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Measurements of the Mass, Total Width and Two-photon Partial Width of the η_c Meson

CLEO Collaboration

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Abstract

Using 13.4 fb^{-1} of data collected with the CLEO detector at the Cornell Electron Storage Ring, we have observed 300 events for the two-photon production of ground-state pseudo-scalar charmonium in the decay $\eta_c \to K^0_S K^{\mp}\pi^{\pm}$. We have measured the η_c mass to be (2980.4 $\pm$ 2.3 (stat) $\pm$ 0.6 (sys)) MeV and its full width as (27.0 $\pm$ 5.8 (stat) $\pm$ 1.4 (sys)) MeV. We have determined the two-photon partial width of the η_c meson to be (7.6 $\pm$ 0.8 (stat) $\pm$ 0.4 (sys) $\pm$ 2.3 (br)) keV, with the last uncertainty associated with the decay branching fraction.

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In this Letter, we report a study of two-photon production of the ground-state pseudoscalar charmonium, *i.e.*, $\gamma\gamma \to \eta_c$. The two space-like photons are radiated by e^+ and e^- beams, each at an energy of approximately 5.3 GeV. The charmonium spectrum is an ideal testing ground for quantum chromodynamics (QCD) calculations, and producing C-even charmonium states through $\gamma\gamma$ fusion provides a clean environment for this purpose.

The two-photon partial width of the η_c meson can be expressed in next-to-leading order (NLO) perturbative QCD (PQCD), in terms of the e^+e^- partial width of the J/ψ meson, as [1]

$$\frac{\Gamma^{\eta_c}_{\gamma\gamma}}{\Gamma^{\psi}_{ee}} = \frac{4}{3}(1 + 1.96\alpha_s/\pi) \times \frac{|\Psi_{\eta_c}(0)|^2}{|\Psi_{\psi}(0)|^2}. \quad (1)$$

Using the world average value [2] of Γ^{ψ}_{ee}, a value of the strong coupling constant α_s evaluated at the charm mass scale [1] of (0.28 ± 0.02), and the assumption that the two 1S wave functions, Ψ, are the same at the origin, this relationship predicts $\Gamma^{\eta_c}_{\gamma\gamma} = (8.2 \pm 0.6)$ keV.

The total width of the η_c meson can be assumed to be dominated by its two-gluon component, *i.e.*, $\Gamma^{\eta_c}_{\rm tot} \approx \Gamma^{\eta_c}_{gg}$. The ratio of the rates for $\eta_c \to gg$ and $\eta_c \to \gamma\gamma$ is an especially clean prediction of PQCD because the dependencies of these rates on the wave functions and non-perturbative factors are identical in the numerator and denominator. The ratio depends only on the coupling constants and has been calculated in NLO [1],

$$\frac{\Gamma^{\eta_c}_{\rm tot}}{\Gamma^{\eta_c}_{\gamma\gamma}} \approx \frac{9\alpha_s^2}{8\alpha^2} \times \frac{(1 + 4.8\alpha_s/\pi)}{(1 - 3.4\alpha_s/\pi)}. \quad (2)$$

Using the value of $\Gamma^{\eta_c}_{\gamma\gamma}$ estimated in NLO gives $\Gamma^{\eta_c}_{\rm tot}$ as (28 ± 6) MeV; using instead the world average value [2] of $\Gamma^{\eta_c}_{\gamma\gamma}$, one obtains an estimate of $\Gamma^{\eta_c}_{\rm tot}$ as (26 ± 6) MeV. A calculation with fully relativistic decay amplitudes and a sophisticated QCD potential model [3] predicts $\Gamma^{\eta_c}_{\rm tot} \approx 23$ MeV. The current world average [2] of $\Gamma^{\eta_c}_{\rm tot} = 13.2^{+3.8}_{-3.2}$ MeV disagrees with these theoretical expectations. A precise measurement of the full width and two-photon partial width of the η_c is important for the verification of these PQCD calculations and approximations.

In the two-photon process $e^+e^- \to e^+e^-\gamma\gamma \to e^+e^-\eta_c$, the photon propagators dictate that the cross section naturally peaks at low momentum transfer, so the photons are almost real ("on shell"). The incident leptons are scattered at very low angles and continue traveling down the beam pipe undetected. Such "untagged" events typically have low net transverse momentum and low visible energy. The production of the η_c meson in this untagged two-photon process was searched for in the $K^0_S K^{\mp}\pi^{\pm}$ decay mode.

