# THE FEASIBILITY OF POLARIZED $e^+e^-$ COLLISIONS AT SLAC B-FACTORY

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## 1 Abstract

A possibility for both polarized –electron and positron collisions at SLAC *B*-factory considered. For polarized positron production, the circularly polarized photons, generated in a helical wiggler by 50 *GeV* beam, converted into $e^+e^-$ pairs in a thin target. Additional minor investments into hardware for spin conservation in colliding rings required. This yield new high quality experiments in *B*-physics possible.

## 2 Introduction

During my stay at SLAC in 1993 when I was involved in jobs associated with FFTB project, I was invited to make a seminar in one of SLAC's High-Energy Physics group. The talk was mostly about polarized positron production for the Future Linear Collider. It has also a part about implementation of such a system for B-factory, which was planned to build at SLAC. The amount of polarized positrons promised to satisfy requirements of B-factory. It also could be a test of conversion system in general. A group leader made a lot of detailed questions about conversion system. However, the conclusion of that seminar was negative in a sense, that B-Factory project has much more general uncertainties at the moment[2]. It also was mentioned that there are no strong recommendations from theorists about importance of polarization at all[3]. So the polarization might be only a far future option.

A time passed, SLAC B-factory running well, a HEP group leader - Prof. J. Dorfan became a SLAC's new director. Coming back to this idea from time to time I decided to recall this option from memory.

A system for polarized positron generation utilizes a 50 *GeV* Linac existence. In that sense it could not be realized somewhere else for the cost comparable with what may be for SLAC. This system uses a 50 *GeV* beam of electrons for generation of circularly polarized photons in a helical undulator. Further these photons are converted into electron-positron pairs in a thing[4] target. At the high-energy edge positrons are polarized longitudinally. Reasonable yield of positrons defined by complex trade between primary beam emittance and period of undulator. Minimal period itself defined by technology of undulator fabrication and strong relation between period and aperture, keeping the feeding current within realistic value. It is easy to build a small-aperture *helical* undulator with 2*mm* period and much harder to build an undulator with period of

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[2] At that time decision to build a B-factory at SLAC was not ready. There was continuing competition with Cornell.

[3] By the way, SLC successful operation obliged, mostly, to the only one of the beams polarized.

[4] In terms of radiation length.

a fraction of this value even with reduced aperture. For the emittance like SLAC has, the energy of 50 *GeV* is, possibly, the minimal one what could be used in this method.

Other competitive way to obtain polarized gammas–to convert a circularly polarized laser radiation on incoming electron beam (Compton back scattering) looks much more expensive. Natural polarization of electrons/positrons in a storage ring requires a long time and is unpractical here[5].

## 3 Importance of polarization

Sometimes it is not easy to prove that polarization is a crucial tool for high-energy physics. Theorists themselves are not educated well on the subject how much efforts accelerator physicists require to prepare collisions with defined helicity and do not developed much this subject. I think this is a key issue. Traditions move theorists to go on easiest way–averaging over initial states and summarizing over final states. The techniques for this is well developed. Indeed, analyzes of polarized states required much more efforts. From the other hand all intermediate states have definite polarization. To combine this state wanted, each particle inside the first bunch sees only half of the partners in the incoming bunch. So one can conclude that usually the yield of useful reactions is half of possible if both electrons and positrons are polarized. The last means that for a fixed time one can make twice more different experiments with the same statistics that without any polarization. With both polarized electrons and positrons it is possible to prepare the states with total spin equal to zero or unity. A lot of new experiments become easy distinguished. Drastic reduction of background is also obvious here. For the events with non-conservative parity the polarized beam is a crucial tool. From general consideration the cross-section with polarized particles could be written as [1]

$$\sigma( P_z^+ , P_z^- ) = \frac{4\pi\alpha^2}{3s} \cdot ( A_1 S_1 + A_2 S_2 ), \qquad (1)$$

$$S_1 = 1 + P_z^+ P_z^- \qquad\qquad S_2 = P_z^+ + P_z^- .$$

Specific values of $A_1$ and $A_2$ depend on the process[6]. One can see from (1) that existence of polarization gives additional information if both beams are polarized, even partially.

So if one smart mind will show us crucial use of polarization, we must be ready to say that we know how to do this.

## 4 Rate of positron production required

As we mentioned above, polarized electrons could be prepared with techniques existing at SLAC and could be injected into HER ring directly after acceleration in Linac. Existing storage ring with equilibrium spin orbit could be optionally used here to increase the longitudinal phase density of polarized electron beam. So this part is absolutely the same as polarized *electron* business for SLC. Polarized electron productivity is capable to satisfy the B-Factory. This could be easily seen from the SLC successful operation: SLC is a single impact machine meanwhile B-

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[5] A rate of self-polarization in a ring is $P(t) = \frac{8}{5\sqrt{3}}(1-\exp(-t/\tau_p))$ where the characteristic time of polarization is $\tau_p = 8|\rho|^3 \alpha/(5\sqrt{3} r_0^2 \gamma^5 c)$ [2]. Here $\rho -$ is a bending radius. Time of radiation damping is $\tau_{rad} = \frac{3}{2}\rho^2/(r_0\gamma^3 c)$. So, the ratio of these times is $\tau_p/\tau_{rad} = 16|\rho|\alpha/(15\sqrt{3} r_0 \gamma^2)$.

