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Note on the Extraction of $|V_{ub}|$ using Radiative B Decays

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Abstract

The element $|V_{ub}|$ of the quark mixing matrix can be extracted with small theoretical uncertainties by combining weighted integrals over the endpoint regions of the lepton spectrum in $B \to X_u\, l\, \nu$ decays and the photon spectrum in $B \to X_s \gamma$ decays. The perturbative corrections to this determination are computed at next-to-leading order including operator mixing, which has an important impact. The effect of Sudakov resummation is shown to be numerically insignificant.

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The extraction of the Cabibbo–Kobayashi–Maskawa matrix element $|V_{ub}|$ from charmless semileptonic B decays is complicated by the fact that, over most of phase space, there is a large background from decays into final states containing a charm hadron. Tight experimental cuts (e.g., on the charged-lepton energy or the hadronic invariant mass) must be applied to isolate the signal from $b \to u$ transitions. Accounting for such cuts theoretically is difficult, because inclusive decay spectra close to the kinematic endpoint are more susceptible to nonperturbative strong-interaction effects than fully inclusive decay rates. To describe the decay spectra near the boundary of phase space, the operator product expansion for inclusive B decays must be replaced by a twist expansion [1, 2, 3, 4]. At leading power in $\Lambda_{\rm QCD}/m_b$, bound-state effects in the B meson are incorporated by a shape function accounting for the "Fermi motion" of the b quark. The presence of this function introduces additional hadronic uncertainties.

It was suggested long ago that $|V_{ub}|$ could be extracted with small theoretical uncertainties by combining weighted integrals over the endpoint regions of the lepton spectrum in $B \to X_u\, l\, \nu$ decays and the photon spectrum in $B \to X_s \gamma$ decays [2]. The underlying idea is that the soft QCD interactions affecting these two spectra are the same and can be canceled by taking an appropriate ratio of weighted integrals. The result is

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 \simeq \left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 = \frac{3\alpha}{\pi}\, K_{\rm pert}\, \frac{\widehat{\Gamma}_u(E_0)}{\widehat{\Gamma}_s(E_0)} + O(\Lambda_{\rm QCD}/M_B)\,, \tag{1}$$

where $\alpha = 1/137.036$ is the fine-structure constant, and the quantities

$$\begin{aligned} \widehat{\Gamma}_u(E_0) &= \int\limits_{E_0}^{M_B/2} dE_l\, \frac{d\Gamma(B \to X_u\, l\, \nu)}{dE_l}\,, \\ \widehat{\Gamma}_s(E_0) &= \frac{2}{M_B} \int\limits_{E_0}^{M_B/2} dE_\gamma\, w(E_\gamma, E_0)\, \frac{d\Gamma(B \to X_s \gamma)}{dE_\gamma} \end{aligned} \tag{2}$$

are to be determined from experiment. At tree-level, the weight function appearing in the definition of $\widehat{\Gamma}_s(E_0)$ is a straight line, $w(E_\gamma, E_0) = E_\gamma - E_0$ [2]. It was shown in this paper that at order α_s there are perturbative corrections resulting in a matching constant $K_{\rm pert}$ at the scale $\mu \sim m_b$ and a logarithm, which was conjectured to result from renormalization-group evolution between the scales $\mu \sim m_b$ and $\mu \sim (\Lambda_{\rm QCD}\, m_b)^{1/2}$. The precise nature of this logarithm was clarified in [4], where the factorization of hard, collinear and soft contributions to the decay rate was proven to all orders of perturbation theory, and Sudakov logarithms were summed using the renormalization group. In [5], this resummation was implemented in momentum (rather than moment) space.

Recently, the relation (1) has been evaluated for the first time using experimental data [6]. It is therefore timely to update the calculation of the QCD corrections. In this note, we report the complete next-to-leading order (NLO) expression for $K_{\rm pert}$ including the effects of operator mixing. This is crucial for obtaining a renormalization-group invariant (i.e., scale and scheme-independent) answer. We study the residual renormalization-scale dependence of the result and its sensitivity to the charm-quark mass, account for power corrections of order $(\Lambda_{\rm QCD}/m_c)^2$, and comment on the numerical significance of the summation of Sudakov logarithms.

