Spectrum and decay matrix elements of B and D-mesons in lattice NRQCD

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We discuss recent results on the excitation spectra of B and D-mesons obtained in the framework of non-relativistic lattice QCD in the quenched approximation. The results allow for the determination of the $\overline{\rm MS}$-mass of $m_{b,\overline{\rm MS}}(m_{b,\overline{\rm MS}}) = 4.34(7)$ GeV in $\mathcal{O}(\alpha_s^3)$ in the perturbative matching. The determination of the decay constants f_{B_s} and f_{D_s} is discussed in detail. Results for the matrix elements of semi-leptonic B to D decays are shown.

1. MOTIVATION

In the standard model of elementary particle physics, CP-violation is caused by a single phase in the CKM-matrix. At the present time it is experimentally observed only within the Kaon system. Establishing CP-violation also in the B-meson system is one of the major goals for the B-factory experiments, such as the newly running BaBar, Belle and CLEO-III experiments as well as the future hadron collider experiment LHCb. Failure of all CP-violating processes to be described by the single CKM-phase would provide direct evidence for *new physics* beyond the standard model.

The extraction of the elements of the CKM-matrix from the above experiments is complicated due to the hadronic nature of the initial and final states. Here accurate knowledge of hadronic quantities, such as QCD form factors of the B-meson, is needed. Due to confinement these quantities are genuine non-perturbative. Lattice gauge theory provides a means to determine such properties of hadronic states based on first principles in a model independent way. The approximations made in present day calculations are expected to be relaxed in future calculations.

2. LATTICE NRQCD

For the results presented here we use non-relativistic QCD (NRQCD) [1,2] to formulate heavy b and c-quarks on a lattice with a lattice spacing a which is not negligible against its Compton wave length. For heavy-light mesons, this results in a systematic expansion of the Hamiltonian in powers of the inverse heavy quark mass m_Q^{-1}

$$H = H_0 + \delta H + \delta H_{disc}\,. \qquad (1)$$

The leading kinetic term is given as

$$H_0 = -\frac{\mathbf{D}^2}{2m_Q}\,, \qquad (2)$$

where $\mathbf{D}$ denotes a covariant lattice derivative. We also include the following relativistic corrections

$$\begin{aligned}\delta H = &\\ &-\frac{g}{2m_Q}\boldsymbol{\sigma}\cdot\boldsymbol{B} + \frac{\mathrm{i}g}{8m_Q^2}(\mathbf{D}\cdot\boldsymbol{E} - \boldsymbol{E}\cdot\mathbf{D})\\ &-\frac{g}{8m_Q^2}\sigma\cdot(\mathbf{D}\times\boldsymbol{E} - \boldsymbol{E}\times\mathbf{D}) - \frac{(\mathbf{D}^2)^2}{8m_Q^3}\,. \end{aligned} \qquad (3)$$

To reduce the dependence on the lattice spacing, the following discretisation corrections are also included

$$\delta H_{disc} = a^2\frac{\sum_i \mathbf{D}_i^4}{24m_Q} - a\frac{(\mathbf{D}^2)^2}{16nm_Q^2}\,. \qquad (4)$$

The first term on the right-hand side corrects in spatial, the second one in temporal direction. The n denotes the stability parameter used in the evolution equation to generate the heavy quark propagators. The inclusion of the improvement terms is important due to the non-renormalisability of

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NRQCD. The action has to be improved at finite values of the lattice spacing such that the results become independent of a within the achieved accuracy. We did not include radiative corrections to the prefactors of the individual terms of this expansion. However, the dominant tadpole contributions to this coefficients are removed by mean field theory [3].

The light quarks are simulated with the clover action [4] and the standard plaquette action is used for the gluon background. All the results reported here are calculated in the quenched approximation. This means the neglection of vaccum polarisation effects due to the light sea quarks. For some of the results also partly unquenched results exist. These calculations include two flavours of light sea quarks. This will be pointed out in the individual subsections.

In the calculations the value of the lattice spacing is determined from the mass of the ρ-meson. This is senseful, since the here discussed B and D-mesons have a typical momentum scale of $\Lambda_{\rm QCD}$ as does the ρ. In the quenched theory the strong coupling constant runs differently from nature. This procedure minimises the quenching error compared to using a quantity of a different scale.

3. MESON SPECTROSCOPY

Due to heavy quark symmetry the excitation spectra of the B and D-mesons are highly related. The lowest lying states are experimentally well understood [5], however the situation with respect to radially and orbitally excited states is not as satisfactory. In this situation we can learn about the accuracy of the approximations done in the present calculation, as well as predict masses for so far undiscovered states.

