V}^{-1} , \tag{25}$$

which transforms under the $SU(1,1)$ as a singlet.

All these fields are associated with a myriad of Bianchi identities. As is typical in supergravity theories, there is an immense amount of superfluous symmetries in the superspace formalism. Some of these can be fixed conventionally; and using the Bianchi identities, relations can be derived relating the various other components. We will be very brief in reviewing this formalism, as our focus will be the string sigma model. Instead of reproducing the full set of equations that determine the IIB theory, we present only those statements that are of direct relevance to the worldsheet theory. Throughout this work, we accord closely to the conventions and notation of [23]; the reader may refer at any point to that work to complement his/her understanding.

From the point of view of the IIB string sigma model, the following combination of the scalars turns out to play an important role

$$\omega = u - \overline{v} . \tag{26}$$

Requiring κ symmetry on the worldsheet leads to the condition

$$\omega = \bar{\omega} . \tag{27}$$

This is a choice that is unconventional from the point of view of the supergravity formalism, but it is natural from the perspective of the string sigma model.

We parameterize the scalar superfields as [24, 25]

$$u = \frac{1 + \bar{W}}{\sqrt{2(W + \bar{W})}} e^{-2i\theta} , \tag{28}$$

$$v = -\frac{1-W}{\sqrt{2(W+\bar{W})}} e^{2i\theta} \ , \tag{29}$$

with the three variables W, $\bar{W}$ and θ parameterizing the $SU(1,1)$. The gauge choice (27) then corresponds to

$$\theta = 0 \ , \tag{30}$$

This leads to

$$\omega = \sqrt{\frac{2}{W+\bar{W}}} \ . \tag{31}$$

And

$$Q_A = \frac{\bar{P}_A - P_A}{4i} \ . \tag{32}$$

Finally, the field strengths are given in terms of W by

$$P = \frac{dW}{W+\bar{W}} \ , \quad Q = \frac{i}{4}\frac{d(W-\bar{W})}{W+\bar{W}} \ . \tag{33}$$

To make contact with the IIB theory's field content (16), we need to specify the map between the superfields (18) and (19) and the physical fields. Each superfield involves an expansion in the fermionic superspace coordinates θ. At zeroth order in this expansion, we have

$$W|_0 = \tau = e^{-\phi} + i\chi \ , \tag{34}$$

Similarly, the zeroth components of the Λ superfield is

$$\Lambda_\alpha|_0 = \lambda_\alpha \ . \tag{35}$$

In the Wess-Zumino gauge, the supervielbein's zeroth component is

$$E_m^\alpha|_0 = \psi_m^\alpha \ . \tag{36}$$

At this point, we can simplify the discussion significantly by choosing to set all background fermionic fields to zero

$$\lambda_\alpha \to 0 \ , \quad \psi_m^\alpha \to 0 \ . \tag{37}$$

This identifies the class of backgrounds which is of most interest to us and that arises most frequently in the literature. Given this, the zeroth components of the other fields are

$$\hat{\mathcal{F}}_{abc}|_0 = \mathcal{F}_{abc} \equiv \frac{h^{(1)}_{abc}}{2} + i\frac{h^{(2)}_{abc}}{2} \ , \tag{38}$$

$$G_{abcde}|_0 = g_{abcde} \ . \tag{39}$$

We also define $\hat{F}_{abc}|_0 \equiv F_{abc}$. And, for completeness, we write the full form of the supervielbein

$$E_M^A|_0 = \begin{pmatrix} \hat{e}_m^a & 0 & 0 \\ 0 & \delta_\mu^\alpha & 0 \\ 0 & 0 & -\delta_{\bar{\mu}}^{\bar{\alpha}} \end{pmatrix} \ ; \tag{40}$$

with the zeroth components of the connection

$$\Omega_{cA}^B|_0 = \omega_{c,A}^B + \mathrm{U}(1) \text{ connection} \ ; \tag{41}$$

$$\Omega_{\alpha,A}^B|_0 = \Omega_{\bar{\alpha},A}^B|_0 = 0 \ ; \tag{42}$$

and the other combinations of indices being zero.

For reasons that will become apparent later, the fields appearing in (3) and (12) are further rescaled with respect to the ones presented here, as in

$$F_{abc} \equiv \omega f_{abc} \ , \quad Q_a|_0 \equiv \omega^2 q_a \ , \quad P_a|_0 \equiv \omega^2 p_a \ . \tag{43}$$

In addition, we will need the zeroth components of the Riemann and torsion superfields, as well as various spinorial components of all the superfields. To make things even worse, various first and second order spinorial derivatives of the superfields will also be needed; *i.e.* some of the higher order terms in the superfield expansions appear in the sigma model. These can be systematically, albeit sometimes tediously, obtained by juggling the superspace Bianchi identities. The needed set turns out not to be exhaustive in terms of determining the IIB supergravity. We will present the relevant pieces as we need them, instead of cataloging an incomplete set of lengthy equations out of context.

