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Hot Little String Correlators: A View From Supergravity

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Abstract

We study the propagation of a massless minimally coupled scalar in the near horizon geometry of non-extremal NS5-branes. Using the holographic principle for dilatonic backgrounds we compute the two-point function of an operator in Little String Theory at the Hagedorn temperature. We then comment on relations with correlation functions in two dimensional string theory.

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1 Introduction

The worldvolume description of N coincident Neveu-Schwarz five-branes is a Poincare invariant non-gravitational field theory in $5+1$ dimensions, referred to as the Little String Theory (LST). This theory is defined [1] to be the decoupled theory on the world-volume in the limit of vanishing string coupling $g_s \to 0$ (for a review see [2]). Even though the string coupling is taken to zero, the theory retains nontrivial dynamics and is nonlocal [3]. The decoupling limit for NS5-branes ensures that these theories are intrinsically strongly coupled, for they have no continuous dimensionless coupling constant, instead being characterized by a single integer N, the number of NS5-branes. In the infrared limit with sixteen supersymmetries, these theories flow to the $(2,0)$ superconformal field theory in $5+1$ dimensions (in the Type IIA description) and to IR free $5+1$ SYM with $(1,1)$ supersymmetry (in the Type IIB description).

Despite having successfully defined decoupled non-local theories on the world-volume of NS5-branes, it has proven quite hard to extract substantial information about the dynamics of the theory. Efforts in this direction include formulation of a light-cone description [4], which has proved useful in extracting the chiral operator spectrum and the formulation of a holographic dual in terms of string theories in linear dilaton backgrounds [5]. The holographic description has been used to extract some correlation functions [7] and has been extended to more general theories with fewer supersymmetries [8, 9, 10].

LSTs can also be defined at finite temperature. A useful technique to study the dynamics of such a system is to construct a holographic dual to the LST at a given temperature, which typically is a black hole geometry. At finite temperature, the decoupling limit for NS5-branes is defined by taking the asymptotic value of the string coupling g_s to zero while keeping fixed the string scale, $l_s = \sqrt{\alpha'}$. By tuning the number N of coincident five-branes and the dimensionless energy density above extremality μ on the branes, the holographic dual supergravity becomes a valid description for the parameter range $\mu \gg N \gg 1$ [11]. It turns out however that this simple holographic dual exists only for a particular value of the temperature which happens to coincide with the Hagedorn temperature of the little strings. It was shown in [11] that LST at the Hagedorn temperature is holographically dual to string theory on the CHS tube [12] capped off by a horizon (the two-dimensional Euclidean cigar geometry), along with an $SU(2)$ WZW model with level prescribed by the number of NS5-branes and a free CFT for the longitudinal directions of the brane.

Correlation functions in LST at zero temperature were calculated in [7] by studying the propagation of a minimal scalar in the supergravity background of N coincident IIA NS5-branes. Since the geometry in the Type IIA description is the linear dilaton background, the growing of the dilaton as we descend down the CHS tube invalidates the use of ten dimensional supergravity all through. Instead what one does is to use the M-theory lift of the configuration: start with a stack of coincident M5-branes evenly arrayed along the 11th direction $\mathbf{S}^1$. In the limit of a large number of M5-branes, one can trust the 11-dimensional supergravity results. The full geometry has the linear dilaton throat interpolating between asymptotically flat space and the near horizon geometry of a stack of M5-branes *i.e.*, $AdS_7 \times \mathbf{S}^4$. In the decoupling limit the asymptotic flat region is pushed away to infinity. One prescribes boundary conditions for the scalar at some hypersurface $r = \Lambda$, and computes the Euclidean action as a function of the boundary values, in the general spirit of holography. The resulting correlation function for the operator that couples to the scalar was found to reduce consistently to that of the $(2,0)$ superconformal theory in the IR. One interesting aspect of the analysis was that the absorption cross-section was found to be non-zero in the decoupling limit for modes with energies larger than $m_s/\sqrt{N}$ (which in the large N limit is much lower than the string scale), the mass gap in the throat.