[6] In [1] was shown that for $\sqrt{s} = 500GeV$, for reaction $e^+e^- \to \mu^+\mu^-$: $A_1$=1.13, $A_2$=0.08; for $e^+e^- \to$ *hadrons*, $A_1$=6.45, $A_2$=3.03 —importance both polarized beams is obvious here.

Factory uses positrons during theirs lifetime. Meanwhile, how many positrons required? According to the latest figures the beam lifetime at the working current *J*=2.15*A* ($5.9{\cdot}10^{10}$ particles/bunch in 1658 bunches or $9.85\cdot10^{13}$ total) is something around 200 *min*. Let us suggest that the luminosity could drop 20% of it's peak value. Then, the conversion system must deliver $\sim10^{13}$ polarized positrons to the orbit (i.e. 10% of circulating) every 20 *min*. If the repetition rate of positron production system is 60*Hz*, then the number of the filling pulses here will be $60Hz\times20\times60\sec = 7.2\cdot10^4$. The last means that the number of polarized positrons generated by conversion system by a single pulse must be $10^{13}/7.2\cdot10^4 \cong 1.4\cdot10^8$. SLAC's beam as a primary one, has $N_{e-}\cong 5\cdot10^{10}$, so the efficiency must be not less than $\cong 1.4\cdot10^8/5\cdot10^{10}\cong 3\cdot10^{-3}$ or 0.3% only. We will see lower, that the system proposed, can deliver this efficiency.

Other important item –how to keep polarization in the rings unperturbed with longitudinally polarized collisions. Well-known techniques with Siberian snakes could be used here [3]. For elimination of coupling, introduced by solenoids, the *skew quadrupoles* could be used here. At CESR the skew quadrupole technique is in use for this purpose for years[7]. Each ring requires separate hardware for this purpose.

## 5 General picture

Polarized *positrons* generated during conversion of circularly polarized gammas in a thin target. General layout is represented in Fig.1. The gammas are generated by 50 *GeV* electron beam passing through *helical undulator*. Unpolarized electrons (usual gun) serve as primary ones.

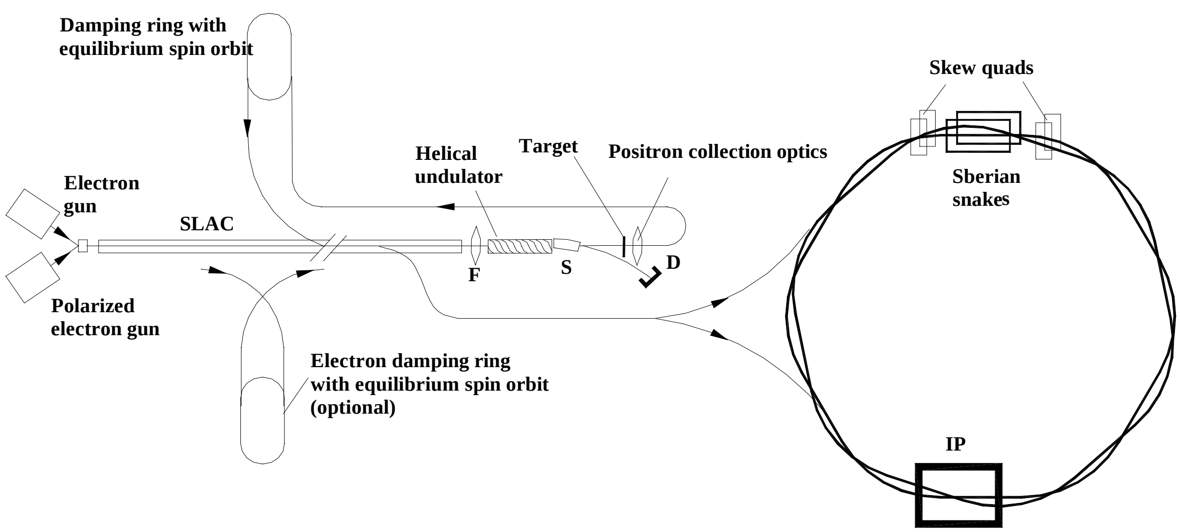


**FIGURE 1**: General layout of polarized positron production scheme. 50 *GeV* electron beam after passing helical undulator is dumped in a special device **D** similarly as it made for FFTB. **F** are the focusing elements for generation of crossover in undulator. **S** is a soft bend for 50 *GeV* beam to deflect it from target. Drawing is not in scale. Length of the helical undulator is 4 meters only. Devices for the spin conservation and handling are not shown, except Siberian snakes.