Finally, to relate these fields to the ones that arise in the modern literature, we write down the equations of motion of the zero component fields as they appear in the relations above:

$$(\tau+\bar{\tau})\,\nabla^2\tau - 2\,(\nabla\tau)^2 + \frac{1}{12}\,(\tau+\bar{\tau})\left(h^{(2)} - i\tau h^{(1)}\right)^2 = 0 \ ; \tag{44}$$

$$\nabla^p\left(\frac{2\tau\bar{\tau}h^{(1)}_{mnp} + i(\bar{\tau}-\tau)h^{(2)}_{mnp}}{\tau+\bar{\tau}}\right) + \frac{1}{6}g^{abcde}h^{(2)}_{cde} = 0 \ ; \tag{45}$$

$$\nabla^p\left(\frac{2h^{(2)}_{mnp} + i(\bar{\tau}-\tau)h^{(1)}_{mnp}}{\tau+\bar{\tau}}\right) - \frac{1}{6}g^{abcde}h^{(1)}_{cde} = 0 \ ; \tag{46}$$

which conform, for example, to those in [26] with $i\bar{\tau} \to \lambda$, $h^{(1)} \to H^{(1)}$, $h^{(2)} \to H^{(2)}$.

2.2 The IIB string worldsheet in superspace

The action of the IIB string in a background represented by the superfields listed above was written in [27]

$$I = \int d^2\sigma \, \left\{ \frac{1}{2}\sqrt{-h}h^{ij}\Phi V_i^{\ a} V_j^{\ b}\eta_{ab} + \frac{1}{2}\varepsilon^{ij} V_i^{\ B} V_j^{\ A} \mathcal{B}_{AB} \right\} \ , \tag{47}$$

with

$$V_i^{\ A} \equiv \partial_i z^M E_M^A = \{V_i^{\ a}, V_i^{\ \alpha}, V_i^{\ \bar{\alpha}}\} \ , \tag{48}$$

and

$$d\mathcal{B} = \hat{\mathcal{F}} + \bar{\hat{\mathcal{F}}} \ , \tag{49}$$

$$\Phi = \omega = \bar{\Phi} \ . \tag{50}$$

The last statement is needed to assure that the action is κ symmetric. The task is to expand this action in component form. This is generally a messy matter, which, however, can be achieved using the algorithm of normal coordinate expansion.

2.3 The method of normal coordinate expansion in superspace

Normal coordinate expansion, as applied to bosonic sigma models, was first developed in [18]. In these scenarios, the method helped to unravel some of the dynamics of highly non-linear theories approximately, as expanded near a chosen point on the target manifold. In the superspace incarnation, the technique is most powerful when used to expand an action only in a submanifold of the target superspace. In particular, expanding in the fermionic variables only, with the space coordinate left arbitrarily, the expansion truncates by virtue of the Grassmanian nature of the fermionic coordinates; leading to an exact expression for the action in component form. This can also be applied of course to the action or equations of motion for the background superfields as well, and the technique has been demonstrated in this context in many examples. As for the IIB sigma model, the expansion has been applied in [28], to expand however the action in all of superspace, leading to a linearized approximate form that can be used to study quantum effects. Our interest is to get to an exact expression for (47) in component form, by fixing the κ symmetry and leaving the space coordinates arbitrary. This approach was applied to the Heterotic string in [20]. There, the absence of RR fields made the discussion considerably simpler. Our approach will probe in this respect a new class of couplings by the use of this method. However, many simplifications and techniques we will use are direct generalizations of the corresponding methods applied in [20]. First, we briefly review the normal coordinate expansion method in superspace. The reader is referred to [19, 20] for more information.

The superspace coordinates are written as

$$Z^M = Z_0^M + y^M \ . \tag{51}$$

We choose

$$Z_0^M = (x^m, 0) \ , \quad y^M = (0, y^\mu) \ , \tag{52}$$

hence expanding only in the fermionic submanifold. The action is then given by

$$I[Z] = e^\Delta I[Z_0] \ , \tag{53}$$

with the operator Δ defined by

$$\Delta \equiv \int d^2\sigma \ y^A(\sigma) D_A(\sigma) \ , \tag{54}$$

and D_A being the supercovariant derivative. Here, we use the supervielbein to translate between tangent space and superspacetime indices

$$D_A \equiv E_A^N(Z_0) D_N \ , \quad y^N \equiv y^A E_A^N \ . \tag{55}$$

For our choice of expansion variables, we then have

$$y^a = 0 \ , \quad y^\alpha = y^\mu \delta_\mu^\alpha \equiv \theta^\alpha \ , \quad y^{\bar\alpha} = y^{\bar\mu} \delta_{\bar\mu}^{\bar\alpha} \equiv \theta^{\bar\alpha} \ . \tag{56}$$

The power of this technique is that it renders the process of expansion *algorithmic*. A set of rules can be taught say to any well-trained mammal; in principle, human intervention (for that matter the same mammal may be used again) is needed only at the final stage when Bianchi identities may be used to determine some of the expansion terms. The rules are as follows:

- Due to the definition of the normal coordinates, we have

$$\Delta y^A = 0 \ . \tag{57}$$

- Using super-Lie derivatives, it is straightforward to derive

$$\Delta V_i^A = D_i y^A + V_i^C y^B T_{BC}^A \ . \tag{58}$$

- And the following identity is needed beyond second order

$$\Delta \left(D_i y^A \right) = y^B V_i^D y^C R_{CDB}^A \ . \tag{59}$$

- Finally, when we apply Δ to an arbitrary tensor with tangent space indices, we get simply

$$\Delta X_{BC..}^{DE..} = y^A D_A X_{BC..}^{DE..} \ . \tag{60}$$

In the next section, we outline the process of applying these rules to (47).

