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Aspects of QCD Factorization[1]

Matthias Neubert

Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA

Abstract. The QCD factorization approach provides the theoretical basis for a systematic analysis of nonleptonic decay amplitudes of B mesons in the heavy-quark limit. After recalling the basic ideas underlying this formalism, several tests of QCD factorization in the decays $B \to D^{(*)}L$, $B \to K^*\gamma$, and $B \to \pi K, \pi\pi$ are discussed. It is then illustrated how factorization can be used to obtain new constraints on the parameters of the unitarity triangle.

INTRODUCTION

In many years of intense experimental and theoretical investigations the flavor sector of the Standard Model has been explored in great detail by studying mixing and weak decays of B mesons and kaons. CP violation has been observed in K–$\bar{K}$ mixing (1964), $K \to \pi\pi$ decays (1999), and most recently in the interference of mixing and decay in $B \to J/\psi K$ (2001). There is now compelling evidence that the Cabibbo–Kobayashi–Maskawa (CKM) mechanism accounts for the dominant source of CP violation in low-energy hadronic weak interactions. Most notably, the discovery of a large CP asymmetry in the B system has established that CP is not an approximate symmetry of Nature. Rather, the smallness of CP-violating effects in kaon (and charm) physics reflects the hierarchy of CKM matrix elements.

Measurements of $|V_{cb}|$ and $|V_{ub}|$ in semileptonic B decays and of the magnitude and phase of V_{td} in K–$\bar{K}$ mixing, $B_{d,s}$–$\bar{B}_{d,s}$ mixing, and $B \to J/\psi K$ decays has helped to determine the parameters of the unitarity triangle $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ with good accuracy. The current values obtained at 95% confidence level are $\bar{\rho} = 0.21 \pm 0.12$, $\bar{\eta} = 0.38 \pm 0.11$ for the coordinates of the apex of the (rescaled) triangle, and $\sin 2\beta = 0.74 \pm 0.15$, $\sin 2\alpha = -0.14 \pm 0.57$, $\gamma = (61 \pm 16)^\circ$ for its angles [1]. These studies have established the existence of a CP-violating phase in the top sector of the CKM matrix, i.e., $\mathrm{Im}(V_{td}^2) \neq 0$. The next step in testing the CKM paradigm must be to explore the CP-violating phase in the bottom sector, i.e., $\gamma = \arg(V_{ub}^*) \neq 0$. In the Standard Model the two phases are, of course, related to each other. However, there is still plenty of room for New Physics to affect the magnitude of flavor violations in both mixing and weak decays (see, e.g., [2]). In particular, the present upper bound on γ is derived from the experimental limit on B_s–$\bar{B}_s$ mixing, which has not yet been seen experimentally and could well be affected by New Physics.

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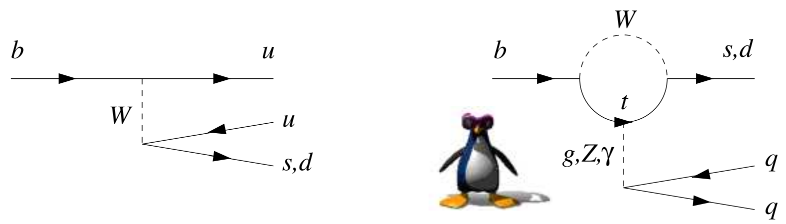


FIGURE 1. Tree and penguin topologies in charmless hadronic B decays.

Common lore says that measurements of γ are difficult. Several "theoretically clean"[2] determinations of this phase have been suggested (see, e.g., [3, 4]), which are extremely challenging experimentally. Likewise, "clean" measurements of $\alpha = \pi - \beta - \gamma$ [5, 6] are very difficult. It is more accessible experimentally to probe γ (and α) via the sizeable tree–penguin interference in charmless hadronic decays such as $B \to \pi K$ and $B \to \pi\pi$. The basic decay topologies contributing to these modes are shown in Figure 1. Experiment shows that the tree-to-penguin ratios in the two cases are roughly $|T/P|_{\pi K} \approx 0.2$ and $|P/T|_{\pi\pi} \approx 0.3$, indicating a sizeable amplitude interference. It is important that the relative weak phase between the two amplitudes can be probed not only via CP asymmetry measurements ($\sim \sin\gamma$), but also via measurements of CP-averaged branching fractions ($\sim \cos\gamma$). Extracting information about CKM parameters from the analysis of nonleptonic B decays is a challenge to theory, since it requires some level of control over hadronic physics, including strong-interaction phases. Such challenges, combined with the importance of the issue, is what triggers theoretical progress.

