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Hadronic Mass Moments in Inclusive Semileptonic B Meson Decays

CLEO Collaboration

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Abstract

We have measured the first and second moments of the hadronic mass-squared distribution in $B \to X_c \ell \nu$, for $P_{lepton} > 1.5$ GeV/c. We find $\langle M_X^2 - \bar{M}_D^2 \rangle = 0.251 \pm 0.066$ GeV^2, $\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle = 0.576 \pm 0.170$ GeV^4, where $\bar{M}_D$ is the spin-averaged D meson mass. From that first moment and the first moment of the photon energy spectrum in $b \to s\gamma$, we find the HQET parameter λ_1 ($\overline{MS}$, to order $1/M_B^3$ and $\beta_0 \alpha_s^2$) to be -0.24 ± 0.11 GeV^2. Using these first moments and the B semileptonic width, and assuming parton-hadron duality, we obtain $|V_{cb}| = 0.0404 \pm 0.0013$.

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The heavy quark limit of QCD [1] is potentially a very useful tool for relating measured inclusive properties in B meson decay, such as semileptonic branching fractions, to fundamental CKM parameters like V_{cb} and V_{ub}. The expressions for inclusive observables are expansions in inverse powers of the B meson mass M_B [2–4]. At order $1/M_B$, the non-perturbative parameter $\overline{\Lambda}$ enters, and at order $1/M_B^2$ two more parameters, λ_1 and λ_2 appear. Intuitively, these parameters may be thought of as the energy of the light quark and gluon degrees of freedom ($\overline{\Lambda}$), the average momentum-squared of the b quark ($-\lambda_1$), and the energy of the hyperfine interaction of the spin of the b quark with the light degrees of freedom (λ_2/M_B). The parameter λ_2 can be extracted directly from the $B^* - B$ mass splitting [3]. The other two parameters can be obtained from inclusive measurement or calculated theoretically with techniques capable of handling non-perturbative effects, such as lattice QCD [5].

There are two problems associated with the interpretation of measured inclusive properties, one associated with the convergence of the expansion, and another with the validity of the assumptions underlying the expansion. The inclusive observables are expansions in powers of $1/M_B$, and at each order more non-perturbative parameters appear. By order $1/M_B^2$ there are three parameters and at order $1/M_B^3$ another six parameters, ρ_1, ρ_2, $\mathcal{T}_1 - \mathcal{T}_4$. Without good estimates for the additional parameters we must rely on the rapid convergence of the expansion. The other problem is the validity of the assumption of parton-hadron duality implicit in this approach, and its potential for introducing additional uncertainties not included in the present estimates [6]. Thus, the experimental determination of $\overline{\Lambda}$ and λ_1 with several different methods is necessary to support the validity of parton-hadron duality [7].

Much interest has been raised by the possibility of estimating $\overline{\Lambda}$ and λ_1 using hadronic spectral moments in semileptonic B decays [2–4]. In this Letter we report a measurement of the first and second moments of the distribution in the hadronic mass-squared in the inclusive semileptonic decay $b \to c\ell\nu$. For this analysis, the leptons are restricted to the kinematical region $P_\ell \geq 1.5$ GeV/c. In particular, we report measurements of $\langle M_X^2 - \bar{M}_D^2\rangle$ and $\langle (M_X^2 - \bar{M}_D^2)^2\rangle$, where M_X^2 is the mass-squared of the charmed hadronic system X_c, and $\bar{M}_D$ is the spin-averaged D meson mass, $0.25M_D + 0.75M_{D^*} = 1.975$ GeV. The theoretical expansion for these two observables has been carried out to order $1/M_B^3$ and order $\beta_0\alpha_s^2$ in the $\overline{MS}$ renormalization scheme [2,3]. (Here $\beta_0 = (33 - 2n_f)/3 = 25/3$ is the one-loop QCD beta function.) We also report the second moment taken about the first moment rather than about $\bar{M}_D^2$, i.e., $\langle (M_X^2 - \langle M_X^2\rangle)^2\rangle$, the mean square width of the mass-squared distribution. (The theoretical expansion for this is readily obtained from those for $\langle M_X^2 - \bar{M}_D^2\rangle$ and $\langle (M_X^2 - \bar{M}_D^2)^2\rangle$.) We use the first moment, along with the first moment of the photon energy spectrum in $b \to s\gamma$ [8], to obtain λ_1 and an improved extraction of V_{cb} from the B meson semileptonic width.

