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HOLOGRAPHIC COOLING OF CHARGED PARTICLES

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Abstract

We are proposing to use *induced* radiation instead of spontantenous in a pickup of any feedback system in a storage ring. In the method described, a signal from a source generator is splitted in two parts. First part–a probe one–*induces* radiation of charged particles in a pickup. The second part used as a reference one. After destructive interference (subtraction) of these two signals and amplification, the residual signal goes to appropriately phased kicker. By this way amplification in the feedback loop could be increased in a ratio of induced to spontantenous radiation in the pickup.
This method, implemented into *optical stochastic cooling*, makes μ–meson, proton, ion and electron cooling schemes practical ones.

Introduction

Any feedback system contains three key elements: a pick-up, an amplifier and a kicker. All together they represent a loop, closed by the beam. Usually this loop arranged to cool (dump) the beam oscillations in storage ring. Basic function of pickup in feedback is to acquire a signal, proportional to local deviation of the bunch centroid from equilibrium one. Speed of damping defined by the bandwidth of all these elements and amplification in the feedback loop. As a rule the amplification in feedback loop is limited by sensitivity of the pickup.
Usually charged particles radiate in a pickup spontantenously. From the point of electrodynamics this is well known behavior $\sim\alpha = e^2/\hbar c \cong 1/137$ quants per formation length. When the number of particles in the bandwidth becomes lower, the time of cooling decreases. However, the amplification required in the cooling (feedback) loop becomes higher and higher. In limiting case, when the only single particle resides in the bandwidth, the system needs to amplify the signal from that single particle.
System for stochastic cooling is a feedback system having bandwidth Δf higher, than characteristic bunch frequency $\sim c/\sigma_b$, where *c*–is a speed of light and σ_b–is the bunch length. This bandwidth allows resolving some structure inside the bunch. Indeed, an ordinary feedback system is working with the bandwidth $\Delta\omega << c/\sigma_b$. The scheme with the feedback bandwidth extended to optical values described in [1]. An optical amplifier became a natural analog of microwave amplifier. The analogues for a pick up and a kicker were found to be quadrupole and dipole wigglers respectively. Here radiation in a pickup – a quadrupole wiggler –is ordinary *spontantenous* radiation in periodical wiggler field. This field is a function of particle's displacement in a wiggler.
We considering here a cooling method for charged particles, when they are emitting radiation in pickup, *induced* by external probe source. By this way it becomes possible significantly increase the signal from the pickup in a ratio of induced to spontantenous radiation in the pickup. As the induced radiation is proportional to the external field, the

only limitation of the radiation here is that this radiation must not disturb the longitudinal emittance of the cold beam. Or with the other words, the energy radiated must be small if compared with instantaneous energy spread in the bunch.
We will be interesting mostly in application of this method to *optical stochastic cooling* scheme [1] due to its possible implementation to the μ–meson, proton, ion and electron cooling schemes. All considerations could be applied to any microwave feedback as well.
Let us consider the scheme, represented in Fig.1.

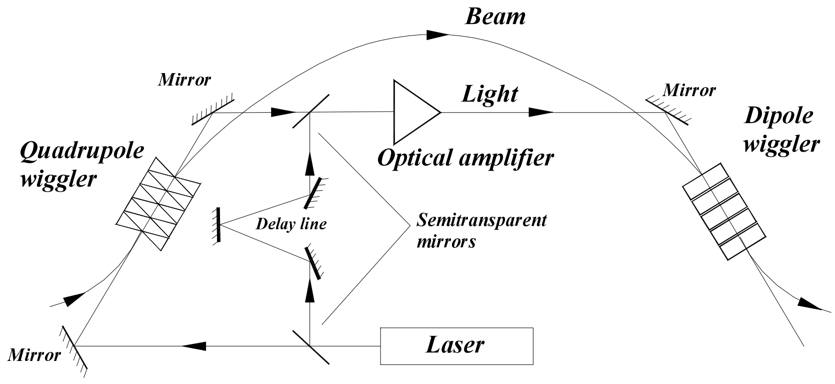

FIGURE 1: The general layout of the system.