In what follows, we compute the two-point function for a minimally coupled scalar in LST at the Hagedorn temperature, by exploiting the aforementioned holographic description. We consider the Euclidean geometry with a cut-off imposed up the tube far from the horizon. This cut-off is equivalent to an ultraviolet cut-off in the dual field theory. We then define a Dirichlet boundary value for the minimal scalar, demanding that the solution go to a constant value at finite distances up the tube as the cutoff is removed. Imposing the constraint of regularity at the horizon, the scalar field can be uniquely solved in the bulk. We then evaluate the Euclidean action on the bulk scalar solution to the Dirichlet problem, which reduces to a cut-off dependent boundary term. The two-point function of the operator dual to the bulk scalar is then computed from the leading finite part after dropping a divergent piece. This is similar to the corresponding computations of two-point functions in AdS backgrounds [13, 14] and their noncommutative counterparts [15].

The basic result of the paper is the two point function of an operator that couples to a massless minimal scalar propagating in the bulk geometry. It agrees with the two-point function (for the momentum sector) obtained from a CFT analysis in double scaled LST [9, 10]. Further, the two-point function contains a factor which is very similar to the "external leg factors" that appear in the context of two dimensional string theory [23, 24]. There are simple poles on the real momentum axis, from these external leg factors.

This paper is organized as follows. In section 2, we describe the supergravity background holographically dual to LST at the Hagedorn temperature. In section 3, we describe the computation of the two-point function. In section 4, we elaborate on properties of the two point function thus obtained and finally in section 5, we discuss certain features that are analogous to correlation functions calculated in two dimensional string theory. Appendix A outlines the computation of the reflection coefficient in the Lorentzian geometry.

2 The Holographic Dual

The metric of the non-extremal NS5-brane in the string frame [11] is given as (suppressing the NS-NS $B_{\mu\nu}$ field that the NS5-branes couple to) [3]

$$
\begin{aligned}
ds^2_{str} &= -f(r)dt^2 + dy_5^2 + A(r)\left(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2\right) \\
e^{2\Phi} &= g_s^2 A(r) \\
f(r) &= 1 - \frac{r_0^2}{r^2} \\
A(r) &= 1 + \frac{N l_s^2}{r^2} \equiv 1 + \frac{r_5^2}{r^2}
\end{aligned}
\tag{1}
$$

The dy_5^2 corresponds to the flat spatial directions along the 5-branes, while the $d\Omega_3^2$ corresponds to the 3-sphere part of the transverse geometry. r_0 is a non-extremality parameter, the location of the outer horizon being $r = r_0$. The geometry transverse to the 5-branes is a long tube which opens up into the asymptotic flat space region with the horizon at the other end. In the extremal limit, *i.e.* $r_0 = 0$, the above geometry factorizes into $\mathbf{R}^{5,1} \times \mathbf{R}_\phi \times \mathbf{S}^3_N$, where $\mathbf{R}_\phi$ stemming from the radial part of the metric represents the linear dilaton tube. The dilaton grows towards the horizon.

The decoupling limit [11] is defined as a double-scaling limit, scaling the asymptotic value of the string coupling and the horizon radius to zero simultaneously, keeping fixed the energy density above extremality in string units *i.e.*,

$$
g_s \to 0, \qquad r_0 \to 0, \qquad \mu = \frac{r_0^2}{g_s^2\, l_s^2} = \text{fixed} \tag{2}
$$

For the purposes of our calculation, we resort to Euclidean space. We will thus work in the Wick-rotated version of the metric (1) and simply drop the 1 in $A(r)$ in the decoupling limit.

To analyse this limit it is convenient to introduce a new coordinate $r = r_0 \cosh\sigma$, which in the scaling limit, gives for the Euclidean near-horizon geometry

[3]Note that this metric is related to that appearing in [16], [17] by a simple coordinate transformation.

$$
\begin{aligned}
ds^2_{str} &= \tanh^2\sigma\ dt^2 + Nl_s^2\ d\sigma^2 + Nl_s^2\ d\Omega_3^2 + dy_5^2 \\
e^{2\Phi} &= \frac{N}{\mu\ \cosh^2\sigma}
\end{aligned} \tag{3}
$$

