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COHERENT SYNCHROTRON RADIATION IN MULTIPLY-CONNECTED VACUUM CHAMBER

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1 Abstract

Coherent synchrotron radiation (CSR) in a singly-connected vacuum chamber is, as it is well known, strongly suppressed for harmonics $n_{cr} \le (R/h)^{3/2}$, where R is the radius of curvature and *h* is characteristic transverse dimension. If, however, the vacuum chamber has a multiply-connected topology, the critical frequency is zero and this restriction is not valid anymore. Estimation for radiated power in a multiply-connected chamber is presented here including CSR in electrostatic separator and at the LHC collider.

2 Introduction

Effect of the coherent synchrotron radiation (CSR) has been discussed in number of publications, for reference see [1]. For a bunch moving along trajectory with bending radius *R*, the CSR in free space takes place for frequencies $\omega\sigma/c<1$, where σ is rms bunch length. Energy loss in this case can be described in terms of the effective longitudinal impedance $Z(\omega)$ which can be obtained directly from the power spectrum of the single particle synchrotron radiation [2], [3]:

$$\mathrm{Re}[Z(\omega)] \cong 1.6 \cdot Z_0 \cdot (kR)^{1/3}, \tag{1}$$

where $k=\omega/c$, and $Z_0 = 120\pi$ Ohms. This result is valid for frequencies $\omega R/c << \gamma^3$, where γ is relativistic parameter. The last condition is, practically, always satisfied for frequencies, which may contribute to coherent effect.
Convoluted with the bunch spectrum, $\sim \exp(-k^2\sigma^2)$, impedance Eq. (1) gives energy loss $U = 2N_b^2 e^2 \kappa$ per turn and per bunch, where N_b is bunch population, and κ is loss factor,

$$\kappa = \int_0^\infty \frac{d\omega}{2\pi} \mathrm{Re}[Z(\omega)] \cdot e^{-k^2\sigma^2} = \frac{3.2}{\sigma} \cdot \Gamma(2/3) \cdot (R/\sigma)^{1/3}. \tag{2}$$

Note $\Gamma(2/3) = 1.354$. Radiated power, $P = U \cdot f_{ref}$, is given in terms of average bunch current I_b:

$$P = \tfrac{1}{2} Z_0 \cdot I_b^2 \kappa R . \tag{3}$$

This result can be obtained directly from the spectrum of synchrotron radiation in magnetic field H [4], $\gamma mc^2 = eHR$,

$$P(\omega) \cong 0.52 \frac{r_0^2 c}{\gamma^2} H^2 n^{1/3} , \tag{4}$$

where $n = kR$ is harmonic number. Integration over n with the bunch spectrum $e^{-n(\sigma / R)^2}$ gives results of Eqs. (2) and (3) for the power radiated by a bunch.

The loss factor Eq. (2) is large. Situation is, however, different for a particle radiating in a beam pipe where the screening effect strongly suppresses the CSR [1]. Due to boundary condition, only waves with the transverse propagating constant $k_\perp \cong k\theta > \pi / h$ can propagate in a beam pipe with the height h. A wave with frequency ω is radiated in the angle $\theta \cong (kR)^{-1/3}$ it is easy to see [2] that boundary conditions can be satisfied only for the waves with frequencies $kR > (Rh)^{3/2}$. Together with the condition $k\sigma < 1$, this imposes very tight limit on the bunch length, $\sigma < h\sqrt{h / R}$.

This may be not true in multiply-connected vacuum chambers, where the cut-off frequency is zero, and propagating modes exist for arbitrary frequency. This eliminates restriction arising on the harmonic number $n_{cr} \le (R / h)^{3/2}$, and CSR for a particle on a trajectory with finite curvature within the multiply-connected vacuum chamber may exist without screening effect.

