Supersymmetry and Strongly Coupled Gauge Theories

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I briefly review how supersymmetry helps in the extraction of exact nonperturbative information from field theories, and then discuss some open problems in strongly coupled gauge theories. (Talk given at "30 Years of Supersymmetry" symposium in Minneapolis, Minnesota on October 15, 2000.)

Unlike many of the other talks at this symposium, this one will not take supersymmetry "seriously": instead of being concerned with how or if supersymmetry enters physics beyond the Standard Model, this talk will treat supersymmetry merely as a technical tool to shed light on the strong coupling behavior of gauge field theories in general. Supersymmetric field theories are especially simple and symmetrical field theories, and as such are particularly "well behaved". We study the unusual or singular behavior of supersymmetric theories with the expectation that whatever can happen in a supersymmetric vacuum, will happen in spades in non-supersymmetric theories.

In the first part of this talk I will briefly outline those features special to supersymmetric theories which together provide a powerful handle on the nonperturbative vacuum structure of these theories. The second part will then describe some of the more unusual or unexpected things we have learned about strongly coupled gauge theories. This discussion will be formulated in the guise of three open questions. Here I restrict myself to Lorentz invariant gauge field theories without gravity in 4, 5, and 6 dimensions with 8 or 16 hermitian conserved supercharges. These restrictions are just to keep this talk short: field theories in lower dimensions and with fewer supersymmetries have an even greater variety of strong coupling behaviors, and likewise for gravitational (*i.e.* string) theories.

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1. HOW SUPERSYMMETRY HELPS

Supersymmetric nonrenormalization (NR) theorems aid in the identification of exactly marginal operators, and ensure exact flat directions giving rise to moduli spaces of vacua. The structure of superconformal algebras aids in the determination of exact anomalous scaling dimensions. And the structure of supersymmetry algebras with central charges can determine the exact mass spectrum of states invariant under a fraction of the supersymmetry generators (the BPS states). We will discuss each of these topics in turn.

1.1. Exactly marginal operators

Knowledge of exactly marginal operators (and their associated dimensionless couplings) is crucial to probing the strong coupling physics of field theories since one way of approaching strong coupling is by appropriately tuning a marginal coupling to a large value. The NR theorems of supersymmetric field theories give information to all orders in perturbation theory (see the talks by Iliopoulos and West) and nonperturbatively (following the work of Seiberg [1]) about certain classes of couplings.

To take the most famous example, in four-dimensional ($d = 4$) $N = 1$ supersymmetric gauge theories, the superpotential and gauge kinetic terms,

$$S \supset \int d^4x \int d^4\theta \left\{ \lambda \mathcal{O}(\Phi) + \frac{1}{g^2} W_\alpha^2 \right\}, \qquad (1)$$

are protected by such an NR theorem. More precisely, the theorem says that the anomalous dimensions γ_i of the chiral superfields Φ_i (coming from their kinetic terms) can only enter into the renormalization group (RG) running of the couplings λ and g^2 in a prescribed manner. Suppose, for instance, that

$$\mathcal{O} \equiv \prod_i \Phi_i^{n_i}. \tag{2}$$

Then the NR theorems imply the exact RG β function relations [2]

$$\begin{aligned} \beta_g &\propto \frac{3}{2}T(\mathbf{adj}) - \frac{1}{2}\sum_i T(\mathbf{R}_i)(1-\gamma_i), \\ \beta_\lambda &\propto 3 - d_{\mathcal{O}} - \frac{1}{2}\sum_i n_i\gamma_i, \end{aligned} \tag{3}$$

where $\mathbf{R}_i$ is the gauge group representation of Φ_i, $T(\mathbf{R}_i)$ is its index, and $d_{\mathcal{O}}$ is the canonical scaling dimension of $\mathcal{O}$.

This NR theorem helps in finding exactly marginal operators in those cases where the choice of representations $\mathbf{R}_i$ and operators $\mathcal{O}$ are such that $\beta_g \propto \beta_\lambda$. In this case one can deduce the existence of linear combinations of couplings with exactly vanishing β functions (see [3] and references therein).

