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Model-independent electroweak penguins in B decays to two pseudoscalars

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Abstract

We study the effects of electroweak penguin (EWP) amplitudes in B meson decays into two charmless pseudoscalars in the approximation of retaining only the dominant EWP operators Q_9 and Q_{10}. Using flavor SU(3) symmetry, we derive a set of model-independent relations between EWP contributions and tree-level decay amplitudes one of which was noted recently by Neubert and Rosner. Two new applications of these relations are demonstrated in which uncertainties due to EWP corrections are eliminated in order to determine a weak phase. Whereas the weak angle α can be obtained from $B \to \pi\pi$ free of hadronic uncertainties, a determination of γ from $B^{0,\pm} \to K\pi^{\pm}$ requires the knowledge of a ratio of certain tree-level hadronic matrix elements. The smallness of this ratio implies a useful constraint on γ if rescattering can be neglected.

I. INTRODUCTION

Nonleptonic weak decays of B mesons into two charmless pseudoscalars provide an important probe of the origin of CP violation in the single complex phase of the CKM matrix [1]. Approximate flavor symmetries of the strong interactions play a useful role in such analysis [2–4]. In one simplified version of such methods the weak phase α is extracted from $B \to \pi\pi$ decays using isospin symmetry [5], and in another case the phase γ is obtained from combining $B \to K\pi$ and $B \to \pi\pi$ amplitudes using flavor SU(3) [6]. Electroweak penguin (EWP) contributions [7], enhanced by the heavy top quark, can spoil such methods. Whereas these contributions are expected to have a small effect on α, they were estimated in a model-dependent manner to have a large effect on the extraction of γ [8–10]. Recently Neubert and Rosner have used Fierz transformations and SU(3) symmetry to include in the latter case the effect of EWP amplitudes in a model-independent way [11,12]. Their method of constraining γ is based on assuming the dominance of two EWP operators (Q_9 and Q_{10}) and relating their matrix elements for the $I = 3/2$ $K\pi$ B decay final state to corresponding tree-level amplitudes. This argument is entirely model-independent, in contrast to previous studies of EWP contributions [8,10,13] which assume certain models for the matrix elements of EWP operators involving factorization and specific form factors.

The purpose of this paper is to generalize the relation proposed by Neubert and Rosner to all matrix elements of EWP operators for nonstrange and strange B mesons and for any two pseudoscalar final state, and to study the consequences of such relations. Sec. II reviews the two alternative descriptions of flavor SU(3), in terms of operator matrix elements on the one hand, and quark diagrams on the other hand. These descriptions are used in Sec. III to derive a complete set of model-independent SU(3) relations between EWP and tree amplitudes for $B \to K\pi$, $B \to \pi\pi$, $B \to K\overline{K}$ and corresponding B_s decays. Using an approximate numerical relation between two ratios of Wilson coefficients, we show in Sec. IV that *all* EWP contributions can be written in terms of tree amplitudes. In Sec. V we demonstrate a few applications of these relations used to eliminate uncertainties due to EWP contributions when determining the weak phases α and γ from $B \to \pi\pi$ and $B \to K\pi$ decays, respectively. Finally, our results are summarized in Sec. VI. An Appendix lists the four-quark operators appearing in the weak Hamiltonian for b decays corresponding to specific SU(3) representations.

II. FLAVOR SU(3) IN B DECAYS

The weak Hamiltonian governing B meson decays is given by (see, e.g., [14])

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \left(\sum_{q'=u,c} \lambda_{q'}^{(q)} [c_1 (\bar{b}q')_{V-A} (\bar{q}'q)_{V-A} + c_2 (\bar{b}q)_{V-A} (\bar{q}'q')_{V-A}] - \lambda_t^{(q)} \sum_{i=3}^{10} c_i Q_i^{(q)} \right) , \quad (1)$$

where $\lambda_{q'}^{(q)} = V_{q'b}^* V_{q'q}$, $q = d, s$, $q' = u, c, t$. Unitarity of the CKM matrix implies $\lambda_u^{(q)} + \lambda_c^{(q)} + \lambda_t^{(q)} = 0$. The first term, involving the coefficients c_1 and c_2, will be referred to as the "tree" part, while the second term, involving c_i, $i = 3 - 10$ is the penguin part. The corresponding Q_i consist of four QCD penguin operators ($i = 3 - 6$) and four electroweak penguin operators ($i = 7 - 10$). Their precise form is not important for our purpose and can be found for example in [14]. In the following we will be only interested in their SU(3) transformation properties, noting that Q_9 and Q_{10} have a $(V-A)(V-A)$ structure similar to the "tree" part. There are two distinct types of QCD penguin operators, with the flavor structure $(q=d,s)$

$$\begin{aligned} Q_{3,5}^{(q)} &= (\bar{b}q)(\bar{u}u+\bar{d}d+\bar{s}s) \ , \\ Q_{4,6}^{(q)} &= (\bar{b}u)(\bar{u}q)+(\bar{b}d)(\bar{d}q)+(\bar{b}s)(\bar{s}q) \ , \end{aligned} \tag{2}$$

and two types of EWP operators

$$\begin{aligned} Q_{7,9}^{(q)} &= \frac{3}{2}\left[(\bar{b}q)(\frac{2}{3}\bar{u}u-\frac{1}{3}\bar{d}d-\frac{1}{3}\bar{s}s)\right] \ , \\ Q_{8,10}^{(q)} &= \frac{3}{2}\left[\frac{2}{3}(\bar{b}u)(\bar{u}q)-\frac{1}{3}(\bar{b}d)(\bar{d}q)-\frac{1}{3}(\bar{b}s)(\bar{s}q)\right] \ . \end{aligned} \tag{3}$$

All four quark operators appearing in (1-3) are of the form $(\bar{b}q_1)(\bar{q}_2q_3)$ and can be written as a sum of $\overline{\mathbf{15}}$, $\mathbf{6}$ and $\bar{\mathbf{3}}$, into which the product $\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}$ can be decomposed [3,4]. Note that the representation $\bar{\mathbf{3}}$ appears twice in this decomposition, both symmetric ($\bar{\mathbf{3}}^{(s)}$), and antisymmetric ($\bar{\mathbf{3}}^{(a)}$) under the interchange of q_1 and q_3.