3.1. Spectrum of the B

The spectrum of the B-meson has been investigated in large detail in [6,7]. In summary these publications provide results for three different values of the lattice spacing a. Within the achieved accuracy, the individual results for the different splittings agree with each other. This is particularly important because of the above discussed non-renormalisability of NRQCD. A sub-

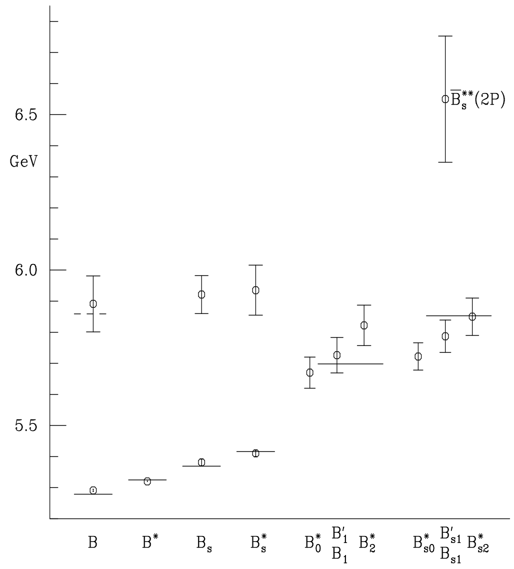

Figure 1. Excitation spectrum of the B-meson.

set of the investigated states are also reported by the JLQCD Collaboration [8] using similar methods. These results are in excellent agreement.

The results of [6,7] are summarised in figure 1. The lattice results are displayed by the octagons, whereas experimental results are given by horizontal lines. We compare the lattice B^{**} and B_s^{**} results to the experimental $B_J^*(5732)$ and $B_{sJ}^*(5850)$ resonances. The dashed line gives a preliminary DELPHI result. The overall agreement between lattice and experiment is very good and the spin-independent spectrum is nicely reproduced. This includes the radial and orbital excitation energies as well as the strange non-strange S-wave splittings. However there are problems with the spin-dependent hyperfine splitting, which turns out to be significantly too small. This might be due to the neglected radiative corrections in eqn. (3), especially in front of the $\boldsymbol{\sigma}\cdot\boldsymbol{B}$ term. Preliminary results [9] indicate this might have an effect of the order of 10% on the hyperfine splitting. Another cause might be the quenched approximation. Similar problems are also observed in quenched light spectroscopy. The hyperfine splitting turns out increasingly too small with increasing quark mass [10]. So far results for the B-meson hyperfine splitting with the inclusion of two flavours of light sea quarks do not show any improvement for fixed lattice spacing [11,12]. However these calculations use quite large values for sea quark mass and improvement might only be seen once more realistic values are used.

For the first time, this calculation gives a result for the radial excited P-states, which is denoted as $B_s^{**}(2P)$ in the figure.

3.2. Spectrum of the D

NRQCD calculations of the D-meson within the here-presented framework are only reliable for larger values of the lattice spacing. The convergence of NRQCD for D-mesons has been investigated in [13]. A detailed analysis of the presented material shows reasonable convergence of the NRQCD expansion for charmed heavy light states [7].

The most complete results to date on the D-meson excitation spectrum have been published in [7]. These results, using a lattice spacing of 0.177 fm, are presented in figure 2. Again the symbols give the lattice results and the horizontal lines the experimental outcome. The dashed line gives a result of the DELPHI collaboration and the shaded region shows a preliminary result of the CLEO collaboration for a wide D_1-resonance. Again the spin-independent spectrum agrees very well with the experimental outcome. Also the experimentally observed increase of the strange non-strange splitting from the B to the D-system is well reproduced. Again the hyperfine splitting turns out to be too small however in this case by not as much as for the B. This is consistent with the observed mass-dependence of the hyperfine splitting for light quarks in the quenched approximation [10].

Some of the splittings in figure 2 have also been computed with heavy clover quarks [14,15] and no significant differences have been found between the heavy clover and the NRQCD result. This is particularly remarkable for the $D_s^* - D_s$ hyperfine splitting, which strongly depends on the details of the applied heavy quark action. For charm quarks, NRQCD and heavy clover quarks behave differently. The details of this comparison are discussed in [7].

