CLNS 99/1632

Rescattering and Electroweak Penguin Effects in Determinations of the Weak Phase γ *

Dan Pirjol†

Floyd R. Newman Laboratory for Nuclear Studies, Cornell University, Ithaca, New York 14853

(August 13, 1999)

Abstract

Determinations of the CKM phase γ from weak nonleptonic B decays are affected by electroweak (EW) penguins and rescattering effects. In this talk I explain how the EW penguin effects can be controlled with the help of SU(3) symmetry, by relating them to tree-level amplitudes. The impact of the final-state interactions on the determination of γ from $B^+ \to K\pi$ decays is studied numerically, showing that they can be important. A few alternative methods are discussed which use additional decays to eliminate their effects.

I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix is one of the most important free parameters of the Standard Model, encoding CP violation as complex phases in some of its matrix elements. These effects manifest themselves only in the couplings of the third generation, which makes the weak decays of bottom flavored hadrons an ideal setting for their study. Preliminary results from CLEO and CDF are already providing tantalizing evidence for nontrivial phases in some of the CKM matrix elements, and more precise determinations are expected soon from the B factories presently being commisioned. Ultimately, studies of weak B decays will help test the Standard Model mechanism of CP violation and explore the possible existence of new physics.

For our purposes, the following approximate form of the CKM matrix given by Wolfenstein will be sufficient

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \tag{1}$$

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*Invited talk given at the Beauty '99 conference, June 21-25, Bled, Slovenia.

†pirjol@mail.lns.cornell.edu

In this parametrization the CP-violating phases are restricted to two matrix elements $V_{ub} = A\lambda^3 R_b e^{-i\gamma}$ and $V_{td} = A\lambda^3 R_t e^{-i\beta}$, where we defined $R_b = \sqrt{\rho^2+\eta^2}$ and $R_t = \sqrt{(1-\rho)^2+\eta^2}$. Finally, a third weak phase α is defined by $\alpha+\beta+\gamma=\pi$. The three weak phases are identical with the angles of the unitarity triangle following from the unitarity condition of the CKM matrix

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\,. \tag{2}$$

Numerically, the best known parameters are $\lambda \simeq |V_{us}| = 0.2196 \pm 0.0023$ and $A \equiv |V_{cb}|/\lambda^2 = 0.819 \pm 0.058$ [2]. The remaining parameters have been estimated from global fits of the unitarity triangle as $R_b = |V_{ub}/V_{cb}|/\lambda = 0.41 \pm 0.07$ and $R_t = |V_{td}/V_{cb}|/\lambda = 1.01 \pm 0.21$ [3]. Although knowledge of the sides of the triangle (2) is sufficient to determine its angles too, one would like to measure the latter directly, which would provide a consistency test of the whole picture.

Several methods have been proposed for determining the weak phases from B decay data, the most popular of which can be divided into two large classes: a) methods using mixing-induced CP violation in neutral (B_d or B_s) decays to CP eigenstates and b) methods using time-independent charged and/or neutral B decay rates (for a discussion of other methods see the contribution by R. Fleischer in these proceedings). The best known methods of type a) include the determination of the weak phase α from $B^0(t) \to \pi^+\pi^-$ decays and of the phase β from $B^0(t) \to J/\psi K_S$ [1]. Such methods are more demanding from a practical point of view, as they require time-dependent measurements of the CP asymmetry.

The second class of methods employs the approximate flavour SU(3) symmetry of the strong interactions [4–6]. The basic idea is that any B decay amplitude is given by a linear combination of (unknown) strong matrix amplitudes T_j times CKM factors ξ_j as $A = \sum_j \xi_j T_j$. The strong amplitudes T_j corresponding to different decays are related by SU(3) symmetry, such that one aims to eliminate them completely by combining sufficiently many physical decay amplitudes, in order to determine the ξ_j factors. A particularly elegant version of this approach is formulated in a graphical language, wherein the weak phases appear as angles in diagrams constructed from physical decay amplitudes [5,7,8]. While simple and attractive from an experimental point of view, this type of methods are fraught with theoretical uncertainties such as SU(3) breaking effects, final state interactions and electroweak penguin effects. We will discuss these issues at length in the following sections.

II. SU(3) FLAVOR SYMMETRY AND NONLEPTONIC B DECAYS

The flavour symmetry of the strong interactions plays an useful role in organizing the structure of weak decay amplitudes of B mesons into two pseudoscalars. The effective weak nonleptonic Hamiltonian responsible for these decays is given by

$$\begin{aligned}\mathcal{H} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \Bigg(&\sum_{q'=u,c} V_{q'b}^* V_{q'q} [c_1 (\bar{b}q')_{V-A} (\bar{q}'q)_{V-A} + c_2 (\bar{b}q)_{V-A} (\bar{q}'q')_{V-A}] \\ &- V_{tb}^* V_{tq} \sum_{i=3}^{10} c_i Q_i^{(q)} \Bigg)\,,\end{aligned} \tag{3}$$

where the eight operators $Q_i^{(q)}$ include four QCD penguin-type and four EW penguin-type operators

$$Q_3^{(q)} = (\bar{b}q)_{V-A} \sum_{q'=u,d,s,c} (\bar{q}'q')_{V-A}\,, \quad Q_4^{(q)} = (\bar{b}_i q_j)_{V-A} \sum_{q'=u,d,s,c} (\bar{q}'_j q'_i)_{V-A} \tag{4}$$

$$Q_5^{(q)} = (\bar{b}q)_{V-A} \sum_{q'=u,d,s,c} (\bar{q}'q')_{V+A}\,, \quad Q_6^{(q)} = (\bar{b}_i q_j)_{V-A} \sum_{q'=u,d,s,c} (\bar{q}'_j q'_i)_{V+A} \tag{5}$$

and

$$Q_7^{(q)} = \frac{3}{2}(\bar{b}q)_{V-A} \sum_{q'} e_{q'} (\bar{q}'q')_{V+A}\,, \quad Q_8^{(q)} = \frac{3}{2}(\bar{b}_i q_j)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_j q'_i)_{V+A} \tag{6}$$

$$Q_9^{(q)} = \frac{3}{2}(\bar{b}q)_{V-A} \sum_{q'} e_{q'} (\bar{q}'q')_{V-A}\,, \quad Q_{10}^{(q)} = \frac{3}{2}(\bar{b}_i q_j)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_j q'_i)_{V-A}\,. \tag{7}$$

