CLNS 00/1687
CLEO 00-16

Study of τ Decays to Six Pions and Neutrino

CLEO Collaboration

(October 11, 2000)

Abstract

The τ decays to six-pion final states have been studied with the CLEO detector at the Cornell Electron Storage Ring. The measured branching fractions are $\mathcal{B}(\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau) = (2.2 \pm 0.3 \pm 0.4) \times 10^{-4}$ and $\mathcal{B}(\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau) = (1.7 \pm 0.2 \pm 0.2) \times 10^{-4}$. A search for substructure in these decays shows that they are saturated by intermediate states with η or ω mesons. We present the first observation of the decay $\tau^- \to 2\pi^-\pi^+\omega\nu_\tau$ and the branching fraction is measured to be $(1.2 \pm 0.2 \pm 0.1) \times 10^{-4}$. The measured branching fractions are in good agreement with the isospin expectations but somewhat below the Conserved-Vector-Current predictions.

A. Anastassov,[1] J. E. Duboscq,[1] E. Eckhart,[1] K. K. Gan,[1] C. Gwon,[1] T. Hart,[1]
K. Honscheid,[1] D. Hufnagel,[1] H. Kagan,[1] R. Kass,[1] T. K. Pedlar,[1] H. Schwarthoff,[1]
J. B. Thayer,[1] E. von Toerne,[1] M. M. Zoeller,[1] S. J. Richichi,[2] H. Severini,[2] P. Skubic,[2]
A. Undrus,[2] S. Chen,[3] J. Fast,[3] J. W. Hinson,[3] J. Lee,[3] D. H. Miller,[3] E. I. Shibata,[3]
I. P. J. Shipsey,[3] V. Pavlunin,[3] D. Cronin-Hennessy,[4] A.L. Lyon,[4] E. H. Thorndike,[4]
C. P. Jessop,[5] V. Savinov,[5] T. E. Coan,[6] V. Fadeyev,[6] Y. Maravin,[6] I. Narsky,[6]
R. Stroynowski,[6] J. Ye,[6] T. Wlodek,[6] M. Artuso,[7] R. Ayad,[7] C. Boulahouache,[7] K. Bukin,[7]
E. Dambasuren,[7] S. Karamov,[7] G. Majumder,[7] G. C. Moneti,[7] R. Mountain,[7] S. Schuh,[7]
T. Skwarnicki,[7] S. Stone,[7] G. Viehhauser,[7] J.C. Wang,[7] A. Wolf,[7] J. Wu,[7] S. Kopp,[8]
A. H. Mahmood,[9] S. E. Csorna,[10] I. Danko,[10] K. W. McLean,[10] Z. Xu,[10] R. Godang,[11]
G. Bonvicini,[12] D. Cinabro,[12] M. Dubrovin,[12] S. McGee,[12] G. J. Zhou,[12] E. Lipeles,[13]
S. P. Pappas,[13] M. Schmidtler,[13] A. Shapiro,[13] W. M. Sun,[13] A. J. Weinstein,[13]
F. Würthwein,[13,*] D. E. Jaffe,[14] G. Masek,[14] H. P. Paar,[14] E. M. Potter,[14] S. Prell,[14]
D. M. Asner,[15] A. Eppich,[15] T. S. Hill,[15] R. J. Morrison,[15] R. A. Briere,[16] G. P. Chen,[16]
W. T. Ford,[17] A. Gritsan,[17] J. Roy,[17] J. G. Smith,[17] J. P. Alexander,[18] R. Baker,[18]
C. Bebek,[18] B. E. Berger,[18] K. Berkelman,[18] F. Blanc,[18] V. Boisvert,[18] D. G. Cassel,[18]
P. S. Drell,[18] K. M. Ecklund,[18] R. Ehrlich,[18] A. D. Foland,[18] P. Gaidarev,[18] R. S. Galik,[18]
L. Gibbons,[18] B. Gittelman,[18] S. W. Gray,[18] D. L. Hartill,[18] B. K. Heltsley,[18]
P. I. Hopman,[18] L. Hsu,[18] C. D. Jones,[18] D. L. Kreinick,[18] M. Lohner,[18] A. Magerkurth,[18]
T. O. Meyer,[18] N. B. Mistry,[18] E. Nordberg,[18] J. R. Patterson,[18] D. Peterson,[18] D. Riley,[18]
A. Romano,[18] J. G. Thayer,[18] D. Urner,[18] B. Valant-Spaight,[18] A. Warburton,[18] P. Avery,[19]
C. Prescott,[19] A. I. Rubiera,[19] H. Stoeck,[19] J. Yelton,[19] G. Brandenburg,[20] A. Ershov,[20]
Y. S. Gao,[20] D. Y.-J. Kim,[20] R. Wilson,[20] T. Bergfeld,[21] B. I. Eisenstein,[21] J. Ernst,[21]
G. E. Gladding,[21] G. D. Gollin,[21] R. M. Hans,[21] E. Johnson,[21] I. Karliner,[21] M. A. Marsh,[21]
M. Palmer,[21] C. Plager,[21] C. Sedlack,[21] M. Selen,[21] J. J. Thaler,[21] J. Williams,[21]
K. W. Edwards,[22] R. Janicek,[23] P. M. Patel,[23] A. J. Sadoff,[24] R. Ammar,[25] A. Bean,[25]
D. Besson,[25] X. Zhao,[25] S. Anderson,[26] V. V. Frolov,[26] Y. Kubota,[26] S. J. Lee,[26]
R. Mahapatra,[26] J. J. O'Neill,[26] R. Poling,[26] T. Riehle,[26] A. Smith,[26] C. J. Stepaniak,[26]
J. Urheim,[26] S. Ahmed,[27] M. S. Alam,[27] S. B. Athar,[27] L. Jian,[27] L. Ling,[27] M. Saleem,[27]
S. Timm,[27] and F. Wappler[27]