[7] Used for compensation of the coupling introduced by solenoid of detector.

As we just mentioned, the helical undulator serves as a source of circularly polarized photons. These photons must have as high energy as possible above the threshold of electron-positron pair creation. The scheme in Fig.1 is working as follows. Polarized electron gun generates electrons, which are going after acceleration directly into HER. For additional cooling a damping ring could be used here. For positron production an ordinary high current electron gun generates the bunch, which accelerated up to 50 GeV in SLAC. At the end of accelerator beam goes into 4-meter long helical undulator. A focusing system **F** in Fig.1 prepares the beta-function about 4-meter at the center of the undulator. Further, after a soft bend **S**, 50 *GeV* beam goes into a dumping device, **D** in Fig.1. Circularly polarized photons irradiate thin target, where they converted into $e^+e^-$ pairs. The particles collected at the high-energy part of the spectrum have longitudinal polarization. Collected particles go to the existing positron storage ring, where they are cooled. After accelerations into linac, they are going into LER.

## 6 First approximation

The energy of the quanta radiated by high-energy electron in an undulator in direction $\vec{n}$ defined by the formula

$$E_k^\gamma = \frac{k\hbar\Omega}{1-(\vec{n}\cdot\vec{\beta})} \cong \frac{k2\gamma^2\hbar\Omega}{1+K^2}\cdot\frac{1}{1+\gamma^2\vartheta^2/(1+K^2)} = \frac{E_{k\max}^\gamma}{1+\gamma^2\vartheta^2/(1+K^2)} \cong , \tag{2}$$

$$\cong \frac{k\cdot 2.48\cdot(\gamma/10^5)^2}{(1+K^2+\gamma^2\vartheta^2)\cdot\lambda_u[cm]}[MeV], \qquad \gamma\vartheta \le 1, \tag{3}$$

where $K = \beta_\perp\gamma = eH_\perp\lambda_u/2\pi mc^2 \cong 93.4\cdot H_\perp[Tesla]\lambda_u[m]$ –is a deflection parameter of undulator, $\Omega = 2\pi c/\lambda_u$, $\lambda_u$– is a period of undulator and $H_\perp$–is a transverse magnetic field, *k*–is a harmonic number, $\vartheta$ is the angle between direction of motion $\vec{\beta}/|\beta|$ and direction of radiation $\vec{n}$. Calculations made (see lower), allow to reach *K*=0.15, $\lambda_u = 2.4mm$ with the inner diameter of undulator chamber about 1*mm*. So, the maximal energy what could be expected here in forward direction, according to (2) is

$$E^\gamma{}_{\max} \cong \frac{2.48}{(1+0.0225)\cdot 0.24} \cong 10.1 MeV .$$

What the chamber diameter required to push the SLAC's bunch through the undulator? For *E*~50*GeV*, $N_{e-} \cong 5\times10^{10}$**,** the emittance of the beam is $\gamma\varepsilon \cong 3\times10^{-3} cm\cdot rad$**.** Let the center of the undulator installed at the beam crossover having the envelope function $\beta^*$. The envelope function behavior could be described by the formula

$$\beta(s) = \beta^* + \frac{z^2}{\beta^*} = \beta^*\cdot(1+\frac{z^2}{\beta^{*2}}), \tag{4}$$

where $z$ – is the distance calculated from the center of undulator in longitudinal direction. Suggesting, that the beam size at the out of the undulator changes only 10%, so $\rightarrow \frac{z^2}{\beta^{*2}} \approx 0.2$, one can obtain for the distance $z$=2$m$ $\rightarrow \beta^* \cong 450cm$. We suggested, that the total length of the undulator is $L = 4m$. Hence, the number of periods in an undulator is $M = L/\lambda_u$ = 4000/2.4=1667.

The beam size for these parameters will be $\sigma \cong \sqrt{\gamma\varepsilon\beta^*/\gamma} = 3.67\times10^{-3}cm$. Ten sigma saving margins yield $\cong 10\sigma \cong 0.37mm$ only. So the *diameter* of the vacuum chamber of the helical undulator could be around 0.8*mm*. Angular spread in the beam for the parameters under discussion $\vartheta_m \cong \sqrt{\gamma\varepsilon/\gamma\beta_u} = \sqrt{3\cdot10^{-3}/10^5/450} \cong 8.15\cdot10^{-6}$, so $\gamma\vartheta_m \cong 0.81$. Angular spread in the primary electron bunch does not affect much the angular distribution of radiated gammas. The size of the photon beam on the target will be $r \cong \sigma + \vartheta_m \cdot z_f$, where $z_f$ is the distance between the target and undulator, see Fig2. For a typical length $z_i \cong 4m$, $r \cong 3.7\cdot10^{-3} + 3.2\cdot10^{-3} \cong 7\cdot10^{-3}cm$ –what is negligibly small.

