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A Biased Review of Tau Neutrino Mass Limits

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After a quick review of astrophysically relevant limits, I present a summary of MeV scale tau neutrino mass limits derived from accelerator based experiments. I argue that the current published limits appear to be too consistent, and that we therefore cannot conclude that the tau neutrino mass limit is as low as usually claimed. I provide motivational arguments calling into question the assumed statistical properties of the usual maximum likelihood estimators, and provide a prescription for deriving a more robust and understandable mass limit.

1. Neutrinos and Cosmology

All particle species, including neutrinos, must have been produced during the Big Bang. Should neutrinos be stable and sufficiently massive, their combined gravitational attraction would be sufficient to collapse the Universe. Our existence therefore allows one to infer properties of invisible but gravitationally interacting matter. This argument was originally advanced by Gershtein and Zel'dovich[1] in 1966 and was used by Cowsik and McClelland[2] to devise a limit on the order of $100(\times h)eV$ for the sum of the masses of stable neutrino species, where h is the scaled Hubble Constant. This argument also supplies a lower bound on extremely heavy neutrinos at the GeV scale. Details of these limits can change substantially if neutrinos decay. The change depends on the equation of state of its decay products - a massive neutrino decaying to massless particles will effect the evolution of the universe differently than one with massive decay products. The effect also depends on the time scale of the neutrino decay relative to other time scales in the Universe, such as that of Big Bang nucleosynthesis. The contribution of decaying neutrinos to supernovae energetics can also be used to derive limits on ν_τ lifetime and mass [3]. Shown in Figure 1 are a set

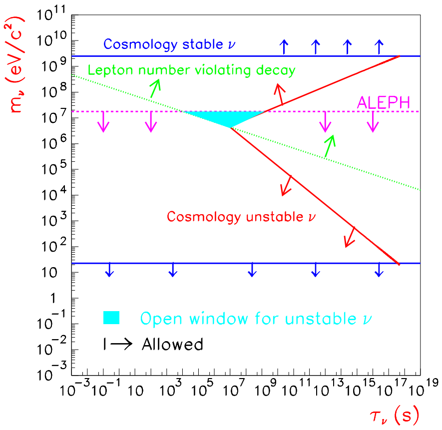

Figure 1. Some cosmological and particle physics limits on the ν_τ mass and lifetime as per [5]

of allowed and excluded regions of mass and lifetime, as supplied by ALEPH [5]. In addition to these cosmological bounds, the figure also shows the limits imposed by the lack of observation of neutrino-less decays such as $\tau \to 3\mu$ - these decays could have massive neutrinos inside of loops and would therefore be sensitive to neutrino properties. In addition, the stringent limit claimed by ALEPH[5] is also plotted in the figure. Note that on such a scale, the difference in mass limits from different experiments would be barely visible. Of interest is that at the MeV mass scale, where accelerators are currently sensitive, many lines cross - this indicates that this could be a region rich with (mis)understanding, where many different approximations might break down.

It would be interesting to review what effect recent indications of a non-zero cosmological constant might have on all these limits. The overlap of the Supernova and Boomerang data released this year indicate that the gravitational energy density of the Universe is to within a factor of a few equal to that used to derive the Cowsik-McClelland limit and that the ensuing argument is still qualitatively valid. One can also recast the argument in terms of the stability of clusters, and this still yields a comparable limit [4].

2. Recent Results of Interest

The DONUT collaboration has recently announced that it has made the first observation of tau neutrino appearance. Although their results (reported elsewhere in these proceedings) do not tell us very much about the mass of this neutrino, they do tell us that it does exist.

The KARMEN collaboration, which had previously reported an anomaly consistent with a weakly interacting neutral particle emitted in pion decays with a mass of 33.9 MeV [6], no longer sees this anomaly in a larger dataset. Their new results are reported elsewhere in these proceedings.