Without laser, the scheme is absolutely the same as described in [1] for optical stochastic cooling. All necessary formulas from there could be used here as well.
By semitransparent mirror (lower in Fig.1) the light from the laser splits in two parts. The first –probe one– goes to the quadrupole wiggler (FODO structure) co-directionally with the beam under cooling. Parameters of the wiggler adjusted so that the probe wave stay in resonance condition with the particles of the beam. Parameters of the wiggler in this case must be the following

$$\lambda \cong \lambda_w \cdot (1+K^2)/2\gamma^2, \tag{1}$$

where λ–is wavelength of laser radiation, λ_w–is the wiggler period (period of FODO structure), $\gamma = E/mc^2$, $K = eH_\perp \lambda_w / 2\pi mc^2 \cong 93.4 \cdot H_\perp[Tesla]\lambda_u[m]$, $H_\perp \cong G(s)\cdot x$ –is actual magnetic field value along the particles trajectory, $G(s) \cong G_0 \cos 2\pi s/\lambda_w$, G–is the gradient in lenses. We will be interesting in case K<<1. Basically λ corresponds to the wavelength of spontantenouse radiation in the wiggler. This is basic principle of coincidence of induced and spontantenous frequencies. Further this signal goes to the input of optical amplifier trough the semitransparent mirror.
The second part of splitted laser signal serves as a reference one. It bypasses the pickup and combines with the probe signal in second semitransparent mirror (see Fig.1). Then the signals are going to the input of amplifier. Adjustable delay line for the reference light signal serves as a tuner. Delay and optical structure in transverse direction of those two signals arranged so that they destructively interfere at the input of amplifier. This means among all, that the matrix of optical transformation for these two light paths is the same for each beam path, but the phase is opposite for each path. The last could be arranged with different number of reflections for these two light paths, for example. This is shown in Fig.1. As the phase of the electromagnetic field is changed to opposite one after reflection,

the number of reflections for two ways have opposite parity in three mirror delay line. Other possibilities discussed lower. This is important only if laser works in pulsed mode, however.
The system is tuned so that in absence of the beam there is no signal at the input of (optical) amplifier. For this amplitudes of probe and reference beams must be equal but the phases must be tuned in opposite. Some adjustable attenuators required are not shown in Fig.1.
We would like to say here, that despite the source is a narrow band laser, so that the length of the photon is longer that the bunch length $\lambda\cdot(\omega/\Delta\omega) >> \sigma_b$, the system is a wide band one. This is due to the fact, that the beam itself modifies the radiation in accordance with fluctuations in its local homogeneity. The length of these pieces modified is of the order of wiggler length reduced by Doppler factor. Namely these modified pieces of radiation carrying information about local beam parameters define the bandwidth.
Due to similarity of this scheme to the techniques used in holographic imaging with reference and probe rays, we called this method *holographic cooling.*

Radiation in a pickup

Spontantenous radiation in microwave pick up.

For differential pickup the energy ΔE deposited in it could be estimated as the following

$$\Delta E = e\Delta U \cong \frac{eq}{D+<x>_S} - \frac{eq}{D-<x>_S} \cong \frac{eq}{D}\frac{2<x>_S}{D}, \tag{2}$$

where D –is the aperture of pickup, $q \cong eN_S$ –is the charge in the bandwidth, i.e. $N_S \sim \frac{N}{\Delta f}\frac{c}{\sigma_B}$, N –is total number of particles in a bunch, $<x>_S$ –is a position of bunch centroid, averaged over bandwidth (subindex *s* means *sample*). This expression means, that the photons, radiated in each individual electrode, *destructively interfere* in the amplifier or in the passive elements (a transformer with two oppositely connected primary coils, for example). The number of photons radiated spontantenously goes to $N_{sp} \cong \Delta E_{sp}/(\hbar c/D)$.

Induced radiation in microwave pickup

The energy losses for induced radiation could be calculated as

$$\Delta E_{ind} \cong eN_s\frac{<x>_S}{D}\cdot U, \tag{3}$$

where U is a voltage applied to the electrodes. The number of the photons radiated in this case is $N_{ind} \cong \Delta E_{ind}/(\hbar c/D)$, so the ratio (3) and (2) goes to

$$\frac{\Delta E_{ind}}{\Delta E_{sp}} = \frac{N_{ind}}{N_{sp}} \cong \frac{U}{e/(D/2)}. \tag{4}$$

The last is the ratio between the potential applied to electrodes and potential, induced by individual particle. The ratio (4) for reasonable parameters: U=300V~1esu, $D \cong 1cm$, goes to $N_{ind}/N_{sp} \cong 2\cdot 10^9$.