3 Unraveling the action

There are three sets of difficulties that arise when attempting to apply the normal coordinate expansion to (47). First, a priori, we need to expand to order 2^5 in θ before the expansion truncates. This problem is remedied simply by fixing the κ symmetry with the light-cone gauge, truncating the action to quartic order in θ, as we will show below. The second problem is that the expansion terms will need first and second order fermionic derivatives of the superfields. This requires us to play around with some of the Bianchi identities to extract the additional information. The process is somewhat tedious, but straightforward. The third problem is computational. Despite the simplifications induced by the light-cone gauge choice, and the algorithmic nature of the process, it turns out that the task is virtually impossible to perform by a human, while still maintaining some level of confidence in the result. On average 10^4 terms arise at various stages of the computation. The use of the computer for these analytical manipulations simplifies matters further. However, we find that, even with this help, the complexity is large enough that computing time is of order of many weeks, unless the task is approached with a set of somewhat smarter computational steps and unless one uses the simplifications that arise from the conditions imposed on the background fields and listed in the Introduction. We do not present all the messy details of these nuances, concentrating instead on the general protocol.

At zeroth order, the action is simply

$$I^{(0)} = I|_0 = \int d^2\sigma \left\{ \frac{1}{2}\sqrt{-h}h^{ij}\omega V_i^{\,a} V_{ja} + \frac{1}{2}\varepsilon^{ij} V_i^{\,b} V_j^{\,a} b^{(1)}_{ab} \right\} \ . \tag{61}$$

Note that this is written with respect to the Einstein frame metric, as determined by the IIB supergravity formalism presented in the previous section. We will rescale it to the string frame at the end.

At first order in Δ, the action becomes

$$\begin{aligned} I^{(1)} &= \Delta I = \int d^2\sigma \left\{ \frac{1}{2}\sqrt{-h}h^{ij}(\Delta\Phi) V_i^{\,a} V_j^{\,b}\eta_{ab} + \sqrt{-h}h^{ij}\Phi(\Delta V_i^{\,a}) V_j^{\,b}\eta_{ab} \right. \\ &+ \left. \frac{1}{2}\varepsilon^{ij} V_i^{\,B} V_j^{\,A} y^C \mathcal{H}_{CBA} \right\} \ , \end{aligned} \tag{62}$$

with

$$\mathcal{H} \equiv d\mathcal{B} \ . \tag{63}$$

This result is not evaluated at zeroth order in θ yet, as further powers of Δ will hit it. The rest is mostly mechanical.

3.1 Fixing the κ symmetry

It simplifies matters if we analyze the form of the action we expect from this expansion, once the κ symmetry is fixed. This will help us avoid manipulating many of the terms that will turn out to be zero in the light-cone gauge. To fix the κ symmetry, we define

$$\sigma^{\pm} \equiv \frac{1}{2}\left(\sigma^0 \pm \sigma^a\right) , \tag{64}$$

where a is some chosen direction in space. For conventions on spinors, the reader is referred to Appendix A and [23]. We choose the spacetime fermions to satisfy the condition [3]

$$\sigma^+\theta = \sigma^+\bar{\theta} = 0 . \tag{67}$$

Consider first all even powers of θ. These will necessarily come within one of the following bilinear combinations

$$A^{ab} \equiv \theta\sigma^-\sigma^{ab}\theta , \quad \bar{A}^{ab} = \bar{\theta}\sigma^-\sigma^{ab}\bar{\theta} ; \tag{68}$$

$$B \equiv \bar{\theta}\sigma^-\theta , \quad B^{ab} \equiv \bar{\theta}\sigma^-\sigma^{ab}\theta , \quad B^{abcd} \equiv \bar{\theta}\sigma^-\sigma^{abcd}\theta . \tag{69}$$

In these expressions, condition (67) has been used, and the Latin indices are necessarily transverse to the light cone directions. We will not keep track of this fact with any special notation as it should be obvious from the context where it arises. Given the symmetry properties of the gamma matrices (see Appendix A), we also have

$$\bar{B} = B , \quad \bar{B}^{ab} = -B^{ab} , \quad \bar{B}^{abcd} = B^{abcd} . \tag{70}$$

We then can contract pairs of spinors and generate relations such as

$$A^{ab}A^{cd} = -\frac{1}{16}\frac{1}{32}A^{ef}A^{gh}\mathrm{Tr}[\sigma^{ab}\sigma_{ef}\sigma^{cd}\sigma_{gh}] . \tag{71}$$

Another example is

$$A^{ab}B = -\frac{1}{16}A^{cd}\left(\frac{1}{32}B^{ef}\mathrm{Tr}[\sigma_{cd}\sigma^{ab}\sigma_{ef}] - \frac{1}{16}B\mathrm{Tr}[\sigma_{cd}\sigma^{ab}] - \frac{1}{16}\frac{1}{4!}B^{efgh}\mathrm{Tr}[\sigma_{cd}\sigma^{ab}\sigma_{efgh}]\right) . \tag{72}$$

[3] Alternatively, we can choose [14]

$$\sigma^{\pm} \equiv \frac{1}{2}\left(\sigma^a \pm i\sigma^b\right) , \tag{65}$$

with a and b being two arbitrary space directions. We can then impose

$$\sigma^+\theta = \sigma^-\bar{\theta} = 0 . \tag{66}$$

It can be seen that this choice leads to a more complicated expansion for the action. It may still be necessary to consider such choices for other classes of background fields than those we focus on in this work.