QCD FACTORIZATION

Hadronic weak decay amplitudes simplify greatly in the heavy-quark limit $m_b \gg \Lambda_{\text{QCD}}$. This statement should not surprise those who have followed the dramatic advances in our theoretical understanding of B physics in the past decade. Many areas of B physics, from spectroscopy to exclusive semileptonic decays to inclusive rates and lifetimes, can now be systematically analyzed using heavy-quark expansions. Yet, the more complicated exclusive nonleptonic decays have long resisted any theoretical progress. The technical reason is that, whereas in most other applications of heavy-quark expansions one proceeds by integrating out heavy fields (leading to local operator product expansions), in the case of nonleptonic decays the large scale m_b enters as the energy carried by light fields. Therefore, in addition to hard and soft subprocesses collinear degrees of freedom become important. This complicates the understanding of hadronic decay amplitudes

[2] In this area of flavor physics many practitioners would consider a method to be "theoretically clean" only if it exclusively relies on elementary geometry (amplitude triangles) and, perhaps, isospin symmetry. We adopt the rationale followed in most other branches of high-energy physics and call a method theoretically clean if it relies on systematic expansions in small parameters. The methods discussed later in this talk are theoretically clean in this wider sense.

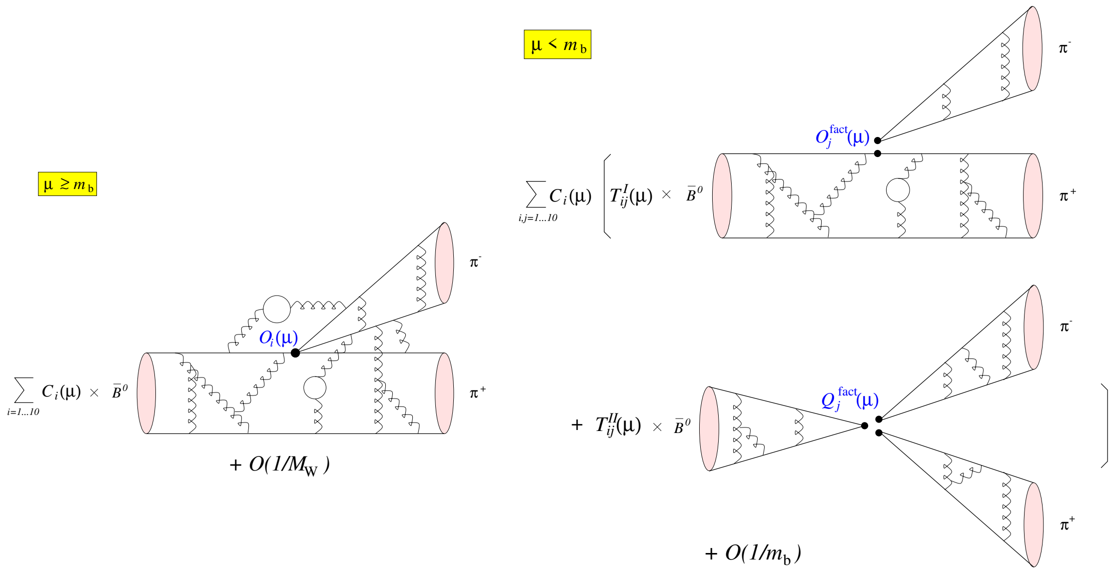


FIGURE 2. Factorization of short- and long-distance contributions in hadronic B decays. Left: Factorization of short-distance effects into Wilson coefficients of the effective weak Hamiltonian. Right: Factorization of hard "nonfactorizable" gluon exchanges into hard-scattering kernels (QCD factorization).

using the language of effective field theory. (Yet, very significant progress towards an effective field-theory description of nonleptonic decays has been made recently with the establishment of a "collinear–soft effective theory" [7]. The reader is referred to these papers for more details on this important development.)

The importance of the heavy-quark limit is based on the physical idea of color transparency [8, 9, 10]. A fast-moving light meson (such as a pion) produced in a point-like source (a local operator in the effective weak Hamiltonian) decouples from soft QCD interactions. More precisely, the couplings of soft gluons to such a system can be analyzed using a multipole expansion, and the first contribution (from the color dipole) is suppressed by a power of $\Lambda_{\mathrm{QCD}}/m_b$. The QCD factorization approach provides a systematic, model-independent implementation of this idea [11, 12]. It gives rigorous results in the heavy-quark limit, which are valid to leading power in $\Lambda_{\mathrm{QCD}}/m_b$ but to all orders of perturbation theory. Having obtained control over nonleptonic decays in the heavy-quark limit is a tremendous advance. We are now able to talk about power corrections to a well-defined and calculable limiting case, which captures a substantial part of the physics in these complicated processes.

