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Comments on Color-Suppressed Hadronic B Decays

Matthias Neubert and Alexey A. Petrov
Newman Laboratory of Nuclear Studies, Cornell University
Ithaca, NY 14853, USA

Abstract

Recent experimental results on the color-suppressed nonleptonic decays $\bar{B}^0 \to D^{(*)0}\pi^0$ provide evidence for a failure of the naive factorization model and for sizeable relative strong-interaction phases between class-1 and class-2 $\bar{B} \to D^{(*)}\pi$ decay amplitudes. The allowed regions for the corresponding ratios of (complex) isospin amplitudes and a_2/a_1 parameters are determined. The results are interpreted in the context of QCD factorization for the related class-1 amplitudes in the heavy-quark limit.

The problem of understanding nonperturbative strong-interaction effects in exclusive nonleptonic weak decays of hadrons has always been a challenge to theorists. Only in a few cases model-independent results based on controlled expansions in QCD can be obtained. In the absence of a quantitative theoretical description various attempts have been made to obtain simple, predictive parameterizations of decay amplitudes based on simple phenomenological assumptions.

The most common of these approaches is the "naive" (or "generalized") factorization model, in which the decay amplitudes are estimated by replacing hadronic matrix elements of four-quark operators in the effective weak Hamiltonian by products of current matrix elements determined in terms of meson decay constants and semileptonic form factors. "Nonfactorizable" strong-interaction effects are parameterized by phenomenological coefficients a_i, which depend on the color and Dirac structure of the operators but otherwise are postulated to be universal [1, 2, 3]. One distinguishes class-1 and class-2 decay topologies, which refer to the cases where a charged (class-1) or a neutral (class-2) final-state meson can be produced from the quarks contained in the four-quark operators of the effective Hamiltonian. For instance, in decays based on the quark transition $b \to c\bar{u}d$ mesons with quark content $(\bar{u}d)$ or $(\bar{u}c)$ can be produced in that way. The decay $\bar{B}^0 \to D^+\pi^-$ is a class-1 process, in which the pion can be generated at the weak vertex, whereas $\bar{B}^0 \to D^0\pi^0$ is a class-2 process, in which the D^0 meson can be directly produced. The corresponding amplitudes are then expressed as

$$\begin{aligned} \mathcal{A}(\bar{B}^0 \to D^+\pi^-) &= i\,\frac{G_F}{\sqrt{2}}\,V_{cb}V_{ud}^*\,(m_B^2 - m_D^2)\,f_\pi\,F_0^{B\to D}(m_\pi^2)\,a_1(D\pi)\,, \\ \sqrt{2}\,\mathcal{A}(\bar{B}^0 \to D^0\pi^0) &= i\,\frac{G_F}{\sqrt{2}}\,V_{cb}V_{ud}^*\,(m_B^2 - m_\pi^2)\,f_D\,F_0^{B\to\pi}(m_D^2)\,a_2(D\pi)\,, \end{aligned} \tag{1}$$

where $F_0^{B\to M}(q^2)$ are $B \to M$ form factors at momentum transfer q^2. In other processes such as $B^- \to D^0\pi^-$ both topologies can contribute and interfere. (Such processes are sometimes called class-3 decays.) In fact, isospin symmetry implies that

$$\mathcal{A}(B^- \to D^0\pi^-) = \mathcal{A}(\bar{B}^0 \to D^+\pi^-) + \sqrt{2}\,\mathcal{A}(\bar{B}^0 \to D^0\pi^0)\,. \tag{2}$$

The large-N_c counting rules of QCD show that $a_1(D\pi) = O(1)$ and $a_2(D\pi) = O(1/N_c)$, which is why the class-2 decays are often referred to as "color suppressed".