Each term in the Hamiltonian (3) contains a product $\bar{q}\bar{q}q$ which transforms as $\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} = \overline{\mathbf{15}} \oplus \mathbf{6} \oplus \bar{\mathbf{3}} \oplus \bar{\mathbf{3}}$ under flavour SU(3). When expressed in terms of well-defined SU(3) transformation properties, the tree part of the Hamiltonian (3) reads (without a factor of $G_F/\sqrt{2}$)

$$\begin{aligned}\mathcal{H}_T &= \lambda_u^{(s)}[\frac{1}{2}(c_1 - c_2)(-\bar{\mathbf{3}}^{(a)}_{I=0} - \mathbf{6}_{I=1}) + \frac{1}{2}(c_1 + c_2)(-\overline{\mathbf{15}}_{I=1} - \frac{1}{\sqrt{2}}\overline{\mathbf{15}}_{I=0} + \frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0})] \\ &+ \lambda_u^{(d)}[\frac{1}{2}(c_1 - c_2)(\mathbf{6}_{I=\frac{1}{2}} - \bar{\mathbf{3}}^{(a)}_{I=\frac{1}{2}}) + \frac{1}{2}(c_1 + c_2)(-\frac{2}{\sqrt{3}}\overline{\mathbf{15}}_{I=\frac{3}{2}} - \frac{1}{\sqrt{6}}\overline{\mathbf{15}}_{I=\frac{1}{2}} + \frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=\frac{1}{2}})]\,.\end{aligned} \tag{8}$$

We denoted here the combinations of CKM factors $\lambda_{q'}^{(q)} = V^*_{q'b}V_{q'q}$. There are two $\bar{\mathbf{3}}$ operators, which were chosen to be symmetric, respectively antisymmetric under permutations of the two q fields in $qq\bar{q}$. The explicit form of the operators in (8) can be found in [9].

The final state in the decay consists of two octet pseudoscalar Goldstone bosons. Bose symmetry constrains its flavour wave function to be symmetric, which allows only certain representations $[\mathbf{8} \otimes \mathbf{8}]_S = \mathbf{27} \oplus \mathbf{8} \oplus \mathbf{1}$. The most general form of the decay matrix element induced by the Hamiltonian (8) is given by the Wigner-Eckart theorem, which is written in tensor language as (omitting the CKM factors)

$$\begin{aligned}\mathcal{H} &= \langle \mathbf{27} \| \overline{\mathbf{15}} \| \mathbf{3} \rangle \bar{M}^{i_1 i_2}_{j_1 j_2} H^{j_1}_{i_1 i_2} B^{j_2} + \langle \mathbf{8} \| \overline{\mathbf{15}} \| \mathbf{3} \rangle \bar{M}^{i_1}_{j_1} H^{j_1}_{i_1 i_2} B^{i_2} \\ &+ \langle \mathbf{8} \| \mathbf{6} \| \mathbf{3} \rangle \epsilon_{abc} \bar{M}^a_i H^{ib} B^c + \langle \mathbf{8} \| \bar{\mathbf{3}}^{(a)} \| \mathbf{3} \rangle \bar{M}^i_j H_i B^j + \langle \mathbf{1} \| \bar{\mathbf{3}}^{(a)} \| \mathbf{3} \rangle \bar{M} H_i B^j\,.\end{aligned} \tag{9}$$

We denoted here with M the possible tensors which can be formed from the usual matrix of octet pseudoscalar $P^i_j = 1/\sqrt{2}\pi^a\lambda^a$, corresponding to the mentioned symmetric representations of SU(3), and with H tensors appearing in the SU(3) decomposition of the weak Hamiltonian. The expansion of (9) gives any B decay amplitude into two pseudoscalars as a linear combinations of reduced SU(3) matrix elements. The results are tabulated in an easy to use form in the Appendix of [6].

There exists an equivalent description of SU(3) amplitudes in terms of quark diagrams [5], wherein a decay amplitude is decomposed into contributions which can be associated with certain quark topologies. There are six graphical amplitudes, denoted with T (tree), C (color-suppressed), A (annihilation), E (W-exchange), P (penguin) and PA (penguin annihilation). Factorization approximation combined with quark models for the form-factors can be used to determine these graphical amplitudes (see, e.g. [10–12]). In this way a hierarchy emerges, according to which the dominant amplitude is T, followed by C which is smaller by a factor $a_2/a_1 \simeq 0.2$. The annihilation-type amplitudes A and E are predicted to be further suppressed by a factor $f_B/m_B \simeq 0.05$ relative to T (they can be however enhanced by rescattering effects [18,19,17,20,22,21,24,33,40]). The QCD penguin amplitude P contributes to $\Delta S = 0$ decays at the same order as C, and the PA amplitude is suppressed relative to it as in the case of A and E. This additional dynamical information makes the graphical method more predictive than the group-theoretical approach discussed above.