The data used in this study correspond to an integrated luminosity of 13.4 fb^{-1} and were collected with two configurations (CLEO II [4] and CLEO II.V [5]) of the CLEO detector at the Cornell Electron Storage Ring (CESR). Approximately one third of the data were taken with the CLEO II configuration. The detector components most useful for this study were the concentric tracking devices for charged particles, operating in a 1.5T superconducting solenoid. For CLEO II, this tracking system consisted of a 6-layer straw tube chamber, a 10-layer precision drift chamber, and a 51-layer main drift chamber. The main drift chamber also provided measurements of the specific ionization loss, dE/dx, used for particle identification. For CLEO II.V, the straw tube chamber was replaced by a 3-layer, double-sided silicon vertex detector and the gas in the main drift chamber was changed from a

50:50 mixture of argon-ethane to a 60:40 helium-propane mixture. These changes gave rise to significant improvements in the momentum and dE/dx resolutions for charged tracks. Photons were detected using the high-resolution electromagnetic calorimeter consisting of 7800 CsI crystals. The Monte Carlo simulation of the CLEO detector response was based upon GEANT [6]. Simulated events were processed in the same manner as the data to determine the $\eta_c \to K^0_S K^{\mp}\pi^{\pm}$ detection efficiency and the $K^0_S K^{\mp}\pi^{\pm}$ mass resolution at the η_c mass.

The K^0_S vertex was reconstructed from its decay to $\pi^+\pi^-$ and was required to be displaced from the e^+e^- interaction point; the amount of this displacement varied with detector configuration but was $\approx$1.5 mm. The K^0_S candidate was also required to be within 4 standard deviations (σ) of the known K^0_S mass [2]; here σ was determined on an event-by-event basis from the momenta measurements. Furthermore, the K^0_S momentum vector was required to point back to the interaction point. Of the two remaining charged tracks, the $K^{\mp}$ and $\pi^{\pm}$ candidates, the one with lower momentum was typically uniquely identified using the particle's specific ionization (dE/dx). This fixed the identity of the only remaining unidentified track, because the presence of the K^0_S dictated that exactly one of these two be a kaon to conserve strangeness in the event.

For the mass measurement only, we restricted ourselves to those events in which the $K^{\mp}$ and $\pi^{\pm}$ daughters of the η_c candidate traversed all layers of the tracking volume. The K^0_S daughter pions were not required to satisfy the same criterion, in that the kinematic fitting of the K^0_S decay corrected for any possible momentum mis-measurement of the daughter tracks. We did not make any such requirements while measuring the width and two-photon partial width, because these quantities are relatively insensitive to precise measurements of the track momenta.

The background from processes other than two-photon production was suppressed by requiring that the η_c candidate have net transverse momentum less than 0.6 GeV/c and that visible energy in the event be less than 6 GeV. Also, because the final state had no expected energy deposits in the calorimeter from neutral particles, the total calorimeter energy in the event not matched to charged tracks was required to be less than 0.6 GeV.

We fitted the background with a power law function ($A \cdot W^n_{\gamma\gamma}$, with $W_{\gamma\gamma}$ the $K^0_S K^{\mp}\pi^{\pm}$ invariant mass and A a multiplicative constant) and the signal with a spin-0 relativistic Breit-Wigner function (describing the natural line shape) convolved with a double Gaussian function (to take into account the detector resolution). The parameters for this double Gaussian were obtained from a Monte Carlo sample that had the η_c intrinsic width set to zero. We performed a simultaneous, binned, maximum-likelihood fit to the invariant mass distributions for the CLEO II and CLEO II.V datasets, constraining the physical variables M_{η_c}, $\Gamma^{\eta_c}_{\rm tot}$, and $\Gamma^{\eta_c}_{\gamma\gamma}$ in the two datasets to be the same. The invariant mass resolution was approximately 9 MeV in CLEO II and 7 MeV in CLEO II.V. The bin width for fitting was chosen as approximately the average of these two resolutions.

Two separate sets of fits were performed, one for the measurements of $\Gamma^{\eta_c}_{\rm tot}$ and $\Gamma^{\eta_c}_{\gamma\gamma}$ and another for the determination of the η_c mass. The full width and yield were obtained from the distributions shown in Fig. 1, with the total observed yield being $N_{obs} = 300 \pm 32$. The fit to the width gives $\Gamma^{\eta_c}_{\rm tot} = (27.0 \pm 5.8)$ MeV, with the uncertainty being only statistical. The two-photon partial width was determined by first correcting for the detector efficiency, ϵ, and then dividing by the number of events expected, N_1, for a two-photon partial width

of 1 keV:

$$\Gamma^{\eta_c}_{\gamma\gamma} = N_{obs}/(\epsilon \cdot N_1). \tag{3}$$

The quantity N_1 was determined using

$$N_1 = \mathcal{L} \cdot \mathcal{B}_{\eta_c} \cdot \mathcal{B}_{K_S} \cdot \sigma_{e^+e^- \to e^+e^-\eta_c}; \tag{4}$$