For the number of photons, which have energy in the interval $\{E^{\gamma}_{1\max}, s\cdot E^{\gamma}_{1\max}\}$, so $E^{\gamma}_1 = sE^{\gamma}_{1\max}$ or ($s = \omega/\omega_{\max}$) we have the formula [4]

$$\Delta N^{\gamma}_1/N_e \cong \int_s^1 \frac{dN^{\gamma}_1}{ds}ds \cong 4\pi\alpha M\frac{K^2}{1+K^2}\cdot\frac{1}{6}(1-s)\cdot(2-s+2s^2)-... \ . \tag{5}$$

So if we would like to collect positrons in the interval $7.5\pm2.5MeV$ generated by 10 *MeV* -photon beam, we may take $s \cong 0.5$ and the number of photons will be $N_{\gamma}/N_{e-} \cong 0.56$. Formally, the number of photons radiated by single particle is less, than one. As the frequency of radiation does not depend on magnetic field for small *K* at all, see (2), the intensity drops down if magnetic field decreasing. So that the energy, radiated during the pass is not enough to build up the energy of the quanta. This doesn't mean that the process of radiation is going in quantum regime. The measure of how close radiation to the quantum regime is $H_{\perp}/H_0 << 1$, where $H_0 = m^2c^3/e\hbar \cong 4.4\cdot10^{13}G$ is well satisfied here. Explanation of this is the following. As the length of formation of radiation is $L_f \cong \rho/\gamma$, (where $\rho = \lambda\!\!\!^{-}_u\gamma/K$ is a bending radius of the trajectory in magnetic field of undulator), becomes longer, than period of undulator for *K*<<1, $L_f = \lambda\!\!\!^{-}_u/K$, radiation is forming at few periods. With other words, for small *K* the deflection angle $\vartheta_d \cong eH_{\perp}\lambda_u/pc \cong K/\gamma$ becomes less than the angle of radiation, the last is $\vartheta \cong \sqrt{1+K^2}/\gamma \cong 1/\gamma$. In this case the details of magnetic field behavior on the length of formation are important, and radiation becomes proportional to the $\propto H^2_{\perp} \propto K^2$, (5). This is a direct manifestation of Landau-Pomeranchuk-Migdal effect. So each particle radiates one photon with probability about 56% with energy around 10*MeV* losing about 10*MeV* of energy with fractional energy loss about $2\cdot10^{-4}$ [8].

The number of positrons generated by these photons could be estimated as the following. Total cross-section per one atom is

$$\sigma_{tot} \cong \int_0^1 \frac{A}{N_0X_0}G(x)dx \cong \frac{7}{9}\frac{A}{N_0X_0}, \tag{6}$$

[8] So one can suggest a possibility for secondary *etc* usage of electron beam is series of undulators and further combining positrons in, say, longitudinal phase space.

where $N_0$ –is the Avohadro number, $A$–is atomic number and $X_0$–is a radiation length, $G(x) = x^2 + (1-x)^2 + \frac{2}{3}x\cdot(1-x) - \frac{1}{9}x\cdot(1-x)/Ln(183\cdot Z^{-1/3})$. The number of atoms $N_a$ in the volume $d \times 1cm^2$ is

$$N_a = N_0 \frac{g[g/cm^3]\cdot 1cm^2 \cdot d[cm]}{A[g]} \tag{7}$$

where $g$–is a specific weight of the target material, $d$–is the thickness of the target in $cm$. The number of positrons at the exit of the target is

$$N_+ \cong N_\gamma \sigma_{tot} N_a \cong \frac{7}{9} N_\gamma \frac{gd}{X_0} = \frac{7}{9} N_\gamma \delta\,, \tag{8}$$

where $\delta = \dfrac{gd}{X_0}$ –is the target thickness, measured as a fraction of the radiation length. Let $\delta \cong 0.28$, and suggesting that only $\eta \cong 1/5$ of all positrons carry the necessary level of polarization, one can obtain from (8), that $N_+/N_\gamma \approx \frac{7}{9}\cdot\frac{1}{5}\cdot 0.28 = 0.045$, or 4.5%. We suggested also that the phase volume of the positrons defined mostly by multi-scattering in a target, so the particles could be accepted by appropriate positron collecting system, much similar as SLC has now[9]. For obtaining closed formula for the number of positrons generated by a single bunch one can substitute (5) into (8)

$$\frac{N_+}{N_{e-}} \cong 4\pi\alpha M \cdot \frac{1}{6}\cdot(1-s)(2-s+2s^2)\frac{K^2}{1+K^2}\cdot\frac{7}{9}\delta\cdot\eta\,. \tag{9}$$

For parameters under discussion ($\delta \cong 0.28$, $\eta \cong 1/5$, $K$=0.15) total number of polarized positrons expected will be

$$N_+ \cong N_{e-}\frac{4}{6}\cdot\pi\alpha\cdot 1667\cdot\frac{0.022}{1.02}\cdot\frac{7}{9}\cdot 0.28\frac{1}{5} \cong 0.028\cdot N_{e-} \tag{10}$$

So, the resulting efficiency obtained as a result of these simple estimations is about 2.8%.