The spacetime geometry is smooth in the parameter range $\mu \gg N \gg 1$. Further in this range, string perturbation theory is also good. The geometry in this case becomes $\mathbf{R}^5 \times \mathcal{M}_{2dbh} \times \mathbf{S}^3_N$. $\mathcal{M}_{2dbh}$ represents the spacetime corresponding to the Euclidean 2D black hole. The 3-sphere is of constant radius $\sqrt{N\alpha'}$. Both string frame and Einstein frame tube lengths, computed using (3), are infinite in the scaling limit. Thus in the decoupling limit, the geometry consists of a semi-infinite tube of constant size capped by the black hole horizon at one end. In other words, the asymptotic flat space region is infinitely far away from the horizon.

The Hawking temperature of this spacetime can be calculated by the usual methods, *i.e.*, by calculating the periodicity of the temporal part of the Wick-rotated metric. In other words, we demand that the Euclidean metric does not have a conical singularity at the origin. This gives

$$
T_H = \frac{1}{2\pi\sqrt{N\alpha'}} \tag{4}
$$

The Hawking temperature is independent of the horizon size r_0 and therefore of the energy density μm_s^6. It only depends on the number N of 5-branes. This independence of the temperature on the energy implies that the entropy is proportional to the energy, giving rise to a vanishing free energy at leading order in supergravity. Basically the thermal ensemble is degenerate, we shall return to this point in the discussion.

3 Scalar propagation

We want to study the issues related to scattering of minimally coupled complex scalar particles in the geometry of the non-extremal NS5-brane. The classical action for a massless minimally coupled complex scalar is given as

$$
S = \frac{\kappa}{g_s^2}\int d^{10}x\ \sqrt{G_E}\ \ |\partial\phi|^2 \tag{5}
$$

We can think of a massless minimally coupled scalar as being a longitudinally polarized mode of the graviton. The normalization factor κ will be fixed at a later stage.

For an s-wave scalar with no momentum along the longitudinal directions and $\phi(t,r,y) = e^{i\omega t + ik\cdot y}\phi(r)$, the wave equation reduces to

$$
\frac{f(r)}{r^3}\,\partial_r\left(r^3\ f(r)\partial_r\phi\right)\ -\ A(r)\left(\omega^2 + k^2 f(r)\right)\phi(r) = 0 \tag{6}
$$

As mentioned previously, we work in the Wick rotated version of the non-extremal metric in (1) and since we will be interested in setting boundary conditions in the CHS tube, we shall drop the 1 from the harmonic function $A(r)$. Incidentally the very same equation was obtained in the analysis of [18] and [19]. These authors were interested in the computation of greybody factors for scattering off 5-dimensional black holes. We shall use the results of the latter as they work in the regime more suited to our analysis.

What we are really after is the two-point function of an operator that couples to the scalar ϕ in the form given by

$$S_{int} = \int d^6 p \ \left(\phi^*(p,\Lambda)\mathcal{O}(p) + \phi(p,\Lambda)\mathcal{O}(p)^*\right), \tag{7}$$

where $r = \Lambda$ is the UV cut-off in the dual field theory.

The two-point function of the operator $\mathcal{O}$ in momentum space is defined to be $\langle \mathcal{O}(p)\mathcal{O}^*(-p')\rangle = \Pi(p)\delta(p-p')$. Given a solution to the equations of motion implied by (5) we evaluate the Euclidean action on the solution. It is imperative that we pick the right boundary conditions for the scalar field. The boundary conditions we wish to impose on the solution are the following:

- The solution has to be regular at the horizon *i.e.*, at the tip of the cigar in the Euclidean case.
- We will impose a cut-off at some distance along the linear-dilaton tube, say at $r = \Lambda$. We require that the solution be constant at this surface as we take the cut-off to infinity.

The boundary condition at the horizon is physically reasonable. The boundary condition at the cut-off surface is chosen in accord with the intuition gained in the analysis of massive scalars in AdS, *cf.*, [20], and a very analogous scenario seen in the case of the holographic dual to the non-commutative gauge theories [15]. The point is the following: As we shall see explicitly below, demanding regularity at the horizon leads to a solution that has an asymptotic behaviour with a growing and a damped piece far up the CHS tube. In the general spirit of holography one would like to impose a Dirichlet boundary condition at the cut-off surface. A natural way to implement the same in the present set-up is to have the scalar field to remain constant at large radii as we take the cut-off to infinity. This is a renormalization prescription that turns out to depend on the momenta of the scalar, reflecting the non-local nature of the theory [6, 7].