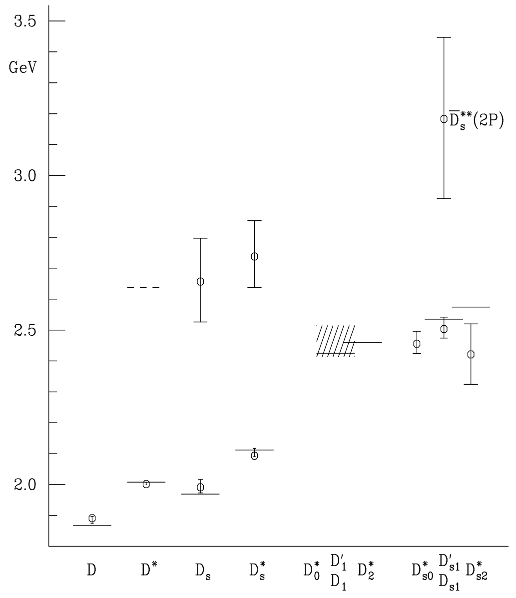

Figure 2. Excitation spectrum of the D-meson for $a^{-1} = 1.116$ GeV.

4. $\overline{\text{MS}}$-MASS OF THE b-QUARK

The mass of the b-quark is a fundamental parameter of the standard model. Based on the results of [6,7] it is possible to determine m_b in NNNLO perturbation theory. Here preliminary results of a forthcoming publication are presented [16].

4.1. Calculation of the b-mass

The calculation is done in two steps. First the pole mass is calculated, which is converted into the $\overline{\text{MS}}$-mass in a second step. The calculation of the pole mass turns out to be most precise using

$$m_{b,\text{pole}} = m_{\overline{B}} - \overline{E}_{\text{sim}} + E_0 \,. \tag{5}$$

The mass $m_{\overline{B}}$ of the spin-average of the B and the B^* is taken from experiment [5] and the simulation energy $\overline{E}_{\text{sim}}$ from lattice simulation. Here

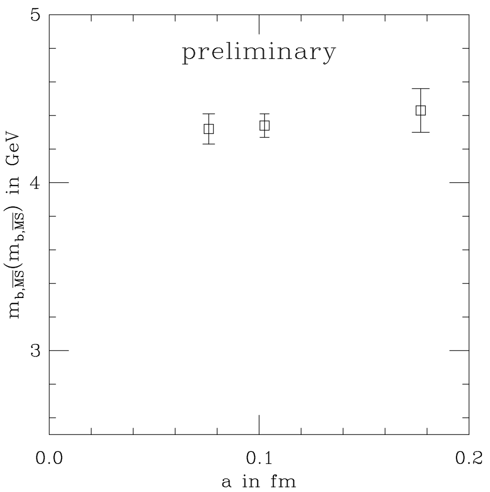

Figure 3. Dependence of $m_{b,\overline{\mathrm{MS}}}(m_{b,\overline{\mathrm{MS}}})$ on the lattice spacing a.

the use of the spin-averages eliminates the problems encountered with the hyperfine splitting in the spectrum calculation. The self-energy E_0 of the b-quark is known from perturbation theory. The latter is available most precise in the limit of infinite quark mass. It has been analytically calculated to $\mathcal{O}(\alpha_s^2)$ [17] and numerically to $\mathcal{O}(\alpha_s^3)$ for the quenched case [18,19]. The here-presented analysis is based on [18]. The simulation energies of [6,7] have been extrapolated to the static limit.

To convert the pole mass to the $\overline{\mathrm{MS}}$ scheme, we use the renormalisation constant to $\mathcal{O}(\alpha_s^3)$ [20]. The result for three different values of the lattice spacing is shown in figure 3. The different results agree within their statistical accuracy and the most precise of them is quoted as the final result

$$m_{b,\overline{\mathrm{MS}}}(m_{b,\overline{\mathrm{MS}}}) = 4.34(7)\ \mathrm{GeV}\,. \tag{6}$$

The uncertainty is still dominated by the uncertainties of E_0. It further includes the uncertainties arising from the conversion to the $\overline{\mathrm{MS}}$-scheme, statistics of $\overline{E}_{\mathrm{sim}}$, the lattice spacing and the discretisation. Corrections due to the finite mass of the b-quark are expected to be of $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b)$ of $\overline{E}_{\mathrm{sim}}$, which is about 0.05 GeV.