Possibly the most talked about recent results are those coming from the SuperKamiokande collaboration, also reported elsewhere in these proceedings. They report that their data is consistent with the mixing of muon type neutrinos into a different state, most probably τ neutrinos. Should these results hold up, they are a strong argument against a massive neutrino in the allowed MeV mass range. Unfortunately, SuperKamiokande has yet to take into account the possibility that neutrinos are massive and can decay. Given that all other known particles that oscillate also decay, one might legitimately ask whether this is also the case in the neutrino sector. One could hope that low mass, long lived electron type neutrinos, medium lived muon neutrinos and short lived massive tau neutrinos might help explain the solar and atmospheric neutrino puzzles. Until such possibilities are ruled out it might be premature to assume that mixing has been proven and that the MeV mass scale for the tau neutrino is no longer interesting. Simply showing consistency with a given model does not necessarily rule out other models.

3. Accelerator Based Non-Endpoint Limits

At electron-positron colliders, it is possible to produce mono-energetic tau pairs. When these decay to hadrons and exactly one neutrino each, the tau flight direction on each side must have been along a cone around the direction of the hadronic decay products. The size of this cone depends on the tau and neutrino mass. Requiring that the common τ flight direction is on the overlap of the two cones allows one to measure $m_\tau^2 - m_\nu^2$. This method was used by CLEO[7], resulting in the mass distribution shown in Figure 2. The sharp drop near 1.8 GeV provides the required mass. Combining this result with the BES result for the τ mass[8], CLEO derives an upper limit of 60 MeV on the neutrino mass at 95% C.L.

If the tau neutrino were more massive than the tau, the tau could not decay: the mass of the neutrino restricts the allowed phase space and decay rate. One can set a limit on the neutrino mass by carefully measuring branching ratios of the tau. The theoretically best predicted decays are the fully leptonic decays $\tau \to l\nu_l\nu_\tau$ where $l = e, \mu$, with the main uncertainties coming from the tau mass and lifetime. These are measured at the per cent level. Knowing the pion and kaon form factors, once can also use the single body hadronic decays of the tau. Using the Particle Data Group best fit values for all these parameters, Swain and Taylor[9] derives a 95 % confidence level upper limit on the tau neutrino mass at 68 MeV.

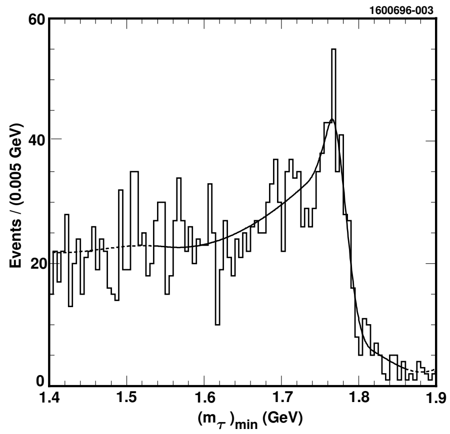

Figure 2. Two Cone Mass for $m_\tau^2 - m_\nu^2$ determination from [7] - the location of the sharp drop determines the limit.

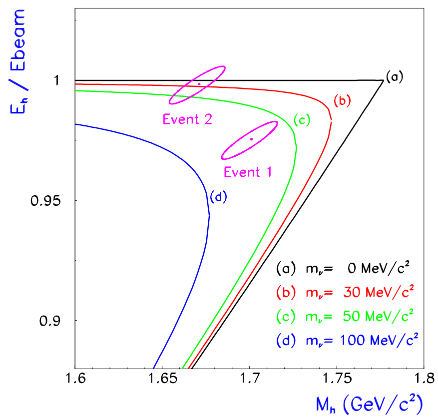

Figure 3. A typical scaled hadronic energy vs mass distribution (from [5]). Note the kinematically allowed boundaries for different neutrino masses and the typical error ellipses around the events.