The workings of QCD factorization can best be illustrated with the cartoons shown in Figure 2. The first graph shows the well-known concept of an effective weak Hamiltonian obtained by integrating out the heavy fields of the top quark and weak gauge bosons from the Standard Model Lagrangian. This introduces new effective interactions mediated by local operators $O_i(\mu)$ (typically four-quark operators) multiplied by calculable running coupling constants $C_i(\mu)$ called Wilson coefficients. This reduction in complexity (nonlocal heavy particle exchanges $\to$ local effective interactions) is exact up to corrections suppressed by inverse powers of the heavy mass scales. The resulting picture at scales at or above m_b is, however, still rather complicated, since gluon exchange is possible between any of the quarks in the external meson states. Additional simplifications occur when the renormalization scale μ is lowered below the scale m_b. Then color transparency comes to play and implies systematic cancellations of soft and collinear gluon exchanges. As a result, all "nonfactorizable" exchanges, i.e., gluons connecting the light meson at the "upper" vertex to the remaining mesons, are dominated by virtualities of order m_b and can be calculated. Their effects are absorbed into a new set of running couplings $T_{ij}^{I,II}(\mu)$ called hard-scattering kernels, as shown in the two graphs on the right-hand side. What remains are "factorized" four-quark and six-quark operators $O_j^{\rm fact}(\mu)$ and $Q_j^{\rm fact}(\mu)$, whose matrix elements can be expressed in terms of form factors, decay constants and light-cone distribution amplitudes. As before, the reduction in complexity (local four-quark operators $\to$ "factorized" operators) is exact up to corrections suppressed by inverse powers of the heavy scale, now set by the mass of the b quark.

The factorization formula is valid in all cases where the meson at the "upper" vertex is light, meaning that its mass is much smaller than the b-quark mass. The second term in the factorization formula (the term involving "factorized" six-quark operators) gives a power-suppressed contribution when the final-state meson at the "lower" vertex is a heavy meson (i.e., a charm meson), but its contribution is of leading power if this meson is also light. Aspects of this power counting will be discussed in more detail later.

Factorization is a property of decay amplitudes in the heavy-quark limit. Comparing the magnitude of "nonfactorizable" effects in kaon, charm and beauty decays, there can be little doubt about the relevance of the heavy-quark limit to understanding nonleptonic processes [13]. Yet, for phenomenological applications it is important to explore the structure of at least the leading power-suppressed corrections. While no complete classification of such corrections has been given to date, several classes of power-suppressed terms have been analyzed and their effects estimated. These estimates (with conservative errors) have been implemented in the phenomenological applications to be discussed later in this talk. Specifically, the corrections that have been analyzed are "chirally-enhanced" power corrections [11], weak annihilation contributions [12, 14], and power corrections due to nonfactorizable soft gluon exchange [15, 16, 17]. With the exception of the "chirally-enhanced" terms, no unusually large power corrections (i.e., corrections exceeding the naive expectation of 5–10%) have been identified so far. Nevertheless, it is important to refine and extend the estimates of power corrections. Fortunately, the QCD factorization approach has a wide range of applicability and makes many testable predictions. Ultimately, therefore, the data will give us conclusive evidence on the relevance of power-suppressed effects. Many tests can, in fact, already be done using existing data. Several examples will now be discussed in detail.

Tests of Factorization in $B \to D^{(*)}L$ Decays

In B decays into a heavy–light final state, when the light meson is produced at the "upper" vertex, the factorization formula assumes its simplest form. Then only the form factor term (the first graph on the right-hand side in Figure 2) contributes at leading power. This is also the place where QCD factorization is best established theoretically. In [12], the systematic cancellation of soft and collinear singularities was demonstrated explicitly at two-loop order. The proof of these cancellations has recently been extended to all orders in perturbation theory [18]. In order to complete a rigorous proof of factorization one would still have to show that the hard-scattering kernels are free of endpoint singularities stronger than $1/x$ or $1/(1-x)$ as one of the quarks in the light meson becomes a soft parton. It has been demonstrated that the kernels tend to a constant (modulo logarithms) at the endpoints in the so-called "large-β_0 limit" of QCD, i.e., to order $\beta_0^{n-1}\alpha_s^n$ for arbitrary n in perturbation theory [17]. However, it is an open question whether such a smooth behavior persists in higher orders of full QCD.