The strip-lines is an example of the vacuum chamber with the multiply-connected topology. The TEM propagating in a strip-lines of finite length have the longitudinal electric field only at the ends of a strip-line. They exponentially decay outside of the strip-line in the regular part of the beam pipe. A bunch with transverse velocity can excite these modes only at the ends of the strip-line. The CSR provides additional mechanism of the energy loss for a particle on a trajectory of finite curvature within the multiple connected wave guide. This effect is different from excitation of the TEM wave due to non-zero transverse component of particle velocity. Indeed, the CSR disappears for a straight bun tilted trajectory while the excitation by the transverse velocity does not vanish.

A beam separator at Cornell CESR is an example of such a case. We will discuss this and then consider some other applications.

3 Beam separators

The CESR machine at Cornell utilizes electrostatic separators which arrange pretzel-like electron and positron orbits [5]. Basically, the separator is a pair of strip-line electrodes with characteristic impedance Z_s and applied high voltage. The electrodes are terminated by some impedance Z_m at their ends. The kick angle α defines the effective radius of curvature $R \cong L / \alpha$, where L is the length of electrodes along the beam trajectory. The impedance of the strip-line is

$$Z(\omega) = \frac{\Delta\phi}{2\pi} \cdot \frac{1 - e^{-i2kL}}{Z_m^{-1} + Z_s^{-1}}, \tag{5}$$

where $\Delta\phi$ is effective azimuthal width of the strip-line. Z_s has only active part, while Z_m must be reactive to allow high voltage application. However, $Z_m \cong Z_s$ for the frequencies of interest $\sim c/\sigma$. The loss-factor under this assumption can be evaluated as

$$\kappa = \frac{2\sqrt{\pi}}{\sigma} \frac{Z_s}{Z_0} (\frac{\Delta\phi}{2\pi})^2. \tag{6}$$

The power losses could be calculated as

$$P = \tfrac{1}{2} Z_s \cdot I_b^2 \kappa R + \frac{\Delta\phi}{2\pi} I_b \cdot V \frac{\Delta r}{d} \ , \tag{7}$$

where V is a voltage applied to the electrodes, d is the distance between electrodes and Δr –is transverse offset of particle trajectory from the axis of symmetry. The first term describes losses due to the geometric beam coupling impedance. The second term proportional to the electric field V/d is caused by the interference between field generated in the chamber by high voltage electrodes and the field induced by the beam. It is assumed here that voltage V does not produce the longitudinal electrical field on the axis of symmetry and, therefore, power is proportional to Δr. For an estimate of the first term let us take $(\Delta\phi/2\pi) \cong 1/2$, and for CESR parameters $\sigma \cong 1.5$ cm, I_b =5 mA, $2\pi R = 768.4$ m, $Z_s \cong 50$ Ohms. Then $\kappa = 0.07$V/pC, and power Eq. (3) is $P \cong 5$W per bunch.

This estimate does not take into account coherent radiation of a deflected bunch. The impedance of the CSR can be obtained from the energy $U = 2N_b^2 e^2 \kappa$ radiated by a bunch in one turn on the circular trajectory with radius R written as

$$U = 0.26 \cdot \frac{e^2}{\sigma} \frac{L_{tr}}{L_{rad}} \cdot \Gamma(2/3), \tag{8}$$

where $L_{tr} = 2\pi R$ is the length of the trajectory and $L_{rad} = \sigma \cdot (R/\sigma)^{2/3}$ is radiation length. The radiation length is estimated here as $L_{rad} \cong Rn^{-1/3}$, where the mode numbers $n = kR \cong R/\sigma$ for the frequencies $\omega \cong c/\sigma$ giving the main contribution to the power of CSR.

The energy radiated in a strip-line with the length L can be obtained from Eq. (8) replacing L_{tr} by L and L_{rad} by

$$L_{rad} = \sigma \cdot (\frac{L}{\alpha\sigma})^{3/2}, \tag{9}$$

where $\alpha \cong L/R$ is a deflecting angle. This defines the loss factor

$$\kappa = \frac{0.26}{\sigma} \cdot (\frac{L\alpha^2}{\sigma})^{1/3} \cdot \Gamma(2/3) \ , \tag{10}$$

which corresponds to the impedance

$$\mathrm{Re}[Z(\omega)] = 0.8(\frac{Z_0}{2\pi})\alpha^{2/3}(kL)^{1/3}. \quad (11)$$