[1]Ohio State University, Columbus, Ohio 43210
[2]University of Oklahoma, Norman, Oklahoma 73019
[3]Purdue University, West Lafayette, Indiana 47907
[4]University of Rochester, Rochester, New York 14627
[5]Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
[6]Southern Methodist University, Dallas, Texas 75275
[7]Syracuse University, Syracuse, New York 13244
[8]University of Texas, Austin, TX 78712
[9]University of Texas - Pan American, Edinburg, TX 78539
[10]Vanderbilt University, Nashville, Tennessee 37235
[11]Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

[*]Permanent address: Massachusetts Institute of Technology, Cambridge, MA 02139.

[12]Wayne State University, Detroit, Michigan 48202
[13]California Institute of Technology, Pasadena, California 91125
[14]University of California, San Diego, La Jolla, California 92093
[15]University of California, Santa Barbara, California 93106
[16]Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
[17]University of Colorado, Boulder, Colorado 80309-0390
[18]Cornell University, Ithaca, New York 14853
[19]University of Florida, Gainesville, Florida 32611
[20]Harvard University, Cambridge, Massachusetts 02138
[21]University of Illinois, Urbana-Champaign, Illinois 61801
[22]Carleton University, Ottawa, Ontario, Canada K1S 5B6
and the Institute of Particle Physics, Canada
[23]McGill University, Montréal, Québec, Canada H3A 2T8
and the Institute of Particle Physics, Canada
[24]Ithaca College, Ithaca, New York 14850
[25]University of Kansas, Lawrence, Kansas 66045
[26]University of Minnesota, Minneapolis, Minnesota 55455
[27]State University of New York at Albany, Albany, New York 12222

The decays of the τ lepton provide a good test of the Standard Model predictions for the hadronic weak current. The six-pion branching fractions are related to the isovector part of the e^+e^- annihilation cross section by the Conserved Vector Current (CVC) hypothesis. Isospin symmetry relates the relative branching fractions of the decays $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$, $\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau$, and $\tau^- \to \pi^-5\pi^0\nu_\tau$. Therefore, the study of six-pion decays can be used to test the CVC hypothesis and isospin predictions. A better understanding of the resonance substructure in the decays is also of particular interest because of the potential application in suppressing the hadronic background in the measurement of the τ neutrino mass. In this Letter, we present a study of the decays [1] $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$ and $\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau$. This includes measurements of the branching fractions and the search for resonance substructure. The latter results are used to identify the vector and axial-vector current contributions to the inclusive branching fractions and allow the proper comparison of the results with the CVC and isospin symmetry predictions.