## 7 Second approximation

Formula (5) requires some comments to be given. It counts the photons radiated at first harmonic and having energy in interval $\{E^\gamma_{1\max}, s\cdot E^\gamma_{1\max}\}$. The photon radiated into angle $\vartheta_c \cong \sqrt{1+K^2}/\gamma \cong 1/\gamma$ has half of its maximal energy. If this photon creates a pair, the maximal possible energy of the positron in this act could be only half of the maximal possible $E^\gamma_{\max}$ also. This positron will carry polarization of the photon fully, but the last one is zero. Selection system, however, will transfer this positron, if it tuned to pass positrons in energy interval 50% from its maximal possible. As $\gamma\vartheta_m \cong 0.81$ for SLAC, there is no possibility to select photons at the entrance of the target by the angle, using a diaphragm of the small size target[10].

[9] The thickness of the target in our case is $0.1X_0$ instead of ~$5X_0$ for SLC, so the angular spread is much less here. Effective thickness of the target for the multi-scattering is about 1/3 $\delta$ for this case, so $<\vartheta^2> \cong 21/pc\cdot\sqrt{\delta/3}$

[10] This is possible, however, for NLC.

The real geometry of irradiation for the target needs to be taken into account also. This is represented in Fig. 2 below. Space between undulator and target required for soft bend of 50 *GeV* primary beam.

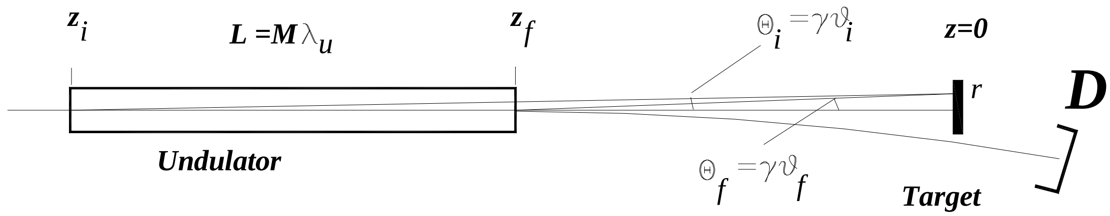


**FIGURE 2**: The target located at some distance $z_f$ from the end of the helical undulator. Smooth bend of 50 GeV beam shown as an arc trajectory to the beam dump D.

In the same geometry, the number of positrons could be calculated as a convolution of (11) with spectral density of positrons created in Tungsten target, function $G(x)$ in (6). The number of positrons per initial particle becomes [4]

$$\Delta N_1^+ \cong 3\cdot 10^{-2}\cdot \kappa^2 \cdot M\cdot \delta \frac{K^2}{1+K^2}\frac{z_f}{z_i}\cdot(1-\xi_1)\,, \qquad (11)$$

where $\kappa = 2K\sqrt{s(1-s)/(1+K^2)} \le 1$, $\xi_1 = \dfrac{E_1^+ - 2mc^2}{sE_{\max}^{\gamma} - 2mc^2}$, $E_1^+$ is the lower energy of positron which could be accepted by collection optics, $E_{\max}^{\gamma}$ is the maximal energy of the quanta on the first harmonics. $\xi_1$ – is the energy range of collected positrons ($\xi_1$=0 $\rightarrow$ *full* energy interval). $z_f$, $z_i$ are $z$ coordinates of the end and the beginning of the undulator respectively ($z_i > z_f$), see Fig.2. Basically $\kappa \cong \theta_s$, the fraction of $\gamma\vartheta$ (1/3 for $s$=0.9), $\delta$ – is thickness of the target defined earlier. Substitute in formula (12) $M$=1667, $\theta_s$=1/2, $\delta$=0.28, $K$=0.15, $\xi_1$=0.5, $z_f / z_i \cong 1/2$, one can obtain $\Delta N_+ \cong 3\cdot 10^{-2} 0.25\cdot 1667\cdot 0.28 \dfrac{0.0225}{1+0.0225}\cdot(1-0.5)\cdot 0.5$ =0.019, i.e. $N_+ / N_{e-} \cong 1.9\%$.

Numerical calculations were carried also to precise the angular density of positrons created as a function of the target thickness. The numerical code takes into account all details of interaction of gammas with matter [5]. For gammas with $E_\gamma = 10 MeV$ and with Tungsten target the energy of positrons collected was around $E_+ = 7.5 \pm 2.5 MeV$[11]. Result of calculations is represented in Table 1. Calculation show, that optimal thickness of the target is $\delta \approx 0.28$ in terms of radiation length, what is about 1*mm*. Angle of capture could be suggested $\vartheta_c \approx \pm 0.7 rad$.