In the naive factorization model one postulates that for a large class of energetic, two-body (or quasi two-body) B decays the coefficients a_1 and a_2 are process-independent phenomenological parameters. These parameters are assumed to be real, ignoring the possibility of relative strong-interaction phases between class-1 and class-2 amplitudes. Surprisingly, despite their crudeness these assumptions seemed to be supported by experimental data [2, 3]. Within errors, the class-1 decays $\bar{B}^0 \to D^{(*)+}M^-$ with $M = \pi, \rho, a_1, D_s, D_s^*$ can be described using a universal value $|a_1| \approx 1.1 \pm 0.1$, whereas the class-2 decays $\bar{B} \to \bar{K}^{(*)}M$ with $M = J/\psi, \psi(2S)$ suggest a nearly universal value $|a_2| \approx 0.2$–0.3 (which is more uncertain due to the uncertainty in the $B \to K^{(*)}$ form factors). Moreover, the class-3 decays $B^- \to D^{(*)0}M^-$ with $M = \pi, \rho$, which are sensitive to the interference of the two decay topologies, could be explained by a real, positive ratio $a_2/a_1 \approx 0.2$–0.3, which seemed to agree with the determinations of $|a_1|$ and $|a_2|$ from other decays. The missing link in this line of argument was a direct measurement of $|a_2|$ in the related class-2 decays $\bar{B}^0 \to D^{(*)0}M^0$.

Recently, the idea of factorization in the class-1 decays $\bar{B}^0 \to D^{(*)+}L^-$, where L is a *light* meson, was put on a more rigorous footing. It was shown that the corresponding decay amplitudes can be systematically calculated in QCD in the limit where the decaying b quark is considered a heavy quark [4, 5, 6] (see [7, 8, 9] for related earlier work). To leading order in Λ/m_b (with Λ a typical hadronic scale), but to all orders of perturbation theory, nonfactorizable strong-interaction effects can be described in terms of convolutions of hard-scattering kernels with the leading-twist light-cone distribution amplitude of the light meson L. The resulting QCD factorization formula allows us to compute the magnitude and phase of the parameters $a_1(D^{(*)}L)$ systematically up to power corrections in Λ/m_b. The values of a_1 in different class-1 decays are not universal, but the process-dependent corrections turn out to be numerically small [5]. Several types of power corrections to the a_1 parameters have been estimated and found to be small [5, 10, 11, 12]. Hence, for the cases where L is a light meson there is now a solid theoretical understanding of the near-universal value $|a_1| \approx 1.1$ observed experimentally.

On the other hand, if the charm quark is treated as a heavy quark, then the QCD factorization formula does *not* apply for the class-2 decays $\bar{B}^0 \to D^{(*)0}L^0$, and so the magnitude and phase of the $a_2(D^{(*)}L)$ parameters are not calculable. The only nontrivial prediction in this case is that the class-2 amplitudes are power suppressed with respect to the corresponding class-1 amplitudes [5]. The apparent universality of the $|a_2|$ values extracted from experiment, and the absence of sizeable relative strong-interaction phases between the various a_1 and a_2 parameters suggested by the data, remained a theoretical puzzle. (Even before the advent of QCD factorization various authors had presented arguments against the universality of nonfactorizable effects in class-2 decays; see, e.g., [13, 14, 15].) New experimental data announced by the CLEO and Belle Collaborations [16, 17] change the picture significantly, in a way that is entirely in line with QCD expectations. As we will illustrate below, these data provide compelling evidence for *process-dependent* a_2 values, and for large *relative strong-interaction phases* between related class-1 and class-2 amplitudes. This shows that the "naive" factorization model is too simple to account for the data.

Because QCD factorization cannot be justified for the class-2 decays, it is in some sense misleading to parameterize the $\bar{B}^0 \to D^0\pi^0$ decay amplitude as done in (1). Although the naive factorization contribution $a_2 = C_2 + \zeta\, C_1$ (with ζ a nonperturbative parameter of order $1/N_c$) is certainly present, it is not a leading contribution to the decay amplitude in any consistent limit of QCD. For instance, there exists a weak annihilation contribution to a_2 which scales like a power of m_b/Λ in the heavy-quark limit and thus formally dominates over the naive factorization piece. (Weak annihilation is mentioned here only as an example of a leading contribution to the class-2 amplitude. Model calculations suggest that the annihilation contribution in $\bar{B} \to D\pi$ decays is nevertheless small [5]. Another example of a leading contribution is charge-exchanging rescattering from the dominant class-1 channel [18, 19].) Also, for class-2 decays naive factorization

Table 1: Heavy-quark scaling laws in the schemes where the charm quark is treated as a heavy quark (left) or as a light quark (right)