A simple example [3] illustrating this is the $d = 4$ $N = 1$ $SU(n)$ theory with three adjoint chiral multiplets Φ_1, Φ_2, and Φ_3, with superpotential

$$\begin{aligned} \mathcal{W} &= \mathrm{tr}\big[a\Phi_1\Phi_2\Phi_3 + b\Phi_3\Phi_2\Phi_1 \\ &\qquad + c(\Phi_1^3 + \Phi_2^3 + \Phi_3^3)\big]. \end{aligned} \tag{4}$$

From the symmetry under cyclic permutations of the Φ_i we see that the γ_i are all equal, to γ, say. Then (3) implies $\beta_g \propto \beta_a \propto \beta_b \propto \beta_c \propto \gamma$. Thus, in this four complex dimensional space of couplings $\{g, a, b, c\}$ we would generically expect to find a three dimensional submanifold of exactly marginal couplings; furthermore, since this submanifold goes through the origin (*i.e.* weak coupling) we are assured that it indeed exists. The precise equation for the manifold of marginal couplings $\gamma(a, b, c, g) = 0$ is in general not known exactly away from the origin. (In this particular example, though, a one dimensional subspace is, namely $\{c = 0, a = -b = g\}$ where there is an enhanced $N = 4$ supersymmetry.)

1.2. Moduli spaces of vacua

Another way of probing the strong coupling behavior of field theories is by tuning the vacuum expectation values (vevs) of scalars to approach a strongly coupled vacuum. This can only be done if the scalars are *moduli* of the theory—*i.e.* have exactly flat potentials.

The nonperturbative control over potential terms implied by the NR theorem discussed above allows us in the case of 4 conserved supercharges (*e.g.* $d = 4$ $N = 1$ supersymmetry) to tune the couplings to ensure the existence of flat directions. In the case of 8 or more supercharges (*e.g.* $d = 4$ $N \geq 2$ or $d = 6$ $N \geq (1, 0)$) the flat directions are generic—no tuning is necessary.

A well known example is the $d = 4$ $N = 2$ $SU(n)$ superQCD theory. Here the complex scalar ϕ in the vector multiplet has an $(n-1)$ complex dimensional moduli space, the Coulomb branch, where $\langle\phi\rangle$ generically breaks $SU(n) \to U(1)^{n-1}$. In the cases where the superQCD theory is asymptotically free (AF) with strong coupling scale Λ, the Coulomb branch breaking for $\langle\phi\rangle \gg \Lambda$ is just a weakly coupled Higgs mechanism. On the other hand, sending $\langle\phi\rangle \to 0$ probes strongly coupled $SU(n)$ physics. As pioneered in [4,5], using analytic continuation in ϕ together with some qualitative physical assumptions allows an exact determination of the nonperturbative low energy effective action on the Coulomb branch for all values of ϕ.

More generally, this program of analytic continuation on a moduli space of vacua is greatly aided by the fact that the geometry of the moduli space is constrained by supersymmetric selection rules. For example, as discussed in the talks of De Wit and Ferrara, $d = 4$ $N = 1$ supersymmetry implies that the moduli space is a Kahler manifold; $d = 4$ $N = 2$ implies a kind of product of rigid special Kahler and hyperKahler manifolds; while $d = 4$ $N = 4$ theories have locally flat moduli spaces with orbifold singularities.

Finally, it is sometimes useful to note that the existence of exactly marginal couplings and exactly flat directions are not unrelated. By embedding a given theory, A, in a larger AF theory, B, marginal couplings in A can often be realized as vevs along a submanifold of the moduli space of B. Indeed, in gravitational (string or M) theories there are no couplings: all field theory marginal couplings are lifted to flat directions for some scalars.

1.3. Superconformal algebras

The structure of superconformal algebras (discussed in the talks by Ferrara and West) allows the determination of the exact anomalous scaling dimensions of certain operators, which often provide useful "diagnostics" for strongly coupled vacua.