We quote for later use the $B^+ \to K\pi$ decay amplitudes in quark diagram language.

$$\begin{aligned} &A(B^+ \to K^0\pi^+) = \qquad\qquad (10)\\ &\quad \lambda_u^{(s)}(P_u + A) + \lambda_c^{(s)} P_c + \lambda_t^{(s)}(P_t + P_t^{EW}(B^+ \to K^0\pi^+)) \;, \\ &\sqrt{2}A(B^+ \to K^+\pi^0) = \qquad\qquad (11)\\ &\quad -\lambda_u^{(s)}(T + C + P_u + A) - \lambda_c^{(s)} P_c + \lambda_t^{(s)}(-P_t + \sqrt{2}P_t^{EW}(B^+ \to K^+\pi^-)) \;. \end{aligned}$$

The unitarity of the CKM matrix can be used to eliminate the charm penguin term P_c with the help of the relation $\lambda_c^{(s)} = -\lambda_u^{(s)} - \lambda_t^{(s)}$ by absorbing it into $P_{uc} \equiv P_u - P_c$ and $P_{tc} \equiv P_t - P_c$.

A. U-spin symmetry

At the first sight, the weak Hamiltonian (8) appears to contain all possible SU(3) representations allowed by the quark structure of the four-quark operators, which would imply that no special symmetry relations exist among decay amplitudes. In fact, an examination of the quark content of the Hamiltonian (3) shows that it transforms as a doublet under U-spin symmetry (the subgroup of SU(3) exchanging d and s quarks). Although the $\overline{\mathbf{15}}$ representation contains both $U = 1/2, 3/2$ components, the $U = 3/2$ piece cancels in the specific combinations $\overline{\mathbf{15}}_{I=1} + \frac{1}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}$ and $\overline{\mathbf{15}}_{I=3/2} + \frac{1}{2\sqrt{2}}\overline{\mathbf{15}}_{I=1/2}$ appearing in (8). The most useful amplitude relations to be used in the following are consequences of this symmetry property.

The weak Hamiltonian (8) can be written as

$$\mathcal{H}_W = \left(V_{ub}^* V_{ud} \mathcal{T}^{(-\frac12)} + V_{tb}^* V_{td} \mathcal{P}^{(-\frac12)}\right) - \left(V_{ub}^* V_{us} \mathcal{T}^{(+\frac12)} + V_{tb}^* V_{ts} \mathcal{P}^{(+\frac12)}\right) \;, \tag{12}$$

with $\mathcal{T}^{(U_3)}$ and $\mathcal{P}^{(U_3)}$ two $U = 1/2$ operators standing for "tree" and "penguin" contributions respectively. The latter includes both strong and electroweak penguin operators.

From the point of view of U-spin symmetry, the octet of pseudoscalar Goldstone bosons contains one U-spin triplet $\mathcal{U}_1$, two doublets $\mathcal{U}_2$, $\mathcal{U}_3$ and one singlet $\mathcal{U}_4$. Their components are

$$\mathcal{U}_1 = \begin{pmatrix} K^0 \\ \frac{\sqrt{3}}{2}\eta_8 - \frac12\pi^0 \\ -\bar{K}^0 \end{pmatrix} \;, \quad \mathcal{U}_2 = \begin{pmatrix} K^+ \\ -\pi^+ \end{pmatrix} \;, \quad \mathcal{U}_3 = \begin{pmatrix} \pi^- \\ -K^- \end{pmatrix} \tag{13}$$

and $\mathcal{U}_4 = \frac{\sqrt{3}}{2}\pi^0 + \frac12\eta_8$.

To demonstrate the power of U-spin symmetry we derive a triangle relation [13] connecting the "tree" contributions to the $\Delta S = 1$ and $\Delta S = 0$ B^+ decay amplitudes

$$A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0) = \frac{V_{us}}{V_{ud}}\sqrt{2}A(B^+ \to \pi^+\pi^0)\,. \tag{14}$$

This relation (more precisely its extension including EWP contributions) plays an important role in certain methods of bounding [14] or determining [15,35,9,16] the weak phase γ from $B \to K\pi$ decays. The strong penguin component in $\mathcal{P}$ does not contribute to either side of this relation because of isospin constraints. However, the electroweak penguin components with $I = 1$ and $I = 3/2$ respectively do contribute [28,31], which will introduce a correction to Eq. (14). This will be discussed in the next section.

The final states on the left-hand side of this relation have $U_3 = +1/2$ and can be obtained by combining $\mathcal{U}_1 \otimes \mathcal{U}_2$ to a total U-spin 1/2 or 3/2, or by combining $\mathcal{U}_1 \otimes \mathcal{U}_4$:

$$|K^0\pi^+\rangle = -\frac{1}{\sqrt{3}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{3}{2}}\rangle - \sqrt{\frac{2}{3}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{1}{2}}\rangle \tag{15}$$

$$|K^+\pi^0\rangle = -\frac{1}{\sqrt{6}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{3}{2}}\rangle + \frac{1}{2\sqrt{3}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{1}{2}}\rangle + \frac{\sqrt{3}}{2}|[\mathcal{U}_1 \otimes \mathcal{U}_4]_{\frac{1}{2}}\rangle \tag{16}$$

$$|K^+\eta_8\rangle = \frac{1}{\sqrt{2}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{3}{2}}\rangle - \frac{1}{2}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{1}{2}}\rangle + \frac{1}{2}|[\mathcal{U}_1 \otimes \mathcal{U}_4]_{\frac{1}{2}}\rangle\,. \tag{17}$$

In the strangeless sector one has the $U_3 = -1/2$ states

$$|\pi^0\pi^+\rangle = \frac{1}{\sqrt{6}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{3}{2}}\rangle + \frac{1}{2\sqrt{3}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{1}{2}}\rangle - \frac{\sqrt{3}}{2}|[\mathcal{U}_1 \otimes \mathcal{U}_4]_{\frac{1}{2}}\rangle \tag{18}$$

$$|K^+\bar{K}^0\rangle = -\frac{1}{\sqrt{3}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{3}{2}}\rangle + \sqrt{\frac{2}{3}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{1}{2}}\rangle \tag{19}$$

$$|\pi^+\eta_8\rangle = -\frac{1}{\sqrt{2}}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{3}{2}}\rangle - \frac{1}{2}|[\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{1}{2}}\rangle - \frac{1}{2}|[\mathcal{U}_1 \otimes \mathcal{U}_4]_{\frac{1}{2}}\,. \tag{20}$$

The initial state in B^+ decays is a U-spin singlet. Using the above expressions for the final states, the relation (14) follows simply as a consequence of the absence of a $U = 3/2$ term in the weak Hamiltonian.