Let us first consider the decays $\bar{B}^0 \to D^{(*)+}L^-$, where L denotes a light meson. In this case the flavor content of the final state is such that the light meson can only be produced at the "upper" vertex, so factorization applies. One finds that process-dependent "nonfactorizable" corrections from hard gluon exchange, though present, are numerically very small. All nontrivial QCD effects in the decay amplitudes are then described by a quasi-universal coefficient $|a_1(D^{(*)}L)| = 1.05 \pm 0.02 + O(\Lambda_{\rm QCD}/m_b)$ [12]. For a given decay channel this coefficient can be determined experimentally from the ratio [8]

$$\frac{\Gamma(\bar{B}^0 \to D^{*+}L^-)}{d\Gamma(\bar{B}^0 \to D^{*+}l^-\nu)/dq^2|_{q^2=m_L^2}} = 6\pi^2|V_{ud}|^2 f_L^2\,|a_1(D^{(*)}L)|^2\,.$$

Using CLEO data one obtains $|a_1(D^*\pi)| = 1.08 \pm 0.07$, $|a_1(D^*\rho)| = 1.09 \pm 0.10$, and $|a_1(D^*a_1)| = 1.08 \pm 0.11$, in good agreement with theory. This is a first indication that power corrections in these modes are under control, but more precise data are required for a firm conclusion. Other tests of factorization in B decays to heavy–light final states have been discussed in [12, 19, 20].

Recently, the experimental observation of unexpectedly large rates for color-suppressed decays such as $\bar{B}^0 \to D^{0(*)}\pi^0$ [21, 22] has attracted some attention. QCD factorization does not allow us to calculate the amplitudes for these processes in a reliable way. It predicts that these amplitudes are power-suppressed with respect to the corresponding $\bar{B}^0 \to D^{+(*)}\pi^-$ amplitudes, but only by one power of $\Lambda_{\rm QCD}/m_c$. Specifically, the prediction is that a certain ratio of isospin amplitudes approaches unity in the heavy-quark limit: $A_{1/2}/(\sqrt{2}A_{3/2}) = 1 + O(\Lambda_{\rm QCD}/m_c)$ [12]. Considering that charm is not a particularly heavy quark, we find that this scaling law is respected by the experimental data, which give $A_{1/2}/(\sqrt{2}A_{3/2}) = (0.70 \pm 0.11)\,e^{\pm i(27\pm7)^\circ}$ for $B \to D\pi$ and $(0.72 \pm 0.08)\,e^{\pm i(21\pm8)^\circ}$ for $B \to D^*\pi$ [13]. Assuming the hierarchy $m_b \gg m_c > \Lambda_{\rm QCD}$, a rough theoretical estimate of the amplitude ratio, $A_{1/2}/(\sqrt{2}A_{3/2}) \sim 0.75\,e^{-15^\circ i}$, had been obtained prior to the observation of the color-suppressed decays [12]. It anticipated the correct order of magnitude of the deviation from the heavy-quark limit.

Tests of Factorization in $B \to K^*\gamma$ Decays

The QCD factorization approach not only applies to nonleptonic decays, but also to other exclusive processes such as $B \to V\gamma$ and $B \to V\,l^+l^-$ (where $V = K^*, \rho, \dots$ is a vector meson) [23, 24]. The resulting factorization formula is similar (but simpler) to that for B decays into two light mesons. Therefore, the study of exclusive radiative transitions not only extends the range of applicability of the method, but also provides a new testing ground for the factorization idea.

Interestingly, the analysis of isospin-breaking effects in radiative B decays gives a direct probe of power corrections to the factorization formula. Experimentally, it is found that [25, 26, 27]

$$\Delta_{0-} \equiv \frac{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) - \Gamma(B^- \to \bar{K}^{*-}\gamma)}{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) + \Gamma(B^- \to \bar{K}^{*-}\gamma)} = 0.11 \pm 0.07\,,$$

indicating (albeit with a large error) that isospin-breaking effects could be as large as 10% at the level of the decay amplitudes. Such effects are absent in the heavy-quark limit. A detailed theoretical analysis of the leading power-suppressed contributions leads to the prediction $\Delta_{0-} = (8.0^{+2.1}_{-3.2})\% \times (0.3/T_1^{B\to K^*})$ [28], where $T_1^{B\to K^*}$ is a tensor form factor, whose value is expected to be close to 0.3. By far the largest contribution to the result comes from an annihilation contribution involving the $(V-A)\otimes(V+A)$ penguin operator O_6 in the effective weak Hamiltonian. Therefore, the quantity Δ_{0-} is a sensitive probe of the magnitude and sign of the ratio $C_6/C_{7\gamma}$ of Wilson coefficients.

The above discussion shows that in the Standard Model one indeed expects a sizeable isospin breaking in the $B \to K^*\gamma$ decay amplitudes, in agreement with the current central experimental value. If the agreement persists as the data become more precise, this would not only test the penguin sector of the effective weak Hamiltonian, but also provide a quantitative test of factorization at the level of power corrections.