The data used in this analysis were collected with the CLEO detector [2] at the Cornell Electron Storage Ring (CESR) at center-of-mass energy of 10.6 GeV. The sample corresponds to a total integrated luminosity 13.5 fb^{-1} and contains 12.3×10^6 $\tau^+\tau^-$ events [3].

For the decay $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$ ($3\pi^-2\pi^+\pi^0\nu_\tau$) we select events with four (six) charged tracks and zero net charge. The momentum of each track must be greater than 100 MeV and the polar angle θ of each track with respect to the beam must satisfy $|\cos\theta| < 0.90$. The track must be consistent with originating from the e^+e^- interaction point. This vertex requirement also suppresses the background from τ events with a K_S or photon conversion. The K_S background is further reduced by rejecting events containing a detached vertex with a $\pi^+\pi^-$ mass consistent with the nominal K_S mass.

We define two exclusive sets of photon candidates in the barrel calorimeter ($|\cos\theta| < 0.80$): high quality (HQ) and low quality (LQ) photons. The selection criteria for HQ photons are designed to minimize the contamination of fake showers from hadronic interactions, while the acceptance of LQ photons ensures high event detection efficiency. A HQ photon must have an energy $E_\gamma > 120$ MeV and a lateral profile of energy deposition consistent with that expected for a photon. The Fisher discriminant technique [4] is used to differentiate a real photon from a fake photon. The discriminant function is a linear combination of the energy of the shower and its distance to the intersection of the nearest charged track with the calorimeter surface. We use Monte Carlo simulated events (see below) to obtain the discriminant function that maximizes the separation between real and fake photons. A LQ photon is defined as a shower that does not pass the HQ photon requirements but has $E_\gamma > 30$ MeV. The Fisher discriminant for differentiating a real photon from a fake photon is only used in the decay $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$ due to the higher photon multiplicity. HQ photons in the endcap calorimeter ($0.80 < |\cos\theta| < 0.95$) are selected as in the barrel.

Each event is divided into two hemispheres using the plane perpendicular to the thrust axis [5], calculated using both charged tracks and photons. There must be one charged track in one hemisphere (tag) recoiling against three or five charged tracks in the other hemisphere (signal), depending on the decay mode.

In the tag hemisphere, the total invariant mass of charged tracks and photons must satisfy $M_{tag} < 0.5$ GeV. In the signal hemisphere, there should be at least six (two) photons forming three (one) π^0 candidates for the decay $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$ ($3\pi^-2\pi^+\pi^0\nu_\tau$). The π^0 candidates are reconstructed from photon pairs in the barrel calorimeter. All HQ photons in the barrel must be used in the reconstruction and no HQ photon in the endcap is allowed. In case of multiple entries, we select the photon combination with smallest $\chi^2 = \sum_{i=1}^{3(1)}[S^2_{\gamma\gamma}]_i$, where $S_{\gamma\gamma} = (m_{\gamma\gamma} - m_{\pi^0})/\sigma_{\gamma\gamma}$ ($\sigma_{\gamma\gamma}$ is the mass resolution calculated from the energy and angular resolution of each photon). The signal region for the π^0 candidates is defined as $-3.5 < S_{\gamma\gamma} < 2.5$. In the case of $\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau$ we use sideband subtraction to estimate the fake π^0 background, with sidebands defined as $-8.5 < S_{\gamma\gamma} < -5.5$ and $4.5 < S_{\gamma\gamma} < 7.5$. The total invariant mass of the hadronic system in the signal hemisphere must satisfy $M_{6\pi} < M_\tau = 1.777$ GeV [6]. The signal hemisphere must have a positive pseudo-neutrino mass:

$$M^2 = M_\tau^2 + M_{6\pi}^2 - 2M_\tau E^*_{6\pi} > 0 \quad ,$$

where $E^*_{6\pi}$ is the energy of the six-pion system in the τ rest frame, assuming that the τ has the full beam energy and that the τ direction is given by the momentum vector of the six-pion system. This cut selects events with tau-like kinematics, suppressing both the hadronic background and the τ migration background from lower multiplicity decays where the six-pion momentum is not as good of an approximation of the τ direction. The background from τ decays with Dalitz π^0 decays or photon conversion in inner detector material is suppressed by requiring that any pairs of oppositely charged tracks have invariant mass exceeding 120 MeV when at least one of the pair is identified as an electron and assigning the electron mass to both tracks. The electron identification requires that a track is matched to a shower with shower energy to momentum ratio exceeding 0.85, and, if available, specific ionization in the drift chamber consistent with that expected for an electron. Two-photon backgrounds are eliminated by requiring the direction of the missing momentum of the event to satisfy $|\cos\theta_{missing}| < 0.9$.

The detection efficiency and τ migration background are calculated with a Monte Carlo technique. We use the KORALB/TAUOLA program [3] for the τ event simulation. The decay $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$ is modeled using a mixture of $\tau^- \to 2\pi^-\pi^+\eta\nu_\tau$, $\tau^- \to \pi^-2\pi^0\eta\nu_\tau$, and $\tau^- \to \pi^-2\pi^0\omega\nu_\tau$. The other decay $\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau$ is modeled using a mixture of $\tau^- \to 2\pi^-\pi^+\eta\nu_\tau$ and $\tau^- \to 2\pi^-\pi^+\omega\nu_\tau$. The relative mixtures are determined from the measured branching fractions presented in this and a previous Letter [7]. We assume that the $3\pi\eta$ decays proceed through πf_1 with a spectral function dominated by the form factor of the $a_1(1260)$ resonance [8]. The $3\pi\omega$ system is modeled assuming dominance of the $\rho(1700)$ resonance. The detector response is simulated using the GEANT program [9]. The hadronic background is calculated empirically using a sample of high-mass tagged events ($1.8 < M_{tag} < 2.8$ GeV) obtained from the data with the assumption that M_{tag} and $M_{6\pi}$ are not correlated. The invariant mass spectra of the six-pion hadronic systems after applying all selection criteria are shown in Fig. 1. There is good agreement between the data and expectation. The signal, background, and detection efficiency are summarized in Table I. The τ background includes the decays with $K_S \to \pi^+\pi^-$ and $\pi^0\pi^0$.

The six-pion τ decays can proceed through the η or ω intermediate hadronic states. Figure 2 shows the invariant mass spectra of 3π combinations in the η mass region. To improve the resolution, the π^0 candidates have been kinematically constrained to the nominal π^0 mass. In the search for $\eta \to \pi^+\pi^-\pi^0$, we reduce the combinatoric background by rejecting events with $3\pi^0$ invariant mass within 20 MeV ($\sim 3\sigma$) of the nominal η mass. There are clear enhancements at the η mass in all three spectra. The invariant mass spectra of 3π

TABLE I. Summary of the results for $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$ and $\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau$. All errors are statistical, except the second errors in the branching fractions, which are systematic.

Decay mode	$2\pi^-\pi^+3\pi^0\nu_\tau$	$3\pi^-2\pi^+\pi^0\nu_\tau$
Data (events)	139.0 ± 11.8	231.0 ± 18.8
$q\bar{q}$ bg (events)	15.1 ± 3.1	25.8 ± 5.9
τ bg (events)	35.2 ± 3.4	19.4 ± 5.5
Efficiency (%)	1.65 ± 0.03	4.45 ± 0.06
$\mathcal{B}$ (10^{-4})	$2.2 \pm 0.3 \pm 0.4$	$1.7 \pm 0.2 \pm 0.2$

TABLE II. Summary of the results for the intermediate states with an η meson. All errors are statistical, except the second errors in the branching fractions, which are systematic.