Another useful application of the U-spin symmetry consists in the existence of pairs of processes which are described by the same strong amplitudes, multiplied with different CKM factors. This is the case, e.g. with the B^+ decays into $K^0\pi^+$ (15) and $K^+\bar{K}^0$ (19), for which the final states contain the same $U = 1/2$ U-spin multiplet. This gives [17,24]

$$A(B^+ \to K^0\pi^+) = V_{ub}^*V_{us}A + V_{tb}^*V_{ts}P \tag{21}$$

$$A(B^+ \to K^+\bar{K}^0) = V_{ub}^*V_{ud}A + V_{tb}^*V_{td}P\,, \tag{22}$$

with A and P the reduced matrix elements of the operators $\mathcal{T}$ and $\mathcal{P}$ in (12). Knowledge of the ratio of charge-averaged rates for such a pair can be used to constrain the ratio of strong amplitudes entering both of them

$$|A/P| < \lambda\sqrt{\frac{B(B^\pm \to K^0\pi^\pm)}{B(B^\pm \to K^\pm \bar{K}^0)}}\,. \tag{23}$$

Also, the CP asymmetries of two such processes are equal and of opposite sign [24]. Similar relations have been used for the pair of decay amplitudes $(B^0, B_s) \to J/\psi K_S$ [25] and for $A(B^0 \to \pi^+\pi^-)$ and $A(B_s \to K^+K^-)$ [26,27].

III. ELECTROWEAK PENGUIN EFFECTS

The contributions of the electroweak penguin operators Q_{7-10} (6)-(7) are suppressed relative to those of the strong penguins Q_{3-6} (4)-(5) by roughly a factor of $\alpha_{\rm e.m.}/(\alpha_s \sin^2\theta_W) \simeq 0.17$ [30,29], which is not negligibly small. They are especially significant in penguin-dominated decays like $B \to K\pi$, where the magnitude of the EWP amplitudes is comparable to that of the tree amplitudes. Therefore, a precise control over their effects is important for an understanding of these decays.

The Wilson coefficients c_{7-10} have been computed to next-to-leading order (see [32] for a review) with the results (at the m_b scale)

$$(c_7,\, c_8,\, c_9,\, c_{10}) \;=\; (-0.002, 0.054, -1.292, 0.263)\alpha_{\rm e.m.}\,. \tag{24}$$

Neglecting the small contributions of the operators $c_{7,8}$ leads to important simplifications [33,34], as the remaining EWP operators $Q_{9,10}$ are related by a Fierz transformation to the current-current operators $Q_{1,2}$. Performing a SU(3) decomposition one obtains the following expression for the EWP Hamiltonian in terms of the $(V-A)\times(V-A)$ operators introduced in (8)

$$\begin{aligned}
\mathcal{H}_{EWP} \simeq \frac{G_F}{\sqrt{2}}&\left\{-\lambda_t^{(s)}\left(c_9 Q_9^{(s)} + c_{10} Q_{10}^{(s)}\right) - \lambda_t^{(d)}\left(c_9 Q_9^{(d)} + c_{10} Q_{10}^{(d)}\right)\right\} = \qquad (25)\\
\frac{G_F}{\sqrt{2}}&\left\{-\frac{\lambda_t^{(s)}}{2}\left(\frac{c_9-c_{10}}{2}(3\cdot \mathbf{6}_{I=1} + \bar{\mathbf{3}}^{(a)}_{I=0}) + \frac{c_9+c_{10}}{2}(-3\cdot\overline{\mathbf{15}}_{I=1} - \frac{3}{\sqrt{2}}\overline{\mathbf{15}}_{I=0} - \frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0})\right)\right.\\
&\left.-\frac{\lambda_t^{(d)}}{2}\left(\frac{c_9-c_{10}}{2}(-3\cdot \mathbf{6}_{I=\frac12} + \bar{\mathbf{3}}^{(a)}_{I=\frac12}) + \frac{c_9+c_{10}}{2}(-\sqrt{\frac{3}{2}}\cdot\overline{\mathbf{15}}_{I=\frac12} - 2\sqrt{3}\cdot\overline{\mathbf{15}}_{I=\frac32} - \frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=\frac12})\right)\right\}\;.
\end{aligned}$$

Now the SU(3) methods discussed in Sec. 2 can be applied to express the EWP amplitude corresponding to any B decay in terms of "tree" amplitudes alone. The results have been presented in [9] in a quark diagram language, which has the advantage of allowing an immediate insight into the relative size of different contributions. In particular, this justifies the color-suppression of certain EWP contributions conjectured in [30,34].

The U-spin formalism discussed in Sec. 2.1 can be used to give a simple derivation of the correction to the triangle relation (14) arising from EWP effects [14]. As mentioned, these corrections appear because the EWP Hamiltonian contains $I = 1$ operators in the $\Delta S = 1$ sector and $I = 3/2$ operators in the $\Delta S = 0$ sector respectively, as one can see from (25). Their matrix elements can be related thanks to the special structure of the weak Hamiltonian (12) written in U-spin symmetric form:

$$\mathcal{T} = \frac{1}{2}(c_1+c_2)\mathcal{D}_1 + \frac{1}{2}(c_1-c_2)\mathcal{D}_2 \tag{26}$$

$$\mathcal{P}^{EWP} = \frac{1}{2}(c_9+c_{10})[-\frac{3}{2}\mathcal{D}_1 + \mathcal{D}_3] + \frac{1}{2}(c_9-c_{10})\mathcal{D}_4\,, \tag{27}$$