Tests of Factorization in $B \to \pi K, \pi\pi$ Decays

The factorization formula for B decays into two light mesons is more complicated because of the presence of the two types of contributions shown in the graphs on the right-hand side in Figure 2. The finding that these two topologies contribute at the same power in $\Lambda_{\rm QCD}/m_b$ is nontrivial [14] and relies on the heavy-quark scaling law $F^{B\to L}(0) \sim m_b^{-3/2}$ for heavy-to-light form factors. Whereas this scaling law has been obtained from several independent studies (see, e.g., [29, 30, 31]), it is not as rigorously established as the corresponding scaling law for heavy-to-heavy form factors. In the QCD factorization approach the kernels $T_{ij}^I(\mu)$ are of order unity, whereas the kernels $T_{ij}^{II}(\mu)$ contribute first at order α_s. Numerically, the latter ones give corrections of about 10–20% with respect to the leading terms. This is consistent with being of the same power but down by a factor of α_s. Therefore, the scaling laws that form the basis of the QCD factorization formula appear to work well empirically.

The factorization formula for B decays into two light mesons can be tested best by using decays that have negligible amplitude interference. In that way any sensitivity to the value of the weak phase γ is avoided. For a complete theoretical control over charmless hadronic decays one must control the magnitude of the tree topologies, the magnitude of the penguin topologies, and the relative strong-interaction phases between trees and penguins. It is important that these three key features can be tested separately. Once these tests are conclusive (and assuming they are successful), factorization can be used to constrain the parameters of the unitarity triangle. (Of course, alternative schemes such as pQCD [32] and "charming penguins" [33] must face the same tests.)

Magnitude of the Tree Amplitude. The magnitude of the leading $B \to \pi\pi$ tree amplitude can be probed in the decays $B^\pm \to \pi^\pm\pi^0$, which to an excellent approximation do not receive any penguin contributions. The QCD factorization approach makes an absolute prediction for the corresponding branching ratio [14],

$$\mathrm{Br}(B^\pm \to \pi^\pm\pi^0) = \left[5.3\,^{+0.8}_{-0.4}\,(\mathrm{pars.}) \pm 0.3\,(\mathrm{power})\right] \cdot 10^{-6} \times \left[\frac{|V_{ub}|}{0.0035}\,\frac{F_0^{B\to\pi}(0)}{0.28}\right]^2 ,$$

which compares well with the experimental result $(5.6 \pm 1.5) \times 10^{-6}$ (see Table 7 in [14] for a compilation of the experimental data on charmless hadronic B decays). The theoretical uncertainties quoted are due to input parameter variations and to the modeling of the leading power corrections. An additional large uncertainty comes from the present error on $|V_{ub}|$ and the semileptonic $B \to \pi$ form factor. The sensitivity to these quantities can be eliminated by taking the ratio

$$\frac{\Gamma(B^\pm \to \pi^\pm\pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+ l^- \bar{\nu})/dq^2|_{q^2=0}} = 3\pi^2 f_\pi^2 \underbrace{|a_1^{(\pi\pi)} + a_2^{(\pi\pi)}|^2}_{1.33^{+0.20}_{-0.11}\,(\mathrm{pars.})\pm 0.07\,(\mathrm{power})} = (0.68\,^{+0.11}_{-0.06})\,\mathrm{GeV}^2 .$$

This prediction includes a sizeable ($\sim 25\%$) contribution of the hard-scattering term in the factorization formula (the lower graph on the right-hand side in Figure 2). Unfortunately, this ratio has not yet been measured experimentally.

Magnitude of the T/P Ratio. The magnitude of the leading $B \to \pi K$ penguin amplitude can be probed in the decays $B^\pm \to \pi^\pm K^0$, which to an excellent approximation do not receive any tree contributions. Combining it with the measurement of the tree amplitude just described, a tree-to-penguin ratio can be determined via the relation

$$\varepsilon_{\mathrm{exp}} = \left|\frac{T}{P}\right| = \tan\theta_C\,\frac{f_K}{f_\pi}\left[\frac{2\mathrm{Br}(B^\pm \to \pi^\pm\pi^0)}{\mathrm{Br}(B^\pm \to \pi^\pm K^0)}\right]^{\frac{1}{2}} = 0.223 \pm 0.034 .$$

The quoted experimental value of this ratio is in good agreement with the theoretical prediction $\varepsilon_{\mathrm{th}} = 0.24 \pm 0.04\,(\mathrm{pars.}) \pm 0.04\,(\mathrm{power}) \pm 0.05\,(V_{ub})$ [14], which is independent of form factors but proportional to $|V_{ub}/V_{cb}|$. This is a highly nontrivial test of the QCD factorization approach. Recall that when the first measurements of charmless hadronic decays appeared several authors remarked that the penguin amplitudes were much larger than expected based on naive factorization models. We now see that QCD factorization reproduces naturally (i.e., for central values of all input parameters) the correct magnitude of the tree-to-penguin ratio. This observation also shows that there is no need to supplement the QCD factorization predictions in an ad hoc way by adding enhanced phenomenological penguin amplitudes, such as the "nonperturbative charming penguins" introduced in [33]. In their most recent paper [34], the advocates of charming penguins parameterize the effects of these animals in terms of a nonperturbative "bag