$\mathcal{L}$ is the integrated luminosity and the cross section for $e^+e^- \to e^+e^-\eta_c$ was obtained from Monte Carlo simulation, using the formalism of Budnev *et al.* [7] and setting $\Gamma^{\eta_c}_{\gamma\gamma} = 1$ keV. Also, $\mathcal{B}_{\eta_c} \equiv \mathcal{B}(\eta_c \to K^0_s K^{\mp}\pi^{\pm})$ and $\mathcal{B}_{K_S} \equiv \mathcal{B}(K^0_S \to \pi^+\pi^-)$, with the world average values [2] used. This procedure gives $\Gamma^{\eta_c}_{\gamma\gamma} = (7.6 \pm 0.8)$ keV, with this uncertainty coming from statistics only.

The mass was obtained from fits to the more restrictive set of events, as described above, with a total signal size of 195 ± 24, yielding $M_{\eta_c} = (2980.4 \pm 2.3)$ MeV, the uncertainty being statistical only.

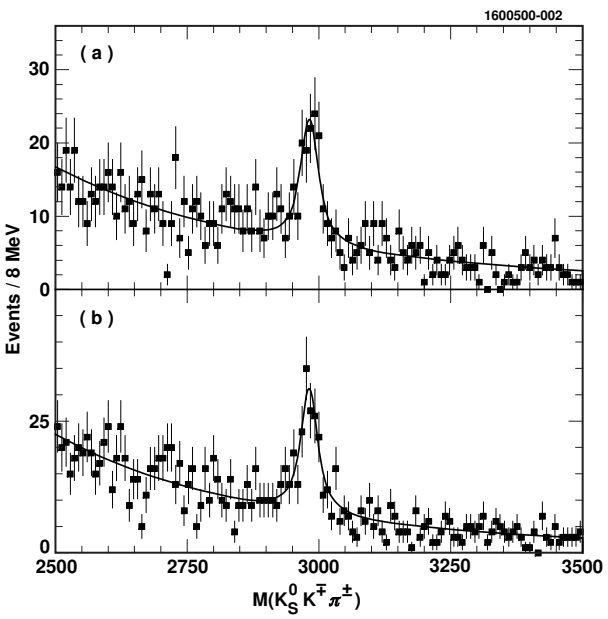

FIG. 1. Results of the simultaneous fit to η_c candidates in (a) CLEO II and (b) CLEO II.V for width and yield measurements, with a combined χ^2/d.o.f. = 226/243.

Possible sources of systematic uncertainty for the measured mass, full width, and two-photon partial width were studied. The results are summarized in Table I, in which the individual uncertainties are added in quadrature to obtain the total systematic uncertainty.

The mass calibration of our detector was checked by measuring the masses of the well known K^0_S, ϕ, D, and J/ψ mesons using decay modes involving only charged tracks. The measurements were found to be in good agreement with their respective world averages when we limited ourselves to events in which the charged tracks traversed all layers of the tracking volume. Our particle identification procedure did not introduce any significant systematic bias to the mass measurement. The measured mass was also insensitive to variations in the signal shape used to fit the η_c resonance.

The systematic uncertainties in the measurement of the full width were dominated by effects due to the mass resolution, the particle identification procedure, and the signal shape used in the fit. Our ability to predict the actual mass resolution was tested by studying the reconstructed D mesons in Monte Carlo simulation and data; the agreement was found to be better than 0.1 MeV.

The particle identification procedure was unable to distinguish between a charged kaon and a charged pion if the track momentum was above 0.8 GeV/c, for which the expected ionization losses are nearly equal for the two species. This led to a broadening of the reconstructed resonance and was taken into account by the wider Gaussian of the double Gaussian resolution function. This effect was limited to less than 5% of the events. We estimated the possible uncertainty due to this mis-assignment of particle species by completely removing the fraction of events having two possible η_c candidates and assigning the corresponding change in the measured width as the systematic error.

The accuracy of the fitting method in extracting the Breit-Wigner width of the resonance was checked by extracting the η_c widths from sets of simulation events generated with different intrinsic widths. We varied the parameters of the signal shape used to fit the η_c resonance within their uncertainties, derived from a comparison of the fit to D meson decays in Monte Carlo simulation and data, to estimate the effect on the measured width. The measured width from the more restrictive set of events used for mass measurement was within 0.2 MeV of the corresponding measurement using the full sample, and no significant correlation was found between the measured mass and full width.