Spontantenous radiation from a quadrupole wiggler

Quadrupole wiggler is a series of quadrupoles with alternating sign of gradient installed along straight line (FODO structure) having period λ_w =2L, where L –is the distance between focusing and defocusing lenses. Such arrangement increases intensity of radiation in the bandwidth. Magnetic field in the aperture of the lens could be represented as $H_y(x) = G \cdot x$, $H_x(y) = G \cdot y$, where G – is a gradient, x, y –are the transverse coordinates of the particle in the wiggler. For a particle, which is going off the central trajectory, the field picture looks like a field in ordinary dipole wiggler with magnetic field proportional to the displacement. Betatron motion modulates the field with the frequency of transverse oscillations. We will be interesting in case, when betatron wavelength is much longer, than λ_w. Detailed consideration of radiation in quadrupole wiggler with arbitrary betatron wavelength is given in [2]. In dipole approximation, $\vec{\beta}_\perp \gamma \le 1$, $\vec{\beta} = \vec{\beta}_\parallel + \vec{\beta}_\perp \cong \vec{k}\beta$, where $\vec{k}$ –is a unit vector in longitudinal direction, electrical field in far zone can be represented as [3]

$$\vec{E}(t) = -\frac{e}{cR}\frac{\vec{n}\times((\vec{n}-\vec{\beta})\times\dot{\vec{\beta}})}{(1-\vec{n}\vec{\beta})^3}\Bigg|_{t'=t-R(t')/c} \cong -\frac{e}{cR}\frac{\dot{\vec{\beta}}_\perp}{(1-\beta(\vec{n}\vec{k}))^2}\Bigg|_{t'=t-R(t')/c}, \tag{5}$$

where $\vec{n}$ –is unit vector in direction of observation, R—is the distance to the observer at the moment of radiation. All parameters at right side are taken at the moment $t' = t - R(t')/c \cong t - R_0/c + \vec{n}\vec{r}(t')/c \cong t - R_0/c + \vec{n}\vec{\beta}t'$, where R_0 is the distance from the center of the wiggler to the observer. One can see from (6), that the time dependence of electric field strength $\vec{E}(t)$ copies the time dependence of the particle's acceleration in a scale $dt/dt' = 1-(\vec{\beta}\vec{n})$, what is Doppler factor. Fig.2. So the frequency of *spontantenous* radiation ω defined as shifted frequency of transverse oscillations

$$\omega = \frac{\Omega}{1-\vec{\beta}\vec{n}} \cong \frac{\Omega}{1-\beta_\parallel \cdot (1-\frac{\vartheta^2}{2})} \cong \frac{\Omega}{1-\beta\cdot\left[1-\frac{\vartheta^2}{2}-\frac{\beta_\perp^2}{2}\right]} \cong \frac{2\Omega\gamma^2}{1+K^2+\gamma^2\vartheta^2} = \frac{\omega_{\max}}{1+\frac{\gamma^2\vartheta^2}{1+K^2}} \tag{6}$$

ϑ –is the observation angle, calculated from the forward direction, between $\vec{k}$ and $\vec{n}$. $\omega_{\max} = \frac{2\Omega\gamma^2}{1+K^2}$ corresponds to the *forward* direction, $\vartheta = 0$ in (6), $\Omega \cong 2\pi c/\lambda_w$–is the frequency of transverse oscillations. Transverse acceleration can be represented as

$$\left|\dot{\beta}_{\perp i}\right| \cong \left|\frac{e\cdot\vec{k}\times\vec{H}_i}{mc\gamma}\right| = \frac{e\cdot G(s)x_i}{mc\gamma}, \tag{7}$$

where x_i –is a transverse coordinate of particle, s –is its longitudinal coordinate. For harmonic motion in periodic field, $G(s) \cong G_o \cos(2\pi s/\lambda_w)$, G_0 –is amplitude of gradient in the lenses, (7) gives $K_i = \beta_{\perp i}\gamma = \frac{eG_0\lambda_w x_i}{2\pi mc^2}$. We suggested that betatron motion have a wavelength much longer, than λ_w. With help of (7), (5) can be rewritten as

$$\left|\vec{E}_i(t)\right| \cong -\frac{e}{mc^2\gamma R_0}\cdot\frac{4\gamma^4 eG_0}{(1+K^2+\gamma^2\vartheta^2)^2}\cdot x_i\Bigg|_{t'=t-R(t')/c} = E_o.x_i(t'). \tag{8}$$

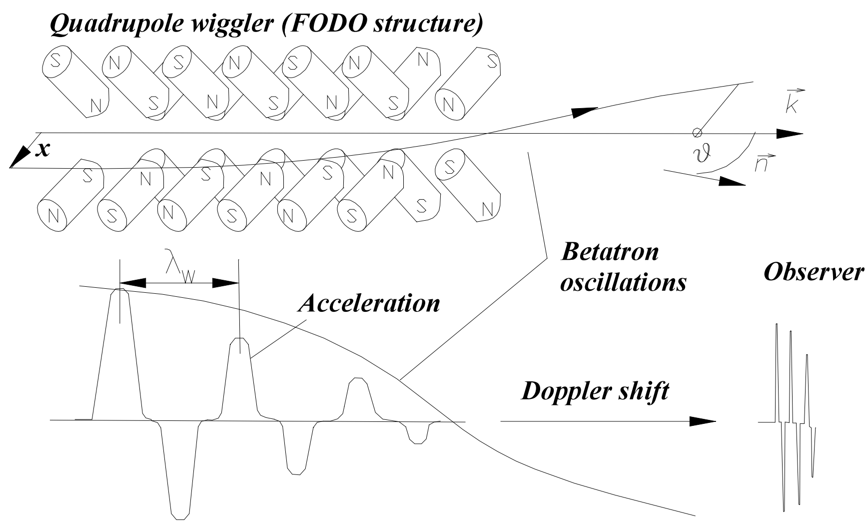

FIGURE 2: Radiation from quadrupole wiggler.