The tree part of the Hamiltonian (1) can be expressed in terms of operators with definite SU(3) transformation properties:

$$\begin{aligned} \mathcal{H}_T = \frac{G_F}{\sqrt{2}}\Big(&\lambda_u^{(s)}[\frac{1}{2}(c_1-c_2)(-\bar{\mathbf{3}}^{(a)}_{I=0}-\mathbf{6}_{I=1})+\frac{1}{2}(c_1+c_2)(-\overline{\mathbf{15}}_{I=1}-\frac{1}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}+\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0}) \\ &+\lambda_u^{(d)}[\frac{1}{2}(c_1-c_2)(\mathbf{6}_{I=\frac{1}{2}}-\bar{\mathbf{3}}^{(a)}_{I=\frac{1}{2}})+\frac{1}{2}(c_1+c_2)(-\frac{2}{\sqrt{3}}\overline{\mathbf{15}}_{I=\frac{3}{2}}-\frac{1}{\sqrt{6}}\overline{\mathbf{15}}_{I=\frac{1}{2}}+\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=\frac{1}{2}})\Big) \ . \end{aligned} \tag{4}$$

The operators $\bar{\mathbf{3}}^{(s)}$ and $\bar{\mathbf{3}}^{(a)}$ appear in the two lines in the same combination. This fact is essential for relating $|\Delta S|=1$ to $\Delta S=0$ amplitudes with the help of SU(3) symmetry. The operators with well-defined SU(3) transformation properties appearing in (4) are given in the Appendix in terms of four-quark operators.

The contribution of the EWP operators (3) is given by

$$\begin{aligned} \mathcal{H}_{EWP} &\simeq -\lambda_t^{(s)}\left(c_9Q_9^{(s)}+c_{10}Q_{10}^{(s)}\right)-\lambda_t^{(d)}\left(c_9Q_9^{(d)}+c_{10}Q_{10}^{(d)}\right) = \\ &-\frac{\lambda_t^{(s)}}{2}\left(\frac{c_9-c_{10}}{2}(3\cdot\mathbf{6}_{I=1}+\bar{\mathbf{3}}^{(a)}_{I=0})+\frac{c_9+c_{10}}{2}(-3\cdot\overline{\mathbf{15}}_{I=1}-\frac{3}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}-\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0})\right) \\ -\frac{\lambda_t^{(d)}}{2}&\left(\frac{c_9-c_{10}}{2}(-3\cdot\mathbf{6}_{I=\frac{1}{2}}+\bar{\mathbf{3}}^{(a)}_{I=\frac{1}{2}})+\frac{c_9+c_{10}}{2}(-\sqrt{\frac{3}{2}}\cdot\overline{\mathbf{15}}_{I=\frac{1}{2}}-2\sqrt{3}\cdot\overline{\mathbf{15}}_{I=\frac{3}{2}}-\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=\frac{1}{2}})\right) \ , \end{aligned} \tag{5}$$

where we made the approximation of keeping only contributions from Q_9 and Q_{10} [7,11]. This is justified by the tiny Wilson coefficients of the remaining two operators Q_7 and Q_8 [14]. In this approximation the operators appearing in (5) are of the $(V-A)(V-A)$ type and can be related to those appearing in the tree Hamiltonian (4). It is this fact which will allow us to express EWP contributions in terms of tree-level decay amplitudes.

Before proceeding to obtain these relations, let us recall the equivalent description of SU(3) amplitudes in terms of quark diagrams [3]. There are six topologies, representing tree

(T), color-suppressed (C), annihilation (A), W-exchange (E), penguin (P) and penguin-annihilation (PA) amplitudes. The six amplitudes appear in five distinct combinations, separately for $\Delta S = 0$ and $\Delta S = 1$ transitions. For convenience, we define these amplitudes such that they don't include the CKM factors. For example, a typical $|\Delta S| = 1$ transition amplitude is

$$A(B^+ \to K^0\pi^+) = \lambda_u^{(s)}(P_u + A) + \lambda_c^{(s)} P_c + \lambda_t^{(s)}(P_t + P_t^{EW}(B^+ \to K^0\pi^+)) \ , \tag{6}$$

where P_u, A and P_c are contributions from the four-quark operators in the first term of (1), while P_t and P_t^{EW} originate from the second term. In a similar way, a typical $\Delta S = 0$ transition amplitude has the form

$$\begin{aligned} A(B^0 \to \pi^+\pi^-) = {} & \lambda_u^{(d)}(-P_u - T - E - PA_u) + \lambda_c^{(d)}(-P_c) \\ & + \lambda_t^{(d)}(-P_t - PA_t + P_t^{EW}(B^0 \to \pi^+\pi^-)) \ . \end{aligned} \tag{7}$$

Despite their name, P_u and P_c originate purely from "tree-level" four-quark operators, . Note that in the SU(3) symmetric limit, the same hadronic parameters $P_u, T, C, A, PA_u, P_c, P_t$ appear in $|\Delta S| = 1$ and $\Delta S = 0$ transitions.