4. Accelerator Based Endpoint Limits

The method that has given the most stringent limits on the tau neutrino mass attempts to fit the two dimensional spectrum of hadronic decay product mass and energy for τ's produced at electron-positron colliders. As shown in Figure 3, near the endpoint, the kinematically allowed contour depends very strongly on the neutrino mass. Not only does the contour depend on the neutrino mass, but for any fixed energy, the hadronic mass spectrum shape also depends on the neutrino mass: points away from the limit contain some information on the neutrino mass. However, for a fixed hadronic mass, the derivative of the hadronic energy with respect to the neutrino mass is effectively flat except for a delta function at the contour limit[1]. Finite smearing dilutes further the neutrino mass sensitivity, as do initial and final state radiation. Note that ISR/FSR cause points to drop along the energy axis in the figure, but do not smear them along the mass direction. These effects are calculable however.

One can distinguish two different types of decays used for this kind of analysis with different sensitivities to systematic errors. Decays to many pions (e.g. $\tau \rightarrow 5\pi\nu$) tend to produce events near the sensitive endpoint but suffer from small branching ratios and low reconstruction efficiencies. Decays with few particles in the final state (e.g. $\tau \rightarrow 3\pi\nu$) tend to produce events far away from the endpoint, but have efficiencies and branching ratios that are large - one hopes that the efficiencies are large enough that a rare event near the endpoint will be reconstructed.

The most general likelihood function is of the

[1]This means that in the limit of a detector with no smearing, only those points at the contour limit would provide information on the neutrino mass in the hadronic energy direction, while along the mass axis all points would provide neutrino mass information. The mass limit could potentially be set by one point with mis-measured energy - this problem is far less severe along the mass axis where all points contain information. This approximation points out that the systematic error estimation due to smearing has to be done very carefully. There is also a need for a careful evaluation of the analytic and statistical properties of the likelihood function in the presence of an underlying discontinuous estimation parameter dependence. No such examination has been done explicitly in any of the neutrino mass literature.

form:

$$\mathcal{L}(M_{\nu_\tau}) = \mathcal{P}\left(N_{obs}, M_{\nu_\tau}\right) \prod_{Data} \left(\alpha \mathcal{L}_{Signal} + (1-\alpha)\mathcal{L}_{BGD}\right)$$

The first term, used only by [10] and [12], is a Poisson term relating the number of events near the endpoint as a function of the number of events far from the endpoint and the neutrino mass. One uses this term as a compromise to avoid fitting over many points which do not have much neutrino mass information, and also lessen the dependence on the explicit form of the physics function in the low mass region.

The product term is taken over all accepted data events, and is composed of a signal term $\mathcal{L}_{Signal}$ and a background term $\mathcal{L}_{BGD}$. The signal term is composed of a convolution of the expected detector smearing, efficiency and the differential decay width. The neutrino mass enters only through the decay width and the limits of integration for the convolution - again, the energy dependence on neutrino mass in this term comes in only as an integration limit, and not a slope. The differential decay width depends on the V-A nature of the weak interaction, the underlying physics of the event and the radiative corrections, including ISR and FSR. The background term is a parameterization of events which are not signal, whether originating from (mis-reconstructed) tau decays or other physics. These terms in general do not depend on the neutrino mass - this is true to a high degree even for the mis-reconstructed τ events since they tend to be highly smeared over the plot. This lack of dependence on neutrino mass makes this type of likelihood divergent when all neutrino masses are considered so one introduces a prior distribution, cutting off neutrino masses greater than say 100 MeV. The background shape is generally determined either from Monte Carlo or directly from non-tau decay data. The relative weight of signal and background, α, is usually determined by fitting for the number of events above the endpoint region.

The likelihood is then used to determine an upper limit on the mass of the neutrino. This is generally done by integrating[2] the likelihood to a 95th percentile above 0 MeV or by finding where the log of the likelihood drops by 1.92 from its peak. Neither method is more correct than the other, rather each expresses a different philosophy on the meaning of the upper limit. In all cases published so far the application of either method gives a consistent answer to within $\approx \mathcal{O}(5\text{MeV})$.