The radiated power is $P = N_b^2 e^2 \kappa/(L/c)$, or

$$P = 0.26\cdot\Gamma(2/3)\cdot cN_B^2\cdot e^2(\frac{\alpha}{L\sigma^2})^{2/3}. \quad (12)$$

The loss factor Eq. (10) and power Eq. (12) increases rapidly for short bunches. For CESR parameters, $P \cong 413$ W. The average power per turn <*P*> is smaller by the factor $L/(2\pi R)$,

$$< P >= Z_0\cdot I_B^2\cdot(\frac{\kappa R}{2}),\ \kappa = \frac{\Gamma(2/3)}{2\sigma}\cdot(\frac{L\alpha^2}{\sigma})^{1/3}. \quad (13)$$

For CESR, <*P*> =0.8 W/turn/bunch.

Here we assumed that the strip lines are longer, than radiation length of coherent radiation Eq. (9), $L\alpha^2/\sigma > 1$. Otherwise, L_{rad} has to be replaced by *L*. The main contribution to power is given by modes with frequencies $k \approx 1/\sigma$. Such modes are radiated in the angle $\theta \cong (kR)^{-1/3} \cong [L/(\sigma\alpha)]^{-1/3}$. For CESR parameters, this angle is large, $\theta \cong 2.4$ and radiation power produces mostly local heating.

4 Implications for other machines

The NLC detuned-damped linac structure (DDT) has damping manifold. In smooth approximation, the curvature of the trajectory is a result of the betatron oscillations. Deflection angle in this case is $\alpha \cong \sigma_\perp/\beta_\perp$ and $L = \beta_\perp/\pi$. Here $\sigma_\perp$ and $\beta_\perp$ are transverse rms and beta-function, respectively. They are related to transverse emittance $\varepsilon_\perp = \sigma_\perp^2/\beta_\perp$. More accurately, particles move on a straight trajectory between quadrupoles, which bend trajectory proportionally to their transverse displacement. This produces the same result for α. Radiation from quadrupoles may interfere with the accelerating field of the structure and penetrate the manifold. This may lead to a small additional beam loading and, what may be more important, affect the HOM measurements and HOM alignment of the structure. Effects, however, are small. The loss factor per unit length is

$$\frac{\kappa}{L} = \frac{0.5\cdot\Gamma(2/3)}{\sigma}\cdot\frac{\pi}{\beta_\perp}\cdot(\frac{\varepsilon_\perp}{\pi\sigma})^{1/3}. \quad (14)$$

For NLC parameters, $\sigma = 100\mu$ m, $\beta_\perp = 50$ m, $\varepsilon_\perp$=2pm, the loss factor is $\kappa/L = 2$ mV/pC/m only

Holes in perforated beam pipe make the beam pipe in LHC multiply-connected. Calculations of the impedances of the perforated beam chamber, carried out recently in [5] do not take into account the curvature of the trajectory.

The loss factor due to CSR for a bunch on the trajectory with curvature 1/*R* is

Given by Eq. (2). The power loss per unit length is $dP/ds = Z_0 \cdot I_B^2 \cdot \kappa/(4\pi)$. For LHC parameters, $2\pi R = 26.6$ km, bunch spacing s_B=7.5m and beam current I_{beam}=0.54A, the power deposited by the beam *dP*/*ds*=5.1W/m. However, only small part of this power is radiated through the perforated wall and can propagate in doubly-connected region arranged by perforated liner and cold inner tube. The attenuation coefficient in a thin liner is of the order of $a^6/(b^2\sigma^4)$ per hole where *a* and *b* are the hole and inner chamber radii respectively. This fraction of radiation includes low harmonics, which can not propagate within the perforated walls. The fraction is of the order 10^{-7} for the hole radius $a \cong 1$ mm and beam pipe radius $b \cong 1.75$, $\sigma = 1.75$ cm. Estimating the number of the holes per one meter as $N_{holes} \sim 10^3$ one can conclude that resulting power could reach $5 \cdot 10^{-4}$ W/m. All of the rest power, which would be radiated in a free space propagates with the bunch and does not contribute to losses.