One can see from Table 1, that starting from thickness 1*mm* the yield does not grow. For small thickness, angular spread is small, but the number of positrons is small also. Probably thickness somewhere in between $0.5mm < \delta X_0 < 1mm$ needs to be found experimentally, taking into account details of capturing optics. These calculations yield the positron production efficiency around $N_+ / N^\gamma \cong$ 1.1%. Resulting efficiency is about $N_+ / N_{e-} \cong 0.56 \times 1.1 \cong$ 0.6%.

[11] This interval could be narrowed in code also.

In Table 2 the row for $\vartheta_c$=0.7 is represented with values of average polarization in the positron beam. The number of the photons in this modeling was about ten thousand.

**TABLE 1**

| | | EFFICIENCY $N_+/N_\gamma$, % | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $\vartheta_c$, *rad* | $\vartheta_c$ | $\vartheta_c$, | $\vartheta_c$, | $\vartheta_c$ | $\vartheta_c$ | $\vartheta_c$ |
| Thickness *mm* | Thickness $1/X_0$ | 1 | 0.7 | 0.5 | 0.4 | 0.3 | 0.2 | 0.1 |
| 0.1 | 0.028 | 0.34 | 0.34 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 |
| 0.25 | 0.07 | 0.74 | 0.73 | 0.72 | 0.71 | 0.68 | 0.67 | 0.65 |
| 0.5 | 0.14 | 1.13 | 1.05 | 0.90 | 0.82 | 0.74 | 0.65 | 0.58 |
| 1.0 | 0.28 | 1.20 | 1.11 | 0.96 | 0.88 | 0.79 | 0.65 | 0.59 |
| 1.5 | 0.43 | 1.23 | 1.11 | 0.97 | 0.87 | 0.77 | 0.58 | 0.52 |
| 2.0 | 0.57 | 1.23 | 1.12 | 0.90 | 0.78 | 0.67 | 0.58 | 0.52 |
| 2.5 | 0.71 | 1.10 | 0.94 | 0.78 | 0.66 | 0.60 | 0.53 | 0.44 |

**TABLE 2**

| Thickness, mm | 0.1 | 0.25 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
|---|---|---|---|---|---|---|---|
| Thickness/$X_0$ | 0.028 | 0.07 | 0.14 | 0.28 | 0.43 | 0.57 | 0.71 |
| $N_+/N_\gamma$,% | 0.34 | 0.73 | 1.05 | 1.11 | 1.11 | 1.12 | 0.94 |
| Polarization,% | 78 | 79 | 79 | 79 | 79 | 78.6 | 79.8 |

Statistical uncertainty is about 10% in $N_+/N_\gamma$ numbers. Polarization was calculated, suggesting that incoming photon beam has 100% circular polarization. The only fractional energy of positrons with respect to the incoming photon energy matters here. If the energy interval becomes narrower, the polarization becomes higher on expense of the number of positrons.

The soft bend **S** in Fig.1 must deflect the 50 *GeV* beam so that it passes by the target at reasonable distance $r_{off}$. That defines the bending radius $\rho \cong z_f^2/r_{off}$. The characteristic energy of synchrotron quanta in this bend will be $\hbar\omega \cong \frac{3}{2}\hbar c\gamma^3/\rho = \frac{3}{2}\hbar c\gamma^3 r_{off}/z_f^2$. For $r_{off} \cong 5$ *mm*, $z_f \cong$ 4*m*, $\hbar\omega/e \cong 0.9$ *MeV*. This value is out of pair production range. The length of formation $L_f \cong \rho/\gamma \cong z_f^2/\gamma r_{off}$. The number of quants from the bend will be $N_{bend}^{\gamma} \cong \alpha z_f/L_f \cong \alpha\gamma r_{off}/z_f$, where $\alpha \cong 1/137$ is a fine structure constant. In our case $N_{bend}^{\gamma} \cong 0.7$, *i.e.* roughly 70% of each 50 *GeV* particle will radiate the 0.9 *MeV* quants. Total energy radiated, hence, will be $5\cdot 10^{10} \times 0.9\,MeV$, or $\sim 4.5\cdot 10^{-3}$ *J/pass/bunch*. So there is no problems here. In principle the distance between undulator and target could be slightly increased. The $r_{off}$ in its turn could be decreased a little.

So, the conclusion is that the efficiency about 0.6% could be expected here, while only 0.3% required. These figures give an idea about safety margins. With improving lifetime of the beams, these requirements could be relived proportionally. Repetition rate is other extensive factor for possible improvement. Other possibility- to use few electron-bunches as the primary ones. In addition, few targets could be used here.