If a scale invariant supersymmetric vacuum is also conformally invariant, then the theory is invariant under the superconformal extension of the supersymmetry algebra. Thus, heuristically, if the supersymmetry algebra is

$$\{Q_n, Q_m\} = \delta_{nm}\, \not{P} \tag{5}$$

where n, m run over the different supersymmetry generators in extended supersymmetry, then the superconformal algebra has double the number of fermionic generators, with a superconformal generator S_n for each Q_n, satisfying

$$\{S_n, S_m\} = \delta_{nm}\, \not{K} \tag{6}$$

where K_μ is the generator of special conformal transformations. Associativity of the algebra then implies

$$\{Q_n, S_m\} = \delta_{nm} D + R_{nm} + \cdots \tag{7}$$

where D is the generator of dilatations and R_{nm} are the R-symmetry generators. These algebras are tightly constrained by associativity; for example, for $d > 4$ they only exist for $N = 1$ in $d = 5$ and $N = (1,0)$ or $(2,0)$ in $d = 6$, and not at all if $d \geq 7$ [6].

The classification of unitary (or "positive energy") representations of the superconformal algebras [7–9] imply inequalities on the eigenvalues of D and R_{nm} following essentially from the positivity of operators like $[(Q_n + Q_n^\dagger) + (S_n + S_n^\dagger)]^2$ and the algebra (5–7). These inequalities are saturated for certain chiral fields (annihilated by some of the Q_n and S_n generators). Knowledge of the R-symmetry charges of these operators then allows one to deduce their D charges, *i.e.* their exact scaling dimensions [10,11].

1.4. BPS states

BPS states are supersymmetric particle states whose mass can be determined exactly from the structure of the supersymmetry algebra. They can thus provide another "diagnostic" of (non-scale-invariant) strongly coupled vacua.

The argument is analogous to that determining scaling dimensions from the superconformal algebra. In the non-conformal case the R-charges do not enter the algebra, but a set of $U(1)$ charges—the central charges Z_{nm}—commuting with all other generators can enter:

$$\{Q_n, Q_m\} = \delta_{nm}\, \not{P} + Z_{nm}. \tag{8}$$

Positivity of operators like $(Q_n + Q_n^\dagger)^2$ then give inequalities between the eigenvalues of P_μ and Z_{mn}, *i.e.* between particle masses and their $U(1)$ charges. For certain states—the BPS states—which are annihilated by some fraction of the Q_n, these inequalities are saturated and the mass is determined by the central charge [12]. In certain cases the supersymmetric selection rules and analytic continuation techniques mentioned in Section 1.2 above can be used to determine the central charges and thus the BPS spectrum as functions of the couplings and vevs at strong coupling; *e.g.* [4,5].

2. THREE QUESTIONS

Instead of attempting a survey of the kinds of strongly coupled phenomena that have been found using the tools outlined in the last section, I will focus on three open questions concerning strongly coupled phenomena which I think are both important and on which progress can be made.

2.1. "Ultra-strong" coupling

Different values of the exactly marginal complex gauge coupling

$$\tau \equiv \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}, \tag{9}$$

where g^2 is the gauge coupling and θ the theta angle in a scale invariant $d = 4$ gauge theory, are typically identified under S-duality (also known as Montonen-Olive duality or strong-weak coupling duality) transformations. The prototypical example of this [13,14,12,15,16] is the $SL(2,\mathbf{Z})$ group of S-duality identifications in $d = 4$ $N = 4$ superYang-Mills. This group is generated by the theta angle 2π rotation $\tau \to \tau + 1$ and by inversion of the gauge coupling $\tau \to -1/\tau$. This S-duality group relates weak $(g \to 0)$ to ultra-strong $(g \to \infty)$ couplings, but does not "solve" the strong coupling behavior of $N = 4$ superYang-Mills. This is illustrated in Figure 1 which shows a fundamental domain in the complex τ upper half plane under these identifications. This domain includes a weak coupling limit $(g \to 0)$ at $\tau = +i\infty$ as well as some special strong coupling vacua at $\tau = i$ and $e^{i\pi/3}$ (where $g \sim 1$). These special strong coupling points have enhanced $\mathbf{Z}_2$ and $\mathbf{Z}_3$ discrete global symmetries, respectively, but apparently no other more striking behavior. A similar story holds for scale invariant $d = 4$ $N = 2$ $SU(2)$ superQCD with four fundamental hypermultiplets [4,5].