Using such relations, it is straightforward to show that

$$A^{ab}A^{cd} = 0 \ , \quad \bar{A}^{ab}\bar{A}^{cd} = 0 \ . \tag{73}$$

And

$$BA^{ab} = 0 \ , \quad B^{ab}A^{cd} = 0 \ , \quad B^{abcd}A^{ef} = 0 \ ; \tag{74}$$

$$B\bar{A}^{ab} = 0 \ , \quad B^{ab}\bar{A}^{cd} = 0 \ , \quad B^{abcd}\bar{A}^{ef} = 0 \ . \tag{75}$$

While we have

$$A^{ab}\bar{A}^{cd} \neq 0 \ . \tag{76}$$

These immediately imply that all powers of θ higher and including five are zero. This does not yet truncate the action to quartic order. One more piece of argument is needed.

3.2 The expected form of the action

First, we note that, given that all background fermions (λ and ψ_m) are zero, only even powers of θ can appear in the expansion. Next, we assume that all background fields have only non-zero components that are either transverse to the light-cone directions, or that the light-cone indices in them come in pairs; and that all the fields depend only on the transverse coordinates. For example, denoting the light-cone directions by $'+'$ and $'-'$, and all transverse coordinates schematically by r, all fields can only depend on r; and a tensor $X_{abc..}$ can be non-zero only if either all $a, b, c, ..$ are transverse; or if $'+'$ and $'-'$ come as in $X_{-+bc..}$ with $b, c..$ transverse or other light-cone pairs. These conditions are satisfied by all backgrounds of particular interest to us. And it leads to a dramatic simplification of the expansion. In particular, given that a $'-'$ index is to appear in all even powers of fermion bilinears, as in (68) and (69), we must pair each bilinear with a V_i^a to absorb the light-cone index $'-'$.

Let Θ represent either θ or $\bar{\theta}$. For example, schematically $\Theta^2 \sim \theta^2, \bar{\theta}\theta, \bar{\theta}^2$. The action consists then of terms of form $\Theta^{2n}V_i^aV_j^b$, $(D\Theta)\Theta^{2n-1}V_i^a$ and $(D\Theta)(D\Theta)\Theta^{2n}$. From the expansion algorithm outlined above, with the use of equations (57)-(60), it is easy to see that

$$\text{number of V's} + \text{number of } D\Theta\text{'s} = 2$$

in each term. Let's then look at each class of terms separately:

- For terms of the form $\Theta^{2n}V^aV^b$, the only non-zero combinations are $\Theta^2V_i^+V_j^a$ and $\theta^2\bar{\theta}^2V_i^+V_j^+$. This means in particular that the Wess-Zumino term involving $\mathcal{H}$ in (62) does not contribute at quartic order.

- Terms of the form $(D\Theta)\Theta^{2n-1}V_i^a$ are zero unless $n = 1$, because, otherwise, there is shortage of Vs to absorb all light-cone indices.

- Terms of the form $(D\Theta)(D\Theta)\Theta^{2n}$ are zero for all n for the same reason as above.

Hence, the action must have the form

$$I \sim \Theta D\Theta + \Theta^2 + \theta^2\bar{\theta}^2 V^+ V^+ \ , \tag{77}$$

with the last piece receiving contributions only from the first two terms of (62). And we focus on expanding only the relevant parts.

Let us also note that for a typical class of D-brane backgrounds, the natural choice that leads to the simplifications we outlined corresponds to aligning the light-cone direction parallel to the worldvolume of the D-brane. The transversality condition on the fields is then satisfied.

3.3 The quadratic terms

As we expand (53), the quadratic terms in θ are very simple to handle, and can be done by hand. On finds that zeroth components of $D\omega$ and $D^2\omega$ are needed. For these, we note the relation

$$d\omega = -\frac{\omega}{2}\left(P + \bar{P}\right) \ . \tag{78}$$

Using the results of [23], we get

$$D_\alpha\omega|_0 = D_{\bar{\alpha}}\omega|_0 = 0 \ . \tag{79}$$

$$D_\alpha D_\beta\omega|_0 = -\omega\frac{i}{24}\sigma^{abc}_{\alpha\beta}F_{abc} \ , \quad D_{\bar{\alpha}} D_{\bar{\beta}}\omega|_0 = -\omega\frac{i}{24}\sigma^{abc}_{\alpha\beta}\bar{F}_{abc} \ , \tag{80}$$

$$D_{\bar{\alpha}} D_\beta\omega|_0 = -\omega\frac{i}{2}\sigma^{a}_{\alpha\beta}P_a|_0 \ , \quad D_\alpha D_{\bar{\beta}}\omega|_0 = -\omega\frac{i}{2}\sigma^{a}_{\alpha\beta}\bar{P}_a|_0 \ . \tag{81}$$

Note that the supercovariant derivative D_A is associated with the standard supergravity superconnection plus the $U(1)$ piece, as discussed in [23]. In these and subsequent equations, a Latin index runs over all directions, the transverse and the light-cone ones. In the Wess-Zumino term, we need $D_\alpha\mathcal{H}_{\beta ab}|_0$ and $D_{\bar{\alpha}}\mathcal{H}_{\beta ab}|_0$. These are found

$$D_\alpha\mathcal{H}_{\beta ab}|_0 = i\frac{\omega}{2}\sigma_{ab\beta}{}^{\gamma}\sigma^{c}_{\alpha\gamma}\bar{P}_c|_0 \tag{82}$$

$$D_{\bar{\alpha}}\mathcal{H}_{\beta ab}|_0 = i\frac{\omega}{24}\sigma_{ab\beta}{}^{\gamma}\sigma^{cde}_{\bar{\alpha}\gamma}\bar{F}_{cde} \ . \tag{83}$$