where $\mathcal{D}_2^{(-\frac{1}{2})}$, $\mathcal{D}_3^{(-\frac{1}{2})}$ and $\mathcal{D}_4^{(-\frac{1}{2})}$ are $I = 1/2$ operators and only $\mathcal{D}_1^{(-\frac{1}{2})}$ has $I = 3/2$. This special property can be used [14] to prove that, although $\mathcal{D}_3^{(+\frac{1}{2})}$ and $\mathcal{D}_4^{(+\frac{1}{2})}$ contain $I = 1$ pieces, they do not contribute to the LHS of (14). Therefore the EWP contribution to the LHS of (14) can be expressed solely in terms of the amplitude $A(B^+ \to \pi^+\pi^0)$ induced by $\mathcal{D}_1$. One obtains in this way the following generalization of (14) including the contributions of the EW penguin effects

$$\begin{aligned} &A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0) = \\ &\quad \frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\sqrt{2}A(B^+ \to \pi^+\pi^0)\left(1 - \frac{c_9+c_{10}}{c_1+c_2}\frac{3}{2R_b\lambda^2}e^{-i\gamma}\right)\,. \end{aligned} \tag{28}$$

In this relation one has neglected the EWP contribution to the decay amplitude $A(B^+ \to \pi^+\pi^0)$. They can be included in a model-independent way too [35,9], although their numerical impact turns out to be rather small, in accordance with earlier estimates [30]. The ratio $f_K/f_\pi \simeq 1.22$ accounts for factorizable SU(3) breaking in the leading tree amplitude.

It is interesting to note that there exist SU(3) amplitude relations which are not affected by EWP effects. One of them has been noted by Deshpande and He [28], who based on it a different method for determining γ (see also [36]). This relation follows from the absence of a $U = 3/2$ term in the weak Hamiltonian:

$$\begin{aligned} 0 &= \langle [\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{3}{2}}, U_3 = +\frac{1}{2}|H_W|B^+\rangle \\ &= \sqrt{2}A(B^+ \to K^+\pi^0) - \sqrt{6}A(B^+ \to K^+\eta_8) + 2A(B^+ \to K^0\pi^+)\,. \end{aligned} \tag{29}$$

Its analog for $\Delta S = 0$ decays has been used in [37] and relates B^+ decay amplitudes into strangeless final states

$$\begin{aligned} 0 &= \langle [\mathcal{U}_1 \otimes \mathcal{U}_2]_{\frac{3}{2}}, U_3 = -\frac{1}{2}|H_W|B^+\rangle \\ &= A(B^+ \to K^+\bar{K}^0) + \sqrt{\frac{3}{2}}A(B^+ \to \pi^+\eta_8) - \frac{1}{\sqrt{2}}A(B^+ \to \pi^+\pi^0)\,. \end{aligned} \tag{30}$$

IV. DETERMINING THE WEAK PHASE γ USING $B \to K\pi$ DECAYS

A method for determining the weak phase γ has been proposed in [13], requiring the B^+ decay rates into $K^0\pi^+, K^+\pi^0, \pi^+\pi^0$ and their charge conjugates. This method, subsequently improved in [15] by including EW penguin effects, rests on the following assumptions:

a) flavor SU(3) symmetry, implied in the triangle relation (14), respectively its version (28) including EWP effects.

b) the absence of a term with nontrivial weak phase in the amplitude $A(B^+ \to K^0\pi^+)$. This amplitude has been given in (10) and can be rewritten as

$$A(B^+ \to K^0\pi^+) = -A\lambda^2 P \left(1 + \varepsilon_A e^{i\phi_A} e^{i\gamma}\right) \tag{31}$$

with ε_A, ϕ_A parametrizing the magnitude and phase of the annihilation contribution relative to the dominant penguin one.

Neglecting the annihilation amplitude ($\varepsilon_A \simeq 0$), the SU(3) triangle (14) and its CP conjugate can be represented together as shown in Fig. 1. The circle has radius δ_{EW} in units of $\sqrt{2}A(B^+ \to \pi^+\pi^0)$, with $\delta_{EW} = -\frac{3}{2\lambda^2 R_b}\frac{c_9+c_{10}}{c_1+c_2} = 0.66 \pm 0.15$. The relative orientation of the two triangles is fixed together with the weak phase γ by requiring the equality of the two angles denoted 2γ in Fig. 1.

There are several sources of theoretical errors affecting this determination. First, there are uncertainties in the value of δ_{EW} from SU(3) breaking effects and the imprecisely known value of the ratio $|V_{ub}/V_{cb}|$. The former have been computed in the factorization approximation [14,16] and they lower δ_{EW} by $(6 \pm 6)\%$ compared to its SU(3) value, although non-factorizable SU(3) breaking, which could be significant [35], remains unknown. At present the latter dominate the error on δ_{EW} but they are likely to decrease as the ratio of CKM matrix elements is better measured.

We will focus in the following on another intrinsic uncertainty of this method, arising from rescattering effects (assumption (b) above). As explained above, the naive factorization approximation suggests that the component with weak phase γ in the amplitude (31) is suppressed by a factor $f_B/m_B \simeq 0.05$ and is thus negligibly small. However, dynamical calculations [18–20,22,23] suggest that rescattering effects can induce a nonnegligible value for ε_A. For example, elastic rescattering through a color-allowed intermediate state as in $B^+ \to \{K^+\pi^0\} \to K^+\pi^0$ can conceivably enhance the annihilation contribution.

The U-spin relation (23) can be used to give an upper bound on the magnitude of these effects $\varepsilon_A < 0.18$, which is not yet very stringent. We used here the CLEO results [38] $B(B^\pm \to K^\pm K^0) < 0.9 \cdot 10^{-5}$ (at 90% CL) and $B(B^\pm \to K^0\pi^\pm) = (1.4 \pm 0.5 \pm 0.2) \cdot 10^{-5}$. We will adopt in our following estimates the value $\varepsilon_A = 0.1$.