In expression (9) we ignored time dependence $R(t')$ in denominator. So what is important, is that the electrical field is proportional to the particle's *displacement* in the lens. To obtain the full radiation from the bunch with finite transverse dimensions, one needs to sum the fields (9) arisen from all particles. So the expression for the field becomes

$$\vec{E}_x(t) \cong -E_0 \cdot \sum_{i=1}^{N} x_i(t')\bigg|_{t'=t-R(t')/c} = -E_0 \cdot < x >_{t'}\,, \quad \vec{E}_y(t) \cong -E_0 \cdot < y >_{t'}\,. \tag{9}$$

The sum gives a *dipole* moment of transverse distribution. One can see from here that radiation in the forward direction interferes *destructively*, Fig.3.

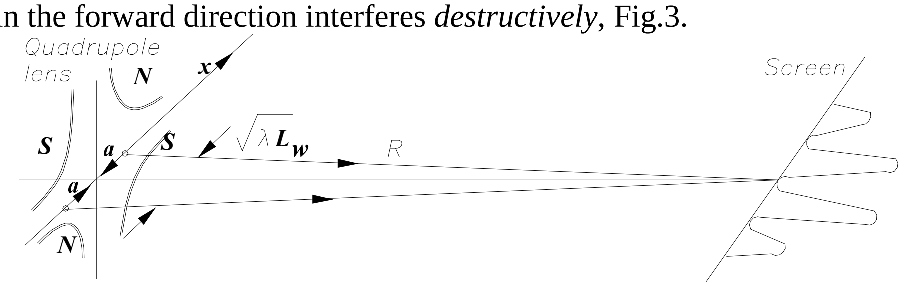

FIGURE 3: Destructive interference of radiation in a quadrupole lens from two particles.

In the same manner one can consider the radiation in *sextupole and so on wiggler*. This provides an information about *higher* moments in transverse beam distribution.

We suggested, that the size of coherence $\sqrt{\lambda L_w}$, is bigger than the transverse dimension of the beam

$$\sqrt{\lambda L_w} \geq \sqrt{\varepsilon \beta}\,, \tag{10}$$

where ε, β –are emittance and envelope function of the beam respectively. So, radiation in *forward* direction occurs only due to fluctuation of the gravity center in a wiggler, similarly to the microwave pick up, described above.

Spectral angular density can be calculated as usual [3]

$$\frac{\partial^2 I}{\partial S \partial \omega} = c\left|\vec{E}(\omega)\right|^2 , \qquad (11)$$

where $dS = R^2 do \cong \pi R_0^2 d\vartheta^2$ –is a spherical area, $E(\omega) = \int_{-\infty}^{+\infty} E(t) e^{i\omega t} dt = \frac{e}{c}\frac{e^{ikR_0}}{R_0}\frac{\vec{n}\times((\vec{n}-\vec{\beta})\times\dot{\beta}_\perp)}{(1-\vec{n}\vec{\beta})^2}$.

Using (6), the factor $1/(1-\vec{n}\vec{\beta})$ could be expressed as ω/Ω. So

$$dE_\omega \cong \frac{e^2}{2\pi c^3}\left(\frac{\omega}{\Omega}\right)^4 \left|\vec{n}\times((\vec{n}-\vec{\beta})\times\dot{\beta}_\perp)\right|^2 \frac{\pi d\vartheta}{2\pi}.$$