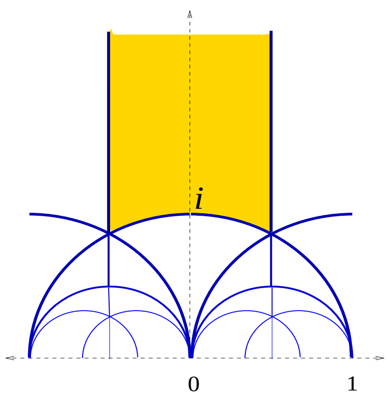

Figure 1. The shaded region is a fundamental domain of the action of $SL(2,\mathbf{Z})$ in the complex τ plane.

However, the story changes qualitatively for $d = 4$ $N = 2$ superQCD with gauge groups with rank ≥ 2. For the generic such theory, the S-duality relations no longer identify all ultra-strong coupling points with weak coupling points. This is illustrated in Figure 2 which shows a fundamental domain for the S-duality group of $SU(n)$ $n \geq 3$ $d = 4$ $N = 2$ superQCD [17] which is an index three subgroup of $SL(2,\mathbf{Z})$. Note that a point $(\tau = 0)$ with $g = \infty$ is included in the domain.

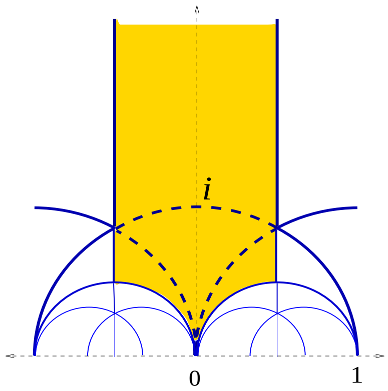

Figure 2. The shaded region is a fundamental domain of the action of an index 3 subgroup of $SL(2,\mathbf{Z})$ in the complex τ plane.

The question thus arises as to what is the nature of the physics at these ultra-strongly coupled theories with $\tau = 0$. That there is some qualitatively new physics (as compared to the merely strongly coupled theories) can be deduced from the behavior of the low energy $U(1)^{n-1}$ effective action on the Coulomb branch of these theories, where we observe that as $\tau \to 0$ the effective action becomes singular, indicating the appearance of new light degrees of freedom. Moreover, these new singularities appear everywhere on the Coulomb branch and not just on a submanifold of it.

There are a number of possibilities for the answer to this question. Perhaps these ultra-strong coupling points are in fact the weak coupling limits of some other theory. If so this would raise the possibility of a "web of dualities" connecting different field theories similar to what is seen in Calabi-Yau compactifications of string theories. However in the simplest case of $SU(3)$ $N = 2$

superQCD an explicit survey of all known scale-invariant theories with rank 2 gauge groups finds no weak coupling candidate which matches onto the $SU(3)$ theory. For example, a promising candidate (from hints from a IIA string construction of the $SU(3)$ theory) is the scale invariant $SU(2) \times SU(2)$ theory with 2 fundamental hypermultiplets in each factor and one bifundamental hypermultiplet; an analysis [18] of the coupling space of this theory shows no ultra-strong coupling points.

A second possibility is that due to an ambiguity in the overall scaling of operators as one approaches the ultra-strong point, the singularity in the effective action is really just a kind of coordinate singularity—*i.e.* a place where no new light states occur, but where the coupling space joins smoothly to that of another theory.

A final, and most likely, possibility is that the ultra-strong coupling points signal a genuinely new strongly coupled "phase" of gauge theories. Perhaps a hint of this comes from the IIA brane construction [19] for the $N = 2$ theories, where the ultra-strong points correspond to brane configurations with coincident NS5 branes. We will discuss the $d = 6$ physics of stacks of NS5 branes in Section 2.3 below.