Perforated wall can change $\omega(q)$ dependence, where q is propagating constant. That may lead to additional propagating modes. The surface impedance of the perforated wall [6] is $\varsigma = -ik \cdot (\alpha_e + \alpha_m) \cdot n_h$, where $k = \omega / c$, $\alpha_{e,m}$ are electric and magnetic polarizabilities of a hole and n_h is number of the holes per unit surface area. For azimuthally symmetric modes propagating in the beam pipe with radius *b*, the boundary condition $E_t = \varsigma H_t$ defines the q by dispersion equation $q^2 = k^2 - \kappa^2$. The longitudinal component of electric field is not zero if κ is a solution of $J_0(\kappa b) = i\varsigma(k/\kappa)J_1(\kappa b)$, where $J_{0,1}$ –are Bessel functions. For the perfectly conducting wall, $\varsigma \to 0$, the first solution corresponds to TM waves and cut-off frequency $k_c = \nu_0 / b$, $\nu_0 \cong 2.4$. The finite ς lowers the cut-off frequency to $k = k_c[1-(\alpha_e + \alpha_m)n_h / b]$. The shift $(\alpha_e + \alpha_m)n_h / b < a/b$ is too small to allow a new propagating mode at low frequencies $k \cong 1/\sigma << k_c$ and does not affect the estimate of the CSR given above.

5 Conclusion

The usual definition of the impedance implies particles going along a straight trajectory parallel to the beam pipe axes with, generally, constant nonzero offset. If the trajectory has a substantial bend due to the leading magnetic field or due to deflecting components of the ring (separators, kickers, etc.), there is additional radiation and, if the radiation is coherent, the additional impedance. Intensity of the synchrotron radiation of a single particle can be easily rewritten to get the intensity of single-particle radiation for an arbitrary cause of deflection on the deflection angle $\alpha >> \gamma$. For the multiply-connected beam pipe there is no screening effect of finite beam pipe aperture and radiation at low frequencies $k\sigma < 1$ is coherent. The estimate of the broadband impedance due to this effect in CESR shows that effect may cause some additional heating in the CESR beam separators. Related effects considered for the LHC and NLC are small.

Effect of CSR in multiply-connected structures could be included in the well known numerical codes such as MAFIA and GdfidL [7] if the computing engine there were modified to allow bunch motion along curved trajectory.

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6 References

[1] R.L. Warnock and P.Morton, Part. Accel., 25, 113-151, 1990.
R.L. Warnock, SLAC_PAB-5523, 1991.
L.B. Murphy, S. Krinsky, R.L. Glukstern, *Longitudinal effects for Synchrotron Radiation*, PAC95.
Y. Derbenev, J. Rossbach, E.Saldin, V. Shiltsev, *Radiative Microbanch Tail-Head Interaction*, Preprint TESLA-FFI, 95-05, 1995.

[2] S. Heifets, A. Mikhailichenko, *On the Impedance due to Synchrotron Radiation*, SLAC/AP-83, 12/1990.

[3] S. Heifets, M. Zolotorev, *Bunch Heating by Coherent Synchrotron Radiation*, PAC98.

[4] L.D. Landau, E.M. Lifshitz, *The Classical Theory of Fields*, Pergamon Press.

[5] R. Littauer, *Multibunch Operations of CESR*, IEEE NS-1610 (1985).

[6] S. Petracca, *Coupling Impedance for Perforated Beam Pipes with General Shape from Impedance Boundary Condition*, CERN-SL-99-003 AP.

[7] The MAFIA collaboration, *User's guide MAFIA Version 4*, CST GmbH, Lauteschlagerstr. 8, D-64289 Darmstadt, Germany.
W. Bruns, *GdfidL: A Finite difference program for arbitrary small perturbations in rectangular geometries*, IEEE Trans. Magn. Vol. 32, No 3 May 1996.