	$m_c \sim m_b \sim m_Q$	$m_c \sim \Lambda \ll m_b$
$\lvert A_{1/2}/\sqrt{2}A_{3/2}\rvert$	$1 + O(\Lambda/m_Q)$	$O(1)$
$\delta_{1/2} - \delta_{3/2}$	$O(\Lambda/m_Q)$	$O[\alpha_s(m_b)]$

does not emerge in the large-N_c limit. It is then more appropriate to employ an alternative parameterization of the decay amplitudes in terms of isospin amplitudes $A_{1/2}$ and $A_{3/2}$ corresponding to transitions into $D\pi$ final states with $I = \frac{1}{2}$ and $\frac{3}{2}$, respectively. It is given by

$$\begin{aligned} \mathcal{A}(\bar{B}^0 \to D^+\pi^-) &= \sqrt{\frac{1}{3}}\, A_{3/2} + \sqrt{\frac{2}{3}}\, A_{1/2}\,, \\ \sqrt{2}\,\mathcal{A}(\bar{B}^0 \to D^0\pi^0) &= \sqrt{\frac{4}{3}}\, A_{3/2} - \sqrt{\frac{2}{3}}\, A_{1/2}\,, \\ \mathcal{A}(B^- \to D^0\pi^-) &= \sqrt{3}\, A_{3/2}\,. \end{aligned} \tag{3}$$

An identical decomposition holds for other decays such as $\bar{B} \to D^*\pi$ and $\bar{B} \to D^{(*)}\rho$. It follows from QCD factorization that the ratio of isospin amplitudes is [5]

$$\frac{A_{1/2}}{\sqrt{2}\,A_{3/2}} = 1 + O(\Lambda/m_Q)\,, \tag{4}$$

which also implies that the relative strong-interaction phase $\delta_{1/2} - \delta_{3/2} = O(\Lambda/m_Q)$. Here m_Q represents either one of m_c and m_b. Note that the corrections to the "1" in (4) are also formally suppressed by a power of $1/N_c$, and it has been argued that perhaps this color suppression may be more relevant to factorization than the heavy-quark limit [20]. However, an identical $1/N_c$ argument would apply to other nonleptonic decays such as $D \to \bar{K}^{(*)}\pi$ and $K \to \pi\pi$, for which color suppression is clearly not operative. Apart from trivial substitutions of quark flavors, the only difference between, say, $\bar{B} \to D^{(*)}\pi$ and $D \to \bar{K}^{(*)}\pi$ decays is the larger energy release in the decay of a heavy b quark, which leads to color transparency and thus is the basis of QCD factorization.

The deviation of the ratio $A_{1/2}/(\sqrt{2}\,A_{3/2})$ from 1 is a measure of the departure from the heavy-quark limit. When contemplating about the expected magnitude of this effect, it is important to realize that the *power suppression* of the corrections in (4) relies on the heaviness of the charm quark, not only the b quark. In order to illustrate this fact it is instructive to consider two different power-counting schemes for the heavy-quark expansion. The most natural scheme, which underlies our discussion so far, is to consider both beauty and charm as heavy quarks with their mass ratio m_c/m_b fixed in the heavy-quark limit. Then the deviation of $A_{1/2}/(\sqrt{2}\,A_{3/2})$ from 1 is power suppressed in Λ/m_Q,

Table 2: Experimental data for the $\bar{B} \to D^{(*)}\pi$ and $D \to \bar{K}^{(*)}\pi$ branching ratios (in units of 10^{-3}), isospin amplitudes, and related quantities