5. Current Results

Since only one new result has been published since the last conference, readers desiring a full review of previous results should refer to R. McNulty's review in the previous set of proceedings [11]. These results are briefly summarized in Table 1. The new result is from CLEO[12] which has examined the neutrino mass using the decay $\tau \to 3\pi\pi^0\nu$ for the first time. This sample is composed of 29,000 decay events, as shown in Figure 4. The fit includes an explicit term accounting function for mis-identified tau decays, corresponding to 7% of the event sample. Some 3% of the sample is accounted for by a non-tau background term. The spectral function in this decay was fit below the neutrino mass sensitive

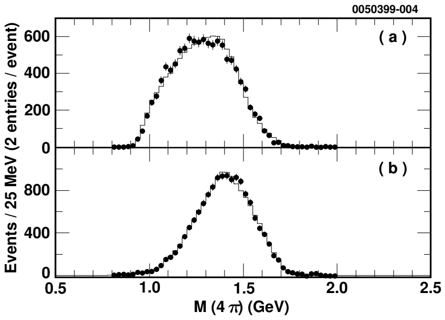

Figure 4. Four pion invariant masses for events with an ω (top) and events with out an ω (bottom) for CLEO's $\tau \to 3\pi\pi^0\nu$ sample. The histogram shows the Monte Carlo expectation.

[2] Note that strictly speaking it is a product of the likelihood function and a top hat prior distribution which is being integrated, as per [13]

region to a combination of non-resonant, rho and omega subcomponents. This fit was then extended into the endpoint region. The resulting likelihood distribution is shown in Figure 5. The

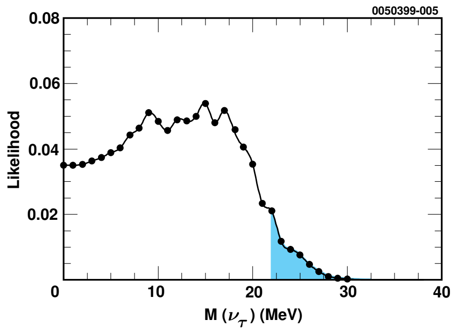

Figure 5. The neutrino mass likelihood distribution for CLEO's $\tau \to 3\pi\pi^0\nu$ sample. Note the raw 95th percentile at 22 MeV.

95th percentile above 0 MeV is at 22 MeV for the raw data, and the total systematic error, added in linearly to conform with the standard set by previous studies, sets the final limit at 28 MeV. The main systematic error comes in through the spectral function dependence, the π^0 energy scale, as well as the charged track momentum reconstruction.

6. General Remarks on Published Results

A glance at Table 1 summarizing all published results shows that all limits are generally within the same range, in spite of a very large range of event sample sizes.

The typical upper limit in this table is in the 30 MeV range. One would naively expect that the scatter in the most likely neutrino mass for each experiment should be on the order of $30MeV/1.64 = 18MeV$ - this is manifestly not the case. For the purposes of the following, let us imagine that the true neutrino mass is 0 MeV, and that each experiment has a resolution of 18 MeV on the neutrino mass. I believe that 7 of the

Table 1: Summary of Neutrino Mass Limits

	Decay Mode	Events	$<\sigma_M>$ (MeV)	M_{95} (MeV)	$\approx \hat{m}(MeV)$
ALEPH98[5]	$5\pi(\pi^0)$	55	15	23	0
CLEO98[10]	$5\pi, 3\pi2\pi^0$	≈ 450	15	30	15
OPAL98[14]	5π	22	25 (?)	43.2	0
ALEPH95[15]	5π	25	15	24	0
OPAL95[16]	5π	5	25	74	0
CLEO93[17]	$5\pi, 3\pi2\pi^0$	113	10	32.6	0
ARGUS92[18]	5π	19	10 (?)	31	(*)
CLEO00[12]	$3\pi\pi^0$	$\approx 17,000$	20	28	0
ALEPH98[5]	3π	$\approx 2,900$	15	22	5
OPAL97[19]	3π	≈ 2500	$\sigma_{M^2_{miss}} \approx 0.1GeV^2$	29.9	0
DELPHI97[11]	3π	$\approx 12,000$	20	28(68)	-10