Evaluation of the Euclidean action (5) on a solution with the aforementioned boundary conditions leads to a boundary term

$$
\begin{aligned}
S &= \frac{\kappa V_5 \Omega_3}{2g_s^2}\ \left[r^3 f(r)\phi^*(r)\partial_r\phi(r)\right]|_{r_0}^{\Lambda} + \text{c.c.} \\
&= \frac{\kappa V_5 \Omega_3}{2g_s^2}\ f(\Lambda)\Lambda^3\ \phi^*(\Lambda)\ [\partial_r\phi(r)]\ |_{r=\Lambda} + \text{c.c.}
\end{aligned} \tag{8}
$$

This can be evaluated and after discarding a piece that diverges as the cut-off is taken to infinity we are left with a well-defined two-point function for the operator $\mathcal{O}$.

The equation (6) is exactly solvable in terms of Hypergeometric functions, *cf.*, [19]. Substituting $z = f(r)$ we obtain

$$
z\frac{d}{dz}z\frac{d}{dz}\phi(z) + \frac{r_5^2}{4(1-z)^2}(\omega^2 + k^2 z)\phi(z) = 0. \tag{9}
$$

which is solved by

$$
\begin{aligned}
\phi(z) &= Az^{\alpha_+}(1-z)^{\beta}F(\alpha_+ + \beta, \alpha_+ + \beta; 1 + 2\alpha_+; z) \\
&\quad + Bz^{\alpha_-}(1-z)^{\beta}F(\alpha_- + \beta, \alpha_- + \beta; 1 + 2\alpha_-; z).
\end{aligned} \tag{10}
$$

for constants A and B, where,

$$
\begin{aligned}
\alpha_\pm &= \pm\frac{\omega r_5}{2} = \pm\frac{\sqrt{s}}{2} \\
\beta &= \frac{1 \pm \sqrt{1 + r_5^2(\omega^2 + k^2)}}{2} = \frac{1 \pm \sqrt{1 + s + k^2 r_5^2}}{2},
\end{aligned} \tag{11}
$$

defining $s = \omega^2 r_5^2 = \omega^2 N\alpha'$, the energy squared in units of the mass gap. Note that in the Euclidean black hole background, ω is quantized in units of $1/\sqrt{N\alpha'}$ which is related to the Hawking temperature T_H (see sec.2). Thus s takes values of integer squared.

The first boundary condition causes us to reject the solution with the negative root for α (For $z \to 0$, which is the location of the horizon, the hypergeometric function goes to 1). We shall henceforth drop the subscript in α_+ and also choose the smaller root for β (this choice turns out to be arbitrary since choosing the larger root does not change our results).

The solution $\phi(z) = z^{\alpha}(1-z)^{\beta}F(\alpha+\beta, \alpha+\beta, 2\alpha+1; z)$ asymptotes at large r ($z \to 1$) to

$$
\begin{aligned}
\phi(r) &\sim \mathcal{N}(p)\left[\left(\frac{r_0}{r}\right)^{2\beta} + B(p)\left(\frac{r_0}{r}\right)^{2(1-\beta)}\right] \\
&\sim \frac{r_0}{r}\mathcal{N}(p)\left[\left(\frac{r_0}{r}\right)^{-\sqrt{1+s+k^2 r_5^2}} + B(p)\left(\frac{r_0}{r}\right)^{\sqrt{1+s+k^2 r_5^2}}\right],
\end{aligned} \tag{12}
$$

where

$$
\begin{aligned}
B(p) &= \frac{\Gamma(2\beta-1)}{\Gamma(1-2\beta)} \frac{\Gamma^2(1+\alpha-\beta)}{\Gamma^2(\alpha+\beta)} \\
&= \frac{\Gamma(-\sqrt{1+s+k^2r_5^2})}{\Gamma(\sqrt{1+s+k^2r_5^2})} \frac{\Gamma^2(\frac{1+\sqrt{s}}{2}+\frac{\sqrt{1+s+k^2r_5^2}}{2})}{\Gamma^2(\frac{1+\sqrt{s}}{2}-\frac{\sqrt{1+s+k^2r_5^2}}{2})}.
\end{aligned} \tag{13}
$$