	$\bar{B} \to D\pi$	$\bar{B} \to D^*\pi$		$D \to \bar{K}\pi$	$D \to \bar{K}^*\pi$
$\bar{B}^0 \to D^{(*)+}\pi^-$	3.0 ± 0.4	2.76 ± 0.21	$D^0 \to K^{(*)-}\pi^+$	38.3 ± 0.9	50 ± 4
$\bar{B}^0 \to D^{(*)0}\pi^0$	0.27 ± 0.05	0.17 ± 0.05	$D^0 \to \bar{K}^{(*)0}\pi^0$	21.1 ± 2.1	31 ± 4
$B^- \to D^{(*)0}\pi^-$	5.3 ± 0.5	4.6 ± 0.4	$D^+ \to \bar{K}^{(*)0}\pi^+$	28.9 ± 2.6	19.0 ± 1.9
$\lvert A_{1/2}/\sqrt{2}\,A_{3/2}\rvert$	0.70 ± 0.11	0.72 ± 0.08	$\lvert A_{1/2}/\sqrt{2}\,A_{3/2}\rvert$	2.71 ± 0.14	3.97 ± 0.25
$\lvert\delta_{1/2} - \delta_{3/2}\rvert$	$(27 \pm 7)^\circ$	$(21 \pm 8)^\circ$	$\lvert\delta_{1/2} - \delta_{3/2}\rvert$	$(90 \pm 6)^\circ$	$(104 \pm 13)^\circ$
$x\,\lvert a_2/a_1\rvert$	0.42 ± 0.05	0.35 ± 0.05	$x\,\lvert a_2/a_1\rvert$	1.05 ± 0.05	1.11 ± 0.08
$\arg(a_2/a_1)$	$(56 \pm 20)^\circ$	$(51 \pm 20)^\circ$	$\arg(a_2/a_1)$	$(149 \pm 2)^\circ$	$(160 \pm 2)^\circ$
$x\,a_2^{\rm eff}/a_1^{\rm eff}$	0.25 ± 0.12	0.23 ± 0.08	$x\,a_2^{\rm eff}/a_1^{\rm eff}$	-0.53 ± 0.02	-0.66 ± 0.02
$a_2^{\rm eff}/a_1^{\rm eff}$	≈ 0.28	≈ 0.25	$a_2^{\rm eff}/a_1^{\rm eff}$	≈ -0.44	≈ -0.35

where $m_Q \sim m_b \sim m_c$. It is not calculable without model dependence. Alternatively, one may consider the charm quark as a light quark with its mass kept fixed in the heavy-quark limit [5]. In such a scheme the class-2 amplitude becomes calculable and of leading power in the limit $m_b \gg \Lambda$ [21]. The leading deviation of the isospin amplitude ratio from 1 is then computable in terms of Wilson coefficient functions $C_i(m_b)$ and short-distance corrections proportional to $\alpha_s(m_b)$. The scaling of the relevant quantities in these two versions of the heavy-quark limit is summarized in Table 1. The conventional scheme is, perhaps, somewhat closer to reality. However, considering that the charm quark is not very heavy in the real world, we may expect a sizeable deviation of the amplitude ratio from 1.

We now investigate to what extent the prediction (4) is supported by the data on $\bar{B} \to D^{(*)}\pi$ decays. For comparison, it will be instructive to analyze the related charm decays $D \to \bar{K}^{(*)}\pi$ in parallel. The upper portion in Table 2 summarizes the experimental data on the various branching ratios. The color-suppressed B decays have just been observed for the first time experimentally [16, 17]. The preliminary results for the $\bar{B}^0 \to D^0\pi^0$ branching ratio are $(2.6 \pm 0.3 \pm 0.6) \times 10^{-4}$ (CLEO) and $(2.9\,^{+0.4}_{-0.3} \pm 0.6) \times 10^{-4}$ (Belle), while those for the $\bar{B}^0 \to D^{*0}\pi^0$ branching ratio are $(2.0 \pm 0.5 \pm 0.7) \times 10^{-4}$ (CLEO) and $(1.5\,^{+0.6}_{-0.5}\,^{+0.3}_{-0.4}) \times 10^{-4}$ (Belle). We have averaged these results to obtain the entries shown in the table. All other numbers are taken from [22]. By combining the measurements of the three branching ratios for each mode, taking into account the lifetime ratios $\tau(B^-)/\tau(\bar{B}^0) = 1.068 \pm 0.016$ [23] and $\tau(D^+)/\tau(D^0) = 2.547 \pm 0.036$ [22], one can extract the magnitude and phase of the ratio of isospin amplitudes. The results are shown in the middle portion of the table. A clear qualitative difference between beauty and charm decays emerges. Whereas the relative phases are close to maximal in D decays and the amplitude ratios are far from the asymptotic value in (4), the corrections to the

