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Mass renormalisation for improved staggered quarks

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Improved staggered quark actions are designed to suppress flavour changing strong interactions. We discuss the perturbation theory for this type of actions and show the improvements to reduce the quark mass renormalisation compared to naïve staggered quarks. The renormalisations are of similar size as for Wilson quarks.

1. INTRODUCTION

The naïve discretisation of the fermionic action leads to additional poles in the corners of the Brillioun zone. Such a theory describes several flavours of degenerate quarks propagating through the lattice. In the case of gauge interactions this leads to flavour changes by exchange of highly virtual gauge bosons with one or several momentum components $aq_\mu \approx \pi$. For QED and QCD flavour changing interactions are unphysical. These interactions lift the degeneracy of the pions associated with the different quark flavours. They also lead to large renormalisations, originating from the high q-region in loop integrals involving fermion propagators. Due to the doubling, for high q the fermions are almost on-shell, leading to excessive contributions from this momentum region.

The staggered quark action reduces the doubling without completely eliminating it. Here we like to discuss the effect of improved staggered quark actions with suppressed flavour changing interactions on the renormalisation of the mass of staggered quarks. For a discussion on the splitting of the pions see for example D. Toussaint's review [1] and the references therein.

2. FAT LINKS

By fattening the links in the action, the coupling of the quarks to highly virtual gluons can be suppressed [2,3]. This is part of the Symanzik improvement program for staggered quarks. The gauge link variables $U_{x,\nu}$ have to be replaced with a fat-link $V_{x,\nu}$; Δ_ν denoting a covariant second lattice derivative

$$V_{x,\mu} := \prod_{\nu\neq\mu} \left(1 + \frac{\Delta_\nu}{4}\right)\bigg|_{\text{symm.}} U_{x,\mu}\,. \tag{1}$$

This replacement removes the coupling to high momentum gluons at tree-level. The fat links introduce a flavour conserving $\mathcal{O}(a^2)$ artifact. This can be removed by further replacing $V_{x,\mu} \to V'_{x,\mu}$

$$V'_{x,\mu} = V_{x,\mu} - \frac{1}{4}\sum_{\substack{\nu \\ \nu\neq\mu}} (\nabla_\rho)^2 U_{x,\mu}\,. \tag{2}$$

The $\mathcal{O}(a^2)$ improvement of the staggered quark action is completed by the addition of the Naik term to the derivative. Our results including the Naik term are still preliminary. Further improvements removing flavour changing interactions from the action at $\mathcal{O}(\alpha_s)$ are reported in [4].

3. QUARK-ANTIQUARK-GLUON VERTEX

The Feynman rules display nicely how the fattening suppresses the coupling between the quark-antiquark pair and a single gluon with large transverse momentum. Let us consider the quark-antiquark-gluon vertex

$$\mathcal{V}_1 \;=\; \mathrm{i}g\Bigg[\gamma_\mu \cos(\tfrac{1}{2}(p+q)_\mu) E^{(1)}_\mu(q-p)\,,$$

$$+\sum_{\nu\neq\mu}\gamma_\nu\cos(\tfrac{1}{2}(p+q)_\nu)\,G^{(1)}_{\mu,\nu}(q-p)\Bigg]T^b\,. \tag{3}$$

For the naïve action one gets $E^{(1)}_\mu = 1$ and $G^{(1)}_{\mu,\nu} = 0$ independent of momentum. For fat links according to eq. (1) we obtain

$$E^{(1)}_\mu(k) = \prod_{\substack{\nu\\ \nu\neq\mu}}\cos^2(\tfrac{1}{2}k_\nu)\,, \tag{4}$$

$$G^{(1)}_{\nu,\mu}(k) = \sin(\tfrac{1}{2}k_\nu)\sin(\tfrac{1}{2}k_\mu)\Bigg[\frac{1}{3}\prod_{\substack{\sigma\\ \sigma\neq\mu,\nu}}\cos^2(\tfrac{1}{2}k_\sigma) + \frac{1}{6}\sum_{\substack{\sigma\\ \sigma\neq\mu,\nu}}\cos^2(\tfrac{1}{2}k_\sigma) + \frac{1}{3}\Bigg]\,. \tag{5}$$

Including the improvement term in eq. (2) this becomes

$$E^{(1)\prime}_\mu(k) = E^{(1)}_\mu(k) + \frac{1}{4}\sum_{\substack{\nu\\ \nu\neq\mu}}\sin^2(k_\nu)\,, \tag{6}$$

$$G^{(1)}_{\sigma,\mu}{}'(k) = G^{(1)}_{\sigma,\mu}(k) - \frac{1}{2}\sin(\tfrac{1}{2}k_\mu)\sin(k_\sigma)\cos(\tfrac{1}{2}k_\sigma)\,. \tag{7}$$

Fig. 1 shows the momentum behaviour of the coefficient functions $E^{(1)}_\mu$ and $G^{(1)}_{\sigma,\mu}$ for the different actions. In the case of fat links $E^{(1)}_\mu \approx 1$ for small transverse momenta, which is the physically relevant region. For the large flavour changing momenta $ak_\nu \approx \pi$ the coupling vanishes as intended. The figure also shows the improvements for $k_\nu \approx 0$ arising from using $V'_{x,\mu}$.