The result of applying these relations can be grouped into several parts. A Weyl term

$$I_{Weyl} = \frac{i}{96}\sqrt{-h}h^{ij}\omega F_{abc}\theta\sigma^{abc}\theta V_{dj}V_i^d - \frac{i}{8}\sqrt{-h}h^{ij}\omega P_a\bar{\theta}\sigma^a\theta V_{bi}V_j^b \to 0 \tag{84}$$

is zero by the transversality condition on the background fields. The kinetic term takes the standard form

$$I_{Kin} = -\frac{i}{2}\omega V_{ai}\left(\varepsilon^{ij}\theta\sigma^a D_j\theta + \sqrt{-h}h^{ij}\bar{\theta}\sigma^a D_j\theta\right) \ , \tag{85}$$

with

$$D_m\theta^\alpha \equiv \partial_m\theta^\alpha + \frac{1}{4}\sigma_\beta^{ab\alpha}\omega_{m,ab}\theta^\beta + iQ_m\theta^\alpha \ . \tag{86}$$

And the quadratic terms look like

$$\begin{aligned} I_{quad} &= 2\omega Z_{abcde}\Sigma_1^{cde,ij}V_i^aV_j^b - \frac{\omega}{2}Z_{bcdef}\Sigma_1^{acdef,ij}V_{ai}V_j^b \\ &+ \frac{i}{8}\omega\bar{F}_{bcd}\Sigma_2^{acd,ij}V_{ai}V_j^b - \frac{i}{8}\omega\bar{F}_{abc}\Sigma_2^{c,ij}V_i^aV_j^b - \frac{i}{48}\omega\bar{F}_{cde}\Sigma_2^{abcde,ij}V_{ai}V_{bj} \\ &+ I_P + \text{c.c} \end{aligned} \tag{87}$$

with

$$I_P \equiv \frac{i}{8}\omega\bar{P}_c|_0\Sigma_1^{abc,ij}V_{ai}V_{bj} \ . \tag{88}$$

And we have defined

$$\Sigma_1^{(r),ij} \equiv \varepsilon^{ij}\theta\sigma^{(r)}\theta + \sqrt{-h}h^{ij}\bar{\theta}\sigma^{(r)}\theta \ ; \tag{89}$$

$$\Sigma_2^{(r),ij} \equiv \varepsilon^{ij}\bar{\theta}\sigma^{(r)}\theta + \sqrt{-h}h^{ij}\theta\sigma^{(r)}\theta \ . \tag{90}$$

We separated the piece in (88) since it involves a coupling to the gradient of the dilaton. This term can be absorbed into the connection by rescaling the metric from the Einstein frame to the string frame

$$e_m^a = \sqrt{\omega}\hat{e}_m^a \Rightarrow G_{mn} = \omega g_{mn} \ . \tag{91}$$

The covariant derivative then becomes

$$D_j\theta^\alpha = \hat{D}_j\theta^\alpha + \frac{1}{4}\sigma_\beta^{ab\alpha}\theta^\beta V_{aj}\partial_b(\ln\omega) + iV_j^aQ_a\theta^\alpha \ , \tag{92}$$

with $\hat{D}$ defined in (5). In that equation, $\Omega_{m,ab}$ is the connection associated with the string frame metric G_{mn}. Massaging these equations back into (85), and making use of the fact that the complex conjugate is also to added to everything to make the action real, we get

$$\begin{aligned} I_{kin} + I_P &= -\frac{i}{2}\omega V_{ai}\left(\varepsilon^{ij}\theta\sigma^a\hat{D}_j\theta + \sqrt{-h}h^{ij}\bar{\theta}\sigma^a\hat{D}_j\theta\right) \\ &- \frac{\omega}{4}Q_c|_0\Sigma_1^{abc,ij}V_{ai}V_{bj} + \frac{\omega}{2}V_{ai}V_j^bQ_b|_0\sqrt{-h}h^{ij}\bar{\theta}\sigma^a\theta \ . \end{aligned} \tag{93}$$

This is not yet the final form. Each tensor involves powers of the vielbein, as in

$$X_{abc..} = \hat{e}_a^m \hat{e}_b^n \hat{e}_c^p X_{mnp...} \ , \tag{94}$$

and hence there are additional powers of ω from these as the vielbein is rescaled. In particular, we have $Z \to \omega^{5/2} Z$, $F \to \omega^{3/2} F$, $P \to \omega^{1/2} P$, and $Q \to \omega^{1/2} Q$. Also, each V_i^b absorbs a $\sqrt{\omega}$, *i.e.* $V^a \to \omega^{-1/2} V^a$. Finally, we normalize the kinetic term by rescaling θ as $\omega^{1/4}\theta \to \theta$. This does not introduce any derivatives of the dilaton because of the symmetry properties of the gamma matrices and the form of the action. The final step involves rewriting some of the fields as in (43), so as to express them in tune with more conventional choices for the IIB fields. Equations (2)-(12) have these changes applied to them; and the indices have been expanded so that all Latin labels on tensors refer to transverse directions only. Note that some of our choices of field redefinitions are different from the ones used in [20] in the case of the Heterotic string.

3.4 The quartic terms

At quartic order in θ, the action is much more difficult to find. Indeed, the use of computation by machine becomes particularly helpful. We do not present all the details, but only the relations that are needed to check the results.