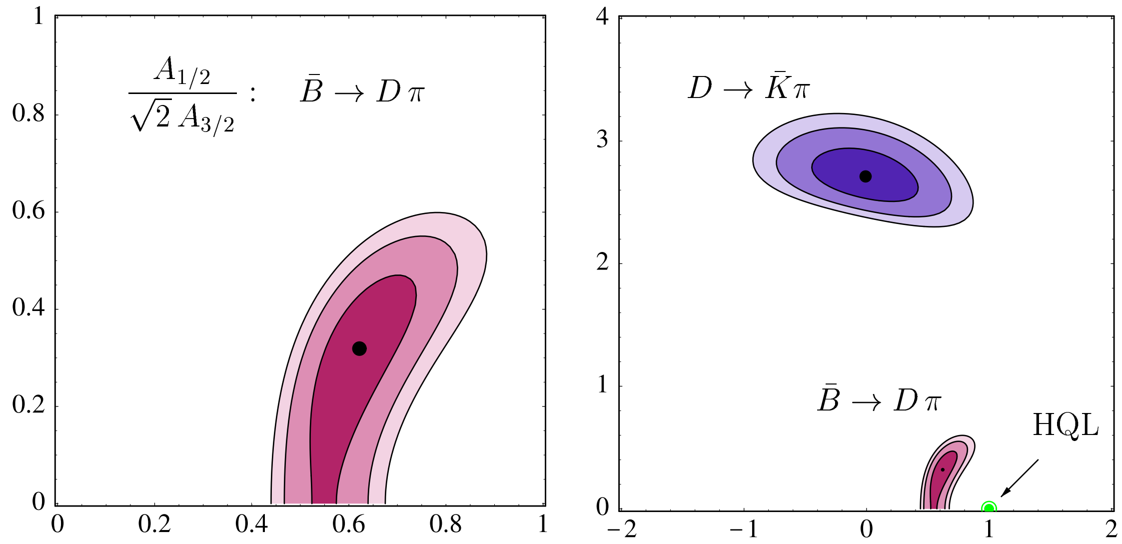

Figure 1: Allowed regions in the complex $A_{1/2}/(\sqrt{2}\,A_{3/2})$ plane obtained at 68%, 95% and 99% confidence level. The sign of the imaginary part is undetermined by the data. The black dots show the central values.

heavy-quark limit appear to be much smaller in B decays.

A more careful analysis of the experimental data for $\bar{B} \to D\pi$ and $D \to \bar{K}\pi$ decays is shown in Figure 1, which gives the allowed regions for the ratio $A_{1/2}/(\sqrt{2}\,A_{3/2})$ obtained at different confidence levels. To derive these regions we find, for each value of the isospin amplitude ratio, the minimum of the χ^2 function for the three measured branching ratios. We then plot contours of minimum χ^2 in the complex plane corresponding to a given confidence level (for two degrees of freedom). Very similar constraints can be derived for $\bar{B} \to D^*\pi$ and $D \to \bar{K}^*\pi$ decays. Note that even at the level of one standard deviation (68% confidence level) the relative strong phase of the isospin amplitudes for B decays shown in the left plot may be zero, in contrast with the naive error propagation in Table 2. In the right plot we compare the results in D and B decays and also indicate the value corresponding to the strict heavy-quark limit (HQL). We conclude that the data on the ratio of isospin amplitudes in $\bar{B} \to D^{(*)}\pi$ decays is compatible with the heavy-quark scaling laws discussed earlier. The comparison of charm to beauty decays shows a clear progression towards the heavy-quark limit as the mass of the decaying quark increases. The remaining deviation of the amplitude ratio from 1 is compatible with a correction whose suppression is governed by the large charm-quark mass.

We have mentioned earlier that the parameterization of the class-2 amplitude in (1) is somewhat misleading, because the naive factorization contribution is in no way the leading term in a controlled expansion of the decay amplitude. Nevertheless, it is instructive to extract the parameter a_2 defined via the second relation in (1) and compare it with the values of the a_2 parameters obtained from other decays, such as