The data used in this analysis were taken with the CLEO detector [9] at the Cornell Electron Storage Ring (CESR), and consist of 3.2 fb^{-1} at the $\Upsilon(4S)$ resonance and 1.6 fb^{-1} at a center-of-mass energy 60 MeV below the resonance. The sample contains 3.4 million $B\bar{B}$ pairs. We select events containing a lepton – μ or e – with momentum between 1.5 and 2.5 GeV/c. We "reconstruct" the neutrino in the event using energy and momentum conservation of the entire event, exploiting the hermiticity of the CLEO detector. The neutrino energy is taken as the difference of twice the beam energy and the sum of the energies of all detected particles, while the neutrino momentum is the negative of the vector sum of the momenta of all detected particles. Considerable effort was expended to remove double counting between calorimeter and tracking chamber measurements. To assure a well-measured neutrino, we require: a neutrino mass consistent with zero; no additional leptons in the event; a measured net charge of zero for the event. The "neutrino reconstruction" aspect of this analysis is similar to that of Ref. [10], and is described in detail in Ref. [11]. Event shape requirements are applied to distinguish the jetty event environment typical of $e^+e^- \to q\overline{q}$ light quark pair production from the more isotropic environment of $e^+e^- \to B\overline{B}$ events. We achieve a sample consisting of 89% $e^+e^- \to B\overline{B}$ and 11% from the continuum, with an efficiency for the desired events of $\approx$2%. The desired semileptonic B decays, $b \to c\ell\nu$, represent 95% of the $e^+e^- \to B\overline{B}$ sample while the remaining consists of $(2.8 \pm 0.6)\%$ secondary lepton production (from Monte Carlo simulation) and $(2.1 \pm 1.1)\%$ $b \to u\ell\nu$ (using $|V_{ub}/V_{cb}| = 0.07 \pm 0.02$).

We determine the mass of the hadronic system X in $B \to X_c\ell\nu$ from the lepton and neutrino momentum vectors alone:

$$\begin{aligned} M_X^2 &= (E_B - E_\ell - E_\nu)^2 - (\vec{P}_B - \vec{P}_\ell - \vec{P}_\nu)^2 \\ &= M_B^2 + M_{\ell\nu}^2 - 2E_B E_{\ell\nu} + 2|\vec{P}_B||\vec{P}_{\ell\nu}|\cos\theta_{\ell\nu,B} \ . \end{aligned} \tag{1}$$

For B mesons produced at the $\Upsilon(4S)$, E_B and $|\vec{P}_B|$ are known and constant, but the angle between the B and $\ell\nu$ system varies from event to event, and is not known. Since $|\vec{P}_B|$ is small (300 MeV/c), we approximate M_X^2 by dropping the final term in Eq. 1, writing

$$\widetilde{M_X^2} = M_B^2 + M_{\ell\nu}^2 - 2E_B E_{\ell\nu}. \tag{2}$$

The background-subtracted $\widetilde{M_X^2}$ distribution, consisting of 11900 B meson decays, is shown in Fig. 1. The background from continuum events has been subtracted using the data collected below the $\Upsilon(4S)$ resonance, scaled to the luminosity of the on-resonance data and corrected for the dependence of the production cross section on beam energy. The small backgrounds from secondary lepton sources and from $b \to u\ell\nu$ decays, which we obtain from Monte Carlo simulation, have also been subtracted.

For the purpose of extracting the moments of the M_X^2 distribution, we divide the $b \to c\ell\nu$ decays into three components: $B \to D\ell\nu$, $B \to D^*\ell\nu$, and $B \to X_H\ell\nu$ where X_H represents all the high-mass charmed meson resonances as well as the charmed non-resonant decays. The individual components are shown in Fig. 1. We use measured form factors [12] to model the $B \to D\ell\nu$ and $B \to D^*\ell\nu$ decays. The true M_X^2 distributions for $B \to D\ell\nu$ and $B \to D^*\ell\nu$ are narrow resonances at M_D^2 and $M_{D^*}^2$. The widths of the Monte Carlo predictions in Fig. 1 for these resonances are dominated by neutrino energy-momentum resolution and our neglect of the last term in Eq. 1. The high-mass contribution, $B \to X_H\ell\nu$, is modeled using six resonances above the D^* with the decay properties specified by ISGW2 form factors [13], and also non-resonant multi-body final states such as $B \to D\pi\ell\nu$ and $B \to D^*\pi\ell\nu$, which are decayed according to the prescription of Goity and Roberts [14].