TABLE 1. Direct CP asymmetries in $B \to \pi K$ decays

	Experiment [35, 36, 37, 38]	Theory Beneke et al. [14]	Keum et al. [32]	Ciuchini et al. [33]
$A_{\rm CP}(\pi^+K^-)$ (%)	-4.8 ± 6.8	5 ± 9	-18	$\pm(17 \pm 6)$
$A_{\rm CP}(\pi^0K^-)$ (%)	-9.6 ± 11.9	7 ± 9	-15	$\pm(18 \pm 6)$
$A_{\rm CP}(\pi^-\bar{K}^0)$ (%)	-4.7 ± 13.9	1 ± 1	-2	$\pm(3 \pm 3)$

parameter" $\hat{B}_1 = (0.13 \pm 0.02)\, e^{i(188\pm 82)^\circ}$ fitted to the data on charmless decays. By definition, this parameter contains the contribution from the perturbative charm loop, which is calculable in QCD factorization. Using the factorization approach as described in [14] we find that $\hat{B}_1^{\rm fact} = (0.09\,^{+0.03+0.04}_{-0.02-0.02})\, e^{i(185\pm 3\pm 21)^\circ}$, where the errors are due to input parameter variations and the estimate of power corrections. The perturbative contribution to the central value is 0.08; the remaining 0.01 is mainly due to weak annihilation. We conclude that, within errors, QCD factorization can account for the "charming penguin bag parameter", which is, in fact, dominated by short-distance physics.

Strong Phase of the T/P Ratio. QCD factorization predicts that (most) strong-interaction phases in charmless hadronic B decays are parametrically suppressed in the heavy-quark limit, i.e., $\sin\phi_{\rm st} = O[\alpha_s(m_b), \Lambda_{\rm QCD}/m_b]$. This implies small direct CP asymmetries since, e.g., $A_{\rm CP}(\pi^+K^-) \simeq -2\,|\frac{T}{P}|\sin\gamma\sin\phi_{\rm st}$. The suppression results as a consequence of systematic cancellations of soft contributions, which are missed in phenomenological models of final-state interactions. In many other schemes the strong-interaction phases are predicted to be much larger, and therefore larger CP asymmetries are expected. Table 1 shows that first experimental data provide no evidence for large direct CP asymmetries in $B \to \pi K$ decays. However, the errors are still too large to draw a definitive conclusion that would allow us to distinguish between different theoretical predictions.

Remarks on Sudakov Logarithms

In recent years, Li and collaborators have proposed an alternative scheme for calculating nonleptonic B decay amplitudes based on a perturbative hard-scattering approach [32]. From a conceptual point of view, the main difference between QCD factorization and this so-called pQCD approach lies in the latter's assumption that Sudakov form factors effectively suppress soft-gluon exchange in diagrams such as those shown in the graphs on the right-hand side in Figure 2. As a result, the $B \to \pi$ and $B \to K$ form factors are assumed to be perturbatively calculable. This changes the counting of powers of α_s. In particular, the nonfactorizable gluon exchange diagrams included in the QCD factorization approach, which are crucial in order to cancel the scale and scheme-dependence in the predictions for the decay amplitudes, are formally of order α_s^2 in the pQCD scheme and consequently are left out. Thus, to the considered order there are no loop graphs that could give rise to strong-interaction phases in that scheme. (However, in [32] large phases are claimed to arise from on-shell poles of massless propagators in tree diagrams. These phases are entirely dominated by soft physics. Hence, the prediction of large direct CP asymmetries in the pQCD approach rests on assumptions that are strongly model dependent.)

The assumption of Sudakov suppression in hadronic B decays is questionable, because the relevant "large" scale $Q^2 \sim m_b \Lambda_{\rm QCD} \sim 1\,{\rm GeV}^2$ is in fact not large for realistic b-quark masses. Indeed, one finds that the pQCD calculations are very sensitive to details of the $p_\perp$ dependence of the wave functions [39]. This sensitivity to infrared physics invalidates the original assumption of an effective suppression of soft contributions. The argument just presented leaves open the conceptual question whether Sudakov logarithms are relevant in the asymptotic limit $m_b \to \infty$. This question has not yet been answered in a satisfactory way.