Demanding that we have

$$
\phi(\Lambda,p) = \left(\frac{r_0}{\Lambda}\right)^{1-\sqrt{1+s+k^2r_5^2}} \phi_0(p) = \left(\frac{r_0}{\Lambda}\right)^{2\beta} \phi_0(p), \tag{14}
$$

we fix

$$
\mathcal{N}(p) \sim \frac{1}{1+B(p)\left(\frac{r_0}{\Lambda}\right)^{2\sqrt{1+s+k^2r_5^2}}}. \tag{15}
$$

We are now well-positioned to evaluate the Euclidean action (8). Using $f(\Lambda) \sim 1$ and

$$
\partial_r\phi \sim \mathcal{N}(p)\left[-\frac{2\beta}{r_0}\left(\frac{r}{r_0}\right)^{-2\beta-1} - B(p)\frac{2(1-\beta)}{r_0}\left(\frac{r}{r_0}\right)^{-2(1-\beta)-1}\right] \tag{16}
$$

implying,

$$
\phi^*(r)\partial_r\phi(r)|_\Lambda \sim |\mathcal{N}(p)|^2 \left[-\frac{2\beta}{r_0}\left(\frac{\Lambda}{r_0}\right)^{-4\beta-1} - 2B(p)\frac{r_0^2}{\Lambda^3}\right]. \tag{17}
$$

The first term in (8) diverges and we drop it, the second term will give (note $\mathcal{N}(p) \sim 1$))

$$
S = -2\kappa V_5\Omega_3 \frac{r_0^2}{g_s^2} B(p)\,\phi_0(p)\phi_0(-p) = -2\kappa V\Omega_3 l_s^2\mu\, B(p)\,\phi_0(p)\phi_0(-p). \tag{18}
$$

Differentiating w.r.t. the sources ϕ_0, we obtain, apart from overall normalization factors, the two-point function

$$
\langle\mathcal{O}(p)\mathcal{O}^*(-p')\rangle \sim \frac{\Gamma(-\sqrt{1+s+k^2r_5^2})}{\Gamma(\sqrt{1+s+k^2r_5^2})} \frac{\Gamma^2(\frac{1+\sqrt{s}}{2}+\frac{\sqrt{1+s+k^2r_5^2}}{2})}{\Gamma^2(\frac{1+\sqrt{s}}{2}-\frac{\sqrt{1+s+k^2r_5^2}}{2})}\,\delta(p-p'). \tag{19}
$$

We can choose κ and the normalization of the operator such that (19) is the correctly normalized two-point function of the operator $\mathcal{O}$ and in the following we shall do so accordingly. So henceforth we shall take the right hand side of (19) to be the appropriately normalized two-point function of the operator $\mathcal{O}$ that couples to a massless minimal scalar.

4 Properties of the two-point function

We have calculated the two-point function $\Pi(p)$ as a function of momentum $p = (\omega, k)$. Note that since the theory is at finite temperature, we have broken Lorentz invariance and thus the spatial momenta k do not appear on the same footing as the energy ω. ω is quantized in units of $1/r_5 = 1/\sqrt{N\alpha'}$, the periodicity of the time variable in the black hole background, which is related to the Hawking temperature (see sec.2). This implies that $s = \nu^2,\ \nu = 0, 1, 2, \ldots$ From (19), we have

$$\Pi(p) = \frac{\Gamma(-\sqrt{1+s+k^2r_5^2})}{\Gamma(\sqrt{1+s+k^2r_5^2})}\ \frac{\Gamma^2(\frac{1+\sqrt{s}}{2}+\frac{\sqrt{1+s+k^2r_5^2}}{2})}{\Gamma^2(\frac{1+\sqrt{s}}{2}-\frac{\sqrt{1+s+k^2r_5^2}}{2})} \tag{20}$$