The $<\sigma_M>$ gives the typical mass smearing for the reconstructed event masses. The $\hat{M}$ column denotes the neutrino mass value which maximizes likelihood as estimated by this author from published likelihoods.
OPAL97[19] is a two dimensional fit of missing mass vs missing energy. DELPHI97[11] notes that its limit depends strongly on the admixture of a possible high mass 3π state, with a 28 MeV limit for no admixture, and a 68 MeV for a large admixture.
ARGUS92[18] did not publish a likelihood curve.

published results in the table are statistically independent, in addition to the unpublished DELPHI result[11]. Once can ask what the Gaussian probability is that all 7 different published experiments could all arrive at the result that the most likely neutrino mass is between 0 MeV and 18 MeV - this is obtained from a one tailed Gaussian distribution and is equal to $0.64^7 = 4\%$. If we include the unpublished DELPHI97[11] result and ask what the probability is that the most likely mass is less than 18 MeV, including negative values, the probability is somewhat more promising, namely $\frac{8!}{7!1!} * 0.84^7 * 0.16^1 = 38\%$. In this case however, the probability of all the maxima being located within $\pm 18 MeV$ of 0 MeV is derived from a two-tailed Gaussian as $0.68^8 = 5\%$. If one argues that in fact the typical resolution is larger than 18 MeV, then these results are even more extreme. These simple considerations lead me to conclude that some subtle series of systematic biases must be driving down the published neutrino mass limit[3].

7. Subtleties

How should one quote the neutrino mass limit? There is no accepted way to judge the relative merits of different limits. The PDG[13] simply quotes the lowest published value for the upper limit. This prescription is unfortunately biased towards finding a massless neutrino.

One can attempt to combine the likelihood functions of different experiments and thus hope to obtain a clear answer. However, should any one of those experiments suffer from an unfortunate fluctuation due to an undetected systematic error towards low m_ν, the resulting combined distribution will also suffer from this. The problem with this method is that all of the experimental results are considered on an equal footing. One should clearly have a mechanism to de-weight less reliable results. Combining limits from different published results is simply quite difficult. Not only is it not clear how to incorporate different systematic errors from different experiments properly, but the fact that the different measurements tend to use slightly different functional forms for their likelihood makes a blind multiplication of likelihood functions difficult to interpret.

One might argue that, in the face of similar experimental resolutions, the experiment with the largest sample should have the most robust result, just like in the case of a branching ratio measurement. Neutrino mass limits differ in some important ways from the limits one typically makes on branching ratios. A branching ratio limit can treat each event as having an equal amount of information. However, the information content of an event in a neutrino mass limit depends on its reconstructed location in the energy vs mass plane and on its associated expected smearing, as well as the particular spectral function of the mode under consideration. This means that the large N limit where we are all typically comfortable in interpreting likelihoods as statistically meaningful depends on what region of neutrino mass one is investigating. "Large N" for a 100 MeV limit can be much smaller than "large N" for a 10 MeV limit[4]. In addition, the range of possible reconstructed masses for an event depends on the value of the true neutrino mass (this is most clearly seen for a perfect detector with delta function resolution.) An important property of maximum likelihood estimators is that they are efficient: if an estimate of minimum variance exists for a parameter, then the maximum likelihood should find it[13]. However, the derivation of this minimum variance bound, known as the Rao-Cramer-Frechet bound, requires that the range of the underlying event probability distribution not depend on the parameter being estimated[13]: this is explicitly not true for the neutrino mass fits. Thus, with the Rao-Cramer-Frechet bound in question, it is not clear that the maximum likelihood estimator can find the estimate with the minimum variance[5]. Of course in the opposite

[3] One might also argue that the Gaussian approximation used in the above is invalid for such small statistics - this of course calls into question the meaning and validity of the quoted upper limits.