In (11) it was taken into account that spectral component of $\beta_\perp$ coincides with its amplitude. Dividing this expression by the energy of quanta with frequency from (6), $\hbar\omega = \hbar\Omega/(1-\vec{\beta}\vec{n})$, and substituting here expression for acceleration (7), one can obtain after integration over ϑ the spectral density of photons radiated by single particle as [4]

$$\frac{dN_i^\gamma}{d(\omega/\omega_{\max})} \cong 4\pi\alpha\frac{< K_i >^2}{1+K_i^2} M \cdot \frac{1}{2}\cdot\left[1-2\frac{\omega}{\omega_{\max}}+2(\frac{\omega}{\omega_{\max}})^2-...\right]\cdot Sinc^2\left(\pi M(1-\frac{\omega}{\omega_{\max}})\right), \qquad (12)$$

where K_i is calculated for each particle, $M = L_w/\lambda_u$ –is the number of periods, $Sinc(x) \equiv \sin x/x$. We wrote this expression in such a manner to attract attention that averaging over particle displacement, <*K*>, is going first. This is required for proper calculation of radiation from *bunch* of particles. In the spectrum region $\omega_{\max} \div \omega_{\max}\cdot(1-1/M)$, the number of photons becomes $N_\gamma \cong 4\alpha < K >^2 /(1+K^2)$. With other words factor *M* disappears. In our case it is about five. Total number of photons, spontantenously radiated by particles in a bandwidth, moving in a periodic field becomes the following

$$N_{sp} \cong 4\pi\alpha N_S \frac{< K >^2}{1+K^2} . \qquad (13)$$

As each quantum carries energy about

$$\hbar\omega \equiv 2\pi\hbar\frac{c}{\lambda} \cong 2\pi\hbar\frac{c}{\lambda_w}\frac{2\gamma^2}{1+K^2} ,$$

total energy radiated by particles goes to

$$\Delta E_{sp} \cong N_{sp}\cdot\hbar\omega = 4\pi\alpha\frac{1}{\lambda_w}4\pi\hbar c\gamma^2 N_S\frac{< K >^2}{(1+K^2)^2} . \qquad (14)$$

Induced radiation from quadrupole wiggler

Let us consider the electromagnetic wave propagating in *s* direction, $\vec{E} = E_\perp\cos(\omega t \mp ks + \varphi)$, where $k = \omega/v_p$, v_p –is the phase velocity of the wave, in vacuum $v_p = c$. Energy variation, what could be treated as *induced absorption*, could be expressed as the integral along particle trajectory

$$\Delta E = \int_0^T P dt = e\int(\vec{v}_\perp\vec{E}_\perp)dt = e\beta_\perp c E_\perp\int Cos\Omega t\cdot Cos(\omega t \mp k v_\parallel\, t+\varphi)dt =$$
$$= \frac{eKcE_\perp}{\gamma}\int Cos\Omega t\cdot Cos\left(\omega t\left(1\mp\frac{v}{v_p}\left(1-\frac{\beta_{\perp x}^2}{2}-\frac{\beta_{\perp y}^2}{2}\right)\right)+\varphi\right)dt = \frac{eKL_w E_\perp}{2\gamma}Cos\varphi \qquad (15)$$

where resonance condition was used here

$$\Omega \cong \omega\left(1 \mp \frac{v}{v_p}\left(1-\frac{\beta_{\perp x}^2+\beta_{\perp y}^2}{2}\right)\right).$$

In case of *co-directional* movement of the wave and the particle, the resonant frequency is the same as for spontantenous radiation. This could be seen easily from here,

$$\Omega \cong \omega\left(1-\frac{v}{v_p}\left(1-\frac{\beta_{\perp x}^2+\beta_{\perp y}^2}{2}\right)\right) \cong \omega\left(\frac{1}{2\gamma^2}+\frac{\beta_{\perp x}^2+\beta_{\perp y}^2}{2}\right)=\frac{\omega}{2\gamma^2}\left(1+K^2\right).$$

$$\lambda \cong \lambda_w \cdot (1+K^2)/(2\gamma^2)\,. \tag{16}$$

For a bunch of particles, the energy acquired from the wave becomes

$$\Delta E_{bunch} \cong \sum_i \Delta E_i = \sum_i \frac{eK(x_i)L_w E_\perp}{2\gamma} = \frac{eL_w E_\perp N_S}{2\gamma} < K >= \frac{e^2 G\lambda_w L_w E_\perp N_s}{4mc^2\gamma} < x > \tag{17}$$

where sum is going over each particle transverse position, so $< x >$ becomes a dipole moment of bunch in a bandwidth.

Ratio of induced/spontantenous radiation

As we could see from (15), the induced change of energy is

$$\Delta E_{ind} \cong \frac{eEL_w K}{2\gamma}\cos\varphi\,, \tag{18}$$

where φ –is relative phase between the wave and particle longitudinal location. The ratio between (18) and (14) goes to

$$\frac{\Delta E_{ind}}{\Delta E_{sp}} \cong \frac{eEL_w K}{2\gamma}\frac{\lambda_w^2}{4\pi\alpha L_w}\frac{1}{4\pi\hbar c}\frac{\cos\varphi}{\gamma^2}\frac{(1+K^2)^2}{K^2} = \frac{E}{2}\frac{\lambda^2\gamma}{\pi\alpha}\frac{e}{4\pi\hbar c}\frac{\cos\varphi}{K}\,. \tag{19}$$

Rewriting the fine structure expression in SI units, and substitute in (19), one can obtain from there

$$\frac{\Delta E_{ind}}{\Delta E_{sp}} \cong \frac{E}{2}\frac{\lambda^2\gamma}{\pi}\frac{\varepsilon_0}{e}\frac{\cos\varphi}{K} \text{ (SI units)}. \tag{20}$$

For the ratio of radiated energy in bandwidth ~1/*M*, the last expression must be multiplied in factor L/λ_w, as it follows from (12), (13) (in favor of induced radiation).