First derivatives of some of the Riemann tensor components arise; particularly, $D_{\bar{\alpha}} \hat{R}^{\gamma_2}_{\beta a \gamma_1}$ and $D_\alpha \hat{R}^{\gamma_2}_{\bar{\beta} a \gamma_1}$. Using the results of [23], it is straightforward to find

$$D_\alpha \hat{R}^{\gamma_2}_{\bar{\beta} a \gamma_1}|_0 = \frac{i}{8} \sigma^{cd\gamma_2}_{\gamma_1} \left(\sigma_{a\bar{\beta}\delta} D_\alpha T^\delta_{cd} + \sigma_{c\bar{\beta}\delta} D_\alpha T^\delta_{ad} + \sigma_{d\bar{\beta}\delta} D_\alpha T^\delta_{ca} \right)|_0 + \frac{i}{2} \delta^{\gamma_2}_{\gamma_1} P_a \sigma^b_{\alpha\bar{\beta}} \bar{P}_b|_0 \ ; \tag{95}$$

$$D_{\bar{\alpha}} \hat{R}^{\gamma_2}_{\beta a \gamma_1}|_0 = -\frac{i}{8} \sigma^{cd\gamma_2}_{\gamma_1} \left(\sigma_{a\beta\bar{\delta}} D_{\bar{\alpha}} T^{\bar{\delta}}_{cd} + \sigma_{c\beta\bar{\delta}} D_{\bar{\alpha}} T^{\bar{\delta}}_{ad} + \sigma_{d\beta\bar{\delta}} D_{\bar{\alpha}} T^{\bar{\delta}}_{ca} \right)|_0 - \frac{i}{2} \delta^{\gamma_2}_{\gamma_1} \bar{P}_a \sigma^b_{\bar{\alpha}\beta} P_b|_0 \ . \tag{96}$$

We note the distinction between R and $\hat{R}$; the latter includes the curvature from the $U(1)$ gauge field, as defined in [23]. To avert confusion, we also note that the covariant derivative D_A is with respect to $\hat{R}$; whereas the one appearing in the introduction as $\hat{D}$ does not involve the $U(1)$ connection and it is the derivative with respect to the *string frame* metric, as mentioned earlier. This aspect of our notation then differs slightly from that of [23].

We need a series of first spinorial derivatives of the torsion. For these, we need to use the Bianchi identity

$$\sum_{(ABC)} D_A T^D_{BC} + T^E_{AB} T^D_{EC} - \hat{R}^D_{ABC} = 0 \ , \tag{97}$$

where the sum is over graded cyclic permutations. We then find

$$D_\alpha T^\delta_{cd}|_0 = R^\delta_{cd\alpha} - D_d T^\delta_{\alpha c} - D_c T^\delta_{d\alpha} + 2T^{\bar{\beta}}_{\alpha[d} T^\delta_{c]\bar{\beta}} - 2T^\beta_{\alpha[d} T^\delta_{c]\beta} + \delta^\delta_\alpha \bar{P}_{[c} P_{d]} \ , \tag{98}$$

and

$$D_{\bar\alpha}T^{\bar\delta}_{bc}|_0 = -D_bT^{\bar\delta}_{c\bar\alpha} - D_cT^{\bar\delta}_{\bar\alpha b} + R^{\bar\delta}_{bc\bar\alpha} + 2T^{\bar\gamma}_{\bar\alpha[c}T^{\bar\delta}_{b]\bar\gamma} - 2T^{\gamma}_{\bar\alpha[c}T^{\bar\delta}_{b]\gamma} + \delta^{\bar\delta}_{\bar\alpha}\bar P_{[b}P_{c]} \ . \tag{99}$$

We also have

$$D_\alpha T^\delta_{\bar\beta\bar\gamma}|_0 = -\frac{i}{24}\sigma^d_{\bar\beta\bar\gamma}\sigma_d^{\delta\beta}\sigma^{abc}_{\alpha\beta}F_{abc} + \frac{i}{24}\delta^\delta_{\bar\beta}\sigma^{abc}_{\alpha\bar\gamma}F_{abc} + \frac{i}{24}\delta^\delta_{\bar\gamma}\sigma^{abc}_{\alpha\bar\beta}F_{abc} \ . \tag{100}$$

In all these and subsequent equations, the right hand sides are to be evaluated as zeroth order in θ.

As if first derivatives are not enough of a mess, two derivatives of the torsion are also needed. For example, $D_\alpha D_\beta T^\delta_{\bar\gamma,a}$ arises and is found

$$\begin{aligned} D_\alpha D_\beta T^\delta_{\bar\gamma a}|_0 = &-\frac{3}{16}\sigma^{de\delta}_{\bar\gamma}\left(-\frac{1}{32}\mathcal{K}_{ade\beta}{}^\gamma D_\alpha D_\gamma\omega + 3P_{[a}\sigma_{de]\beta}{}^\gamma D_\alpha D_\gamma\omega + 3i\sigma_{[a\beta\gamma}D_\alpha T^\gamma_{de]}\right) \\ - &\frac{1}{48}\sigma_{a\bar\gamma}{}^{cde\delta}\left(-\frac{1}{32}\mathcal{K}_{cde\beta}{}^\gamma D_\alpha D_\gamma\omega + 3P_{[c}\sigma_{de]\beta}{}^\gamma D_\alpha D_\gamma\omega + 3i\sigma_{[c\beta\gamma}D_\alpha T^\gamma_{de]}\right) \ , \end{aligned} \tag{101}$$

where we define the matrix

$$\mathcal{K}_{cde} \equiv \sigma_{cdefgh}\bar F^{fgh} + 3\bar F_{[c}{}^{fg}\sigma_{de]fg} + 52\bar F_{[cd}{}^f\sigma_{e]f} + 28\bar F_{cde} \ . \tag{102}$$