It is straightforward to relate the "graphical" hadronic parameters P_u, PA_u, T, C, A, E to SU(3) reduced matrix elements of the operators appearing in (4). This was done in the appendix of [3], and can also be done by computing representative decay amplitudes and expressing them with the help of the relations in the Appendix of [4]. We find the following set of linearly independent relations

$$\begin{aligned} P_u + T &= \frac{3}{2\sqrt{10}} a_2 + \frac{1}{2}\sqrt{\frac{3}{5}} a_3 + \frac{1}{4}\sqrt{\frac{3}{5}} a_4 - \frac{2}{3}\sqrt{\frac{2}{5}} a_5 \ , \\ P_u + A &= \frac{3}{2\sqrt{10}} a_2 - \frac{1}{2}\sqrt{\frac{3}{5}} a_3 - \frac{3}{4}\sqrt{\frac{3}{5}} a_4 + \frac{2}{3\sqrt{10}} a_5 \ , \\ -P_u + C &= -\frac{3}{4}\sqrt{\frac{2}{5}} a_2 - \frac{1}{2}\sqrt{\frac{3}{5}} a_3 - \frac{1}{4}\sqrt{\frac{3}{5}} a_4 - \sqrt{\frac{2}{5}} a_5 \ , \\ P_u + PA_u &= -\frac{1}{2} a_1 + \frac{1}{2\sqrt{10}} a_2 - \frac{1}{2}\sqrt{\frac{3}{5}} a_3 + \frac{3}{4}\sqrt{\frac{3}{5}} a_4 + \frac{1}{6\sqrt{10}} a_5 \ , \\ C - E &= -\sqrt{\frac{3}{5}} a_3 + \sqrt{\frac{3}{5}} a_4 - \sqrt{\frac{2}{5}} a_5 \ . \end{aligned} \tag{8}$$

a_i denote the following combinations of reduced matrix elements (a factor $G_F/\sqrt{2}$ is omitted for simplicity)

$$\begin{aligned} a_1 &= \frac{1}{2}(c_1 + c_2)\frac{1}{\sqrt{2}}\langle \mathbf{1} \| \bar{\mathbf{3}}^{(s)} \| \mathbf{3} \rangle - \frac{1}{2}(c_1 - c_2)\langle \mathbf{1} \| \bar{\mathbf{3}}^{(a)} \| \mathbf{3} \rangle \ , \\ a_2 &= \frac{1}{2}(c_1 + c_2)\frac{1}{\sqrt{2}}\langle \mathbf{8} \| \bar{\mathbf{3}}^{(s)} \| \mathbf{3} \rangle - \frac{1}{2}(c_1 - c_2)\langle \mathbf{8} \| \bar{\mathbf{3}}^{(a)} \| \mathbf{3} \rangle \ , \\ a_3 &= -\frac{1}{2}(c_1 - c_2)\langle \mathbf{8} \| \mathbf{6} \| \mathbf{3} \rangle \ , \\ a_4 &= \frac{1}{2}(c_1 + c_2)\langle \mathbf{8} \| \overline{\mathbf{15}} \| \mathbf{3} \rangle \ , \\ a_5 &= \frac{1}{2}(c_1 + c_2)\langle \mathbf{27} \| \overline{\mathbf{15}} \| \mathbf{3} \rangle \ . \end{aligned} \tag{9}$$

The normalization of the reduced matrix elements is chosen as in [4]. Relative normalization with respect to the one used in [3] is given in the Appendix.

One can find three combinations of graphical amplitudes which are independent of the reduced matrix elements a_1, a_2. As explained in the next section, they will be useful in relating EWP contributions to tree amplitudes.

$$
\begin{aligned}
T - A &= \sqrt{\frac{3}{5}} a_3 + \sqrt{\frac{3}{5}} a_4 - \sqrt{\frac{2}{5}} a_5 \ , \\
T + C &= -\frac{\sqrt{10}}{3} a_5 \ , \\
C - E &= -\sqrt{\frac{3}{5}} a_3 + \sqrt{\frac{3}{5}} a_4 - \sqrt{\frac{2}{5}} a_5 \ .
\end{aligned}
\tag{10}
$$

These relations can be solved for $a_3\,, a_4$ and a_5

$$
\begin{aligned}
a_3 &= -\frac{1}{2}\sqrt{\frac{5}{3}}(A + C - T - E) \ , \\
a_4 &= \frac{1}{2}\sqrt{\frac{5}{3}}(-A - \frac{1}{5}C - \frac{1}{5}T - E) \ , \\
a_5 &= -\frac{3}{\sqrt{10}}(T + C) \ .
\end{aligned}
\tag{11}
$$

In Sec. IV we will need also the results for the reduced matrix elements a_1 and a_2 expressed in terms of graphical contributions

$$
\begin{aligned}
a_1 &= -\frac{1}{2}T + \frac{1}{6}C - \frac{4}{3}E - \frac{4}{3}P_u - 2PA_u \\
a_2 &= \frac{1}{2}\sqrt{\frac{5}{2}}\left(T - \frac{1}{3}C + A - \frac{1}{3}E + \frac{8}{3}P_u\right) \ .
\end{aligned}
\tag{12}
$$

III. RELATIONS BETWEEN EWP AND TREE AMPLITUDES

Our purpose is to relate in the SU(3) limit EWP contributions to tree amplitudes. We note that the operators $\bar{\mathbf{3}}^{(s)}$ and $\bar{\mathbf{3}}^{(a)}$ occur in (5) in different combinations than in (4). Therefore, for arbitrary values of $c_1,\ c_2,\ c_9$ and c_{10}, symmetry relations for EWP contributions can only be obtained which are independent of the matrix elements of $\bar{\mathbf{3}}^{(s)}$ and $\bar{\mathbf{3}}^{(a)}$. The respective EWP contributions can then be expressed only in terms of tree-level amplitudes T, C, A, E with the help of the relations (11).