Let us make some estimation. Let the laser probe signal represented by power density ~100*W*/*mm*^{2} or $E^2/377 \sim 10^8 W/m^2$. The last gives $E \cong 2\cdot 10^5\ V/m$. Let the energy of electron is 500 *MeV*, $\gamma \cong 10^3$. Let also the wiggler have period $\lambda_w \cong \lambda\frac{2\gamma^2}{1+K^2}$ such the wavelength $\lambda \cong 1\mu m$. For this $\lambda_w \cong 10^{-6}\frac{2\cdot 10^6}{1+K^2} \cong 2m$, $K \cong 0.1$, *M*=5. For these numbers (20) gives

$$\frac{\Delta E_{ind}}{\Delta E_{sp}} \cong \frac{2\cdot 10^5}{2}\frac{10^{-12}10^3}{\pi}\frac{10^{-9}}{36\pi\cdot 1.6\cdot 10^{-19}}\frac{\cos\varphi}{0.1}5 \cong 1.3\cdot 10^5\cos\varphi\,. \tag{21}$$

The last means that energy radiated by single particle as induced one is about hundred thousands times higher, than if radiated spontantenously for suggested power in probe signal. In its turn this yields, that power amplification coefficient required becomes lower in the same ratio, see (30) below.

Energy spread in the beam

Energy spread as a result of induced radiation from example above goes to

$$\Delta E_{ind} \cong \frac{eEL_w K}{2\gamma}\cos\varphi = 8\cdot 10^{-16}eV\ .$$

Typical energy spread in the beam is about 10^{-3}, so the value of probe signal amplitude could be increased sixteen order of magnitude, to be compared with natural energy spread. So there the only limiting parameter is laser power available.

As the bunch has finite length σ_b, the laser must generate in principle the flash of the same duty. In this case laser pulsed power ~100*W/mm*^{2} considered in our example is much below, what is available routinely. If the bunch length is say 10*cm*, then total energy carrying by this laser flash goes to $\sim 100\cdot\sigma_b / c \sim 3\cdot 10^{-8}J$ only.

Let the magnets of the ring be the main source of radiation at the beginning of cooling. If the bending radius in magnets is ρ, then characteristic photon energy in the magnet will be $\hbar\omega \cong \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$. As the length of radiation formation is $l_f \cong \frac{\rho}{\gamma}$, the number of the phonons radiated per revolution is $N_{\gamma turn} \sim \alpha\frac{2\pi\rho}{l_f} \cong 2\pi\alpha\gamma$. The number of turns required for damping is $n \cong \frac{E}{\hbar\omega} = \frac{2}{3}\frac{E\cdot\rho}{\hbar c\gamma^3}$. So the energy spread in the beam could be estimated as

$$\frac{\Delta E}{E} \cong \frac{\hbar\omega}{E}\sqrt{N_{\gamma turn}\cdot n} \cong \sqrt{\frac{3\pi r_0\gamma^3}{\rho}}\ , \tag{22}$$

where $r_0 = e^2 / mc^2$–is a classical electron radius. This energy spread must remain big if compared with energy perturbation induced by probe beam (17). Equations (22) and (17) yield

$$\sqrt{\frac{3\pi r_0\gamma^3}{\rho}} >> \frac{eE_\perp L_w}{2mc^2\gamma^2}\ . \tag{23}$$

The right side of this equation is the ratio between the energy gain in the wiggler divided by energy of the particle. This equation could be easily satisfied for mostly systems.

Reversing the phase

The phase reversal for the reference signal is not a problem. As we mentioned the number of reflections on the way of reference light signal and on the way of probe one must have opposite parity. As the phase of reflected wave has opposite sing this will satisfy the situation.

If the system is working in pulsed mode, the length of the pulse must be longer, than the bunch length. So mismatches at the beginning and the end of the interfered signal is not acting to the bunch. The only possible problem here- saturation of the amplifier-at the beginning and the end of reference signal. Fast optical gates could eliminate this problem.

If, however, the laser is working in DC mode, there is no a problem at all.