## 8 Polarization

Polarization of the positron defined by well-known dependence on energy [7]. Closer to the top energy- higher the polarization. At the top the degree of polarization defined by the degree of polarization of the photon beam. The last is defined by averaging the polarization of individual photons. In its turn the polarization of the photon defined by its angle with respect to the direction of motion. As this angle also defines the energy of the quanta in terms of maximal possible energy (see (2), $\vartheta = 0$) $s = \omega / \omega_{\max}$

$$\xi_{21} = \xi_{22} = \frac{2s-1}{1-2s+2s^2} \quad . \tag{12}$$

That dependence is the same for the first and second harmonics. In is clear from (12) that under the angles $\sqrt{1+K^2}/\gamma$ the polarization becomes linear, $\xi_{11}, \xi_{12} \neq 0$. The photons has half of the maximal possible energy, *s* = ½ and circular polarization is zero $\xi_{21} = \xi_{22} = 0$. Polarization in the photon beam, averaged over al possible angles and/or energies for $K^2 \leq 1$ is defined by averaging (13) with the photon spectral density, $<\xi_{2k}>_s = \dfrac{\int_{s_t}^{1} \xi_{2k}(s) \frac{dN_k^{\gamma}}{ds} ds}{\int_{s_t}^{1} \frac{dN_k^{\gamma}}{ds} ds}$, what gives [5]

$$<\xi_{21}> = \frac{3s_t}{2 - s_t + 2s_t^2}\,, \quad <\xi_{22}> = \frac{5s_t}{1 + 2s_t - 2s_t^2 + 4s_t^3}\,. \tag{13}$$

Here $s_t = 0$ corresponds to the full energy interval, $s_t = 1$ corresponds straightforward direction. For example, $s_t = 0.8$ corresponds to the flux with photons starting from maximal and 20% down from the maximal possible energy of the quanta. Average polarization here will be 96%. Let us remind here formula (5) here, which gives $\Delta N_1^{\gamma} \cong 4\pi\alpha M \dfrac{K^2}{1+K^2} \cdot 0.083 - ...$ .

It is clear that the angular spread in the irradiating photon beam caused by angular spread in the electron beam $\vartheta_m \cong \sqrt{\gamma\varepsilon / \gamma\beta_u}$ , does not affect the average polarization at all, as the polarization of the positron defined by the individual act of interaction the photon with the nuclei. Only energy of this secondary particle defines its polarization [7],

$$\varsigma_2 \cong 1 - 2\left(\frac{sE_{\max}^{\gamma} - E_{+} - 2mc^2}{sE_{\max}^{\gamma} - 2mc^2}\right). \tag{14}$$

The interfering effect is radiation on the second harmonics. This is due to the mechanism of selection: quanta at second harmonic has twice much energy, and has a maximum in angular distribution around $\sqrt{1+K^2}/\gamma$, where it has the same energy as the quants at the first harmonics. As the second harmonics under this angle is linear, and as selection system is sensitive only to the energy of the secondary particle, this process reduces the average level of polarization [5]. But for small *K* the ratio is, [4]

$$\frac{\Delta N_2^{\gamma}}{\Delta N_1^{\gamma}} \cong \frac{6}{5}\frac{K^2}{(1+K^2)}\,. \tag{15}$$

In our case the last ratio is 2.6%. Depolarization effects in the target are small, as the depolarization length is $3X_0$ [7]. Numerical calculations [5] show that the mean path length even less than 1/3, reflecting the total tendency that the particles created at the outside of the target, have more probability to come out of the target. The mean polarization can reach 60-70% total as a multiplication of polarizations of photon flux and the bottom line from Table 2.

## 9 Undulator design

Helical undulator must satisfy the following: $L$=4$m$, $\lambda_u = 2.4mm$, $K$=0.15, diameter of aperture must be at least 0.8$mm$. Field in helical undulator can be represented as

$$\vec{H}_{\perp}(z) \cong \vec{e}_x H_{\perp}(x,y)\cdot Cos\frac{2\pi z}{\lambda_u} + \vec{e}_y H_{\perp}(x,y)\cdot Sin\frac{2\pi z}{\lambda_u}\,, \tag{16}$$

$x$, $y$ – are the transverse coordinates, $z$ is the longitudinal one, $\lambda_u$ –is the period of the undulator, $H_{\perp}$ is the transverse magnetic field amplitude, see (2), (2a).

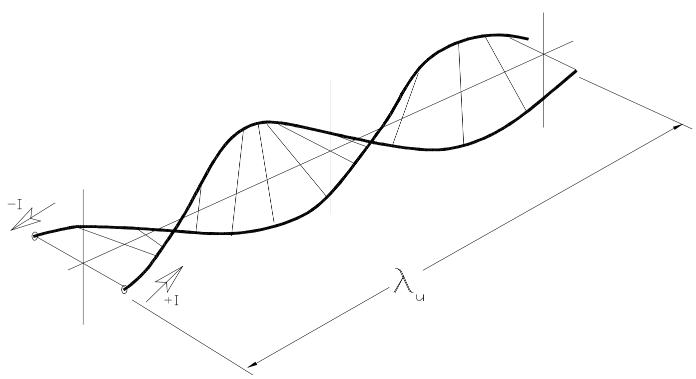


**FIGURE 3**: The concept of corkscrew wiring.

Helical undulator is a bifilar helix winding with currents opposed – "Corkscrew" [6]. The cross-section of undulator with superconducting wiring is represented in Fig. 4.