The complete set of $B^+ \to K\pi$ decay amplitudes is defined by specifying $\varepsilon, \phi_P, \gamma$ and the rescattering parameters ε_A, ϕ_A, where we denote the "tree-to-penguin" ratio

$$\varepsilon = \lambda \frac{f_K}{f_\pi} \sqrt{\frac{B(B^\pm \to \pi^\pm\pi^0)}{B(B^\pm \to K^0\pi^\pm)}} \tag{32}$$

and the relative phase $\phi_P = \mathrm{Arg}(P/(T+C))$. One can simulate sets of decay amplitudes corresponding to given values of these parameters and study the effects of the rescattering effects on the extracted value of γ.

In Fig. 2 are shown the results of such a simulation using the input values $\varepsilon = 0.24$, $\varepsilon_A = 0.1$ and $\gamma = 76°$. In Fig. 2(a) the output value of γ is plotted as a function of ϕ_A at $\phi_P = 60°$ and $\phi_P = 90°$, and in Fig. 2(b) the dependence of γ is shown as function of ϕ_P at $\phi_A = 0°$. The most notable feature of these results is the large deviation of the extracted γ from its physical value for a strong phase ϕ_P around 90°, of about 14°. This example illustrates the possible significance of the rescattering corrections on this method, even for moderate values of $\varepsilon_A \simeq 0.1$.

A modified version of this method for determining γ has been proposed in [16], with the view of minimizing the rescattering effects. This method is formulated in terms of two quantities R_* and $\tilde{A}$ defined by

$$R_* \equiv \frac{B(B^\pm \to K^0\pi^\pm)}{2B(B^\pm \to K^\pm\pi^0)} \,, \tag{33}$$

$$\tilde{A} \equiv \frac{B(B^+ \to K^+\pi^0) - B(B^- \to K^-\pi^0)}{B(B^\pm \to K^0\pi^\pm)} - \frac{B(B^+ \to K^0\pi^+) - B(B^- \to \bar{K}^0\pi^-)}{2B(B^\pm \to K^0\pi^\pm)} \,.$$

These quantities do not contain $\mathcal{O}(\varepsilon_A)$ terms; their dependence on the rescattering parameter ε_A appears only at order $\mathcal{O}(\varepsilon\varepsilon_A)$. Therefore, it was argued in [16], the determination of γ, by setting $\epsilon_A = 0$ in the expressions for R_* and $\tilde{A}$, is insensitive to rescattering effects. This procedure gives two equations for γ and ϕ which can be solved simultaneously from R_* and $\tilde{A}$. Using two pairs of input values for $(R_*, \tilde{A})$ (corresponding to a restricted range for ϕ_A and ϕ_P) seemed to indicate that the error in γ for $\varepsilon_A = 0.08$ is only about 5°.

In Fig. 3 are shown the results of such an analysis carried out for the entire parameter space of ϕ_A and ϕ_P. Whereas the angle ϕ_P can be recovered with small errors, the results for γ show the same large rescattering effects for values of ϕ_P around 90° as in Fig. 2. (A slight improvement is the absence of a discrete ambiguity in the value of γ.) These results indicate that the large deviation of γ from its physical value for $\phi_P = 90^\circ$ is a general phenomenon, common to all variants of this method. Some information about the size of the expected error can be obtained by first determining ϕ_P. Values not too close to 90° would be an indication for a small error.

A. Eliminating the rescattering effects using additional processes

Several modifications of the method discussed above have been proposed [34,35,37,41], which use additional processes in order to completely eliminate the rescattering contributions. All of these methods make use of the decays $B^\pm \to K^\pm K^0$ which are related by U-spin to the amplitude $B^\pm \to K^0\pi^\pm$ affected by rescattering by (21), (22). Using these relations one can see that the rescattering effects cancel out in the difference $A(B^+ \to K^0\pi^+) - \lambda A(B^+ \to K^+\bar{K}^0)$.

This is illustrated in Fig. 4, where in addition to the SU(3) triangles of Fig. 3 the amplitudes $\lambda A(B^+ \to K^+\bar{K}^0)$ and of its CP conjugate are shown as the segments OC and OD respectively. Assuming that the positions of OC and OD are known, then the relative orientation of the $B \to K\pi$ triangles and thereby γ can be fixed by requring the equality of the two angles marked 2γ in Fig. 4. The various existing methods in the literature differ in the way the positions of the OC and OD segments are determined.

A minimal extension has been proposed in [34,35] which requires, in addition to $B^+ \to K\pi, \pi^+\pi^0$ data, only the charge-averaged rate for $B^\pm \to K^\pm K^0$. In the geometrical formulation given in [15], this method works by requiring the equality of the two segments $|YC| = |YD|$ (both considered as functions of γ) in Fig. 4. Due to the smallness of the rescattering contribution relative to the penguin amplitude, this equality is almost automatical for most values of γ, which is to say that small errors in the amplitudes $A(B^+ \to K^+\bar{K}^0)$ are amplified in the extracted value of γ. Also, SU(3) breaking effects introduce large errors, which can be however controlled if the direct CP asymmetry of the $K^+\bar{K}^0$ mode is measured.

An improvement of this approach has been given in [37], where the positions of the segments OC and OD are determined independently of the $B^+ \to K\pi$ data, with the help of the SU(3) relation (30). The uncertainty in the position of the point Y due to SU(3) breaking is naturally small because the sides OC and OD themselves are small, relative to the long side of the triangle (30). Naive dimensional estimates [37] suggest that the SU(3) breaking-induced error on γ is of the order of a few degrees, which is confirmed by a detailed numerical study [39]. One additional problem with this method is introduced by the $\eta - \eta'$ mixing, whose treatment will add some model dependence. This can be avoided by using instead an alternative approach using $B^0 \to K\pi$ and B_s decays [37]. However, it remains to be seen if the statistical errors due to the necessity of combining nine different decay rates will not overweigh the theoretical advantages of this method.