Amplification in a feedback loop

Transverse kick over energy change

According to the idea of stochastic cooling each particle decreases its amplitude only due to interaction with its *own* field amplified. This field is proportional $\sim A/N_S$, where *A*–is amplitude of particle oscillations in the beam. The signal acting to the particle is a sum of this signal and noise signal, $\sim A/\sqrt{N_S}$. The last is much bigger, and considered as heating term, acting incoherently. To kick the same particle after amplification, the radiated electromagnetic wave must be applied to the particle in congruence with initial longitudinal distribution. This could be done if the photons and the beam are moving in the same direction. As the direct transverse kick is not possible under this condition, the transverse kick in optical stochastic cooling arranged through the *energy change* in the place, where dispersion of trajectory has some value $\eta(s)$ [1]. That means the amplitude of transverse betatron oscillations around new closed orbit will change on $\Delta x \cong \eta \cdot (\Delta E/E)$, where $\Delta E/E$–is a relative energy change, see Fig.4. This yields $\Delta\, x \cong A/\sqrt{N_S} \cong \eta(s)\Delta E\,/\,E$.

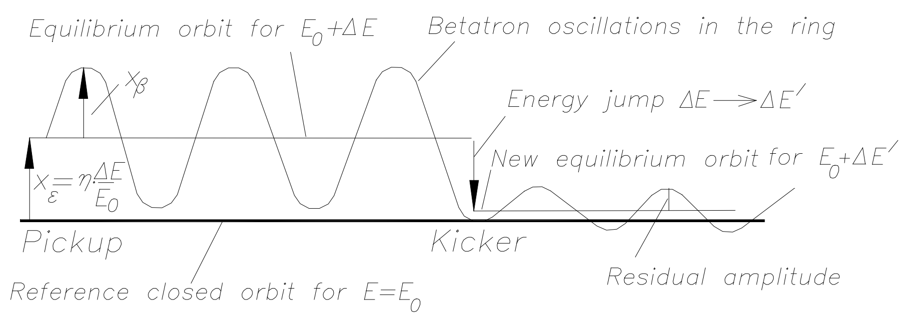

FIGURE 4: Transverse kick through energy change.

If the dipole wiggler has the same period as quadrupole wiggler and the undulatority factor *K*, then relative energy change will be

$$\frac{\Delta E}{E} \cong \frac{eE_\perp K L_w M}{E\gamma} = \frac{A}{\eta\sqrt{N_S}} \quad , \tag{24}$$

where $E_\perp$–is the electrical field strength in the electromagnetic wave after amplification. The last expression allows calculating the field required for optimal cooling as

$$E_\perp \cong \frac{AE\gamma}{eKLM\,\eta\sqrt{N_S}} \tag{25}$$

Amplification

Total energy delivered by optical amplifier must be

$$\Delta E_{kick} \cong E_\perp^2 V/4\pi\,, \tag{26}$$

where V – is an effective volume. One can estimate

$$V \cong \pi A_0^2 \sigma_B \cong \pi 2 L_w M \lambda \sigma_B\,, \tag{27}$$

where transverse cross section chosen corresponding to the size of coherence (10). Combining (26)–(29) and estimating $\eta \cong A_0 \cdot (\Delta E\,/\,E)^{-1}$ and $\Delta f\,/\,f \cong 1/M$, one can obtain the amplification for amplitude of the field required from the optical amplifier as [1]

$$\kappa = \sqrt{\frac{\Delta E_{kick}}{\Delta E_{pick}}} \cong \frac{\gamma\sigma_B}{r_0}\frac{\Delta E}{E}\frac{1}{N}\frac{\Delta f}{f} \cong \frac{\gamma\lambda}{r_0}\frac{1}{N_S}\frac{\Delta E}{E} , \tag{28}$$

where $r_0 = e^2/mc^2$ is classical electron radius, if electrons considered, again, N_S is a number of the particles in the bandwidth $N_S \sim \frac{N}{\Delta f}\frac{c}{\sigma_B} \cong N\frac{M\lambda}{\sigma_B} \cong N\frac{ML\sigma_B}{\gamma^2}$.

Equation for a change of emittance ε by the number of revolutions *n* is the following [1]

$$\frac{d\varepsilon}{dn} = -\frac{\varepsilon}{N_S}, \tag{29}$$

For a single cooling system installed in the ring this gives the cooling times $\tau \cong N_S T$, where T –is period of revolution. For reduction of damping tame one needs to reduce the number of particles in the bandwidth $-N_S$. One can see from (30), that this requires adequate increase of amplification, however. So the additional signal from pickup helps to bring amplification to the low level again. Reduction of the number of particles in a bandwidth could be achieved by increasing the length of the bunch accordingly.