A. $|\Delta S| = 1$ amplitudes

EWP contributions to $B \to K\pi$ decays can be easily computed using the Hamiltonian (5). One obtains

$$
\begin{aligned}
P^{EW}(B^0 \to K^+\pi^-) &= \frac{3}{4\sqrt{10}}b_2 + \frac{1}{4}\sqrt{\frac{3}{5}}b_3 + \frac{3}{8}\sqrt{\frac{3}{5}}b_4 - \sqrt{\frac{2}{5}}b_5 \ , \\
P^{EW}(B^+ \to K^0\pi^+) &= -\frac{3}{4\sqrt{10}}b_2 + \frac{1}{4}\sqrt{\frac{3}{5}}b_3 + \frac{9}{8}\sqrt{\frac{3}{5}}b_4 - \frac{1}{\sqrt{10}}b_5 \ , \\
P^{EW}(B^0 \to K^0\pi^0) &= -\frac{3}{8\sqrt{5}}b_2 - \frac{1}{4}\sqrt{\frac{3}{10}}b_3 - \frac{3}{8}\sqrt{\frac{3}{10}}b_4 - \frac{3}{2\sqrt{5}}b_5 \ , \\
P^{EW}(B^+ \to K^+\pi^0) &= \frac{3}{8\sqrt{5}}b_2 - \frac{1}{4}\sqrt{\frac{3}{10}}b_3 - \frac{9}{8}\sqrt{\frac{3}{10}}b_4 - \frac{2}{\sqrt{5}}b_5 \ .
\end{aligned} \tag{13}
$$

The parameters b_i, analogous to a_i, are defined as

$$
\begin{aligned}
b_1 &= -\frac{1}{2}(c_9 + c_{10})\frac{1}{\sqrt{2}}\langle \mathbf{1}\|\bar{\mathbf{3}}^{(s)}\|\mathbf{3}\rangle + \frac{1}{2}(c_9 - c_{10})\langle \mathbf{1}\|\bar{\mathbf{3}}^{(a)}\|\mathbf{3}\rangle \ , \\
b_2 &= -\frac{1}{2}(c_9 + c_{10})\frac{1}{\sqrt{2}}\langle \mathbf{8}\|\bar{\mathbf{3}}^{(s)}\|\mathbf{3}\rangle + \frac{1}{2}(c_9 - c_{10})\langle \mathbf{8}\|\bar{\mathbf{3}}^{(a)}\|\mathbf{3}\rangle \ , \\
b_3 &= \frac{3}{2}(c_9 - c_{10})\langle \mathbf{8}\|\mathbf{6}\|\mathbf{3}\rangle \ , \\
b_4 &= \frac{1}{2}(c_9 + c_{10})\langle \mathbf{8}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle \ , \\
b_5 &= \frac{1}{2}(c_9 + c_{10})\langle \mathbf{27}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle \ .
\end{aligned} \tag{14}
$$

The EWP contributions satisfy the isospin relation (as do the full amplitudes [15])

$$
\begin{aligned}
&P^{EW}(B^+ \to K^0\pi^+) + \sqrt{2}P^{EW}(B^+ \to K^+\pi^0) = \\
&\qquad \sqrt{2}P^{EW}(B^0 \to K^0\pi^0) + P^{EW}(B^0 \to K^+\pi^-) \ .
\end{aligned} \tag{15}
$$

It is clear now that any combination of P^{EW} amplitudes which is independent of b_1, b_2 can be expressed directly in terms of the tree-level amplitudes T, C, A, E using the relations (11)

$$
\begin{aligned}
b_3 &= -3\frac{c_9 - c_{10}}{c_1 - c_2}a_3 = \frac{c_9 - c_{10}}{c_1 - c_2}\frac{\sqrt{15}}{2}(A + C - T - E) \ , \\
b_4 &= \frac{c_9 + c_{10}}{c_1 + c_2}a_4 = \frac{1}{2}\sqrt{\frac{5}{3}}\frac{c_9 + c_{10}}{c_1 + c_2}(-A - \frac{1}{5}C - \frac{1}{5}T - E) \ , \\
b_5 &= \frac{c_9 + c_{10}}{c_1 + c_2}a_5 = -\frac{3}{\sqrt{10}}\frac{c_9 + c_{10}}{c_1 + c_2}(T + C) \ .
\end{aligned} \tag{16}
$$

One can form two combinations of electroweak penguin contributions in $B \to K\pi$ decays which do not depend on b_1, b_2:

$$
P^{EW}(B^+ \to K^0\pi^+) + \sqrt{2}P^{EW}(B^+ \to K^+\pi^0) = -\sqrt{\frac{5}{2}}b_5 = \frac{3}{2}\frac{c_9 + c_{10}}{c_1 + c_2}(T + C) \ , \tag{17}
$$

$$
\begin{aligned}
&P^{EW}(B^0 \to K^+\pi^-) + P^{EW}(B^+ \to K^0\pi^+) = \frac{1}{2}\sqrt{\frac{3}{5}}b_3 + \frac{3}{2}\sqrt{\frac{3}{5}}b_4 - \frac{3}{2}\sqrt{\frac{2}{5}}b_5 \\
&\qquad = \frac{3}{4}\frac{c_9 - c_{10}}{c_1 - c_2}(A + C - T - E) - \frac{3}{4}\frac{c_9 + c_{10}}{c_1 + c_2}(A - C - T + E) \ .
\end{aligned} \tag{18}
$$

A third combination $P^{EW}(B^0 \to K^0\pi^0) + P^{EW}(B^+ \to K^+\pi^0)$ is not independent of these two in view of the isospin identity (15). The first relation (17) was obtained in [11]. The second one (18) is new.