The effect of the quenched approximation can be estimated at $\mathcal{O}(\alpha_s^2)$ [17], when using $\overline{E}_{\mathrm{sim}}$ with two flavours of light quarks from [11]. In comparison to the quenched case in the same order of perturbation theory, the sea quarks reduce $m_{b,\overline{\mathrm{MS}}}(m_{b,\overline{\mathrm{MS}}})$ by 0.07 GeV. Eqn. (6) is compatible to the result in $\mathcal{O}(\alpha_s^2)$ of [17].

5. LEPTONIC DECAY

The pseudo-scalar decay constant f_M determines the decay of a mesonic state M into a pair of leptons

$$p_\mu f_M = \langle 0|A_\mu|M(p)\rangle\,. \tag{7}$$

For the B^+-meson the leptonic width is highly CKM suppressed and is not expected to be measurable in the near future. Therefore the decay constant cannot be measured directly, however it is an important input parameter in other analysis, for example $B-\bar{B}$-mixing [21]. The decay constant of the B has to be determined from theory. The D_s has a much weaker CKM suppression and its decay constant has been measured experimentally, which provides us with the possibility to check the theoretical calculation.

When using NRQCD for the heavy quarks, the matrix element of the axial current of QCD has to be expanded into matrix elements of the effective theory

$$\begin{aligned}\langle A_0\rangle_{\mathrm{QCD}} &= C_0\langle\bar{q}\gamma_5\gamma_0 Q\rangle \\ &\quad -C_1\frac{\langle\bar{q}\gamma_5\gamma_0(\boldsymbol{\gamma}\cdot\mathbf{D})Q\rangle}{2m_Q} \\ &\quad +C_2\frac{\langle(\mathbf{D}\bar{q}\cdot\boldsymbol{\gamma})\gamma_5\gamma_0 Q\rangle}{2m_Q}\,.\end{aligned} \tag{8}$$

The coefficients C_i on the right-hand side have been determined in 1-loop lattice perturbation theory [22].

5.1. Investigations of f_{B_s}

The dependence of the result of f_{B_s} on the lattice spacing is one of the key issues of [23]. The results have been combined with those of [24] and are reproduced in figure 4. A similar investigation has been performed in [8]. These results are

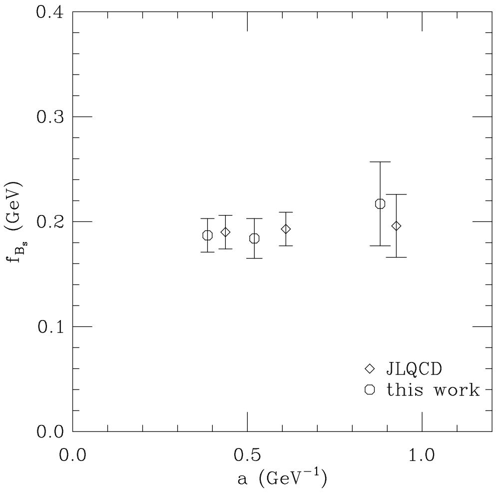

Figure 4. Lattice spacing dependence of f_{B_s}.

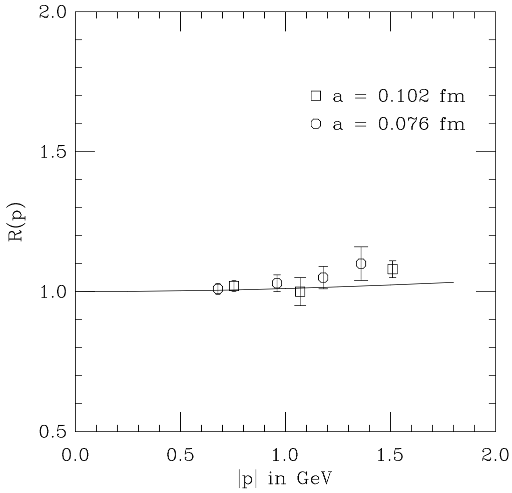

Figure 5. Investigation for momentum dependent discretisation effects on f_{B_s}.

included in the figure as well. The individual results agree well with each other and one can conclude that the results are within the claimed accuracy indeed independent of the value of the lattice spacing a. For the final number we quote

$$f_{B_s} = 187(16)\ \text{MeV}\,. \tag{9}$$

A detailed breakdown of the error can be found in [23]. The result agrees well with the outcome of recent studies using other ways to formulate the heavy quark on the lattice, see [25,26] for reviews. Calculations including two flavours of sea quarks indicate an increase of approximately 30 MeV for f_{B_s} [11,12].