NEW CONSTRAINTS ON THE UNITARITY TRIANGLE

The QCD factorization approach, combined with a conservative estimate of power corrections, offers several new strategies to derive constraints on CKM parameters. This has been discussed at length in [14], to which we refer the reader for details. Some of these strategies will be illustrated below. Note that the applications of QCD factorization are not limited to computing branching ratios. The approach is also useful in combination with other ideas based on flavor symmetries and amplitude relations. In this way, strategies can be found for which the residual hadronic uncertainties are simultaneously suppressed by three small parameters, since they vanish in the heavy-quark limit ($\sim \Lambda_{\rm QCD}/m_b$), the limit of SU(3) flavor symmetry ($\sim (m_s - m_q)/\Lambda_{\rm QCD}$), and the large-$N_c$ limit ($\sim 1/N_c$).

Determination of γ with minimal theory input. Some years ago, Rosner and the present author have derived a bound on γ by combining measurements of the ratios $\varepsilon_{\rm exp} = |T/P|$ and $R_* = \frac{1}{2}\Gamma(B^\pm \to \pi^\pm K^0)/\Gamma(B^\pm \to \pi^0 K^\pm)$ with the fact that for an arbitrary strong phase $-1 \le \cos\phi_{\rm st} \le 1$ [40]. The model-independent observation that $\cos\phi_{\rm st} = 1$ up to second-order corrections to the heavy-quark limit can be used to turn this bound into a determination of γ (once $|V_{ub}|$ is known). The resulting constraints in the $(\bar\rho,\bar\eta)$ plane, obtained under the conservative assumption that $\cos\phi_{\rm st} > 0.8$ (corresponding to $|\phi_{\rm st}| < 37^\circ$) are shown in the left-hand plot in Figure 3 for several illustrative values of the ratio R_*. Note that for $0.8 < R_* < 1.1$ (the range preferred by the Standard Model) the theoretical uncertainty reflected by the widths of the bands is smaller than for any other constraint on $(\bar\rho,\bar\eta)$ except for the one derived from the $\sin 2\beta$ measurement. With present data the Standard Model is still in good shape, but it will be interesting to see what happens when the experimental errors are reduced.

Determination of $\sin 2\alpha$. With the help of QCD factorization it is possible to control the "penguin pollution" in the time-dependent CP asymmetry in $B \to \pi^+\pi^-$ decays, defined such that $S_{\pi\pi} = \sin 2\alpha \cdot [1 + O(P/T)]$. This is illustrated in the right-hand plot in Figure 3, which shows the constraints imposed by a measurement of $S_{\pi\pi}$ in the $(\bar\rho,\bar\eta)$ plane. It follows that even a result for $S_{\pi\pi}$ with large experimental errors would imply a useful constraint on the unitarity triangle. A first, preliminary measurement of the

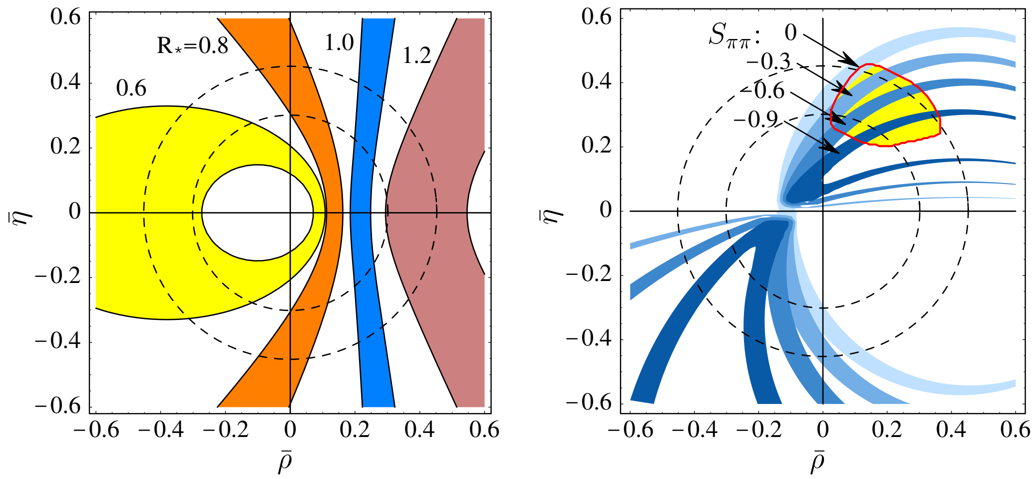
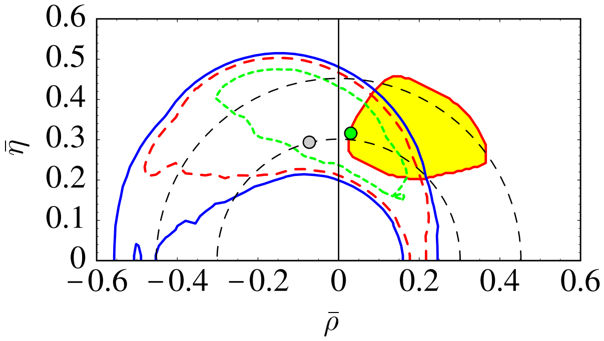