Decay mode	$2\pi^-\pi^+\eta\nu_\tau$	$\pi^-2\pi^0\eta\nu_\tau$	$2\pi^-\pi^+\eta\nu_\tau$
	$\eta \to 3\pi^0$	$\eta \to \pi^+\pi^-\pi^0$	$\eta \to \pi^+\pi^-\pi^0$
Data (events)	32.1 ± 6.7	15.4 ± 5.4	52.4 ± 10.4
$q\bar{q}$ bg (events)	1.9 ± 1.5	$0.2^{+1.0}_{-0.2}$	5.2 ± 3.2
τ bg (events)	0.9 ± 0.9	2.3 ± 1.6	2.5 ± 2.2
Efficiency (%)	1.28 ± 0.05	1.48 ± 0.05	4.18 ± 0.08
$\mathcal{B}$ (10^{-4})	$2.9 \pm 0.7 \pm 0.5$	$1.5 \pm 0.6 \pm 0.3$	$1.9 \pm 0.4 \pm 0.3$

combinations in the ω mass region are shown in Fig. 3. There is also a clear signal at the ω mass. In particular, the decay $\tau^- \to 2\pi^-\pi^+\omega\nu_\tau$ is observed for the first time.

The η and ω signals are extracted by maximum likelihood fits with a dual Gaussian over a polynomial background. The shapes have been constrained to the Monte Carlo expectations. The results are summarized in Tables II and III.

There are several sources of systematic errors. These include the uncertainty in the number of produced $\tau^+\tau^-$ pairs (1.4%), charged track reconstruction (1% per track), π^0 reconstruction (4% per π^0), efficiency (1-5%) and τ migration background (4-12%) estimates due to limited Monte Carlo statistics, hadronic background estimates (4-9%), mixtures of various intermediate states (1-6%) and branching fractions of $\eta \to 3\pi^0$ and $\pi^+\pi^-\pi^0$ and $\omega \to \pi^+\pi^-\pi^0$ [6]. For the extraction of ω and η signals, there are also systematic errors (5-10%) resulting from the choice of combinatorial background shape and fit region. The branching fractions with systematic errors are listed in Tables I-III. The results represent significant improvement in precision over previous measurements [6]. The branching fractions for the two decays with ω in the final states are somewhat smaller than the recent calculations by Gao and Li [10].

The results on the six-pion decays can be compared with the isospin symmetry and CVC predictions, after correcting for the contributions from the axial-vector current $\tau^- \to (3\pi)^-\eta\nu_\tau$, which also violates isospin conservation with the decays $\eta \to 3\pi^0$ and $\pi^+\pi^-\pi^0$. To reduce the uncertainty in the corrections, we use measurements from these decays and

TABLE III. Summary of the results for the intermediate states with an ω meson. All errors are statistical, except the second errors in the branching fractions, which are systematic.

Decay mode	$\pi^-2\pi^0\omega\nu_\tau$	$2\pi^-\pi^+\omega\nu_\tau$
Data (events)	54.3 ± 11.4	110.1 ± 18.6
$q\bar{q}$ bg (events)	$1.0^{+2.4}_{-1.0}$	$3.5^{+4.2}_{-3.5}$
τ bg (events)	8.7 ± 3.4	$2.3^{+3.7}_{-2.3}$
Efficiency (%)	1.39 ± 0.06	4.06 ± 0.14
$\mathcal{B}$ (10^{-4})	$1.5 \pm 0.4 \pm 0.3$	$1.2 \pm 0.2 \pm 0.1$

the decay $\eta \to \gamma\gamma$ [7] to obtain the average branching fractions: $\bar{\mathcal{B}}(\tau^- \to 2\pi^-\pi^+\eta\nu_\tau) = (2.4 \pm 0.5) \times 10^{-4}$ and $\bar{\mathcal{B}}(\tau^- \to \pi^-2\pi^0\eta\nu_\tau) = (1.5 \pm 0.5) \times 10^{-4}$. This yields the vector current branching fractions: $\mathcal{B}_V(\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau) = (1.1 \pm 0.4) \times 10^{-4}$ and $\mathcal{B}_V(\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau) = (1.1 \pm 0.2) \times 10^{-4}$, corresponding to $\sim$50% and $\sim$65%, respectively, of the inclusive six-pion branching fractions.