In a similar way one can compute EWP contributions to B_s decay amplitudes. We find

$$
\begin{aligned}
P^{EW}(B_s \to \pi^+\pi^-) &= -\frac{1}{4}b_1 - \frac{1}{2\sqrt{10}}b_2 - \frac{3}{4}\sqrt{\frac{3}{5}}b_4 + \frac{1}{4\sqrt{10}}b_5 \ , \\
P^{EW}(B_s \to \pi^0\pi^0) &= \frac{1}{4\sqrt{2}}b_1 + \frac{1}{4\sqrt{5}}b_2 + \frac{3}{4}\sqrt{\frac{3}{10}}b_4 - \frac{1}{8\sqrt{5}}b_5 \ , \\
P^{EW}(B_s \to K^+K^-) &= -\frac{1}{4}b_1 + \frac{1}{4\sqrt{10}}b_2 + \frac{1}{4}\sqrt{\frac{3}{5}}b_3 - \frac{3}{8}\sqrt{\frac{3}{5}}b_4 - \frac{7}{4\sqrt{10}}b_5 \ , \\
P^{EW}(B_s \to K^0\bar{K}^0) &= \frac{1}{4}b_1 - \frac{1}{4\sqrt{10}}b_2 + \frac{1}{4}\sqrt{\frac{3}{5}}b_3 - \frac{9}{8}\sqrt{\frac{3}{5}}b_4 - \frac{1}{4\sqrt{10}}b_5 \ . \qquad (19)
\end{aligned}
$$

Eliminating b_1, b_2 gives two relations

$$
P^{EW}(B_s \to \pi^+\pi^-) + \sqrt{2}P^{EW}(B_s \to \pi^0\pi^0) = 0 \ , \qquad (20)
$$

$$
\begin{aligned}
&P^{EW}(B_s \to K^+K^-) + P^{EW}(B_s \to K^0\bar{K}^0) \\
&\qquad = \frac{3}{4}\frac{c_9 - c_{10}}{c_1 - c_2}(A + C - T - E) + \frac{3}{4}\frac{c_9 + c_{10}}{c_1 + c_2}(A + C + T + E) \ . \qquad (21)
\end{aligned}
$$

The first relation is simply a consequence of the absence of $\Delta I = 2$ terms in the EWP Hamiltonian (5).

B. $\Delta S = 0$ amplitudes

For this case the Hamiltonian (5) gives the following results for B and B_s decays

$$
\begin{aligned}
P^{EW}(B^+ \to \pi^+\pi^0) &= -\frac{\sqrt{5}}{2}b_5 \ , \\
P^{EW}(B^0 \to \pi^+\pi^-) &= -\frac{1}{4}b_1 + \frac{1}{4\sqrt{10}}b_2 + \frac{1}{4}\sqrt{\frac{3}{5}}b_3 - \frac{3}{8}\sqrt{\frac{3}{5}}b_4 - \frac{7}{4\sqrt{10}}b_5 \ , \\
P^{EW}(B^0 \to \pi^0\pi^0) &= \frac{1}{4\sqrt{2}}b_1 - \frac{1}{8\sqrt{5}}b_2 - \frac{1}{4}\sqrt{\frac{3}{10}}b_3 + \frac{3}{8}\sqrt{\frac{3}{10}}b_4 - \frac{13}{8\sqrt{5}}b_5 \ , \\
P^{EW}(B^+ \to K^+\bar{K}^0) &= -\frac{3}{4\sqrt{10}}b_2 + \frac{1}{4}\sqrt{\frac{3}{5}}b_3 + \frac{9}{8}\sqrt{\frac{3}{5}}b_4 - \frac{1}{\sqrt{10}}b_5 \ , \\
P^{EW}(B^0 \to K^+K^-) &= -\frac{1}{4}b_1 - \frac{1}{2\sqrt{10}}b_2 - \frac{3}{4}\sqrt{\frac{3}{5}}b_4 + \frac{1}{4\sqrt{10}}b_5 \ , \\
P^{EW}(B^0 \to K^0\bar{K}^0) &= \frac{1}{4}b_1 - \frac{1}{4\sqrt{10}}b_2 + \frac{1}{4}\sqrt{\frac{3}{5}}b_3 - \frac{9}{8}\sqrt{\frac{3}{5}}b_4 - \frac{1}{4\sqrt{10}}b_5 \ , \\
P^{EW}(B_s \to K^-\pi^+) &= \frac{3}{4\sqrt{10}}b_2 + \frac{1}{4}\sqrt{\frac{3}{5}}b_3 + \frac{3}{8}\sqrt{\frac{3}{5}}b_4 - \sqrt{\frac{2}{5}}b_5 \ , \\
P^{EW}(B_s \to \bar{K}^0\pi^0) &= -\frac{3}{8\sqrt{5}}b_2 - \frac{1}{4}\sqrt{\frac{3}{10}}b_3 - \frac{3}{8}\sqrt{\frac{3}{10}}b_4 - \frac{3}{2\sqrt{5}}b_5 \ . \qquad (22)
\end{aligned}
$$

Eliminating b_{1-4} gives the following relations for EWP contributions to $B \to \pi\pi$ decays

$$\begin{aligned}\sqrt{2}P^{EW}(B^+ \to \pi^+\pi^0) &= P^{EW}(B^0 \to \pi^+\pi^-) + \sqrt{2}P^{EW}(B^0 \to \pi^0\pi^0) \\ &= \frac{3}{2}\frac{c_9+c_{10}}{c_1+c_2}(T+C)\ .\end{aligned} \tag{23}$$

This relation, describing decay amplitudes into two pions in a $I = 2$ state, follows from isospin alone. Only the $\Delta I = 3/2$ part of the Hamiltonian contributes to these amplitudes. Comparing the tree-level (4) and EWP (5) Hamiltonians, one observes that their $\Delta I = 3/2$ parts are simply related by

$$\mathcal{H}^{EW}_{\Delta I=3/2} = -\frac{3}{2}\frac{\lambda_t^{(d)}}{\lambda_u^{(d)}}\frac{c_9+c_{10}}{c_1+c_2}\mathcal{H}^{tree}_{\Delta I=3/2}\ . \tag{24}$$

