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HOLOGRAPHIC COOLING

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We propose a new method for cooling a beam of charged particles in a storage ring. The method deals with *stimulated* radiation instead of *spontantenous* radiation in the pickup of a stochastic cooling system. For this purpose the signal from a source generator is split into two parts. One part travels with the beam, preliminary bunched with the period of signal through pickup and stimulates radiation of charged particles. The second part is used as a reference. After destructive interference (subtraction) of these two parts and amplification, the residual signal goes to an appropriately phased kicker. This stimulated radiation in the pickup is higher than spontantenous radiation in a ratio that can reach ~10^6.
This method, implemented into *optical stochastic cooling*, makes muon, proton/antiproton, ion and electron cooling schemes practical ones. The method can be applied to any feedback system as well.
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Any feedback system contains three key elements: a pick-up, an amplifier and a kicker. All together they represent a loop, closed by the beam. The damping rate is determined by the bandwidth of all these elements and the amplification. As a rule the sensitivity of the pickup is a limiting factor.

Usually charged particles radiate in a pickup spontantenously. But as the number of particles in the bandwidth decreased, the damping rate decreases unless the amplification feedback loop is made higher and higher. In limiting case, when the only a single particle resides in the bandwidth, the system needs to amplify the signal from that single particle.

A system for stochastic cooling requires a feedback system having bandwidth $\Delta f >> c/\sigma_b$, where *c*–is a speed of light and σ_b–is the bunch length. This bandwidth allows the resolution of structure inside the bunch. A scheme with the feedback bandwidth extended to optical frequencies is described in [1]. An optical amplifier became the natural analog of a microwave amplifier. The analogues for the pick up and kicker were found to be a quadrupole wiggler and dipole wiggler. In ref. [1] radiation in the pickup –a quadrupole wiggler –is ordinary *spontantenous* radiation in the periodical wiggler field. This field is a linear function of the particle's transverse displacement.

We consider here a cooling method for charged particles, when they emit radiation in the pickup, *stimulated* by an external source. This method allows a significant increase in the signal from the pickup: the ratio of stimulated to spontantenous radiation. As the stimulated radiation is proportional to the external field, the only limitation of the radiation here is that this radiation must not disturb the longitudinal emittance of the cold beam. In other words, the energy radiated must be small compared with the instantaneous energy spread in the bunch.

We will apply this method to the *optical stochastic cooling* scheme [1] due to its possible implementation in muon, proton, ion and electron cooling schemes, although these considerations could be applied to any microwave feedback as well.

Let us consider the scheme, represented in Fig.1.

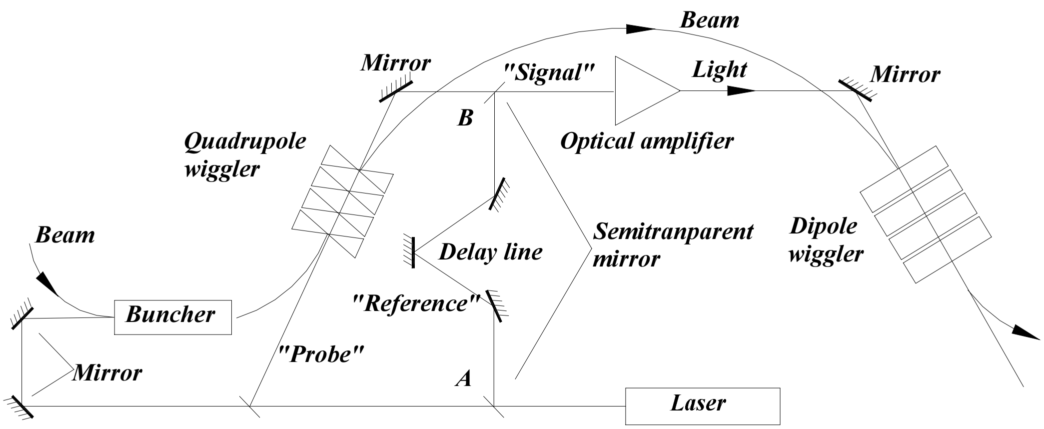

FIGURE 1: The general layout of the system.

Without the laser and buncher, the scheme is absolutely the same as described in [1] for optical stochastic cooling. All necessary formulas from there could be used here as well. Function of the buncher will be specified lower.