V. CONCLUSIONS

Nonleptonic weak decays of the B mesons are a valuable source of information about the elements of the CKM matrix. In particular, the penguin-dominated decays $B \to K\pi$ can provide useful constraints [42,43,14] and determinations [13,15,34,35,9] of the weak phase γ, which complement those from global fits of the unitarity triangle. Although the focus of this talk has been on charged B decays, useful information can be obtained also by combining B^0 with B^+ decay data [31,42,43,34,35].

While the electroweak penguin contributions to the determination of γ from $B^+ \to K\pi, \pi^+\pi^0$ decays can be controlled using SU(3) symmetry, the rescattering effects are potentially significant. Depending on the precise value of a strong phase ϕ_P (which can be determined fairly precisely), the corresponding uncertainty on γ can be as large as $\pm 15^\circ$. Several methods exist which make it possible to completely eliminate these effects with the help of additional decays.

ACKNOWLEDGMENTS

It is a pleasure to thank Michael Gronau and Tung-Mow Yan for discussions and collaboration on the subjects discussed in this talk. This work has been supported by the National Science Foundation.

REFERENCES

[1] I. I. Bigi and A. I. Sanda, Nucl. Phys. B **193** (1981) 85; Phys. Rev. D **29** (1984) 1393.
[2] Review of Particle Physics, Eur. Phys. Jour. C **3** (1998) 1.
[3] J. L. Rosner, hep-ph/9809545.
[4] D. Zeppenfeld, Z. Phys. C**8** (1981) 77.
[5] M. Gronau, O. Hernández, D. London and J. L. Rosner, Phys. Rev. D **50** (1994) 4529.
[6] B. Grinstein and R. F. Lebed, Phys. Rev. D **53** (1996) 6344.
[7] M. Gronau and D. London, Phys. Lett. B **253** (1991) 483.
[8] M. Gronau and D. Wyler, Phys. Lett. B **265** (1991) 172.
[9] M. Gronau, D. Pirjol and T. M. Yan, Phys. Rev. D **60** (1999) 034021.
[10] A. Ali, G. Kramer, C.-D. Lü, Phys. Rev. D **58** (1998) 94009.
[11] Y. H. Chen, H. Y. Cheng and B. Tseng, Phys. Rev. D **59** (1999) 74003.
[12] Y. H. Chen, H. Y. Cheng and K. C. Yang, hep-ph/9903453.
[13] M. Gronau, D. London and J. L. Rosner, Phys. Rev. Lett. **73** (1994) 21.
[14] M. Neubert and J. L. Rosner, Phys. Lett. B **441** (1998) 403.
[15] M. Neubert and J. L. Rosner, Phys. Rev. Lett. **81** (1998) 5076.
[16] M. Neubert, JHEP **9902** (1999) 014.
[17] A. Falk, A. L. Kagan, Y. Nir and A. A. Petrov, Phys. Rev. D **57** (1998) 4290.
[18] B. Blok, M. Gronau and J. L. Rosner, Phys. Rev. Lett. **78** (1997) 3999; **78** (1997) 1167.
[19] J. M. Gérard and J. Weyers, Eur. Phys. J. C**7** (1999) 1.
[20] D. Delépine, J. M. Gérard, J. Pestieau and J. Weyers, Phys. Lett. B **429** (1998) 106.
[21] D. Atwood and A. Soni, Phys. Rev. D **58** (1998) 036005.
[22] A. N. Kamal, Alberta U. report Thy-01-99, hep-ph/9901342.
[23] I. Caprini, L. Micu and C. Bourelly, Marseille report CPT-99-P-3795, hep-ph/9904214.
[24] A. J. Buras, R. Fleischer and T. Mannel, Nucl. Phys. B **533** (1998) 3.
[25] R. Fleischer, CERN report CERN-TH-99-78, hep-ph/9903455.
[26] D. Pirjol, Phys. Rev. D **60** (1999) 54020.
[27] R. Fleischer, CERN report CERN-TH-99-79, hep-ph/9903456.
[28] N. G. Deshpande and X.-G. He, Phys. Rev. Lett. **74** (1995) 26; [E: **74** (1995) 4099].
[29] R. Fleischer, Int. J. Mod. Phys. **A12** (1997) 2459.
[30] M. Gronau, O. Hernández, D. London and J. L. Rosner, Phys. Rev. D **52** (1995) 6374.
[31] R. Fleischer, Z. Phys. C **62** (1994) 81; Phys. Lett. B **321** (1994) 259; Phys. Lett. B **332** (1994) 419.
[32] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68** (1996) 1125.
[33] M. Neubert, Phys. Lett. B **424** (1998) 152.
[34] R. Fleischer, Eur. J. Phys. C **6** (1999) 451.
[35] A. J. Buras and R. Fleischer, CERN report CERN-TH-98-319, hep-ph/9810260.
[36] M. Gronau and J. L. Rosner, Phys. Rev. D **53** (1996) 2516.
[37] M. Gronau and D. Pirjol, Phys. Lett. B **449** (1999) 321.
[38] F. Würthwein, talk given at the DPF99 meeting, Los Angeles, 1999.
[39] M. Gronau and D. Pirjol, Cornell report CLNS-99-1604, hep-ph/9902482.
[40] X.-G. He, Eur. Phys. J. C **9** (1999) 443.
[41] K. Agashe and N. G. Deshpande, Phys. Lett. B **451** (1999) 215.
[42] R. Fleischer and T. Mannel, Phys. Rev. D **57** (1998) 2752.
[43] M. Gronau and J. L. Rosner, Phys. Rev. D **57** (1998) 6843.