Therefore isospin symmetry alone suffices to relate their matrix elements. A similar relation holds for EWP contribution in $B_s \to (\overline{K}\pi)_{I=3/2}$

$$P^{EW}(B_s \to K^-\pi^+) + \sqrt{2}P^{EW}(B_s \to \bar{K}^0\pi^0) = \frac{3}{2}\frac{c_9+c_{10}}{c_1+c_2}(T+C)\ . \tag{25}$$

IV. GRAPHICAL REPRESENTATION FOR EWP

The numerical values of the two ratios of Wilson coefficients appearing in the previous section are very close to each other

$$\frac{c_9+c_{10}}{c_1+c_2} = -1.139\alpha\ , \qquad \frac{c_9-c_{10}}{c_1-c_2} = -1.107\alpha\ . \tag{26}$$

We used here the leading log values of the Wilson coefficients at m_b [14]

$$c_1 = 1.144\ , \quad c_2 = -0.308\ , \quad c_9 = -1.280\alpha\ , \quad c_{10} = 0.328\alpha\ , \tag{27}$$

with $\alpha = 1/129$. The two values in (26) differ by less that 3%. Therefore, they can be taken as having a common value to a very good approximation

$$\frac{c_9+c_{10}}{c_1+c_2} = \frac{c_9-c_{10}}{c_1-c_2} = \kappa\ , \tag{28}$$

where $\kappa \simeq -1.123\alpha$. As a consequence of this approximate equality, all EWP reduced matrix elements (14) are proportional to the corresponding tree amplitudes (9) with a common proportionality constant

$$b_1 = -\kappa a_1\ , \quad b_2 = -\kappa a_2\ , \quad b_3 = -3\kappa a_3\ , \quad b_4 = \kappa a_4\ , \quad b_5 = \kappa a_5\ . \tag{29}$$

These equalities suggest introducing the following six EWP amplitudes, analogous to the ones used to parametrize tree-level decay amplitudes

$$P_i^{EW} = \kappa i\ , \qquad i = T, C, A, E, P_u, PA_u\ . \tag{30}$$

These amplitudes have a direct graphic interpretation in terms of quark diagrams with one insertion of an electroweak penguin operator. Furthermore, the simple proportionality relation (30) guarantees that the P_i^{EW} amplitudes will satisfy the same hierarchy of sizes as the tree-level amplitudes [3,9].

Decay mode	P_T^{EW}	P_C^{EW}	P_A^{EW}	P_E^{EW}	$P_{P_u}^{EW}$	$P_{PA_u}^{EW}$
$B^+ \to \pi^+\pi^0$	$3/2\sqrt{2}$	$3/2\sqrt{2}$	0	0	0	0
$K^+\bar{K}^0$	0	$1/2$	0	-1	$1/2$	0
$B^0 \to \pi^+\pi^-$	0	1	$1/2$	$-1/2$	$-1/2$	$-1/2$
$\pi^0\pi^0$	$3/2\sqrt{2}$	$1/2\sqrt{2}$	$-1/2\sqrt{2}$	$1/2\sqrt{2}$	$1/2\sqrt{2}$	$1/2\sqrt{2}$
K^+K^-	0	0	$1/2$	0	0	$-1/2$
$K^0\bar{K}^0$	0	$1/2$	1	$1/2$	$1/2$	$1/2$
$B_s \to K^-\pi^+$	0	1	0	$-1/2$	$-1/2$	0
$\bar{K}^0\pi^0$	$3/2\sqrt{2}$	$1/2\sqrt{2}$	0	$1/2\sqrt{2}$	$1/2\sqrt{2}$	0

Table 1. EW penguin contributions to $\Delta S = 0$ transitions in terms of the graphical amplitudes P_i^{EW}.

Inserting the relations (29) into (8) one may express the parameters b_i in terms of P_i^{EW}. Using (13), (19) and (22), EWP contributions to any given decay can be written as a linear combination of the P_i^{EW} amplitudes. The results are given in Table 1 for $\Delta S = 0$ transitions and in Table 2 for $|\Delta S| = 1$ decays.

Decay mode	P_T^{EW}	P_C^{EW}	P_A^{EW}	P_E^{EW}	$P_{P_u}^{EW}$	$P_{PA_u}^{EW}$
$B^+ \to K^0\pi^+$	0	$1/2$	0	-1	$1/2$	0
$K^+\pi^0$	$3/2\sqrt{2}$	$1/\sqrt{2}$	0	$1/\sqrt{2}$	$-1/2\sqrt{2}$	0
$B^0 \to K^+\pi^-$	0	1	0	$-1/2$	$-1/2$	0
$K^0\pi^0$	$3/2\sqrt{2}$	$1/2\sqrt{2}$	0	$1/2\sqrt{2}$	$1/2\sqrt{2}$	0
$B_s \to \pi^+\pi^-$	0	0	$1/2$	0	0	$-1/2$
$\pi^0\pi^0$	0	0	$-1/2\sqrt{2}$	0	0	$1/2\sqrt{2}$
K^+K^-	0	1	$1/2$	$-1/2$	$-1/2$	$-1/2$
$K^0\bar{K}^0$	0	$1/2$	1	$1/2$	$1/2$	$1/2$

Table 2. EW penguin contributions to $|\Delta S| = 1$ transitions in terms of the graphical amplitudes P_i^{EW}.