The semitransparent mirror ("A" in Fig.1) splits the light from the laser into two parts. The first part –"probe" one– goes to the quadrupole wiggler (FODO structure) co-directionally with the beam under cooling. Parameters of the wiggler are adjusted so that the probe wave stays in resonance condition with the particles of the beam. Parameters of the wiggler in this case must be the following

$$\lambda \cong \lambda_w \cdot (1+K^2)/2\gamma^2, \tag{1}$$

where λ–is wavelength of laser radiation, λ_w–is the wiggler period (period of FODO structure), $\gamma = E/mc^2$, $K = eH_{\perp}\lambda_w / 2\pi mc^2$, $H_{\perp} \cong G(s)\cdot x$ –is actual magnetic field value along the particles trajectory, $G(s) \cong G_0 \cos 2\pi s/\lambda_w$, G the gradient in lenses, x–is particle's transverse position. We will be interested in the case K<<1. Basically λ corresponds to the wavelength of spontantenouse radiation in the wiggler. This is the basic principle of coincidence of induced and spontantenous frequencies.

The second part of the split laser beam serves as a reference. The reference beam bypasses the pickup and combines with the probe beam in a second semitransparent mirror ("B" in Fig.1). The difference signal goes to the input of an amplifier. The adjustable delay line serves as a tuner for the reference light. The delay and optical structure in the transverse direction of these two beams arranged so that they destructively interfere at the input of the amplifier. This means, that the matrix of optical transformation for these two light paths is the same for each beam path, but the phase is opposite. This last can be arranged with a different number of reflections for these two light paths, for example, as shown in Fig.1. If the laser is pulsed, the length of the pulse must be longer, than the bunch length. A mismatch at the beginning and the end of the interference (or difference) signal does not action to the bunch, but may cause saturation of the amplifier. Fast optical gates could eliminate this problem. If, however, the laser is working in DC mode, there is no a problem at all.

The system is tuned so that in absence of the particle beam there is no signal at the input of the (optical) amplifier. Some adjustable attenuators required are not shown in Fig.1.

We would like to emphasize that despite the source being a narrow band laser, the all system is a wide band one. This is due to the fact, that the particles modify the probe beam in accordance with fluctuations in homogeneity and transverse particle's distribution. The length of these pieces modified is of the order of $\sim \lambda M$ where M –is the number of wiggler periods. Namely these modified pieces of radiation carrying information about local beam parameters define the bandwidth.

Due to similarity of this scheme to the techniques used in holographic imaging with reference and probe rays, we call this method *holographic cooling.*

The energy ΔE_{sp} deposited in differential microwave pickup can be estimated as the difference between the energy deposited in each electrode

$$\Delta E_{sp} = e\Delta U \cong \frac{e^2 N}{D+<x>_S} - \frac{e^2 N}{D-<x>_S} \cong \frac{e^2 N}{D}\frac{2<x>_S}{D}, \tag{2}$$

where D –is the aperture of pickup, N –is total number of particles in a bunch, $<x>_S$ –is a position of bunch centroid, averaged over bandwidth (subindex s means *sample*). The energy losses for stimulated radiation in the same pick up can be calculated as

$$\Delta E_{ind} \cong eN\frac{<x>_S}{D}\cdot U, \tag{3}$$

where U is a voltage applied to the electrodes. The ratio of (3) and (2) goes to

$$\frac{\Delta E_{ind}}{\Delta E_{sp}} \cong \frac{U}{e/(D/2)}. \tag{4}$$

The last is the ratio between the potential applied to electrodes and the potential induced by an individual particle. The ratio (4) for reasonable parameters: U=300V~1esu, $D \cong 1cm$, goes to $\Delta E_{ind}/\Delta E_{sp} \cong 2\cdot 10^9$. The same ratio characterizes the number of radiated photons in each case. This ratio is an expression for the general relation between induced and spontantenous radiation.

Really, stimulated (induced) and spontantenous variation of energy radiated by the bunch can be represented as integrals of instant power $(\vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{v}})$ over time as following

$$\Delta E_{ind} = \sum_{i=1}^{N} \int \vec{\boldsymbol{F}}_{ext}(t)\cdot\vec{\boldsymbol{v}}_i(t)dt, \tag{5}$$

$$\Delta E_{sp} = \sum_{i=1}^{N} \int \vec{\boldsymbol{F}}_i(t)\cdot\vec{\boldsymbol{v}}_i(t)dt + \sum_{i=1}^{N}\sum_{j\neq i}^{N} \int \vec{\boldsymbol{F}}_j(t)\cdot\vec{\boldsymbol{v}}_i(t)dt, \tag{6}$$

where the sum is over all particles in the bunch, $\vec{v}$ –is the speed of particle, $\vec{F}_{ext}$ –is the external force, $\vec{F}_i$ –are the forces arising from radiation of each particle. The two terms in (6) represent *noncoherent* and *coherent* spontantenous radiation respectively.