2.2. Gauge theories with no weak coupling limits

Another interesting class of strongly coupled theories occur as superconformal vacua at special points on a moduli space of vacua. For example, all the $d = 4$ $N = 2$ superconformal field theories with a one complex dimensional Coulomb branch (which we will parameterize as the complex u plane) can be described by the singular degenerations of Seiberg-Witten tori as shown in Table 1.

Here x and y are the auxiliary complex variables describing the Seiberg-Witten Riemann surface. The mass scaling dimensions $D(u)$ of the $N = 2$ chiral superfield U whose lowest component is the modulus u are found using the superconformal algebra as described in Section 1.3 above. The dimensions of the associated coupling λ is $D(\lambda) = 2 - D(u)$ as follows from the form of

Table 1
The physical inequivalent degenerations of Seiberg-Witten tori (as $u \to 0$) describing $d = 4$ $N = 2$ conformal field theories with a single modulus u together with their associated global symmetry groups G and the scaling dimensions $D(u)$ of the modulus superfield.

Curve	G	$D(u)$
$y^2 = x^3 + u$	$U(1)$	$6/5$
$y^2 = x^3 + ux$	$U(2)$	$4/3$
$y^2 = x^3 + u^2$	$U(3)$	$3/2$
$y^2 = x^3 + \lambda u^2 x + u^3$	$SO(8)$	2
$y^2 = x^3 + u^4$	E_6	3
$y^2 = x^3 + u^3 x$	E_7	4
$y^2 = x^3 + u^5$	E_8	6

the $N = 2$ superspace coupling

$$\int d^4x \int d^4\theta \, \lambda U. \qquad (10)$$

Referring to these theories by their global symmetry groups, we see that only the global $SO(8)$ theory has an exactly marginal coupling. This coupling is in fact the gauge coupling of the scale invariant $SU(2)$ theory with four fundamental hypermultiplets described in [5]. The three global $U(n)$ theories were found in [20] by tuning masses and vevs in the global $SO(8)$ theory.

What interests us here are the three E_k theories found in [21] which have no known embedding in $d = 4$ gauge theories. In other words they have not been shown to appear as limits of other AF or scale invariant gauge theories in four dimensions. There is in fact a large class of such "exceptional" or "isolated" $d = 4$ conformal field theories: many can easily be found with 2 complex dimensional Coulomb branches (though no complete classification is known in this case), and evidence from the AdS/CFT correspondence [22] shows that the global E_k theories have relatively benign large N limits (where N refers to the "rank" of the theory, in this case given by the dimension of its Coulomb branch).

Given that these theories have no known embedding in a $d = 4$ theory with a weakly coupled limit, how do we know they actually exist at all as consistent field theories? The evidence for their existence comes from string constructions: they can be realized as compactifications of similar isolated $d = 5$ [23,24] or $d = 6$ conformal theories [25–27] which in turn have known string constructions. For instance, the $d = 5$ theories can be constructed [23] as the field theory on coincident D4 brane probes of a IIA string background with D8 branes and O8 orientifold planes.

The question naturally arises as to what is an intrinsically four dimensional *definition* of these theories. Possible ways of answering this question would be to either embed these theories in some higher rank AF theories (possibly with less supersymmetry); or to find something analogous to the lattice definition of AF gauge theories, *i.e.* a direct nonperturbative definition.

This question is equivalent to asking for a $d = 4$ characterization of the universality classes of these theories. Note that these isolated $d = 4$ conformal field theories should be contrasted to other $d = 4$ theories which are thought to have only higher-dimensional definitions, such as the "scale invariant" $d = 4$ $N = 2$ theories with compact Coulomb branch [28], or the infrared (IR) free $d = 5$ $N = 2$ superYang-Mills theory [29]. Unlike the $d = 4$ conformal field theories we have been discussing, there is no reason to think that these latter theories *have* to have a $d = 4$ (or $d = 5$) definition, since they include irrelevant operators which may indeed be inherited from a higher dimensional theory.