Perturbative corrections to the endpoint region of the charged-lepton energy spectrum in $B\to X_u\,l\,\nu$ decays were computed long ago [7]. On the other hand, the complete NLO result for the photon energy spectrum in $B\to X_s\gamma$ decays was obtained only recently by combining the findings of several authors [8, 9, 10, 11]. Using these results, we have evaluated the corrections to the weighted integrals in (2). We find that at order α_s the weight function in the second integral is

$$w(E_\gamma,E_0)=(E_\gamma-E_0)\left[1-\frac{10}{9}\,\frac{\alpha_s(\mu)}{\pi}\,\ln\!\left(1-\frac{E_0}{E_\gamma}\right)\right], \tag{3}$$

whereas the matching corrections are given by

$$\begin{aligned} K_{\rm pert} &= [C_7^{(0)}(\mu)]^2\left\{1+\frac{\alpha_s(\mu)}{2\pi}\left[-\frac{83}{9}+\frac{4\pi^2}{9}+\frac{32}{3}\ln\frac{m_b}{\mu}+\frac{C_7^{(1)}(\mu)}{C_7^{(0)}(\mu)}\right]\right\}\\ &\quad + C_2^{(0)}(\mu)\,C_7^{(0)}(\mu)\left[\frac{\alpha_s(\mu)}{2\pi}\left(\mathrm{Re}(r_2)+\frac{416}{81}\ln\frac{m_b}{\mu}\right)-\frac{\lambda_2}{9m_c^2}\right]\\ &\quad + C_8^{(0)}(\mu)\,C_7^{(0)}(\mu)\,\frac{\alpha_s(\mu)}{2\pi}\left(\frac{44}{9}-\frac{8\pi^2}{27}-\frac{32}{9}\ln\frac{m_b}{\mu}\right), \end{aligned} \tag{4}$$

where $\mathrm{Re}(r_2)\approx -4.092-12.78\,(0.29-m_c/m_b)$ [9]. C_i are Wilson coefficients appearing in the effective weak Hamiltonian for $B\to X_s\gamma$ decays,[1] which are expanded as

$$C_i(\mu)=C_i^{(0)}(\mu)+\frac{\alpha_s(\mu)}{4\pi}\,C_i^{(1)}(\mu)+\dots\,. \tag{5}$$

Explicit expressions for these coefficients can be found, e.g., in [10, 11]. The terms in the last two lines in (4) result from the interference of the amplitudes corresponding to the operators O_2, O_7 and O_8 in the effective weak Hamiltonian. For completeness, we include a power correction proportional to λ_2/m_c^2 in the result for $K_{\rm pert}$, which represents a long-distance contribution from $(c\bar c)$ intermediate states to the matrix element of the operator O_2 [12, 13, 14, 15, 16]. (Here $\lambda_2\approx 0.12\,\mathrm{GeV}^2$ is the B-meson matrix element of the chromo-magnetic operator [17].) Its numerical effect is, however, very small.

The above expressions for $w(E_\gamma,E_0)$ and $K_{\rm pert}$ are scale and scheme independent at NLO. In this context, it is crucial that the terms arising from operator mixing are included. The μ-dependent terms proportional to $C_2\,C_7$ and $C_8\,C_7$ in (4), together with the μ-dependent term in the first line, are required to cancel the scale dependence of the leading-order coefficient $[C_7^{(0)}(\mu)]^2$. Likewise, the constant terms proportional to $C_2\,C_7$ and $C_8\,C_7$, together with the constant term in the first line, are necessary to compensate the scheme dependence of the NLO coefficient $C_7^{(1)}$. In all previous analyses of the ratio

[1] Strictly speaking, C_7 and C_8 are the so-called "effective" Wilson coefficients $C_7^{\rm eff}$ and $C_8^{\rm eff}$.

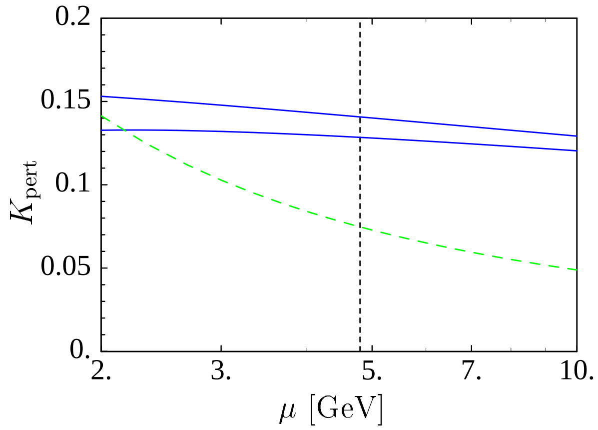

Figure 1: Scale dependence of the coefficient K_{pert} for $m_c/m_b = 0.29$ (lower curve) and 0.22 (upper curve). The dashed line shows the result without operator mixing.