A quadrupole wiggler is a series of quadrupoles with alternating sign of gradient installed along a straight line.

For a particle passing a quadrupole wiggler off the central trajectory, the field picture looks like a field in an ordinary dipole wiggler with magnetic field proportional to the displacement. Betatron motion modulates the field with the frequency of transverse oscillations. We will be interested in the case, when the betatron wavelength is much longer, than λ_w. Detailed consideration of radiation in a quadrupole wiggler with arbitrary betatron wavelength is given in [2].

For calculating the energy change described by the first term in (5), or *stimulated radiation*, let us consider an electromagnetic wave propagating in the longitudinal direction s, $\vec{F}_{ext} = e\vec{E}_{ext}$, where $\left|\vec{\boldsymbol{E}}_{\boldsymbol{ext}}\right| = E_\perp \cos(\omega t - ks + \varphi)$, $k = \omega / v_p$, v_p–is the phase velocity of the wave, in vacuum $v_p = c$. If the particle experience transverse oscillations $\vec{\boldsymbol{\beta}}_\perp = \left|\vec{\boldsymbol{\beta}}_\perp\right| Cos\Omega t$, $\Omega \cong 2\pi c / \lambda_w$–is the frequency of transverse oscillations, then the integral, can be expressed in dipole approximation, $\beta_\perp \gamma \le 1$, $\vec{\boldsymbol{\beta}} = \vec{\boldsymbol{\beta}}_\parallel + \vec{\boldsymbol{\beta}}_\perp \cong \vec{\boldsymbol{k}}\beta$, where $\vec{\boldsymbol{k}}$–is unit vector in longitudinal direction as

$$\Delta E_{ind} = e\beta_\perp c E_\perp \int Cos\Omega t \cdot Cos(\omega t \mp \vec{\boldsymbol{k}}\vec{\boldsymbol{v}}t + \varphi)dt = eE_\perp \frac{L_w K}{2\gamma} Cos\varphi \tag{7}$$

where the value $K = \beta_\perp \gamma$ was introduced. We use in (7) a resonance condition for co-directional motion of wave and particle as

$$\Omega \cong \omega\left(1 - \frac{v}{v_p}\left(1 - \frac{\beta_{\perp x}^2 + \beta_{\perp y}^2}{2} - \frac{\vartheta^2}{2}\right)\right) \cong \frac{\omega}{2\gamma^2}\left(1 + K^2 + \gamma^2\vartheta^2\right), \tag{8}$$

where ϑ–is the observation angle, calculated from the forward direction. In straightforward direction this is the same ratio as mentioned above (1). Phase φ characterizes the relative shift between betatron phase, multiplied by the factor $\sim 2\gamma^2$ and the phase of the electromagnetic wave. The same formula describes interaction between the amplified signal and the particle in the kicker. As for harmonic motion in periodic quadrupole field,

$$\left|\dot{\beta}_{\perp i}\right| \cong \left|\frac{e \cdot \vec{\boldsymbol{k}} \times \vec{\boldsymbol{H}}_{\boldsymbol{i}}}{mc\gamma}\right| = \frac{e \cdot G_0 x_i}{mc\gamma}$$

K becomes $K_i = \beta_{\perp i}\gamma = \dfrac{eG_0\lambda_w x_i}{2\pi mc^2}$, where x_i –is a transverse coordinate of particle. One can see that particles with different K absorb energy from the wave in accordance with their *transverse* position in the quadrupole lens.

For cooling the *bunch* of particles, it needs to be bunched with period of laser radiation. That is the function of the buncher, Fig.1. Using for example, the bunching mechanism of optical klystron type [3], the particle density in half of period could be made higher, than average level on $b \cong J_0(1.8) \cong 0.57$ for a single dispersion section. This means that 57% of particles will be in the one period and the rest particles distributed randomly over period. Parameters of the bunching wigglers are the same as (1). With two bunching sections this fraction could reach $b \cong 0.75$. In case of bunching the energy modulation must be of the order of current energy spread, but while the beam is cooling this value becomes lower and lower. We also would like to stress here, that spontantenous coherent radiation is also small, as the number of the particles within the wavelength is small. So the energy acquired from the wave becomes

$$\Delta E_{ind} \cong \sum_i \frac{eK(x_i)L_w E_\perp Cos\varphi_i}{2\gamma} \cong \frac{r_0 G_0 \lambda_w L_w E_\perp N \cdot b}{4\pi\gamma} < x \cdot Cos\varphi >_S \tag{9}$$

where sum is over each particle transverse position, so $< x >_S$ becomes the dipole moment of the bunch within the bandwidth.