To find $D_{\bar\alpha}D_\beta T^{\gamma_2}_{\gamma_1 a}$, we use the standard statement

$$[D_A, D_B\} = -T^C_{AB}D_C - \hat R^D_{ABC} \ . \tag{103}$$

And we get

$$D_{\bar\alpha}D_\beta T^{\gamma_2}_{\gamma_1 a}|_0 = -T^b_{\bar\alpha\beta}D_bT^{\gamma_2}_{\gamma_1 a} + R^\delta_{\bar\alpha\beta\gamma_1}T^{\gamma_2}_{\delta a} + R^b_{\bar\alpha\beta a}T^{\gamma_2}_{\gamma_1 b} - T^\delta_{\gamma_1 a}R^{\gamma_2}_{\bar\alpha\beta\delta} - D_\beta D_{\bar\alpha}T^{\gamma_2}_{\gamma_1 a} \ . \tag{104}$$

We need $D_\alpha D_{\bar\beta}T^{\gamma_2}_{\gamma_1 a}$, which is

$$\begin{aligned} D_\alpha D_{\bar\beta}T^{\gamma_2}_{\gamma_1 a}|_0 \ = \ & -D_\alpha D_{\gamma_1}T^{\gamma_2}_{\bar\beta a} - T^b_{\bar\beta\gamma_1}D_\alpha T^{\gamma_2}_{ba} - D_\alpha R^{\gamma_2}_{\bar\beta a\gamma_1} \\ - \ & T^{\bar\alpha}_{\gamma_1 a}\sigma^b_{\bar\alpha\bar\beta}\sigma_b^{\gamma_2\delta}D_\alpha D_\delta\omega + T^{\bar\alpha}_{\gamma_1 a}\delta^{\gamma_2}_{\bar\alpha}\delta^\delta_{\bar\beta}D_\alpha D_\delta\omega + T^{\bar\alpha}_{\gamma_1 a}\delta^{\gamma_2}_{\bar\beta}\delta^\delta_{\bar\alpha}D_\alpha D_\delta\omega \ . \end{aligned} \tag{105}$$

Finally, we collect the zeroth order components of some of the superfields that arise in the computation as well. These can be found in [23], but we list them for completeness:

$$T^a_{\alpha\bar\beta}|_0 = -i\sigma^a_{\alpha\beta} \ . \tag{106}$$

$$T^{\bar\gamma}_{a\beta}|_0 = -\frac{3}{16}\sigma^{bc\gamma}_\beta\bar F_{abc} - \frac{1}{48}\sigma_{abcd\beta}{}^\gamma\bar F^{bcd} \ . \tag{107}$$

$$T^{\gamma}_{a\beta}|_0 = i\sigma_{\beta}^{bcde\gamma} Z_{abcde} \ . \tag{108}$$

$$R_{\alpha\beta,ab}|_0 = i\frac{3}{4}\sigma^{c}_{\alpha\beta}\bar{F}_{abc} + \frac{i}{24}\sigma_{abcde\alpha\beta}\bar{F}^{cde} \ . \tag{109}$$

$$R_{\alpha\bar{\beta},ab}|_0 = -\frac{1}{24}\sigma^{cde}_{\alpha\beta} g_{abcde} \ . \tag{110}$$

$$H_{a\beta\gamma}|_0 = -i\omega\sigma_{a\beta\gamma} \ . \tag{111}$$

$$H_{a\bar{\beta}\bar{\gamma}}|_0 = -i\omega\sigma_{a\beta\gamma} \ . \tag{112}$$

All other components as they arise in the expansion are zero. The final result is given in (12), rescaled to the string frame as in (2) and (3).

4 Discussion

There is a great deal of physical information in the various parts of equations (3) and (12). We first note that the terms are linear or quadratic in the string coupling $e^{\phi} = \omega^2$, except for the canonical couplings to the Riemann tensor, which are independent of ω. Some of the significance of this will become clear below, as we look at the NCOS example. The form of the action is such that the fermionic variables θ may acquire a non-trivial vacuum configuration depending on the strengths of the various background fields. There is also an unusual coupling to the derivative of the five-form field strength. And many of the terms vanish when one considers center of mass motion of the closed string within certain ansatzs. An important program is then to arrange for simplified settings and see how turning on the various couplings independently affects the evolution of the bosonic coordinates x^m. This can help us develop intuition about the effect of RR fields on closed string dynamics. We defer such a complete analysis to an upcoming work [30], and content ourselves for now with a few brief observations relevant to the NCOS case.

By substituting into the action the fields describing the near horizon geometry of (N, M) strings, we can study the strong coupling dynamics of two dimensional NCOS theory. A sector of this dynamics was studied in [29], where, by focusing on an ansatz expected to correspond to supersymmetric trajectories, the effects of the RR fields were ignored. For these motions, the fermions had to have zero condensates $\langle\theta\rangle = 0$. While this is reasonable to expect for part of the spectrum, and specially for BPS dynamics generally associated with center of mass motion, we may find that there are regimes or certain subsets of the solutions for which the situation changes. Indeed, the original motivation for the current work was to understand the extent to which this assumption is justified, given the particulars of some of the results of [29]. If there are static non-trivial condensates for the fermions, the dynamics of the bosonic sector of the action will indirectly get affected.