The results in Tables 1 and 2 agree with a previous analysis of the EWP contributions in quark diagram language [9]. The relation between the EWP amplitudes of [9] and our parameters P_i^{EW} is given by

$$
\begin{aligned}
P_{EW} &= -\frac{3}{2}\lambda_t^{(d)} P_T^{EW} , \qquad & P_{EW}^C &= -\frac{3}{2}\lambda_t^{(d)} P_C^{EW} , \\
P'_{EW} &= -\frac{3}{2}\lambda_t^{(s)} P_T^{EW} , \qquad & P'^C_{EW} &= -\frac{3}{2}\lambda_t^{(s)} P_C^{EW} .
\end{aligned} \tag{31}
$$

The improvement over [9] is that these parameters can be simply expressed through (30) in terms of tree-level graphical amplitudes. Thus, the effects of EWP contributions can be included to a good approximation in a model-independent way without encountering any new hadronic amplitudes. One of the consequences of this simplification is that color-suppression of certain EWP amplitudes is identical to the corresponding suppression of tree amplitudes, and does not require further assumptions about hadronic matrix elements of EWP operators.

V. APPLICATIONS

A. Determination of α from $B \to \pi\pi$ decays

It has been proposed in [5] to determine the weak angle α from a combined measurement of the time-dependent decay rate $B^0(t) \to \pi^+\pi^-$ and time-integrated branching ratios for $B^+ \to \pi^+\pi^0$, $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$ and their CP-conjugated modes. As noted in [8–10], this method is affected by uncertainties arising from the presence of EWP contributions. We will show in the following how their effect can be taken into account in a model-independent way [16].

The angle α is measured through the time-dependent decay rate $B^0(t) \to \pi^+\pi^-$ which contains a term of the form

$$|\langle\pi^+\pi^-|B^0(t)\rangle|^2 = \cdots + |A(B^0 \to \pi^+\pi^-)|\,|A(\bar{B}^0 \to \pi^+\pi^-)|e^{-\Gamma t}\sin(2\alpha+\theta)\sin(\Delta m t)\ , \quad (32)$$

Δm being the mass difference between the two neutral B mass eigenstates. The angle θ is due to the presence of QCD penguins in the $B^0 \to \pi^+\pi^-$ amplitude and is defined as $\theta = \mathrm{Arg}(\tilde{A}(\bar{B}^0 \to \pi^+\pi^-)/A(B^0 \to \pi^+\pi^-))$ (with $\tilde{A}(\bar{B} \to \bar{f}) \equiv e^{2i\gamma}A(\bar{B} \to \bar{f})$).

The idea of [5] is to measure θ through a geometrical construction. An essential ingredient of the method is the equality of the following two decay amplitudes

$$A(B^+ \to \pi^0\pi^+) = \tilde{A}(B^- \to \pi^0\pi^-)\ , \quad (33)$$

which can be therefore taken as the common base of two isospin triangles for the decays $B^+ \to \pi^+\pi^0$, $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$ and their CP-conjugate modes. The angle θ is obtained from this construction as $\theta =\mathrm{Arg}(\tilde{A}(\bar{B}^0 \to \pi^+\pi^-)/A(B^0 \to \pi^+\pi^-))$.

The equality (33) is spoiled in the presence of the EWP terms, in which case one has

$$\sqrt{2}A(B^+ \to \pi^0\pi^+) = -\lambda_u^{(d)}(T+C) + \lambda_t^{(d)}\frac{3}{2}\frac{c_9+c_{10}}{c_1+c_2}(T+C)\ . \quad (34)$$

We made use of the isospin relation (23) for the EWP contribution to this decay.

The amplitude (34) and its CP-conjugate are shown in Figure 1, from which two conclusions are immediately apparent: a) the equality between the decay rates for $B^+ \to \pi^0\pi^+$ and its CP-conjugate holds also in the presence of the EWP amplitudes; b) the value of the angle 2ξ between $A(B^+ \to \pi^0\pi^+)$ and $\tilde{A}(B^- \to \pi^0\pi^-)$ is a calculable function of α alone. A simple calculation gives

$$\tan\xi = \frac{x\sin\alpha}{1+x\cos\alpha}\ , \qquad x \equiv \frac{3}{2}\frac{c_9+c_{10}}{c_1+c_2}\left|\frac{\lambda_t^{(d)}}{\lambda_u^{(d)}}\right| = -0.013\left|\frac{\lambda_t^{(d)}}{\lambda_u^{(d)}}\right|\ , \quad (35)$$

where $|\lambda_t^{(d)}/\lambda_u^{(d)}| = |V_{tb}V_{td}/V_{ub}V_{ud}| \approx |V_{td}/V_{ub}|$. Note that the angle ξ depends only on α and on the parameter x which involves some uncertainty in its CKM factor, but is free of any hadronic uncertainty.

Therefore, the method proposed in [5] can be adapted to include the effects of the EWP by defining the modified amplitudes $\tilde{A}'(\bar{B} \to \bar{f}) = e^{2i\xi}\tilde{A}(\bar{B} \to \bar{f})$ in terms of which the equality (33) is restored. The geometrical construction of [5] can be carried through as before and θ is extracted as

$$\theta = \mathrm{Arg}\frac{\tilde{A}'(\bar{B}^0 \to \pi^+\pi^-)}{A(B^0 \to \pi^+\pi^-)} \mp 2\xi(\alpha) \ . \tag{36}$$

The upper (lower) sign in this formula corresponds to the case when the two triangles are drawn on the same (on opposite) side of the common amplitude (33). As in the original version of this method, there is a four-fold ambiguity in the value of α, arising from the above mentioned freedom in the geometric construction and from having to extract α from $\sin(2\alpha + \theta)$.

Numerically the shift in the angle $\Delta\theta = 2\xi$ induced by EWP contributions is seen to be rather small, of the order of 1.5^o. Therefore, in practice these contributions can be neglected and the results of this analysis are not likely to be of immediate relevance for an extraction of α. However, we use this example to demonstrate that, in principle, the effects of EWP terms can be eliminated in a model-independent manner to allow a determination of the weak phase.