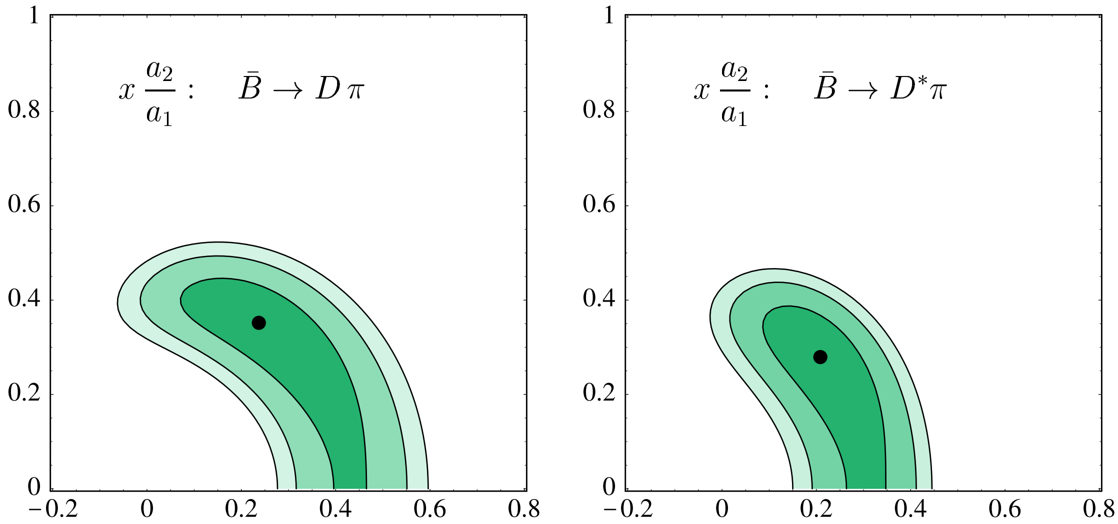

Figure 2: Allowed regions in the complex $x\,a_2/a_1$ plane for $\bar{B} \to D^{(*)}\pi$ decays, obtained at 68%, 95% and 99% confidence level. The sign of the imaginary part is undetermined by the data. The black dots show the central values.

$\bar{B} \to \bar{K}^{(*)} J/\psi$. The ratios of the $\bar{B} \to D^{(*)}\pi$ branching ratios can be expressed in terms of the ratios $x\,a_2/a_1$, where

$$x(D\pi) = \frac{(m_B^2 - m_\pi^2)\, f_D\, F_0^{B\to\pi}(m_D^2)}{(m_B^2 - m_D^2)\, f_\pi\, F_0^{B\to D}(m_\pi^2)} \approx 0.9\,,$$

$$x(D^*\pi) = \frac{f_{D^*}\, F_+^{B\to\pi}(m_{D^*}^2)}{f_\pi\, A_0^{B\to D^*}(m_\pi^2)} \approx 0.9\,. \tag{5}$$

The numerical values have been obtained using $f_D \approx 200\,\text{MeV}$, $f_{D^*} \approx 230\,\text{MeV}$, and $F_0^{B\to\pi}(m_D^2)/F_0^{B\to D}(m_\pi^2) \approx F_+^{B\to\pi}(m_{D^*}^2)/A_0^{B\to D^*}(m_\pi^2) \approx 0.5$. (The corresponding quantities for D decays may be estimated using the BSW model [1], with the result that $x(\bar{K}\pi) \approx 1.2$ and $x(\bar{K}^*\pi) \approx 1.9$.) Figure 2 shows the corresponding allowed regions for $x\,a_2/a_1$ obtained at different confidence levels. The central values are also shown in Table 2. The data prefer values $|a_2| \approx 0.4$–0.5, which are larger by almost a factor 2 than those obtained from $\bar{B} \to \bar{K}^{(*)} J/\psi$ decays (see above).[1] Moreover, the best fits prefer phases of about 50° in the two cases (with large errors), suggesting that strong final-state interactions cannot be neglected and lead to a nontrivial relative phase between class-1 and class-2 amplitudes. Both observation are in conflict with the assumptions underlying the naive factorization model. It now appears that the a_2 coefficients of the class-2 amplitudes are nonuniversal, with magnitudes and phases that may be rather

[1] Preliminary Belle data [17] on related color-suppressed decays support this conclusion. Using the form-factor model of [2], we find that the decays $\bar{B}^0 \to D^{(*)0}\eta$ and $\bar{B}^0 \to D^{(*)0}\omega$ have $|a_2|$ values between 0.4 and 0.5, with experimental errors of about 0.1 and a theoretical uncertainty of about 30%.

different for different types of decays. More precise data will be required to fully explore the pattern of QCD effects in the class-2 and class-3 amplitudes.