A fit of the Monte Carlo to the data $\widetilde{M_X^2}$ distribution determines the relative contributions from $B \to D\ell\nu$, $B \to D^*\ell\nu$ and $B \to X_H\ell\nu$. The relative rates and the generated masses are used to calculate $\langle M_X^2 - \bar{M}_D^2\rangle$ and $\langle (M_X^2 - \bar{M}_D^2)^2\rangle$ of the true M_X^2 distribution. Equation 3 shows the derivation of the average mass squared, M_X^2, from the relative rates.

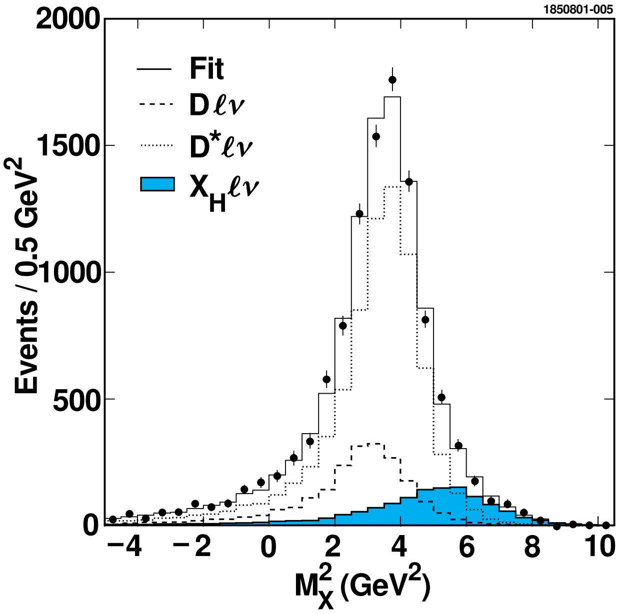

FIG. 1. Measured $\widetilde{M_X^2}$ distributions, for background-corrected data (points), Monte Carlo (solid line), and the three components of the Monte Carlo: $B \to D\ell\nu$ (dashed), $B \to D^*\ell\nu$ (dotted), $B \to X_H\ell\nu$ (shaded). The normalization of each component is derived from a fit to the data.

$$\langle M_X^2 \rangle = r_D \cdot M_D^2 + r_{D^*} \cdot M_{D^*}^2 + r_{X_H} \cdot \langle M_{X_H}^2 \rangle, \tag{3}$$

where r_D is the rate of $B \to D\ell\nu$ production compared to the combined rate of $B \to D\ell\nu$, $B \to D^*\ell\nu$, $B \to X_H\ell\nu$, and similarly for r_{D^*} and r_{X_H}. The individual values obtained for r_D, r_{D^*} and r_{X_H}, while perfectly consistent with world average branching fractions [15], are not well determined and are sensitive to the model chosen for $B \to X_H\ell\nu$. The *moments*, however, are well-determined and stable against model changes, as discussed below. We find $\langle M_X^2 - \bar{M}_D^2 \rangle \equiv M1 = 0.251 \pm 0.023 \pm 0.062$ GeV^2, $\langle (M_X^2 - \bar{M}_D^2)^2 \rangle \equiv M2 = 0.639 \pm 0.056 \pm 0.178$ GeV^4, and $\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle \equiv M2' = 0.576 \pm 0.048 \pm 0.163$ GeV^4, where the errors are statistical and systematic, in that order. The experimental errors on $\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle$ are somewhat smaller than for $\langle (M_X^2 - \bar{M}_D^2)^2 \rangle$ and have a smaller correlation with the first moment. (A correction for final state radiation, not included in the Monte Carlo samples used in our fits, has been applied, using PHOTOS [16].)

The errors on both first and second moments are dominated by systematic errors. The leading contribution is from the simulation parameters that impact neutrino resolution: photon identification efficiency, tracking efficiency, and the rate of additional neutrals such as K_L^0 and additional neutrinos; it amounts to ± 0.058 GeV^2, ± 0.140 GeV^4, and ± 0.129 GeV^4, for $M1$, $M2$, and $M2'$, respectively.

The second leading source of systematic error is from the models for the high-mass contribution to the $\widetilde{M_X^2}$ distribution. We have varied aspects of the high-mass component in order to quantify the sensitivity. The six contributing mass states of the resonant component (above D^*) have been systematically dropped singly, in pairs and in triplets so as to vary the internal structure of the resonant model. Taking the r.m.s. deviations of these variations, we find errors of ± 0.015 GeV^2, ± 0.090 GeV^4, and ± 0.083 GeV^4, for $M1$, $M2$, $M2'$.