Looking at the form of our action as applied to the NCOS case, and looking for configurations near $\langle\theta\rangle = 0$, we focus on the quadratic terms given in (3). We can see that only two terms contribute to the dynamics of interest; these are of the form $\omega^2 f^{-+a}\bar{\theta}\sigma^-\theta$ and $\omega^2 q_a\bar{\theta}\sigma^-\theta$. Given the form of the dilaton in these terms, it may also be seen that the dependence on the D-string charge M cancels, so that the large or infinite M limit is regular on the worldsheet. These two terms are then finite and important to the dynamics. The first term, the one involving f^{-+a}, which is found proportional to the RR 3-form once the complex conjugate piece is added, is necessarily multiplied by a factor V_1^+, which changes sign depending on the orientation of the closed string along the direction parallel to the (N, M) strings. We then expect sensitivity of the effect of the RR fields on the orientation of the strings in the solutions of [29]; in particular, we may hope to find that the case with negatively wound strings, a scenario that was already pointed out in [29] to be pathological, may be consequently ruled out. Another very interesting effect is hidden in the second term, which involves coupling to the axion's field strength. The axion has a non-trivial profile. It is attracted to constants values inside the non-commutative throat and far away from it. In between, a kink profile results in a flux of axion charge at the throat only! The term $q_a\bar{\theta}\sigma^-\theta$ then becomes important at the throat. This may be signalling that RR fields play a crucial role in understanding how to extend the Maldacena duality beyong the near horizon region [29]. We hope to report on definitive conclusions and a detailed analysis of all these issues in [30].

Other future directions include writing down the IIA action in a similar manner, or by using T-duality (see, for example, [21]). Furthermore, given the algebraic complexity of the computations involved in deriving some parts of our action, it can be useful to have some of the details of our results checked independently, preferably with different methods. A most ambitious, yet a very important matter, would be to try to understand the open-closed string duality, for example in the context of the NCOS theory, directly on the worldsheet level. For such a map to exist, knowledge of the couplings to the RR fields is obviously very useful. Finally, it would be helpful to develop general computational techniques that allow us to analyze, at least semi-classically, dynamics of closed strings in arbitrary backgrounds (with the effect of the RR fields we discussed taken into account). In this regard, approximation methods such as expansion about center of mass motion – which is in some respects an extension of the normal coordinate expansion technique we used in superspace – may be used.

5 Appendix A: Spinors and conventions

Our spinors are Weyl but not Majorana. They are then complex and have sixteen components. The associated 16×16 gamma matrices satisfy

$$\left\{\sigma^a, \sigma^b\right\} = 2\eta^{ab} \ , \tag{113}$$

with the metric

$$\eta_{ab} = diag(+1, -1, -1, ..., -1) \ . \tag{114}$$

Note that the signature is different from the standard one in use in modern literature. This is so that we conform to the equations appearing in [23]. Also, the worldsheet metric h^{ij} has signature $(-,+)$ for space and time, respectively. Throughout, the reader may refer to [23] to determine more about the spinorial algebra and identities that we are using. However, we make no distinction between σ and $\hat{\sigma}$ as defined in [23] as this will be obvious from the context.

We note that σ^a, σ^{abcd} and σ^{abcde} are symmetric; while σ^{ab} and σ^{abc} are antisymmetric; and σ^{abcde} is self-dual.

With the choice given in (64), we then have

$$\sigma^+\sigma^- + \sigma^-\sigma^+ = 1 \ . \tag{115}$$

And complex conjugation is defined so that

$$\overline{\sigma^a} = \sigma^a \ . \tag{116}$$

Conjugation also implies

$$\overline{\theta_1\theta_2} = \bar{\theta}_2\bar{\theta}_1 \ . \tag{117}$$

Finally, antisymmetrization is defined as

$$\sigma^{ab} \equiv \sigma^{[a}\sigma^{b]} \ , \tag{118}$$

with a conventional 2! hidden by the braces.

Using the completeness relation and the algebra above, we have, for any matrix $Q_{\alpha\beta}$ with lower indices

$$Q_{\alpha\beta} = \frac{1}{16}\left(\mathrm{Tr}[Q\sigma_a]\sigma^a_{\alpha\beta} - \frac{1}{3!}\mathrm{Tr}[Q\sigma_{abc}]\sigma^{abc}_{\alpha\beta} + \frac{1}{5!}\mathrm{Tr}[Q\sigma_{abcde}]\sigma^{abcde}_{\alpha\beta}\right) \ . \tag{119}$$

This allows us, for example, to rearrange certain combinations such as

$$(\bar{\theta}\sigma^{-(r)}\theta)(\bar{\theta}\sigma^{-(s)}\theta) = \frac{1}{2}\frac{sgn(r)}{16^2}(\bar{\theta}\sigma^-\sigma^{bc}\bar{\theta})(\theta\sigma^-\sigma^{ef}\theta)\mathrm{Tr}[\sigma^{bc}\sigma^{(s)}\sigma^{ef}\sigma^{(r)}] \ , \tag{120}$$

$$sgn(r) \equiv \begin{cases} +1 & \text{for } r=0 \\ -1 & \text{for } r=2 \\ +1 & \text{for } r=4 \end{cases} ; \tag{121}$$

this identity arises repeatedly in the computations. Finally, to avert confusion, we also note the summation convention used

$$U^A V_A = U^a V_a + U^\alpha V_\alpha - U^{\bar{\alpha}} V_{\bar{\alpha}} . \tag{122}$$

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[30] Work in progress.