We should mention that also in $D \to \bar{K}^{(*)}\pi$ decays the extracted values of a_2/a_1 have large phases and are larger in magnitude than those extracted from other D decays (see Table 2). This "failure" of naive factorization is usually attributed to the strong final-state interactions caused by nearby resonances (as signaled by the fact that $|\delta_{1/2} - \delta_{3/2}| \approx 90^\circ$ in these decays). It is then argued that one can only expect to correctly predict the *magnitudes* of the isospin amplitudes but not their relative phase [2]. The ratio of these magnitudes is determined by the ratio of the real, effective $a_1^{\rm eff}$ and $a_2^{\rm eff}$ parameters of the naive factorization model via the relation

$$\frac{|A_{1/2}|}{\sqrt{2}\,|A_{3/2}|} = \frac{2 - x\,a_2^{\rm eff}/a_1^{\rm eff}}{2(1 + x\,a_2^{\rm eff}/a_1^{\rm eff})}\,. \tag{6}$$

The effective parameters so determined are shown in the lower portion of Table 2. In the case of D decays, the physical picture underlying this approach is that of predominantly elastic final-state interactions, which mix the various $\bar{K}^{(*)}\pi$ final states and thereby changes the phases but not the magnitudes of the isospin amplitudes. While this assumption may be questioned even in the case of charm decays [24], is it clearly not justifiable for decays of B mesons, in which rescattering is predominantly inelastic [5, 18]. Therefore, we believe it is a coincidence that the "effective" $a_2^{\rm eff}/a_1^{\rm eff}$ ratios are close to the expectations of the naive factorization model.

Finally, we like to comment on the observation that the ratios x in (5), which govern the relative strength of the class-2 and class-1 amplitudes in naive factorization, exhibit large violations of the scaling $x \sim (\Lambda/m_Q)^2$ expected in the heavy-quark limit. Does this imply a failure of QCD factorization in hadronic B decays? We believe the answer to this question is negative. Consider first the conventional case where the charm quark is treated as a heavy quark. Then the ratios x arise only in naive factorization. The fact that they are not numerically suppressed reflects the well-known failure of the conventional heavy-quark expansion for heavy-light form factors and decay constants, i.e., the empirical fact that the ratios

$$\frac{f_D}{f_\pi} \approx 1.5 \quad [\sim (\Lambda/m_Q)^{1/2}]\,, \qquad \frac{F_0^{B\to\pi}(m_D^2)}{F_0^{B\to D}(m_\pi^2)} \approx 0.5 \quad [\sim (\Lambda/m_Q)^{3/2}] \tag{7}$$

do not scale as expected from the heavy-quark limit of QCD. (The reason for this failure may be related to the "smallness" of f_π, which in turn reflects the smallness of the light quark masses via the relation $m_\pi^2\,f_\pi^2 = -2(m_u + m_d)\,\langle\bar{q}q\rangle$.) However, as we have argued earlier the factorized class-2 contributions appearing in the numerator of the ratios x are not a leading contribution to the class-2 amplitudes in the heavy-quark limit. Other contributions exist that are parametrically larger. It is therefore not clear to what extent the large scaling violations in (7) are relevant to the class-2 amplitudes. In the opposite limit where the charm quark is considered a light quark the naive factorization contribution is the leading contribution to the class-2 amplitudes. In this limit also the ratios x are of leading order in the heavy-quark expansion, which is consistent with the numerical values $x(D^{(*)}\pi) \approx 0.9$. So there is no evidence for a failure of the heavy-quark expansion either.

In summary, we have argued that new experimental results on the color-suppressed nonleptonic decays $\bar{B}^0 \to D^{(*)0}\pi^0$ provide evidence for a failure of the naive factorization model and for sizeable relative strong-interaction phases between class-1 and class-2 $\bar{B} \to D^{(*)}\pi$ decay amplitudes. This resolves a long-standing puzzle created by the apparent universality and small rescattering phases of the class-2 parameters a_2 in B decays. The new data suggest that the a_2 parameters in different types of decays such as $\bar{B} \to D^{(*)}\pi$ and $\bar{B} \to \bar{K}^{(*)}J/\psi$ differ by almost a factor 2 in magnitude, indicating a strong nonuniversality of nonfactorizable effects. This is in agreement with theoretical expectations based on the heavy-quark expansion. We find that the size of corrections to the heavy-quark limit seen in the data is compatible with the expectation that the suppression of the corrections is governed by the large charm-quark mass. We urge our experimental colleagues to produce more precise data on a large variety of hadronic B decays. This will help to explore in detail the pattern of QCD effects in the class-2 and class-3 amplitudes, as well as to further establish the validity of QCD factorization (and hence the applicability of the heavy-quark limit) for class-1 decays.

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Note added: While this Letter was in writing the paper `hep-ph/0107257` by Z.-z. Xing appeared, in which a similar analysis is carried out but different conclusions are obtained.

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