Another contributing uncertainty arises from the lack of knowledge on the amount and shape of non-resonant contribution to the high-mass component. Although we fix the fraction of non-resonant to resonant high-mass states during a fit, we systematically vary this fraction over the limits that the data allow. A one unit variation of fit χ^2 determines a systematic variation of 0.011 GeV^2 for $M1$ and 0.060 GeV^4 and 0.054 GeV^4 for $M2$ and $M2'$. Systematic errors other than those from neutrino resolution simulation, modelling of high-mass resonances, and modelling of non-resonant high-mass decays, such as the subtractions for secondary leptons and for $b \to u\ell\nu$, the final state radiation correction, and the $B \to D^{(*)}\ell\nu$ form factor uncertainties, are negligible by comparison.

As an alternative to the default Goity-Roberts parameterization, we have also used a phase space model to generate the four-body non-resonant decays [11]. This phase space model generates, on average, higher mass states than the Goity-Roberts parameterization but yields hadronic mass moments consistent with those obtained from the Goity-Roberts parameterization. This observation emphasizes the fact that the data essentially constrain the product of the average mass squared and production rate while these quantities may individually vary significantly.

The correlation coefficients between errors of first and second moments are positive, and substantial. They are +0.71 for $M1$ – $M2$ (+0.56 for $M1$ – $M2'$) for the statistical error, +0.50 (+0.34) for the systematic error, and +0.52 (+0.36) for the total error.

The expressions [2,17] for the hadronic mass moments in $B \to X_c\ell\nu$, to order $\beta_0\alpha_s^2$ and $1/M_B^3$, subject to the restriction $P_\ell > 1.5$ GeV/c, are given in Eqs. 4 and 5. (Due to technical difficulties, the coefficients of the $\frac{\bar\Lambda}{M_B}\frac{\alpha_s}{\pi}$ terms were computed *without* the 1.5 GeV lepton energy restriction, and so are only approximate, believed [17] good to ± 50%.)

$$\begin{aligned}\frac{\langle M_X^2 - \bar{M}_D^2\rangle}{M_B^2} = [&0.0272\tfrac{\alpha_s}{\pi} + 0.058\beta_0\tfrac{\alpha_s^2}{\pi^2} + 0.207\tfrac{\bar\Lambda}{\bar{M}_B}(1 + 0.43\tfrac{\alpha_s}{\pi}) + 0.193\tfrac{\bar\Lambda^2}{\bar{M}_B^2} + 1.38\tfrac{\lambda_1}{\bar{M}_B^2} + 0.203\tfrac{\lambda_2}{\bar{M}_B^2}\\ &+ 0.19\tfrac{\bar\Lambda^3}{\bar{M}_B^3} + 3.2\tfrac{\bar\Lambda\lambda_1}{\bar{M}_B^3} + 1.4\tfrac{\bar\Lambda\lambda_2}{\bar{M}_B^3}\\ &+ 4.3\tfrac{\rho_1}{\bar{M}_B^3} - 0.56\tfrac{\rho_2}{\bar{M}_B^3} + 2.0\tfrac{\mathcal{T}_1}{\bar{M}_B^3} + 1.8\tfrac{\mathcal{T}_2}{\bar{M}_B^3} + 1.7\tfrac{\mathcal{T}_3}{\bar{M}_B^3} + 0.91\tfrac{\mathcal{T}_4}{\bar{M}_B^3} + \mathcal{O}(1/\bar{M}_B^4)],\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{\langle (M_X^2 - \langle M_X^2\rangle)^2\rangle}{M_B^4} = [&0.00148\tfrac{\alpha_s}{\pi} + 0.0025\beta_0\tfrac{\alpha_s^2}{\pi^2} + 0.027\tfrac{\bar\Lambda}{\bar{M}_B}\tfrac{\alpha_s}{\pi} + 0.0107\tfrac{\bar\Lambda^2}{\bar{M}_B^2} - 0.12\tfrac{\lambda_1}{\bar{M}_B^2}\\ &+ 0.02\tfrac{\bar\Lambda^3}{\bar{M}_B^3} - 0.06\tfrac{\bar\Lambda\lambda_1}{\bar{M}_B^3} - 0.129\tfrac{\bar\Lambda\lambda_2}{\bar{M}_B^3}\\ &- 1.2\tfrac{\rho_1}{\bar{M}_B^3} + 0.0032\tfrac{\rho_2}{\bar{M}_B^3} - 0.12\tfrac{\mathcal{T}_1}{\bar{M}_B^3} - 0.36\tfrac{\mathcal{T}_2}{\bar{M}_B^3} + \mathcal{O}(1/\bar{M}_B^4)],\end{aligned} \tag{5}$$

In these expressions, $\bar{M}_B$ represents the spin-averaged B meson mass, 5.313 GeV.