Note that with the assignments [4]

$$\sqrt{1+s+k^2r_5^2} = 2j+1, \qquad \sqrt{s} = \nu = -2m = 2\bar{m}, \tag{21}$$

$\Pi(p)$ coincides with the two-point function given in eqn.(3.6) of [10] in the large N limit (with small j),

$$\Pi(j,m) = \frac{\Gamma(-2j-1)}{\Gamma(2j+1)}\ \frac{\Gamma^2(j-m+1)}{\Gamma^2(-j-m)} \tag{22}$$

upto an overall constant, which has to do with the normalization of the operators. Here $(m, \bar{m})$ parametrize momentum and winding around the cigar via

$$m = \frac{1}{2}(-\varrho + wN), \qquad \bar{m} = \frac{1}{2}(\varrho + wN). \tag{23}$$

Note that $\varrho = -p$ in eqn.(2.3) of [10]. Since our computation corresponds to momentum modes on the cigar geometry, the winding number $w = 0$ and we have $m = -\bar{m}$. We do not see the first ratio of Γ-functions in eqn.(3.6) of [10] since our semiclassical analysis holds only for small j/N.

$\Pi(p)$ has a relatively simple analytic structure. There is a tower of simple poles from the first ratio of Γ-functions in (20) for

$$p^2r_5^2 = s + k^2r_5^2 = \nu^2 + k^2r_5^2 = n^2 - 1, \qquad n = 1, 2, \ldots \tag{24}$$

from the Γ-function in the numerator [5]. Since the Γ-function has no branch cuts, the only branch cuts in $\Pi(p)$ arise from the square roots in the arguments of the Γ-functions. The absence of branch cuts (apart from the square roots) in (20) might

[4] We thank D. Kutasov for pointing this out to us.

[5] Note that the possible pole at $n = 0$ is absent because it cancels with a similar singular factor from the denominator Γ-function.

suggest that within the approximations of our calculation, only single-particle states are visible from the couplings to the bulk observables we use, while multi-particle states are not (see also [9]).

Of course, not all of the above are poles of the two-point function, since the squared Γ-function in the denominator too has poles, which compensates. For instance, setting $s = \nu^2 = 0$, the poles are at $1 + k^2 r_5^2 = n^2$. The denominator has $\Gamma^2(\frac{1-n}{2})$ which has double poles when n is an odd integer. So basically one half of the poles actually end up being zeros instead of the two-point function. Nonetheless, there is a tower of poles on the real axis, given by (24) with n being a nonzero even integer, all of whose residues have the same sign. These poles become more and more evenly spaced in $|p|$, as p grows large. The poles do not coincide with the locations of the quantized frequencies.

Since on-shell bulk fields give rise to off-shell correlation functions in the dual field theory, ν and k are independent variables above. They are not related by a six dimensional mass-shell condition.

Note that $\Pi(p)$ is really the renormalized two-point function. The cutoff Λ appears in our boundary condition (14), for large p, as

$$\Lambda^{|p|\sqrt{N\alpha'}} = e^{|p|\sqrt{N\alpha'}\ \log\Lambda}. \tag{25}$$

Consider, for *e.g.*, performing a "wave-function renormalization" in our interaction action (7). If we rescale ϕ by this exponential factor thereby rescaling the operator $\mathcal{O}$ by the inverse factor, the unrenormalized two-point function rescales by the square of this factor, *i.e.* $e^{2|p|\sqrt{N\alpha'}\ \log\Lambda}$, which grows exponentially for large p, corroborating with [6], [7], [3].

It would be interesting to perhaps incorporate our results in modelling these nonlocal nongravitational field theories with a view to obtaining insight into string interactions in these systems (see, for *e.g.* [22] for discussions on extensivity in these systems).