(1), the terms proportional to $C_2\,C_7$ and $C_8\,C_7$ were neglected. In [2], only partial $O(\alpha_s)$ corrections to the $B\to X_s\gamma$ decay rate available at that time were included. (In addition, there is a sign mistake in the expression for η_{QCD} in eq. (58) of that paper.) The NLO coefficient $C_7^{(1)}$ was calculated later in [10]. In [5], only the terms in the first line in (4) were included.

In order to illustrate the importance of the various terms in (4), we quote the result for $\mu = m_b = 4.8\,\mathrm{GeV}$ and $m_c/m_b = 0.29$:

$$K_{\mathrm{pert}} \simeq [C_7^{(0)}(m_b)]^2 \Big(1 - 0.23 + 0.53\,[\text{2-7 mix}] + 0.03\,[\text{8-7 mix}]\Big) \approx 1.33\,[C_7^{(0)}(m_b)]^2\,. \quad (6)$$

Here -0.23 is the sum of the various constants in the square brackets in the first line in (4), and the last two terms are the contributions from operator mixing as shown in the second and third line. It is evident that the mixing contributions are numerically important and of opposite sign than the remaining $O(\alpha_s)$ corrections. Most important is the O_2-O_7 interference term, which is enhanced by the large ratio of Wilson coefficients $C_2^{(0)}(m_b)/C_7^{(0)}(m_b) \approx -3.6$.

It has recently been argued that the $B\to X_s\gamma$ decay rate should not be evaluated as a function of the ratio of pole masses $m_c/m_b \approx 0.29$, but that a more appropriate choice would be to use a running charm-quark mass such that $m_c/m_b \approx 0.22$ [18]. If this is done, the contribution from the O_2-O_7 interference term in the above example increases from 0.53 to 0.65. Since the question of the quark-mass definition can, strictly speaking, only be settled by a NNLO calculation, we will include the variation of the result under the variation of m_c/m_b between 0.22 and 0.29 as part of the theoretical uncertainty.

The two solid lines in Figure 1 show our result for K_{pert} as a function of the renormalization scale, varied between 2 and 10 GeV. The dashed line shows, for comparison, the result obtained ignoring the effects of operator mixing, i.e., retaining only the first line in (4). The vertical dashed line indicates the default value $\mu = m_b$ used in previous analyses. We observe a good stability of the NLO prediction under variation of the

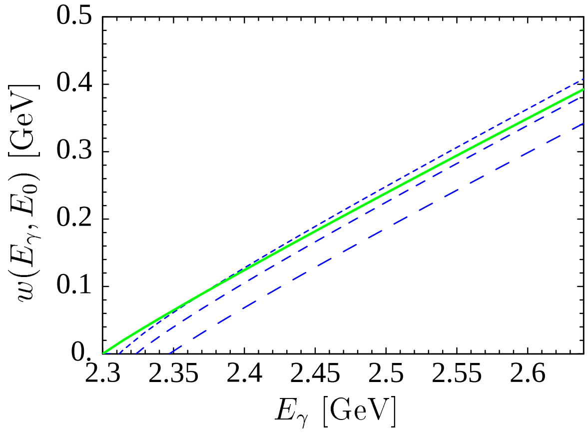

Figure 2: Comparison of the resummed weight function in (8) (dashed lines) with the one-loop function in (3) (solid line) for $E_0 = 2.3\,\text{GeV}$. The singularity of the integrand is avoided by replacing the upper limit of integration by $0.98\,E_\gamma$ (long dashed), $0.99\,E_\gamma$ (dashed), and $0.995\,E_\gamma$ (short dashed).

renormalization scale. The sensitivity to the charm-quark mass implies an uncertainty of about $\pm 5\%$. On the other hand, the result without operator mixing is strongly scale dependent and, for $\mu \approx m_b$, underestimates the correct answer by almost a factor 2.