The $1/M_B^3$ parameters ρ_i, $\mathcal{T}_i$ are estimated [3], from dimensional considerations, to be $\sim (0.5\text{GeV})^3$. Taking values of ρ_2 and $\mathcal{T}_1$ through $\mathcal{T}_4$ to be $0.0 \pm (0.5\text{GeV})^3$, taking ρ_1 (believed to be positive) to be $\frac{1}{2}(0.5\text{GeV})^3 \pm \frac{1}{2}(0.5\text{GeV})^3$, taking $\lambda_2 = 0.128 \pm 0.010$ GeV^2 (appropriate with a calculation to order $1/M_B^3$) [3], and using $\alpha_s(m_b) = 0.220$, the expressions combined with our measurements define bands in $\bar\Lambda - \lambda_1$ space. The band for the first moment is shown in Fig. 2. The dark grey region indicates the error band from the measurement; the light grey extension includes the error from the theoretical expression, in particular from the $\rho_1 - \mathcal{T}_4$ terms and from the scale uncertainty ($\alpha_s(m_b/2) = 0.275$ to $\alpha_s(2m_b) = 0.176$).

In the preceeding Letter [8], we presented measurements of the first and second moments of the photon energy spectrum in $b \to s\gamma$, and gave the OPE expansion expressions for those moments, again valid to order $\beta_0 \alpha_s^2$ and $1/M_B^3$. Again, equation plus measurement defines a band in $\bar{\Lambda} - \lambda_1$ space. The band for the first moment, $\langle E_\gamma \rangle$ is also shown in Fig. 2. The expressions for the second moments converge more slowly in $1/M_B$ than those for the first moments, and the theoretical advice [17] is *not* to put much trust in the bands they define. Consequently we have not shown them in Fig. 2.

The intersection of the two bands from the first moments determines $\bar{\Lambda}$ and λ_1. A $\Delta\chi^2 = 1$ ellipse is shown. The values obtained are

$$\bar{\Lambda} = 0.35 \pm 0.07 \pm 0.10 \text{ GeV} ,$$

$$\lambda_1 = -0.236 \pm 0.071 \pm 0.078 \text{ GeV}^2 .$$

Here, the first error is from the experimental error on the determination of the two moments, and the second error from the theoretical expressions. (Using the information from all four bands, first and second moments, the results differ little, both as to central values and as to errors.) Note that $\bar{\Lambda}$ and λ_1 are scheme and order dependent. The values obtained above are for $\bar{\Lambda}$ and λ_1 to order $1/M^3$, order $\beta_0\alpha_s^2$, in the $\overline{MS}$ renormalization scheme.

Given this determination of $\bar{\Lambda}$ and λ_1, we can use them to improve the determination of $|V_{cb}|$ from the measured $B \to X_c \ell \nu$ semileptonic width. The expression [18,3] for the semileptonic width, to order $\beta_0 \alpha_s^2$ and $1/M_B^3$, is given in Eq. 6.

$$\begin{aligned}\Gamma_{sl} = \tfrac{G_F^2 |V_{cb}|^2 M_B^5}{192\pi^3} 0.3689 [1 &- 1.54\tfrac{\alpha_s}{\pi} - 1.43\beta_0 \tfrac{\alpha_s^2}{\pi^2} - 1.648\tfrac{\bar{\Lambda}}{M_B}(1 - 0.87\tfrac{\alpha_s}{\pi}) - 0.946\tfrac{\bar{\Lambda}^2}{M_B^2} - 3.185\tfrac{\lambda_1}{M_B^2} \\ &+ 0.02\tfrac{\lambda_2}{M_B^2} - 0.298\tfrac{\bar{\Lambda}^3}{M_B^3} - 3.28\tfrac{\bar{\Lambda}\lambda_1}{M_B^3} + 10.47\tfrac{\bar{\Lambda}\lambda_2}{M_B^3} - 6.153\tfrac{\rho_1}{M_B^3} + 7.482\tfrac{\rho_2}{M_B^3} \\ &- 7.4\tfrac{\mathcal{T}_1}{M_B^3} + 1.491\tfrac{\mathcal{T}_2}{M_B^3} - 10.41\tfrac{\mathcal{T}_3}{M_B^3} - 7.482\tfrac{\mathcal{T}_4}{M_B^3} + \mathcal{O}(1/M_B^4)] . \end{aligned} \tag{6}$$