For calculation of *spontantenous radiation* from a quadrupole wiggler the friction force of radiation for the force F_i need to be substituted in (6). This friction force of radiation expressed through wiggler parameters becomes [4]

$$\left|\vec{F}_i\right| \cong -\tfrac{2}{3} r_0^2 \gamma^2 H_\perp^2 \cong -\tfrac{2}{3} r_0^2 \gamma^2 G_0^2 < x_i >^2 \tag{10}$$

where $r_0 = e^2 / mc^2$ is classical electron radius. So (6) now can be expressed as the following

$$\Delta E_{sp} = \sum_{i=1}^{N} \int \vec{F}_i(t) \cdot \vec{v}_i(t) dt \cong -\tfrac{2}{3} r_0^2 G_0^2 N \cdot < x >_S^2 \gamma^2 \cdot L_w \,. \tag{11}$$

The frequency of *spontantenous* radiation, ω, defined as Doppler-shifted frequency of transverse oscillations $\omega = \Omega / (1 - \vec{\beta}\vec{n})$ is the same as (8).

In the same manner one can consider the radiation in a *sextupole or higher multipole wiggler*. This provides an information about *higher* moments in transverse beam distribution.

In general the size of coherence $\sqrt{\lambda L_w}$, is bigger than the transverse dimension of the beam

$$\sqrt{\lambda L_w} \geq \sqrt{\varepsilon \beta}\,, \tag{12}$$

where ε, β–are emittance and envelope function of the beam respectively. So, radiation in the *forward* direction occurs only due to fluctuation of the instantaneous center of gravity within the bandwidth.

As we could see from (7), (9), the ratio between induced and spontantenous radiation is

$$\frac{\Delta E_{ind}}{\Delta E_{sp}} \cong \frac{3}{8\pi} \frac{E_\perp \lambda_w b}{r_0 G_0 < x >_S \gamma^3} = \frac{3}{4\pi^2} \frac{E_\perp \lambda^2 \gamma \cdot b}{e < K >_S}\,, \tag{13}$$

Basically this is a ratio of $E_\perp / E_{sp}$, where E_{sp}–is electric field, generated by a particle what is $E_{sp} \cong F_{rad} / e$. So here again like in (4), the ratio of induced to spontantenouse radiation can be treated as the ratio of external field to the field spontantenously generated by a particle.

For the ratio of radiated energy in bandwidth ~1/*M*, the last expression must be multiplied by factor *M* (in favor of induced radiation).

Let us make some estimation. Let the laser probe beam be of power density ~100*W/mm*^{2} or $E^2/377 \sim 10^8 W/m^2$, giving $E \cong 2 \cdot 10^5\, V/m$. Let the energy of the electron be 500 *MeV*, $\gamma \cong 10^3$. Let also the wiggler have period $\lambda_w \cong \lambda 2\gamma^2/(1+K^2) \cong 2m$, $K \cong 0.1$, such that the wavelength $\lambda \cong 1\mu m$. For these values (13) yields

$$\Delta E_{ind} / \Delta E_{sp} \cong 10^5\,. \tag{14}$$

This means that the induced energy radiated by single particle is about hundred thousands times higher than spontantenous radiation the corresponding gain in pickup signal. Therefore the amplification required is lower in the same ratio (see (15) below).

The energy spread as a result of induced radiation from example above is

$$\Delta E_{ind} \cong eE_\perp L_w K \cos\varphi / 2\gamma = 8 \cdot 10^{-16} eV\,.$$

A typical energy spread in the beam is about 10^{-3}, so the value of probe signal amplitude could be increased sixteen orders of magnitude, to be compared with natural energy spread. Therefore the limiting parameter is available laser power. As the bunch has finite length σ_b, the laser may be pulsed with corresponding length flashes. In this case laser pulsed power ~100W/mm^2 considered in our example is much below what is available routinely. If the bunch length is say 10*cm*, then the total energy of this laser flash is only $\sim 100\cdot\sigma_b / c \sim 3\cdot 10^{-8} J$.