Our result for the perturbative correction factor K_{pert} can be summarized as

$$K_{\text{pert}} = 0.134\,^{+0.007}_{-0.009}\,[\text{scale}]\,^{+0.007}_{-0.006}\,[m_c] \pm 0.010\,[\alpha_s^2]\,. \tag{7}$$

The quoted errors are obtained by scanning the renormalization scale μ between $m_b/2$ and $2m_b$, and the mass ratio m_c/m_b between 0.22 and 0.29. In addition, there is an uncertainty due to the neglect of $O(\alpha_s^2)$ corrections, which we have estimated by squaring the $O(\alpha_s)$ coefficients of the different combinations of Wilson coefficients in (4).

We finally like to comment on the effect of the resummation of Sudakov logarithms for the weight function $w(E_\gamma, E_0)$ in (3). It is well-known that the leading, double-logarithmic contributions cancel in the ratio of the two rates in (1), so only subleading logarithms remain [2]. These terms were resummed in [4, 5] to all orders of perturbation theory. The net effect is to replace the weight function in (3), evaluated at the scale $\mu = m_b$, by the more complicated function

$$w_{\text{res}}(E_\gamma, E_0) = Z^{-1} \int\limits_{E_0}^{E_\gamma} dE\, K\!\left[\frac{2E}{M_B}\,;\,\frac{16}{3\beta_0}\,\ln\left(1 + \frac{\beta_0\alpha_s(m_b)}{4\pi}\,\ln\ln\frac{E_\gamma}{E}\right)\right], \tag{8}$$

where $\beta_0 = 11 - \frac{2}{3}\,n_f$. (We omit a factor $(1 + 3\bar{\Lambda}/M_B)\,(2E_\gamma/M_B)^2 = 1 + O(\Lambda_{\text{QCD}}/M_B)$ introduced by hand in [5], since it is beyond the accuracy of (1) and has nothing to do with resummation effects.) The function $K(x;y)$ is given in eq. (42) of [5], and

$$Z = 1 - \frac{\alpha_s(m_b)}{2\pi}\left(-\frac{83}{9} + \frac{4\pi^2}{9}\right) \tag{9}$$

is defined such that $w(E_\gamma, E_0)$ coincides with the one-loop result in (3) up to terms of order α_s^2. The integrand in (8) is singular at $E/E_\gamma \approx 0.999$, and it was suggested in [5] to avoid this singularity by replacing the upper limit of integration by $0.99\,E_\gamma$. It turns out that the numerical results are rather sensitive to this treatment. In Figure 2, we compare the resummed weight function (8) obtained with three different cutoff prescriptions with the one-loop function in (3). We use $E_0 = 2.3\,\mathrm{GeV}$ corresponding to the energy cutoff employed in the experimental analysis [6]. It is evident that within the intrinsic uncertainty of the resummation procedure (as reflected by the sensitivity to the integration cutoff) the effect of Sudakov resummation is negligible. It is thus a safe approximation for all practical purposes to work with the fixed-order expression (3).

In summary, we have computed the complete NLO perturbative corrections to the ratio of weighted integrals in (1). This provides the basis for a model-independent determination of $|V_{ub}|$ from semileptonic and radiative B decays. We find that NLO corrections from operator mixing are numerically important. Neglecting these terms would not only lead to scale and scheme-dependent predictions, but also introduce a numerical error in the result for $|V_{ub}|$ of as much as 50% (for $\mu = m_b$). We have also shown that the resummation of subleading Sudakov logarithms for the weight function $w(E_\gamma, E_0)$ does not have a numerically significant effect on the result, although it is a conceptual improvement of the calculation.

Adding the various contributions to the error in quadrature, we find $K_{\mathrm{pert}} = 0.134 \pm 0.014$ for the perturbative correction in (1). The potentially most important source of theoretical uncertainty is not the small perturbative error found here, but the presence of unknown first-order power corrections $\sim \Lambda_{\mathrm{QCD}}/M_B$. In practice, the empirical finding that the result for $|V_{ub}|$ obtained from (1) were independent of the threshold E_0 employed in the analysis of the experimental data would give us confidence that the impact of power corrections was not very significant.

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Note added: After this note was submitted the paper [19] appeared, in which the authors suggest to raise the cutoff in the evaluation of (8) from $0.99\,E_0$ to $0.9987\,E_0$, thereby enhancing the numerical effect of Sudakov resummation. Leaving aside the fact that this is an ad hoc prescription, we would not trust "perturbative" resummation effects that result from the immediate vicinity of the Landau pole in a running coupling.

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