5 Discussion

To shed some light on the poles we see in the above two-point function, let us recall a similar feature [6] that is seen in the analysis of 2-dimensional gravity coupled to $c = 1$ matter [23, 24]. Correlation functions of tachyon vertex operators in that model have poles arising from the so called "external leg factors". These leg factors are proportional to $\frac{\Gamma(-|k_i|)}{\Gamma(|k_i|)}$, k_i being the momenta of the tachyon. In the limit of large momenta the argument of the first ratio of Gamma functions in (19) is identical to the leg factors. We of course have other factors in the correlation function, but they are incapable of producing any poles, and as we have seen they can at best project out some of the

[6]We thank I. Klebanov for valuable discussions on these issues.

poles. In the $c=1$ story, the poles occur at integral momenta and are understood to be indicative of special states in the theory occurring only at these momenta [25], [26]. They can be thought of as remnants of the transverse excitations of the string in two spacetime dimensions.

To get some insight into these aspects, realize that we have constructed momentum states in the asymptotic geometry of the supergravity background (1) and computed the reflection coefficient for scattering these states off the background. Since the linear dilaton region of the geometry has a mass-gap of $m_{gap} = \frac{1}{\sqrt{N\alpha'}}$, the argument of the first ratio of Γ-functions in $\Pi(p)$ is the energy of the excitation above the mass gap. In the two-dimensional string case the argument of the Γ-function in the external leg factor can be thought of as a Liouville energy, since the vertex operator satisfying the mass-shell condition is $e^{ikX}e^{(-2+|k|)\Phi}$, Φ being the Liouville coordinate. By Liouville energy we simply mean the factor multiplying Φ in the exponent, modulo a universal factor, which in the units we choose to write the vertex operators happens to be -2.

Now consider trying to formulate the physical state conditions for vertex operators in the case we are studying : the background in string frame is $\mathbf{R}^5 \times \mathbf{S}^3 \times \mathcal{M}_{2dbh}$ where $\mathcal{M}_{2dbh}$ corresponds to the 2D black hole cigar geometry. This geometry asymptotes to $\mathbf{R}^5 \times \mathbf{S}^3 \times \mathbf{R}_\phi \times \mathbf{R}_t$, with R_ϕ corresponding to the linear dilaton direction and in addition we have $g_s^2 \sim e^{-2\sigma/\sqrt{N\alpha'}}$ as can be seen from (3) (σ parametrizes the linear dilaton direction). Consider a graviton vertex operator (with no excitation on the $\mathbf{S}^3$) in the linear dilaton region of the geometry,

$$\xi_{\mu\nu}\psi^\mu\bar{\psi}^\nu e^{ik\cdot X}e^{\alpha\Phi}, \tag{26}$$

ψ^μ being the worldsheet fermions and $\xi_{\mu\nu}$ being the polarization tensor. We have dropped the ghost contributions for simplicity (see eqn.(3.21) of [9], for the vertex operator written in the $SL(2)/U(1)$ coset language). Then the mass shell condition is

$$\frac{1}{2}k^2 - \frac{1}{2}\alpha(\alpha - Q) = 0 \tag{27}$$

where Q is the Liouville charge, which can be determined from the asymptotic behaviour of the dilaton (also *c.f.*, eqn.(2.18) of [21]) to be $Q = \frac{2}{\sqrt{N\alpha'}}$. From the mass-shell condition (27) we can determine the Liouville energy α to be

$$\alpha = \frac{Q}{2} \pm \sqrt{k^2 + \frac{Q^2}{4}} \tag{28}$$

The first term corresponds to the shift in the Liouville energy arising from the background while the second term is exactly the same as the arguments in the first ratio of Γ-functions in $\Pi(p)$ for $Q = 2/\sqrt{N\alpha'}$.

It is therefore tempting to conjecture that the poles in the two-point function (19) correspond to special states in LST at the Hagedorn temperature, whose vertex operators are of the form given in (26). More precisely it is likely that the physical state conditions for the vertex operators of the above form have extra solutions at momenta which satisfy the conditions in (24) in analogy with the situation in two dimensional string theory as was demonstrated in [25]. It would be interesting to analyze the special states in the present case along these lines.

In the analysis of double scaled LST [9, 10] there were additional poles arising from modes that had winding about the asymptotic circle. These were understood to correspond in the semi-classical description to bound states living near the tip of the cigar [27]. The spectrum of the bound states has been known to correspond exactly with the principal discrete series representation of $SL(2,R)$. We however have no winding about the asymptotic circle and therefore are not in a position to reproduce these poles in our analysis.