FIGURES

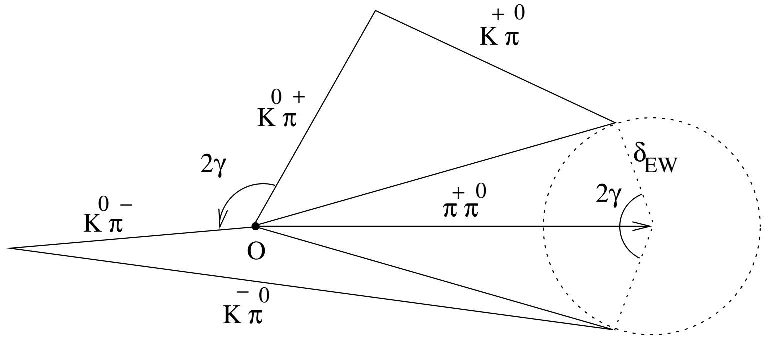
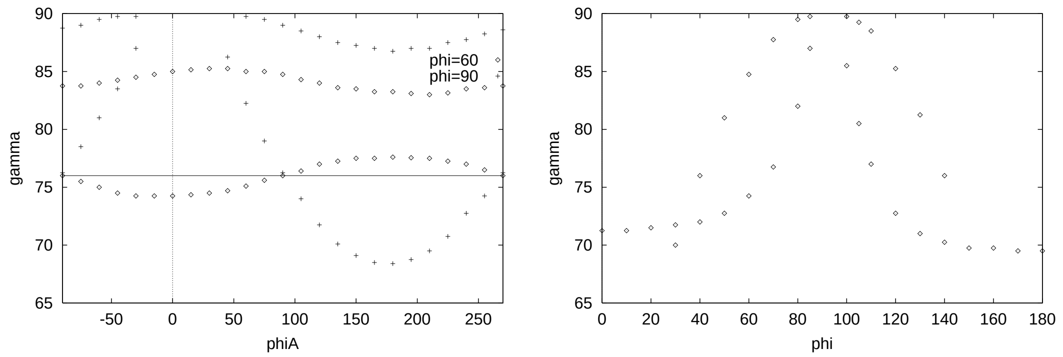
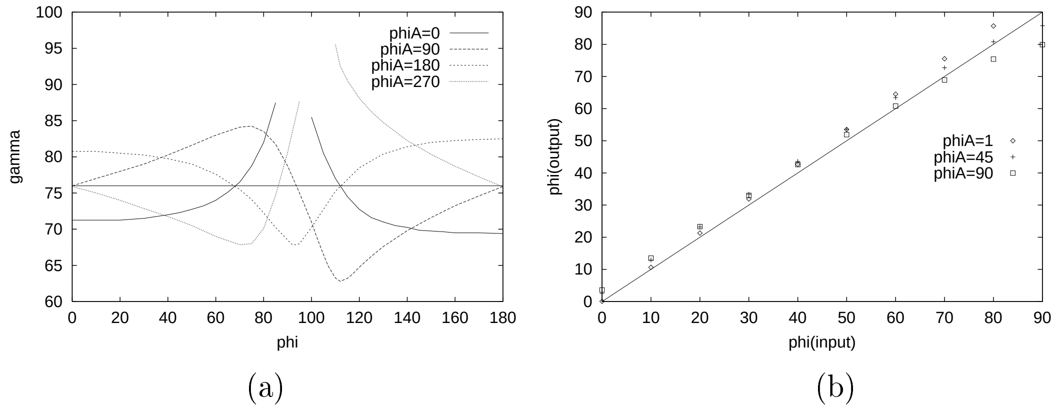


FIG. 1. Graphical representation of the SU(3) amplitude relation (28) and of its CP conjugate used in the determination of the weak angle γ.



(a) (b)

FIG. 2. Rescattering effects on the determination of the weak phase γ from $B^+ \to K\pi$ decays. (a) - the dependence of γ on ϕ_A, for two values of $\phi_P = 60^\circ$ and $\phi_P = 90^\circ$; (b) - the dependence of the solution on ϕ_P, for $\phi_A = 0^\circ$. (both graphs correspond to $\varepsilon_A = 0.1, \gamma_{\text{phys}} = 76^\circ$)



(a) (b)

FIG. 3. (a) - the weak phase γ extracted from the method using the parameters $(R_*, \tilde{A})$, as a function of the strong phase ϕ_P for several values of ϕ_A ($\varepsilon_A = 0.1$). The horizontal line shows the assumed physical value of $\gamma = 76^\circ$. (b) - the strong phase ϕ_P can be reconstructed using the $(R_*, \tilde{A})$ data.

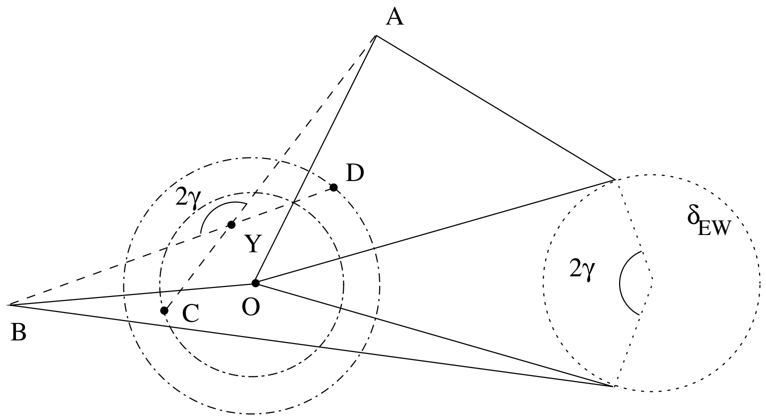


FIG. 4. Eliminating rescattering corrections with the help of $B^{\pm} \to K^{\pm}K^{0}$ decays. The lines OC and OD denote the amplitudes $\lambda A(B^{\pm} \to K^{\pm}K^{0})$.