Reference [23] investigates power law terms and the $\Lambda_{\rm QCD}/m_Q$ corrections of the decay constant in large detail. Due to the different dimensionality of the operators on the right hand side of eqn. (8) the coefficient C_0 develops a $1/am_Q$ divergence, which cancels a similar divergence in the matrix element $\langle\bar{q}\gamma_5\gamma_0(\boldsymbol{\gamma}\cdot\mathbf{D})Q\rangle$. The perturbative expansion of C_0 allows for the explicit investigation of this cancellation in $\mathcal{O}(\alpha_s/am_Q)$. The remaining part of the $\mathcal{O}(1/m_Q)$ matrix elements in eqn. (8), which contains the $\mathcal{O}(\alpha_s^2/am_Q)$ part of the divergence as well as physical $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$ terms, amounts to a few percent of the final result.

The decay constant f_{B_s} is a momentum independent form factor. It allows the study of the effect the lattice discretisation has on the form factor of moving mesons. This is an important consistency check with respect to semi-leptonic decays. In this case the form factors are momentum dependent, and discretisation effects and physical effects are hard to disentangle. In figure 5 we plot the ratio

$$R(\vec{p}) = \frac{\langle 0|A_0|B_s(\vec{p})\rangle/\sqrt{E(\vec{p})}}{\langle 0|A_0|B_s(0)\rangle/\sqrt{E(0)}}\,. \tag{10}$$

This has been investigated for two different values of a [23]. In the absence of discretisation effects this becomes $\sqrt{E(\vec{p})/E(0)}$, which is included as a full line. The results are consitent with the continuum expectation up to momenta of about 1.2 GeV. For the largest momenta the deviations are less than 8%.

5.2. Decay constant of the D_s

The decay constant of the lighter D_s was successfully determined in lattice NRQCD [23] as well. As for the spectrum, this was done on coarse lattices. The result includes all the corrections up to $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$. After cancellation of the power divergence in $\mathcal{O}(\alpha_s/am_c)$ the residual $\mathcal{O}(1/m_c)$ matrix elements contribute only on the 10% level to the final number, indicating good convergence of the NRQCD expansion in the charm region. The final outcome is

$$f_{D_s} = 223(54)\ \mathrm{MeV}\,. \tag{11}$$

This result agrees well with the experimental result of $f_{D_s} = 280(19)(28)(34)$ MeV [27] as well as other lattice calculations in the quenched approximation [25,26].

6. SEMI-LEPTONIC DECAYS $B \to Dl\nu$

Semi-leptonic decays of a B-meson into $D, D^*, D^{**}, D', \ldots$ provide the best way to measure $|V_{cb}|$. A precise measurement of this is required to relate CP-violation as measured from the K to the one measured from the B. In case of pseudo-scalar decay products such as D and D', one needs to determine the matrix element

$$\begin{aligned}\langle B|V_\mu|D\rangle &= \sqrt{m_B m_D}[h^+(\omega)(v_B+v_D)_\mu \\ &\quad +h^-(\omega)(v_B-v_D)_\mu]\,.\end{aligned} \tag{12}$$

As shown, this can be parametrised by two form factors $h^+(\omega)$ and $h^-(\omega)$ with $\omega = v_B\cdot v_D$. In this section we summarise the results of [28], which are still preliminary.

6.1. Elastic scattering

The elastic scattering of a B-meson from a vector current provides the simplest approximation to the matrix element in eqn. (12). In the static limit, $m_B \to \infty$ this becomes the Isgur-Wise function $\xi(\omega)$. The ω-dependence of the elastic form factor has been studied for two different values of the mass of the heavy quark. The result is reproduced in figure 6. The values for $am_Q = 4$ corresponds approximately to a B_s-meson. The figure gives

$$\rho^2_{\rm strange} = 1.5(3)(4) \tag{13}$$

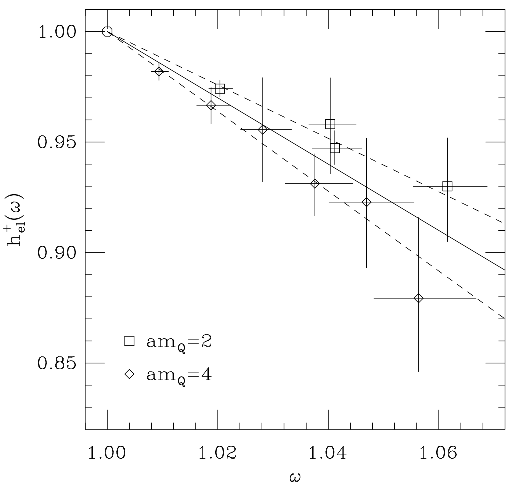

Figure 6. Form factor $h^+_{\rm el}$ in the case of elastic scattering as a function of the recoil parameter ω.

for the slope of the Isgur-Wise function in case of a strange spectator quark. The results have been checked for their dependence on the momentum of the external states.