The isospin model [11] classifies n-pion final states into orthogonal isospin partitions and determines their contributions to the branching fractions. The partitions are labeled by three integers (n_1, n_2, n_3), where n_3 is the number of isoscalar subsystems of three pions, $n_2 - n_3$ is the number of isovector systems of two pions, and $n_1 - n_2$ is the number of single pions. For $n = 6$ there are four partitions: 510 ($4\pi\rho$), 330 (3ρ), 411 ($3\pi\omega$), and 321 ($\pi\rho\omega$), denoted according to the lowest mass states. The isospin model imposes constraints on the relative branching fractions, which can be tested by comparing the fraction

$$\mathrm{f}_{2\pi^-\pi^+3\pi^0} = \frac{\mathcal{B}_V(\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau)}{\mathcal{B}_V(\tau^- \to (6\pi)^-\nu_\tau)}$$

to a similar fraction $\mathrm{f}_{3\pi^-2\pi^+\pi^0}$, where $\mathcal{B}_V(\tau^- \to (6\pi)^-\nu_\tau)$ is the sum of the branching fractions for the three six-pion vector decays. Figure 4 shows $\mathrm{f}_{2\pi^-\pi^+3\pi^0}$ vs. $\mathrm{f}_{3\pi^-2\pi^+\pi^0}$ with our new measurement of the branching fractions. The measurement is presented as a line because $\mathcal{B}_V(\tau^- \to \pi^-5\pi^0\nu_\tau)$ has not been measured yet. The result is consistent with the isospin expectation since the experimental measurement overlaps with the isospin triangle. Our result indicates the 321 ($\pi\rho\omega$) partition is dominant because the decays $\tau^- \to \pi^-2\pi^0\omega\nu_\tau$ and $\tau^- \to 2\pi^-\pi^+\omega\nu_\tau$ saturate the six-pion (vector) decays.

The CVC hypothesis combined with isospin symmetry predicts that the branching fractions for both $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$ and $\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau$ are greater than $(2.5 \pm 0.4) \times 10^{-4}$ [12]. This is somewhat larger than the measured branching fractions for the six-pion vector decays. The discrepancy is even more significant if we compare the predicted inclusive branching fraction $\mathcal{B}_{CVC}(\tau^- \to (6\pi)^-\nu_\tau) = (12.3 \pm 1.9) \times 10^{-4}$ with the sum of the measured six-pion vector branching fractions under the assumption that $\mathcal{B}_V(\tau^- \to \pi^-5\pi^0\nu_\tau)$ is comparable with or smaller than $\mathcal{B}_V(\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau)$ and $\mathcal{B}_V(\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau)$ [13]. This assumption is consistent with the observation that the six-pion vector decays are saturated by intermediate states with an ω meson, which implies a small decay width for the 510 ($4\pi\rho$) state, the only state that contributes to the decay $\tau^- \to \pi^-5\pi^0\nu_\tau$. The discrepancy might be explained by sizable presence of $I = 0$ states in the e^+e^- annihilation data that should be subtracted before calculating the CVC prediction.

In conclusion, the branching fractions for two six-pion decays have been measured with much improved precision. The resonance substructure in the decays has been studied. In particular, the decay $\tau^- \to 2\pi^-\pi^+\omega\nu_\tau$ has been observed for the first time. Within the statistical precision, the decays are saturated by η and ω intermediate states. The measured branching fractions are in good agreement with the isospin expectations but somewhat below the CVC predictions.

We gratefully acknowledge the effort of the CESR staff in providing us with excellent luminosity and running conditions. This work was supported by the National Science Foundation, the U.S. Department of Energy, the Research Corporation, the Natural Sciences and Engineering Research Council of Canada, the A.P. Sloan Foundation, the Swiss National Science Foundation, the Texas Advanced Research Program, and the Alexander von Humboldt Stiftung.