There were several sources of uncertainty for the estimation of the efficiency, which in turn affected the measurement of $\Gamma_{\gamma\gamma}^{\eta_c}$. These were dominated by the uncertainties in the tracking and trigger efficiencies. The effect of a possible presence of resonant substructure ($K^*\overline{K}$) in η_c decay was studied and was found to give an insignificant variation in the detection efficiency. We estimated the uncertainty in the measured partial width from this effect by considering the possibility that all the η_c mesons decay through $K^*\overline{K}$. The selection requirements on total visible energy and unmatched energy clusters were shown by simulation to be essentially 100% efficient for our signal process and free of systematic bias. Possible bias from the transverse momentum requirement was investigated by changing the nature of the form factor in the simulation and shown to also be negligible.

In our analysis, we investigated the possible effects of interference between the $K_S^0 K^{\mp}\pi^{\pm}$ resonant and non-resonant final states. From the distribution of net transverse momentum for events in the sidebands of the signal lineshape, we estimated that one third of the background events were not of the type $\gamma\gamma \to K_S^0 K^{\mp}\pi^{\pm}$; these included events with at least one missing particle (π_0, γ) as well as events of the type $e^+e^- \to$ hadrons, $e^+e^- \to \tau^+\tau^-$, and $\gamma\gamma \to \tau^+\tau^-$. Such events cannot have interference with our signal $K_S^0 K^{\mp}\pi^{\pm}$ events. Study of the helicity angle distributions in the η_c rest frame indicated that the sideband events predominantly have $J = 2$ (or higher) while our signal has $J = 0$. Due to the preferential production of states with natural parity in two-photon un-tagged processes, a majority of the remaining background events were expected to have the natural spin-parity (0^+) compared to the unnatural spin-parity (0^-) of the signal events. The acceptance of our detector is symmetric in polar angle and uniform in azimuth, making the interference between these states of opposite parity vanish. We did not include any possible effects due to interference on the measured mass, full width and two-photon partial width. Further, the signal shape

showed no distortions and the goodness of fit to the hypothesis that ignored interference was very good, as shown in Fig. 1.

In summary, we have measured the mass, full width, and two-photon partial width of the η_c produced in two-photon collisions. The mass measurement of (2980.4 $\pm$ 2.3 (stat) $\pm$ 0.6 (sys)) MeV compares well with the world average [2] of (2979.8 $\pm$ 2.1) MeV. The measured total width of (27.0 $\pm$ 5.8 (stat) $\pm$ 1.4 (sys)) MeV disagrees with the world average [2] of $(13.2^{+3.8}_{-3.2})$ MeV, which consists of measurements with large relative uncertainties (40$-$100%). Our measured width is consistent with theoretical expectations [1,3]. The measured two-photon partial width of the η_c meson of (7.6 $\pm$ 0.8 (stat) $\pm$ 0.4 (sys)) keV agrees well with the world average [2] of $(7.5^{+1.6}_{-1.4})$ keV and theoretically expected values, and is a significant improvement in terms of experimental precision. We use the world average [2] of the $\eta_c \to K\overline{K}\pi$ branching fraction of $(5.5 \pm 1.7)\%$. The uncertainty in $\Gamma^{\eta_c}_{\gamma\gamma}$ due to the uncertainty in this branching fraction is $\pm$2.3 keV and is stated separately from the other contributions. From the ratio of our measured full width and two-photon partial width, we have extracted α_s at the charm mass scale to be 0.285 ± 0.025, for which we have added our sources of uncertainty in quadrature. We have used the NLO calculation in Eq. 2 to estimate α_s, thus making the result dependent on renormalization scheme and scale; we have not included such theoretical uncertainties in our quoted value.

Our measurements of $\Gamma^{\eta_c}_{\rm tot}$ and $\Gamma^{\eta_c}_{\gamma\gamma}$ show that PQCD calculations are able to reliably predict the ratios of the decay rates of a heavy quarkonium system, where non-perturbative effects cancel.

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Source of systematic uncertainty	M_{η_c} (MeV)	$\Gamma^{\eta_c}_{\rm tot}$ (MeV)	$\Gamma^{\eta_c}_{\gamma\gamma}$ (keV)
Mass calibration	0.6	< 0.1	< 0.1
Particle identification	< 0.1	1.3	0.1
Signal shape	< 0.1	0.3	0.1
Detector resolution	< 0.1	0.3	< 0.1
Trigger	-	-	0.2
Tracking	-	-	0.2
Resonant substructure	< 0.1	< 0.1	0.2
Luminosity	-	-	0.1
K^0_S selection	< 0.1	< 0.1	0.1
Event Selection	< 0.1	< 0.1	< 0.1
Total	0.6	1.4	0.4

TABLE I. Systematic uncertainties in the three measurements. The overall value is obtained by adding the individual contributions in quadrature.

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