The coefficient $G^{(1)}_{\sigma,\mu}$ describes an unphysical coupling between a gluon field A_σ to the γ_μ-term in the action. Such terms arises from the sides of the staples when fattening the links. Fortunately this coupling vanishes in the physical region around $k = 0$ as well as in the corner of the Brillioun zone. Flavour changing interactions are not reentering through the back door. Again $V'_{x,\mu}$ gives a substantial improvement for $k \approx 0$. We also worked out the Feynman rules for the quark-antiquark-gluon-gluon Vertex as needed for the tadpole diagram.

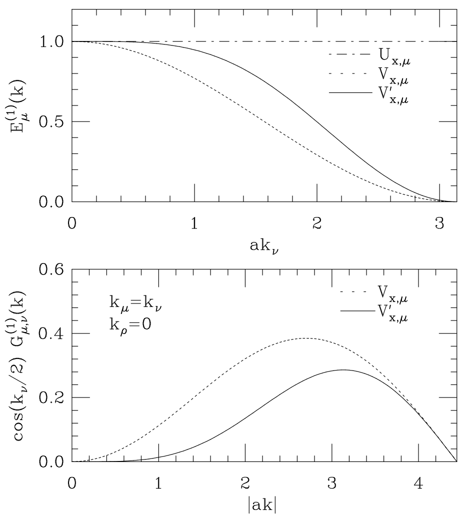

Figure 1. Coeffcient functions for the single gluon vertex

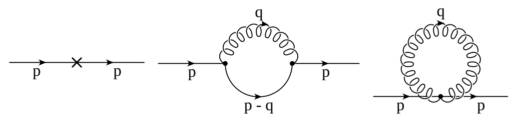

Figure 2. Feynman diagrams for the self-energy: counter term, rainbow and tadpole diagram

4. MASS RENORMALISATION

We use the following definitions to relate the bare mass m in the action and the renormalised pole mass m_{pole}

$$m_{\text{pole}} = m\left(1 + \frac{\delta m^{(0)}}{m} + \frac{g^2}{4\pi}\frac{\delta m^{(2)}}{m} + \mathcal{O}(g^4)\right). \tag{8}$$

For the self energy $\Sigma^{(2)}$ we use the on-shell condition $p_0 = \mathrm{i}(m+\delta m^{(0)})$, $p_m = 0$. The diagrams for $\Sigma^{(2)}$ are shown in fig. 2. The rainbow diagram is the one which is affected by the flavour changing interactions.

Tadpole improvement [5] turns out to be crucial for the fat-link actions. In Feynman gauge we observe an increased tadpole contribution for fat links. This is always matched by similar sized tadpole improvement counter term of opposite sign. To achieve this cancellation, tadpole im-

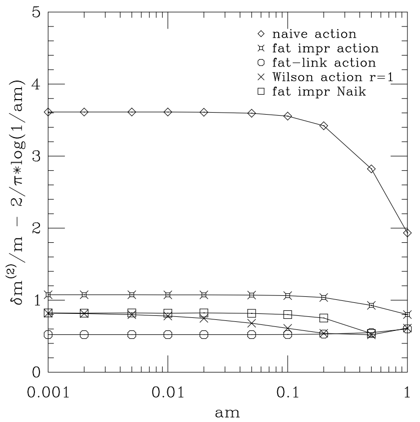

Figure 3. Mass shift $\delta m^{(2)}$ for different actions.

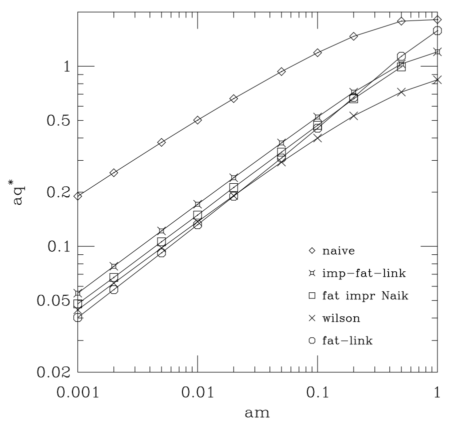

Figure 4. Momentum scale aq^* for different actions.

provement has to be implemented after working out the higher derivatives of the link operators in the action [6]. The total contribution of these two graphs to $\delta m^{(2)}/m$ was about $\frac{1}{2}$ or smaller.

For $am \to 0$ the mass shift $\delta m^{(2)}/m$ diverges as $\frac{2}{\pi}\log(\frac{1}{am})$. Our result for $\delta m^{(2)}/m$ is presented in fig. 3, where the above divergence is subtracted. We used tadpole improvement as defined from the average link in Landau gauge and the gluon propagator from the Wilson gauge action. The naïve action leads to the well known large renormalisation, which is substantially reduced by the introduction of the fat links. The figure also shows the effect of including eq. (2) and the Naik improvement term. These terms have little effect on the result compared to the fattening of the links. The figure also includes standard Wilson quarks. Their renormalisations are shown to be of similar size as for the fat-link quarks. This comparison clearly shows flavour changing interactions to be the cause of the large renormalisations.

In this context it is interesting to have a look at the momentum scale aq^* [5], which can be understood as the "average loop momentum". One observes a significantly larger q^* for the naïve action compared to all other actions. This larger q^* can be traced to the flavour changing interactions in the rainbow diagram.

5. CONCLUSION AND OUTLOOK

Improved staggered quarks suppress strong flavour changing interactions by fattening the links. This improvement reduces the known large mass renormalisations for naïve staggered quarks to the same level as for Wilson quarks. This will allow for a reliable calculation of quark masses using staggered quarks in the future. Preliminary results for current and four quark operators indicate reduced renormalisations, when using improved staggered quarks [4].

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