REFERENCES

[1] We assume all charged particles in the final states are pions. Charge conjugate states are implied throughout this Letter.

[2] Y. Kubota *et al.* (CLEO Collaboration), Nucl. Instrum. Methods Phys. Res., Sec. A**320**, 66 (1992); T.S. Hill, Nucl. Instrum. Methods Phys. Res., Sec. A**418**, 32 (1998).

[3] S. Jadach and Z. Was, Comp. Phys. Comm. **36**, 191 (1985); *ibid.* **64**, 267 (1991); S. Jadach, J. H. Kuhn, and Z. Was, *ibid.* **64**, 275 (1991).

[4] See for example, L. Lebart, A. Morineau, and K. Warwick, Multivariate Descriptive Statistical Analysis, John Wiley & Sons, Inc. (1984).

[5] E. Farhi, Phys. Rev. Lett. **39**, 1587 (1977).

[6] Particle Data Group, Eur. Phys. J. C **3**, 1 (1998).

[7] T. Bergfeld *et al.* (CLEO Collaboration), Phys. Rev. Lett. **79**, 2406 (1997).

[8] B.A. Li, Phys. Rev. D **55**, 1436 (1997).

[9] R. Brun *et al.*, CERN Report No. CERN-DD/EE/84-1, 1987 (unpublished).

[10] J. Gao and B.A. Li, hep-ph/0004097.

[11] A. Pais, Ann. Phys. **9**, 548 (1960).

[12] S.I. Eidelman and V.N. Ivanchenco, Nucl. Phys. B, **55C**, 181 (1997).

[13] Isospin symmetry imposes the constraint $\mathcal{B}_V(\tau^- \to \pi^- 5\pi^0\nu_\tau) \leq \frac{9}{26}[\mathcal{B}_V(\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau) + \mathcal{B}_V(\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau)]$.

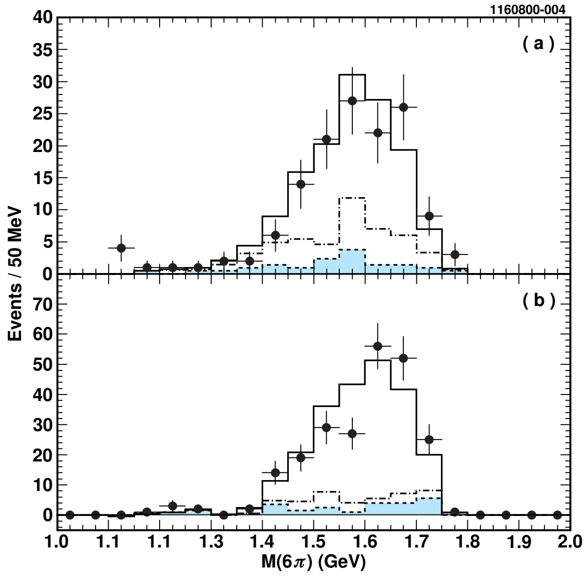

FIG. 1. Mass spectra of the hadronic systems in the decays (a) $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$ and (b) $\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau$. The solid histogram is the sum of the signal Monte Carlo and background (dashed), which includes the τ migration and hadronic (shaded) background.