6.2. Radial excited states

The use of two different interpolating fields with the same quantum numbers makes it possible to observe a signal for the matrix element $\langle B_s|V_0(q)|B'_s\rangle$ involving a radial excited state. The q dependence of the matrix element is shown in figure 7. For both external states the same heavy quark mass is used. The external states are orthogonal at zero recoil and the matrix element vanishes as expected. Once these states get boosted with respect to each other, the matrix element becomes finite. This result is qualitative and demonstrates the feasibility to determine such matrix elements in lattice gauge theory.

6.3. Non-massdegenerate transitions

The case of unequal heavy quark mass for the in and outgoing state has been studied at zero recoil. In this case the vector current is not a con-

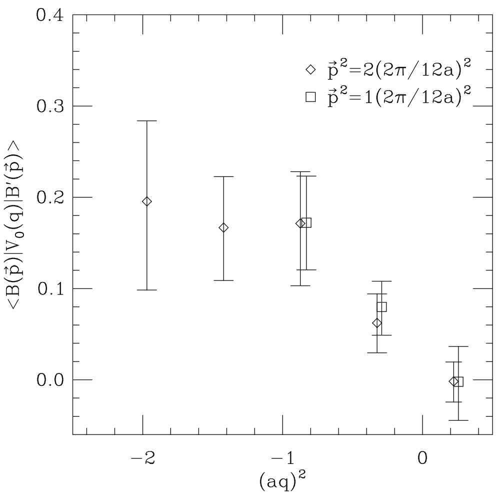

Figure 7. Form factor for transitions between pseudoscalar ground and radially excited state.

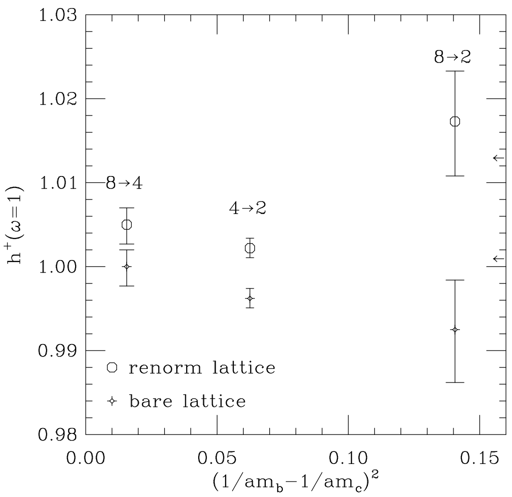

Figure 8. Form factor h^+ at zero recoil.

served current anymore and gets renormalised. The renormalisation constants have been perturbatively calculated to $\mathcal{O}(\alpha_s)$ [29]. Figure 8 gives the result with and without this renormalisation for three different pairs of heavy quark masses. The used heavy quark masses are indicated above the individual results in units of the lattice spacing. Again the value of $am_Q = 4$ corresponds to the B_s-meson. The arrows at the right hand side of the the figure give the upper and lower error bound of the lattice result from [30]. This has been extrapolated to the physical $B \to D$ transition and corresponds to $(1/am_b - 1/am_c)^2 \approx 0.56$.

7. DISCUSSION

This talk summarises our recent results on the physics of heavy light mesonic states. The results include the B and D-meson spectrum as well as the decay constants of the B_s and the D_s. The results for the B and the B_s have been obtained for three different values of the lattice spacing and the results are found to be independent of the lattice spacing within error bars. In the spectrum we observe good agreement to experiment for spin independent splittings. However the spin dependent hyperfine splitting comes out too small. This has to be resolved in future calculations.

The spectrum calculations also allow for the precise determination of the b-quark mass in the $\overline{\text{MS}}$-scheme. The preliminary result is $m_{b,\overline{\text{MS}}} = 4.34(7)$ GeV at its own scale. This result is not affected by the too small hyperfine splitting.

The results for the D-spectrum and the D_s decay constant agree well with experimental results as well as other lattice calculations, employing a lattice discretisation of the Dirac action for the charm quark. To obtain reasonable results for charm quarks is crucial with respect to the calculation of the form factors for semi-leptonic $B \to D$ decays. Recent results for the semi-leptonic form factors have been presented as well.

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