Little String Theories as we have seen are defined holographically at the Hagedorn temperature. At tree level this implies that when thought of as a thermal field theory, the thermodynamics is degenerate, since the energy and the temperature can be tuned independently. A proper treatment of the thermal ensemble would need to ask what changes would be induced by quantum fluctuations. It turns out that a one-loop analysis of the thermodynamics reveals an instability [21]: to be precise, the one-loop specific heat is negative. It was argued in [30] that this instability would imply that the background geometry (3), suffers from a marginal case of Gregory Laflamme instability [28, 29], generalising the ideas of [31].

In light of the instability of the geometry one might be skeptical about the nature of the computation presented above. The basic point is that the geometry (3) is only marginally unstable (or better, neutrally stable). In particular the tree level supergravity analysis ought to reveal only a massless mode and so the geometry is metastable. In particular, our results for the two-point function ought to capture the leading behaviour.

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Appendix A : Reflection coefficient in the tube

In this section, we describe the calculation of the reflection coefficient for scalar modes propagating in the geometry with energies $m_s/\sqrt{N} < \omega << m_s$. This is usually calculated demanding ingoing boundary conditions at the horizon (this is equivalent to regularity in the interior in the Euclidean calculation). In addition, we demand that the solution be constant on a cutoff surface along the linear dilaton tube, as the cutoff is taken to infinity. The solution that results thus is pure ingoing at the horizon and has both ingoing and outgoing components far from the horizon. The action evaluated on this solution is used after subtracting off the action evaluated on free propagation in the tube (this "free" action corresponds to the divergent piece in the Euclidean calculation). This subtracted action is then used to compute the reflection coefficient.

The wave equation for a massless minimal scalar reduces to

$$\frac{f(r)}{r^3}\,\partial_r\left(r^3\ f(r)\partial_r\phi\right)\ +\ A(r)\left(\omega^2+k^2f(r)\right)\phi(r)=0 \tag{29}$$

Using the substitution $z = f(r)$, the resulting equation can be solved to give

$$\phi(z) = z^{\alpha}(1-z)^{\beta}F(\alpha+\beta,\alpha+\beta;1+2\alpha;z). \tag{30}$$

Here we have defined new parameters α and β which are given as

$$\alpha = -i\frac{\sqrt{s}}{2}, \qquad \beta = \frac{1-\sqrt{1-s+k^2r_5^2}}{2} \tag{31}$$

where the lower root for β has been chosen. The negative root for α is chosen since we are interested in an ingoing wave at the horizon. The other independent solution of the second order differential equation above corresponds to the outgoing wave solution and is thus discarded.

This solution asymptotes at large r, *i.e.* $z \to 1$, to (we set $k=0$ here for simplicity)

$$\begin{aligned}\phi(r) &\sim \mathcal{N}(s)\left[\left(\frac{r_0}{r}\right)^{2\beta}+B(s)\left(\frac{r_0}{r}\right)^{2(1-\beta)}\right]\\ &\sim \frac{r_0}{r}\mathcal{N}(s)\left[\left(\frac{r_0}{r}\right)^{-\sqrt{1-s}}+B(s)\left(\frac{r_0}{r}\right)^{\sqrt{1-s}}\right].\end{aligned} \tag{32}$$

where

$$B(s) = \frac{\Gamma(2\beta-1)}{\Gamma(1-2\beta)}\,\frac{\Gamma^2(1+\alpha-\beta)}{\Gamma^2(\alpha+\beta)} \tag{33}$$

Demanding that we have $\phi(\Lambda,\omega) = \left(\frac{r_0}{\Lambda}\right)^{1-\sqrt{1-s}}\phi_0(\omega)$, we fix the normalization $\mathcal{N}(s)$.

The reflection coefficient is obtained from the ratio of the ingoing and the outgoing wave pieces in the solution far from the horizon, *i.e.* $B(s)$. Alternatively, calculating the action from the solution thus obtained and subtracting a divergent piece corresponding to free motion in the tube gives the reflection coefficient $B(s)$ upto other factors.

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