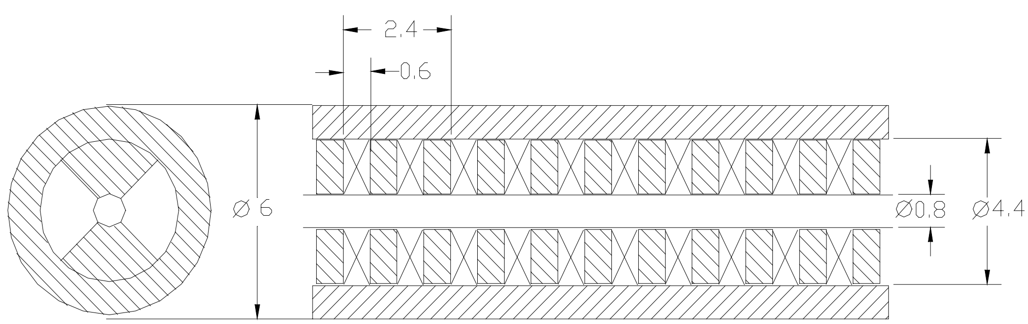


**FIGURE 4**: The cross-section of the undulator. Dimensions in *mm*.

Optimized parameters yield dimensions like in Fig. 4. The dependence on current is shown in Table 3. The cross-section of the window for the wire is $0.6\times1.8mm^2$. Current density is shown for the filling coefficient 65%, i.e. suggesting, that only 65% of this window carry the current.

**TABLE 3**

| Ampere-turns | K | J(65%) *A/mm*$^2$ | $H_\perp$, *G* |
|---|---|---|---|
| 50 | 0.017 | 71 | 770 |
| 100 | 0.0345 | 142 | 1541 |
| 150 | 0.052 | 213 | 2311 |
| 200 | 0.069 | 284 | 3082 |
| 250 | 0.086 | 356 | 3853 |
| 300 | 0.104 | 427 | 4623 |
| 350 | 0.12 | 498 | 5394 |
| 400 | 0.138 | 569 | 6165 |
| 450 | 0.1497 | 641 | 6678 |
| 500 | 0.152 | 712 | 6774 |
| 550 | 0.154 | 783 | 6870 |
| 600 | 0.156 | 854 | 6967 |
| 650 | 0.158 | 925 | 7063 |

At the current around 500 *A*, the iron comes to saturation.

I am not considering here other technical realization of undulators such as with permanent magnets and microwaves[12]. My opinion is that the undulator with superconducting wiring is the cheapest one.

## 10 Conclusion

Technical realization of the method of polarized positron production with an undulator looks feasible. This also can be considered as a test for the future NLC implementation. Some detailed work required for consideration of spin-orbit equilibrium trajectories in the rings and around. The conversion system could be tested stage-by stage. The cost of helical undulator fabrication is reasonable. Experience in short period undulator fabrication is well established[13]. With both polarized electron-positron beams, the collisions at SLAC B-Factory may give additional information about parity non-conserving processes.

I would like to mention here, that at Cornell the plan exists to build a dual bore machine [8]. With CESR as a booster, this brings a possibility to come close to $10^{34}$-$10^{35}$ limit in luminosity scale. Moreover, utilization CESR as a booster makes possible arrangements for both (longitudinally) polarized collisions here also. The method is the following[14]: the beam stacked into the booster and then ramped up to 8 *GeV*[15]. The damping time goes down from 10*ms* at 5 *GeV* to $\tau_{rad} \cong 10ms\cdot(\frac{5}{8})^3 = 2.44\,ms$. Time of self-polarization respectively, goes to $\tau_p = \tau_{rad}16|\rho|\alpha/(15\sqrt{3}r_0\gamma^2) \cong 21min$. After holding at high energy about $2\tau_p$, energy goes down to the working one. Degree of polarization could reach 79.8%. The crossing of spin resonances in the booster arranged with the same well-known technique [3]. Further, the beam, extracted from the booster, goes to the dual bore machine at the energy of experiment. In this new dual bore machine the optics arranged so that polarization at IP is longitudinal.

[12] In that sense an incoming laser radiation is similar to this.
[13] See references in [5].
[14] Detailed description of this method will be given in other place.
[15] CESR was designed for 8 *GeV* long-term operation.

So the continuing scientific competition between Laboratories at SLAC and CESR can bring the accelerator physics to the absolutely new level. Winner is obvious-a High-Energy Physics community.

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