2.3. Little string theories

Perhaps the strangest and most interesting strong coupling behavior yet encountered in supersymmetric field theories are the "little string" theories which can be thought of either as $d = 6$ non-gravitational string theories at a fixed (strong) coupling, or as quasi-local field theories which lack operators more localized than a certain length scale.

These theories are known through their string constructions. Below I will simply list some of the properties of these theories in the simplest case of $d = 6$ with 16 supercharges. Their string construction is as the theory living on a stack of k NS5 branes in either IIA or IIB string theory with fixed string tension $1/\alpha' = M_s^2$ and in the limit as the string coupling vanishes $g_s \to 0$. This limit implies that bulk ($d = 10$) string modes including gravity decouple, and leaves a non-trivial $d = 6$ theory with a scale M_s [30,31].

The IR properties of these theories, *i.e.* at scales below M_s, are as follows (short reviews are [32,33]). At the origin of their moduli spaces one finds either an $N = (1, 1)$ supersymmetric IR free Yang-Mills theory with a general rank k gauge group G_k [34] (from the IIB string construction); or an $N = (2, 0)$ conformal field theory with a $G_k = A_k$-D_k-E_k classification (from the IIA string construction). At a generic point on moduli space the $N = (1, 1)$ theory is broken down to k massless $U(1)$ vector multiplets, while the $N = (2, 0)$ theory is broken down to k self-dual antisymmetric tensor multiplets. These theories both have BPS string states with tensions which approach M_s^2 at the origin in the $N = (1, 1)$ theory and zero in the $(2, 0)$ theory. The density of states at the origin depends on the rank as k^2 in the $(1, 1)$ theory and k^3 in the $(2, 0)$ theory. The long wavelength limit of compactifications of both these theories on a circle gives a $d = 5$ $N = 2$ superYang-Mills theory with gauge group G_k.

The $(2, 0)$ low energy properties strongly hint at some kind of "nonabelian self-dual 2 form gauge theory". However no such local classical field theory has been constructed, despite interesting guesses at the algebraic structure which might play a role analogous to the Lie algebra structure of Yang-Mills theory (for example, [35]), and systematic computations of the self-interactions of 2 form gauge fields from the effective action of 5 branes (for example, [36]). In any case, the IR properties of the little string theories by themselves are consistent with them being some relevant deformation of a $d = 6$ local superconformal field theory. The real surprise is that this is *not* the case: the theory is in fact nonlocal above the scale M_s.

The most striking evidence of this is the fact that these theories inherit the T-duality of their IIA and IIB superstring parents. Thus, the $(2, 0)$ theory compactified on a circle of radius R is equivalent to the $(1, 1)$ theory compactified on a circle of radius $(M_s^2 R)^{-1}$. This sort of behavior is characteristic of closed string degrees of freedom with tension M_s^2. Furthermore, these theories have a Hagedorn density of states $\rho(E) \sim e^{E/T_H}$ with $T_H \sim M_s/\sqrt{k}$ [37]. These high energy properties suggest that there may be a weakly coupled (and non-gravitational) description of these theories as strings, though perhaps only at high enough energy densities.

The question here, as in Section 2.2, is what is an intrinsic or minimal definition of little string theory. Actually, there is already an answer: the discrete light cone quantization of little strings in (uncompactified) $d = 6$ is equivalent to the $N \to \infty$ limit of a certain $SU(N)$ supersymmetric quantum mechanics or a $1+1$ dimensional sigma model [38–40]. Although in principle this gives a nonperturbative definition of the theory, it is very hard to compute with it. Moreover, basic properties of the theory, such as Lorentz invariance and its BPS spectrum are not manifest and are only expected to be recovered in the $N \to \infty$ limit. On the other hand, it has been argued that these theories obey the axioms of "quasi-local" field theory in which correlators exist in momentum space but not in position space on length scales less than M_s^{-1} [41–43]. Perhaps this indicates that there is a more direct definition of little string theories.

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