[4] Similarly "large N" for a mode such as $\tau \to 3\pi\nu$ has to be much larger than "Large N" for the mode $\tau \to 5\pi\nu$.

[5] One might argue that the presence of smearing, which allows events to be reconstructed above the kinematically determined endpoint, makes the above argument less compelling. However, this line of thought leads to the conclu-

extreme, if the smearing is large enough, the information content of any point tends to zero, and the minimum variance bound tends to infinity. These observations point out the need for a serious evaluation of the form of the likelihood used for neutrino mass estimation and its properties.

One should also keep in mind that showing consistency with a massless neutrino as one typically does by quoting an upper limit is not the same as excluding massive neutrinos. One can occasionally get low fluctuations in the neutrino mass limit even for massive neutrinos.[6]

Of course one very important consideration that must be taken into account is the sociological bias towards massless neutrino results. Experimental groups tend not to work as hard to publish results that are not more stringent that their competitor's.

8. Upper Limits Are Not What You Think They Are

As an illustration that neutrino mass limits are not straightforward, consider Table 2. This table summarizes the number of times one gets an upper limit at 27 MeV or lower using the CLEO Monte Carlo for reconstructed data samples of 25 events and 450 events for two different input neutrino masses of 0 MeV and 50 MeV. An experimenter is confronted with an experimentally derived limit, and from this must venture a guess as to what the input (true) neutrino mass might have been. With this in mind, with a small sample of 25 events one is much more likely to mistakenly accept or reject a massive neutrino than one is with a large 450 event sample. Thus with similar detectors at similar energies and the same decay mode, one is more likely to infer the correct conclusion about the neutrino mass using information from the largest reconstructed sample instead of using the lowest estimated upper limit. It is statistics, not lucky events near the endpoint that give discriminatory power. It is also interesting that even with a sample as large as 450 events, the figure of 95% appears nowhere in this table - this indicates that the variance in results is still not small.

sion that a perfect detector with no smearing is less useful for determining a neutrino mass than one with smearing. Such a repugnant thought is not consistent with our understanding of how measurements should work.

[6]See hypothesis testing, and Type I and Type II errors in any introductory statistics book.

9. Exhortations to future limit setters

In light of the observations made above, I would like to counsel future limit setters to heed the following advice:

- Restrict your fit region and use a Poisson Term: Since most events far from the endpoint have little neutrino mass information apart from helping to normalize the expected number of events near the endpoint, it is not worth CPU time to fit these events in detail.

- Use a Background Function: If one wishes to avoid using a background function, one must use tighter and tighter cuts as the accepted luminosity increases. This can result in the paradoxical situation in which an experiment ends up with less sensitivity as its sample size grows. The inclusion of a background function remedies this strange state of affairs.

- Publish the expected reach $< M_{95} >$ (and its variance) for a massless neutrino hypothesis. This will help establish whether the limit from data is meaningful or a dangerous fluctuation.

- Publish the discriminatory power of your experiment: how often does your data derived limit occur for a large (say 50 MeV) neutrino mass?

- Compare the limit derived from mass information only to the limit derived from the energy vs mass distribution: if the two dimensional limit is substantially different from the one dimensional one, then the limit is being driven by a small number of events or a very unlikely event distribution.

- Carefully examine large smearing tails: comments given in the text point out that

Table 2
The probability of obtaining a 95th percentile mass limit of 27 MeV or lower for different input neutrino masses and event sample sizes.

M_ν^{input}	25 Events	450 Events
0 MeV	3%	67%
50 MeV	1%	$\ll 1\%$

The values here are calculated using the CLEO Monte Carlo and detector simulation for $\tau \to 5\pi\nu$ decays. Exact values will vary from experiment to experiment, and decay mode to decay mode.

the sharp cut-off in reconstructed energy as determined by the neutrino mass make the properties of the neutrino mass estimator unclear. At the very least, one should carefully examine the effects of systematic error along the energy axis. Ultimately, as samples grow and the relevant fit region shrinks, fluctuations due to the unknown tails of the smearing functions in both mass and energy must dominate the fits.