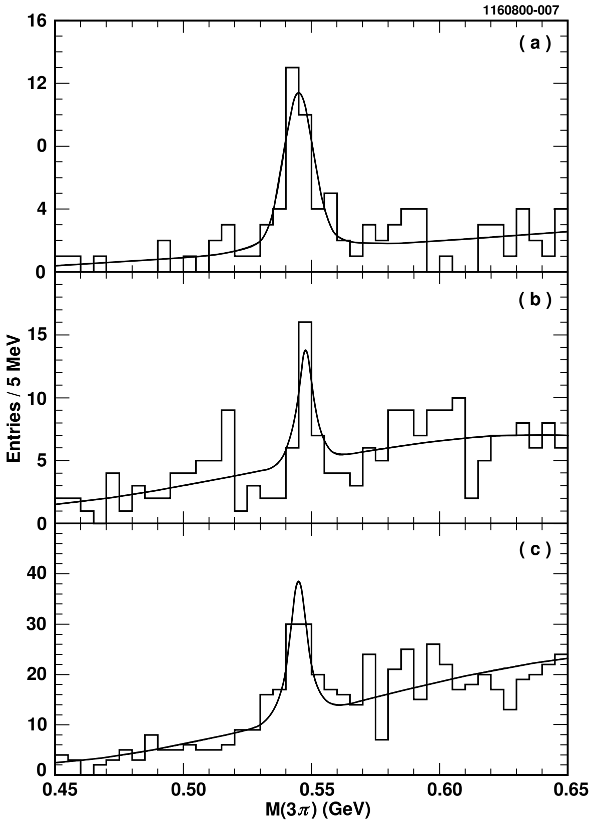

FIG. 2. Three-pion mass spectra of η candidates: (a) $M_{3\pi^0}$ in $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$, (b) $M_{\pi^+\pi^-\pi^0}$ in $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$, (c) $M_{\pi^+\pi^-\pi^0}$ in $\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau$. There are six entries per event in (b) and (c). The solid lines shows fits to the data.

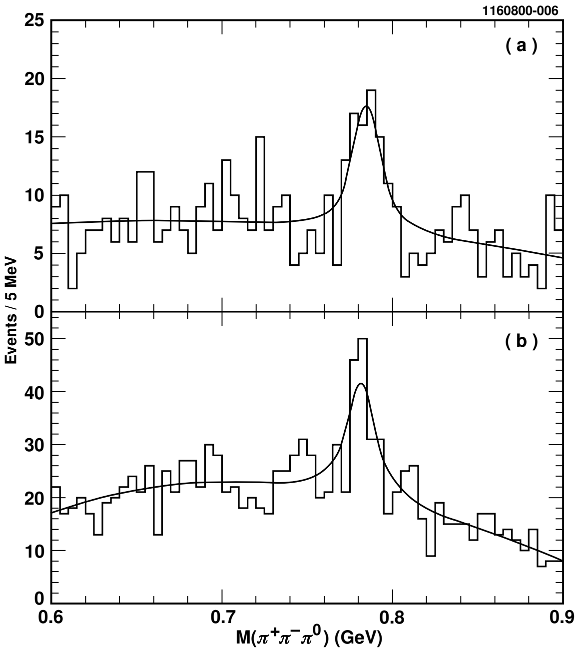

FIG. 3. Three-pion mass spectra of ω candidates (six entries per event) in (a) $\tau^- \to 2\pi^-\pi^+3\pi^0\nu_\tau$ and (b) $\tau^- \to 3\pi^-2\pi^+\pi^0\nu_\tau$. The solid lines shows fits to the data.

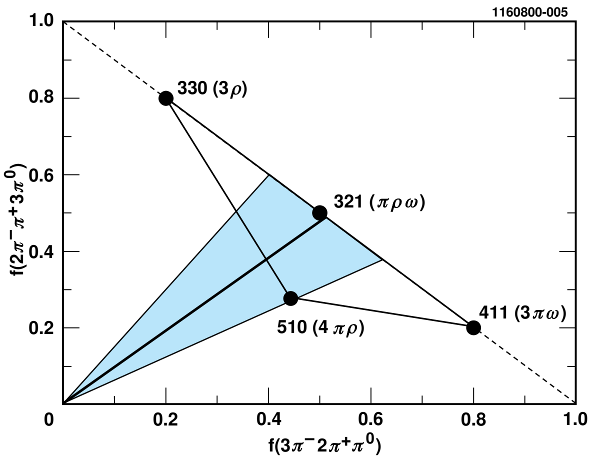

FIG. 4. Decay fractions of $\tau^- \to (6\pi)^-\nu_\tau$. The thick solid line through the origin represents the measurement. The shaded area indicates the one standard deviation region, calculated with correlated errors taken into account. The triangle bounded by the dots shows the isospin expectation.