According to the idea of stochastic cooling [5] each particle decreases its amplitude only due to interaction with its *own* amplified field. To kick the same particle after amplification, the radiated electromagnetic wave must be applied to the particle in congruence with initial longitudinal distribution. This can be done if the photons and the beam are moving in the same direction. As a direct transverse kick is not possible under this condition, the transverse kick in optical stochastic cooling is arranged through an *energy change* in a place, where dispersion is nonzero. As a result, amplification coefficient required can be expressed as the following [1]

$$\kappa = \sqrt{\frac{\Delta E_{kick}}{\Delta E_{pick}}} \cong \frac{\gamma\sigma_B}{r_0}\frac{\Delta E}{E}\frac{1}{N}\frac{\Delta f}{f} \cong \frac{\gamma\lambda}{r_0}\frac{1}{N_S}\frac{\Delta E}{E}\ , \tag{15}$$

where N_S is a number of the particles in the bandwidth $N_S \sim \dfrac{N}{\Delta f}\dfrac{c}{\sigma_B} \cong N\dfrac{M\lambda}{\sigma_B}$, $\Delta E / E$ –is current energy spread in the cooled beam. The equation for a change of emittance ε by the number of revolutions *n* is the following [5]

$$d\varepsilon / dn = -\varepsilon / N_S\ , \tag{16}$$

For a single cooling system installed in the ring this gives the cooling times $\tau \cong N_S T$, where T –is period of revolution. For reduction of damping time one needs to reduce the number of particles in the bandwidth $-N_S$. One can see from (15), that this requires an adequate increase of amplification, however. So the additional signal from the pickup helps to keep required amplification low. Reduction of the number of particles in a bandwidth can be achieved by increasing the length of the bunch accordingly.

The emittance decrease defined by the number of the photons radiated by the particles within bandwidth. In case of spontantenous radiation it is $\cong 1/(\alpha N_s)$ [1]. In case of stimulated radiation it will be lower in the same ratio as the number of photos is bigger in case of induced radiation i.e. 10^5 times lower.

For one example, considered in [1] for electron bunch, with number of the electrons $N \cong 10^{10}$, $\sigma_B \cong 15cm$ at $E \cong 500MeV$, with amplifier having pulsed power 5 *kW* and average ~25W implies $\tau \cong 10ms$. If the number of the particles reduced to 0.3% of its value, still keeping the same bunch length one can reach $\tau \cong \tau\cdot\sqrt{\Delta E_{sp}/\Delta E_{\sin d}} \cong 30\mu s$ with the same amplifier.

Also the number of periods in a quadrupole wiggler could be reduced, opening the possibility to pick up radiation from single quadrupole. This decreases the number of particles in the bandwidth.

For particles with different rest mass the amplification coefficient depends on the classical radius. If the γ –factor is the same, the radiation in the wiggler will be the same if particles with different rest masses go by the same trajectory.

The mass ratio between electron and muon yields an amplification coefficient (15) required by factor ~10^2 more for muons, than for electrons. This could be easily compensated by increased signal from pickup in the same proportion. If one increases the energy in the laser flash to 3 *mJ*, in the same 10 *cm*-long laser flash, then the electric field strength here becomes higher in $\sqrt{3/3\cdot 10^{-8}} \cong 10^4$ times. The number of photons here becomes $\sim 3\cdot 10^8$ times higher than in the case of spontantenous radiation. So cooling times of the order of μs could be reached here with the same amplifier. For cooling in longitudinal phase space one can use as a pickup a dipole wiggler as well. In this case radiation does not depend on transverse size of the beam. According to definition, the radiation within the size of coherence $\sqrt{\lambda L_w}$ could not be resolved. If however the transverse size of the bunch is larger, that this size, one can pick up radiation from different transverse parts of the bunch. This might be the way for further reduction of N_S, potentially opening a possibility for *single muon in a bandwidth*.

Proton cooling now becomes not limiting by optical amplifier. The only limiting factor for damping times becomes the number of protons in a bandwidth. For this the longitudinal density of the particles needs to be reduced either by reducing the number of the particles in a bucket or by lengthening the bunch. *Ion* cooling might be practical if atoms are fully ionized. In this case the charge/mass ratio is about half of the proton. So amplifier is practically the same as required for protons.

In conclusion, the radiation, *stimulated* by a probe beam, if used instead usual spontantenous radiation in the pickup of a feedback loop, makes the signal from the pickup orders of magnitude higher. This opens new possibilities in a beam cooling with the help of optical stochastic cooling as well as to increase the efficiency of working feedback schemes.

This may benefit existing machines (TEVATRON) as well as bring new possibilities in the planning stage (muon collider, LHC).

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