10. How to Set a Limit We Can All Understand

In light of the suspicious properties of the maximum likelihood estimator raised in this review, it would be very useful if experimenters would calibrate their estimators using a Monte Carlo method. The method proposed herein answers the question of "What masses are ruled out by my data ?"

- Generate a large number of Monte Carlo samples with sizes comparable to those obtained in data for different input neutrino masses (m_ν^{in})
- For each of these samples, form the likelihood, and find the neutrino mass which maximizes the likelihood, $\hat{m}_\nu^{MC}$
- Plot $\hat{m}_\nu^{MC}$ vs m_ν^{in}
- Form the likelihood for the data sample and find its maximum $\hat{m}^{Data}$
- Find m_ν^{in} such that $\hat{m}_\nu^{MC}$ is larger than $\hat{m}^{Data}$ 95% of the time
- Set the upper limit to this value

Unfortunately this method does not provide a nice way to combine limits from different experiments. However given the points made in this review, perhaps it is better to suffer from this than to combine likelihoods with unclear statistical properties, and possibly incompatible meanings.

11. Conclusions

I have briefly reviewed the current constraints on an MeV scale tau neutrino mass. These constraints are from astrophysical observations as well as from terrestrial observations and all of them taken at face value allow for the existence of an unstable ν_τ with a mass on an MeV scale. Recent SuperKamiokande results indicate that the (stable) neutrinos participating in atmospheric ν_μ disappearance have a mass well below an MeV and are consistent with stable tau neutrinos. An inspection of the two dimensional limits on the τ neutrino mass derived from experimental data reveal that the ensemble of limits is too consistent. I have also for the first time shown that the usual assumption that the likelihood is well behaved for this technique is questionable. I have ended by recommending what extra information future limit setters should publish to allow others to gauge the believability of their limits, and have also recommended a method more suited to obtaining an answer to the question of what neutrino mass is excluded by the observed data than the methods used up to now.

12. Acknowledgments

The audience would have found this talk far more tedious had it not been for many interesting discussions with Ronan McNulty and many other colleagues at CERN and on CLEO. I'd like to thank the conference organizers for inviting me to give this talk in such a nice location. I also would like to thank the National Science Foundation for its support.

REFERENCES

1. S.S. Gershtein. and Ya.B. Zel'dovich, JETP Lett. **4** (1966) 174.
2. R. Cowsik and J. McClelland, Ap.J. **180** (1973) 7.
3. E. Kolb and M. Turner, The Early Universe, Addison-Wesley, 1990.
4. M. Turner, private communication.
5. B. Barate et al., ALEPH coll., Euro. Phys. J. **C2** (1998) 395.
6. B. Armbruster et al., KARMEN coll., Phys. Lett. **B348** (1999) 19.
7. A. Anastassov et al., CLEO coll., Phys. Rev. **D55** (1997) 2559.
8. J.Z. Bai et al., BES Collaboration, Phys. Rev. **D53** (1996) 20.
9. J. Swain and L. Taylor, Phys. Rev. **D55** (1997) 1.
10. R. Ammar et al., CLEO coll., Phys. Lett. **B431** (1998) 209.
11. R. McNulty, Nuclear Physics B (Proc. Suppl.) 76 (1999) 409.
12. M. Athanas et al., Phys. Rev. **D61** (2000) 052002.
13. The Particle Data Group, Euro. Phys. J. **C15** 2000.
14. K. Ackerstaff et al., OPAL coll., Euro. Phys. J. **C5** (1998) 229.
15. D. Buskulic et al., Phys. Lett. **B349** (1995) 585.
16. R. Akers et al., Z. Phys. **C65** (1995) 183.
17. D. Cinabro et al., Phys. Rev. Lett. **70** (1993) 3700.
18. H. Schroder et al., Mod. Phys. Lett. **A8** (1993) 573.
19. G. Alexander et al., OPAL coll., Z. Phys. **C72** (1996) 231.