FIGURE 3. Left: Allowed regions in the $(\bar{\rho},\bar{\eta})$ plane corresponding to $\varepsilon_{\rm exp} = 0.22$ and different values of the ratio R_* as indicated. The widths of the bands reflect the total theoretical uncertainty. The current experimental values are $\varepsilon_{\rm exp} = 0.22 \pm 0.03$ and $R_* = 0.71 \pm 0.14$. Right: Allowed regions in the $(\bar{\rho},\bar{\eta})$ plane corresponding to different values of the mixing-induced CP asymmetry $S_{\pi\pi}$. The widths of the bands reflect the total theoretical uncertainty. The corresponding bands for positive values of $S_{\pi\pi}$ are obtained by a reflection about the $\bar{\rho}$ axis. The bounded light area is the allowed region obtained from the standard global fit of the unitarity triangle [1].

asymmetry has been presented by the BaBar Collaboration this summer. Their result is $S_{\pi\pi} = 0.03^{+0.53}_{-0.56} \pm 0.11$ [38].

Global Fit to $B \to \pi K, \pi\pi$ Branching Ratios. Various ratios of CP-averaged $B \to \pi K, \pi\pi$ branching fractions exhibit a strong dependence on γ and $|V_{ub}|$, or equivalently, on the parameters $\bar{\rho}$ and $\bar{\eta}$ of the unitarity triangle. From a global analysis of the experimental data in the context of the QCD factorization approach it is possible to derive constraints in the $(\bar{\rho},\bar{\eta})$ plane in the form of regions allowed at various confidence levels. The results are shown in Figure 4. The best fit of the QCD factorization theory to the data yields an excellent $\chi^2/n_{\rm dof}$ of less than 0.5. (We should add at this point that we disagree with the implementation of our approach presented in [34] and, in particular, with the numerical results labeled “BBNS” in Table II of that paper, which led the authors to the premature conclusion that the “theory of QCD factorization ... is insufficient to fit the data”. Even restricting $(\bar{\rho},\bar{\eta})$ to lie within the narrow ranges adopted by these authors, we can find parameter sets for which QCD factorization fits the data with a good $\chi^2/n_{\rm dof}$ of less than 1.5.)

The results of this global fit are compatible with the standard CKM fit using semileptonic decays, K–$\bar{K}$ mixing and B–$\bar{B}$ mixing ($|V_{ub}|$, $|V_{cb}|$, ε_K, Δm_d, Δm_s, $\sin 2\beta$), although the fit prefers a slightly larger value of γ and/or a smaller value of $|V_{ub}|$. The combination of the results from rare hadronic B decays with $|V_{ub}|$ from semileptonic decays excludes $\bar{\eta} = 0$ at 95% CL, thus showing first evidence for the existence of a CP-violating phase in the bottom sector. In the near future, when the data become more precise, this will provide a powerful test of the CKM paradigm.



Decay Mode	Best Fit	Exp. Average
$B^0 \to \pi^+\pi^-$	4.6	4.4 ± 0.9
$B^\pm \to \pi^\pm\pi^0$	5.3	5.6 ± 1.5
$B^0 \to \pi^\mp K^\pm$	17.9	17.2 ± 1.5
$B^\pm \to \pi^0 K^\pm$	11.3	12.1 ± 1.7
$B^\pm \to \pi^\pm K^0$	17.7	17.2 ± 2.5
$B^0 \to \pi^0 K^0$	7.1	10.3 ± 2.5

FIGURE 4. 95% (solid), 90% (dashed) and 68% (short-dashed) confidence level contours in the $(\bar{\rho}, \bar{\eta})$ plane obtained from a global fit of QCD factorization results to the CP-averaged $B \to \pi K, \pi\pi$ branching fractions. The dark dot shows the overall best fit, whereas the light dot indicates the best fit for the default choice of all theory input parameters. The table compares the best fit values for the various CP-averaged branching fractions (in units of 10^{-6}) with the world average data.

OUTLOOK

The QCD factorization approach provides the theoretical framework for a systematic analysis of hadronic and radiative exclusive B decay amplitudes based on the heavy-quark expansion. This theory has already passed successfully several nontrivial tests, and will be tested more thoroughly with more precise data. A new effective field-theory language appropriate to QCD factorization is emerging in the form of the collinear–soft effective theory. Ultimately, the developments reviewed in this talk may lead to theoretical control over a vast variety of exclusive B decays, giving us new constraints on the unitarity triangle.

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The second version of this talk was delivered on 11 September 2001, a few hours after terrorist attacks hit various targets in the U.S. Our thoughts and sympathy are with the many innocent victims of this tragedy.

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