For example considered in [1] for electron bunch, with number of the electrons $N \cong 10^{10}$, $\sigma_B \cong 15cm$ at $E \cong 500MeV$, with amplifier having pulsed power 5 *kW* and average one ~25*W* delivers $\tau \cong 10ms$. If the number of the particles reduced to 0.3% of its value, still keeping the same bunch length one can reach $\tau \cong \tau \cdot \sqrt{\Delta E_{sp}/\Delta E_{\sin d}} \cong \tau \cdot 3 \cdot 10^{-3} \cong 30\mu s$ with the same amplifier.

Also the number of periods in a quadrupole wiggler could be reduced, opening the possibility to pick up radiation from single quadrupole. This decreases the number of particles in the bandwidth.

Implementation to meson/proton/ion cooling

For particles with different rest mass the amplification coefficient depends on the classical radius. Formally if the γ –factor is the same, the wave of radiation in the wiggler will be the same also. Second power in energy dependence on mass appears in requirement that the particles with different rest masses must go by the same trajectory. So resulting coefficient in comparison between electrons and $\mu -$ **meson** has dependence on rest mass in a power two

$$\frac{\kappa_\mu}{\kappa_e} \propto \frac{m_\mu^2}{m_e^2}.$$

The last ratio goes to $\sim 10^4$ what needs be compensated by increased signal from pickup in the same proportion. If one increase the energy in laser flash to 3 *mJ*, in the same 10 *cm*-long laser flash, then the electric field strength here becomes higher in $\sqrt{3/3 \cdot 10^{-8}} \cong 10^4$ times. The number of photons here becomes $\sim 3 \cdot 10^8$ times higher, than in case of spontantenous radiation. So the cooling times of the order of μs could be reached here. For cooling in longitudinal phase space one can use as a pickup a dipole wiggler as well. In this case radiation does not depend on transverse size of the beam. According to definition, the radiation within the size of coherence $\sqrt{\lambda L_w}$ could not be resolved. If however the transverse size of the bunch is larger, that this size, one can pick up radiation from different transverse parts of the bunch. This might be the way for further reduction of N_S, potentially opening a possibility for *single meson in a bandwidth*.

Proton cooling now becomes not limiting by optical amplifier. The only limiting factor for damping times becomes the number of protons in a bandwidth. For this the longitudinal density of the particles needs to be reduced either by reduction the number of the particles in a bucket or by lengthening of the bunch.

Ion cooling might be practical if atoms are fully ionized. In this case the charge/mass ratio is about half of the proton one. So amplifier is practically the same as required for protons.

Stacking during the cooling

Dependence of resonance frequency on particle's transverse position in quadrupole wiggler (6) $\omega \cong 2\Omega\gamma^2/(1+K^2)$ allows arrangement of selective cooling of particles with different amplitudes. Let us consider the situation, when some particles added to the cooled ones in the same RF bucket. Let the number of the cooled particles is N_C. The input in dipole moment of these particles could be $\sim A_C/\sqrt{N_C}$, where A_C is amplitude of cooled particles. New particles will have dipole moment respectively $\sim A_n/\sqrt{N_n}$. As we suggesting that new particles represent only fraction of cooled ones, and $N_C >> N_n$, the signal from these new particles in the bucket will be much higher, than from cooled ones in proportion $\sim (A_n/A_C)\sqrt{N_C/N_n}$. Again if initial amplitude arranged so that dependence on K becomes important, one can tune amplifier so, that it becomes resonant only with particles, having correspondent amplitude.

Conclusion

In conclusion we can say, that the radiation, *induced* by the probe signal, if used instead usual spontantenous radiation in a pickup of the feedback loop, makes the signal from pickup orders of magnitude higher, than in ordinary system with spontantenous radiation. This opens new possibilities in a beam cooling with the help of optical stochastic cooling method as well as increase of efficiency in working feedback schemes.

References

[1] A.A. Mikhailichenko, M.S. Zolotorev, *Optical Stochastic Cooling*, Phys.Rev.Lett.**71**: 4146-4149, 1993.

[2] E.G. Bessonov, A.A. Mikhailichenko, *Alignment of Linac with the help of Radiation from Quadrupoles of the Focusing System*, EPAC94, London, 26 June–1 July 1994, Proceedings, Ed. V. Suller, C. Petite-Jean-Genaz, pp. 2579-2581.

[3] L.D. Landau, E.M. Lifshits, *The Classical Theory of Fields*, Pergamon press, London, 1979, 4th edition.

[4] D.E. Alferov, Yu.A. Bashmakov, E.A. Belovintsev, E.G. Bessonov, P.A. Cherenkov, *The Undulator as a Source of Electromagnetic Radiation*, Particle Acc., 1979, vol.9., p.223.