For the experimental determination of Γ_{sl}, we use: $\mathcal{B}(B \to X_c \ell \nu) = (10.39 \pm 0.46)\%$ [19], $\tau_{B^\pm} = (1.548 \pm 0.032)$ ps [15], $\tau_{B^0} = (1.653 \pm 0.028)$ ps [15], $f_{+-}/f_{00} = 1.04 \pm 0.08$ [20], giving $\Gamma_{sl} = (0.427 \pm 0.020) \times 10^{-10}$ MeV.

Combining the measured semileptonic width with the theoretical expression for it, and using the determination of $\bar{\Lambda}$ and λ_1 from the first moments, we find

$$|V_{cb}| = (4.04 \pm 0.09 \pm 0.05 \pm 0.08) \times 10^{-2} ,$$

where the errors are from experimental determination of Γ_{sl}, from experimental determination of $\bar{\Lambda}$ and λ_1, and from the $1/M_B^3$ terms and scale uncertainty in α_s, in that order. This gives a determination of $|V_{cb}|$ from inclusive processes, with a precision of $\pm 3.2\%$. This result depends on the assumption of global parton-hadron duality, with its unknown uncertainties.

Summarizing, we have measured the first and second moments of the hadronic mass-squared distribution in the B meson semileptonic decay to charm, $B \to X_c \ell \nu$. We find $\langle M_X^2 - \bar{M}_D^2 \rangle = 0.251 \pm 0.023 \pm 0.062$ GeV^2, $\langle (M_X^2 - \bar{M}_D^2)^2 \rangle = 0.639 \pm 0.056 \pm 0.178$ GeV^4, and $\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle = 0.576 \pm 0.048 \pm 0.163$ GeV^4. The measurement of $\langle M_X^2 - \bar{M}_D^2 \rangle$ and the HQET expression for this moment are used, in conjunction with similar information on

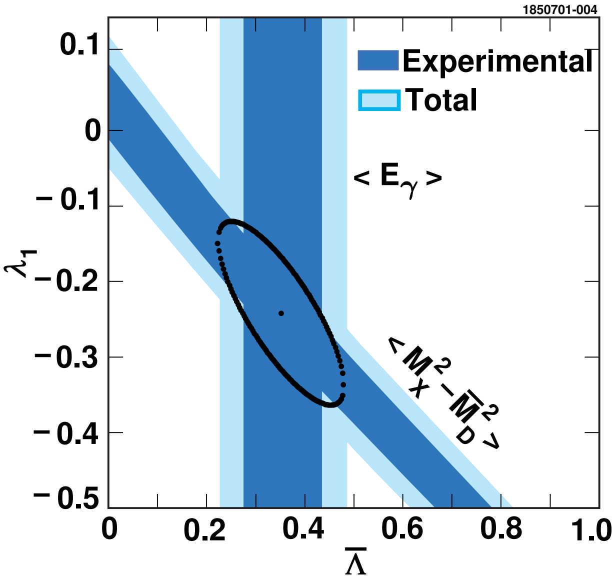

FIG. 2. Bands in $\bar{\Lambda} - \lambda_1$ space defined by $\langle M_X^2 - \bar{M}_D^2 \rangle$, the measured first moment of hadronic mass-squared, and $\langle E_\gamma \rangle$, the first moment of the photon energy spectrum in $b \to s\gamma$ [8]. The inner bands indicate the error bands from the measurements. The light grey extensions include the errors from theory. All bands are derived from $\mathcal{O}(1/\bar{M}_B^3)$ $\mathcal{O}(\beta_0 \alpha_s^2)$ HQET expressions, using the $\overline{MS}$ renormalization scheme.

the first moment of the photon energy spectrum in $b \to s\gamma$, to determine λ_1 and $\overline{\Lambda}$. These in turn are used, along with the B meson semileptonic width, to obtain V_{cb}.

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