B. Constraints on γ from $B \to K\pi$ decays

Recently the SU(3) relation (17) between EWP contributions in $B^+ \to K^0\pi^+$ and $B^+ \to K^+\pi^0$ was obtained by Neubert and Rosner [11], and was used to derive information on γ from the CP-averaged ratio

$$R_*^{-1} = \frac{2[B(B^+ \to K^+\pi^0) + B(B^- \to K^-\pi^0)]}{B(B^+ \to K^0\pi^+) + B(B^- \to \bar{K}^0\pi^-)} \ . \tag{37}$$

Further constraints on the weak phase were shown to be provided by separate B^+ and B^- branching ratio measurements if rescattering effects can be neglected [12]. In the present section we will review the arguments of [11], and then apply Eq.(18), the second relation between EWP amplitudes in $B \to K\pi$, to the ratio [17]

$$R = \frac{B(B^0 \to K^+\pi^-) + B(\bar{B}^0 \to K^-\pi^+)}{B(B^+ \to K^0\pi^+) + B(B^- \to \bar{K}^0\pi^-)} \ . \tag{38}$$

Our purpose here is to possibly eliminate uncertainties in R due to EWP contributions in a model-independent manner. These contributions were argued in [9,18] to be color-suppressed and were calculated in specific model calculations [17,19] to be very small. Assuming that they can be neglected, and that the same applies to certain rescattering effects, one obtains the bound [17] $R \geq \sin^2\gamma$ which can be useful provided that $R < 1$. Furthermore, measuring the CP asymmetry in $B \to K^\pm\pi^\mp$ would constrain γ even if $R \geq 1$ [18]. Here we will attempt to obtain a model-independent generalization of the bound $R \geq \sin^2\gamma$ including EWP effects [20]. The role of rescattering effects [21], and possible limits on such effects [18,22], were discussed elsewhere.

The amplitudes of the two decay processes appearing in R_*^{-1} are given by [3,9]

$$\begin{aligned}\sqrt{2}A(B^+ \to K^+\pi^0) &= -\lambda_u^{(s)}(T + C + P_{uc} + A) - \lambda_t^{(s)}(P_{ct} - \sqrt{2}P^{EW}(B^+ \to K^+\pi^0)) \ , \\ A(B^+ \to K^0\pi^+) &= \lambda_u^{(s)}(P_{uc} + A) + \lambda_t^{(s)}(P_{ct} + P^{EW}(B^+ \to K^0\pi^+)) \ . \end{aligned} \tag{39}$$

The contribution of the QCD penguin amplitude with an internal charm quark was included in $P_{uc} = P_u - P_c$ and $P_{ct} = P_t - P_c$ by making use of the unitarity of the CKM matrix. Using (17), the first amplitude can be written as

$$\sqrt{2}A(B^+ \to K^+\pi^0) = -|\lambda_u^{(s)}|(T + C)(e^{i\gamma} - \delta_{EW}) - \lambda_u^{(s)}(P_{uc} + A) - \lambda_t^{(s)}(P_{ct} + P^{EW}) \ , \tag{40}$$

where $P^{EW} \equiv P^{EW}(B^+ \to K^0\pi^+)$ and

$$\delta_{EW} = -\frac{3}{2}\left|\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}\right|\frac{c_9 + c_{10}}{c_1 + c_2} \ , \tag{41}$$

where $|\lambda_t^{(s)}/\lambda_u^{(s)}| = |V_{tb}V_{ts}/V_{ub}V_{us}| \approx |V_{cb}/V_{us}V_{ub}|$.

Therefore,

$$R_*^{-1} = \frac{|\epsilon e^{i\phi_T}(e^{i\gamma} - \delta_{EW}) + \epsilon_A e^{i\phi_A}e^{i\gamma} - e^{i\phi_P}|^2 + |\epsilon e^{i\phi_T}(e^{-i\gamma} - \delta_{EW}) + \epsilon_A e^{i\phi_A}e^{-i\gamma} - e^{i\phi_P}|^2}{|\epsilon_A e^{i\phi_A}e^{i\gamma} - e^{i\phi_P}|^2 + |\epsilon_A e^{i\phi_A}e^{-i\gamma} - e^{i\phi_P}|^2} \ , \tag{42}$$

where we denote

$$\epsilon e^{i\phi_T} = \frac{|\lambda_u^{(s)}|(T + C)}{|\lambda_t^{(s)}||P_{ct} + P^{EW}|} \ , \qquad \epsilon_A e^{i\phi_A} = \frac{|\lambda_u^{(s)}|(P_{uc} + A)}{|\lambda_t^{(s)}||P_{ct} + P^{EW}|} \ , \tag{43}$$

and $\phi_P = \mathrm{Arg}(P_{ct} + P^{EW})$. To first order in the small parameter $\epsilon \simeq 0.24$ [11], obtained through [6]

$$\epsilon = \sqrt{2}\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\frac{|A(B^+ \to \pi^0\pi^+)|}{|A(B^+ \to K^0\pi^+)|} \ , \tag{44}$$

the ratio R_*^{-1} is independent of the rescattering parameter ϵ_A and is given by

$$R_*^{-1} = 1 - 2\epsilon\cos\Delta\phi(\cos\gamma - \delta_{EW}) + \mathcal{O}(\epsilon^2) \ , \qquad \Delta\phi = \phi_T - \phi_P \ . \tag{45}$$

This then implies the bound [11]

$$|\cos\gamma - \delta_{EW}| \geq \frac{|1 - R_*^{-1}|}{2\epsilon} \ , \tag{46}$$

which can set new constraints on γ if $R_* \neq 1$. The central value of a recent measurement [23], $R_* = 0.47 \pm 0.24$, lies two standard deviations away from one.

We now proceed to study the ratio R. Applying the relation (18) to the corresponding EWP contributions, we find

$$\begin{aligned} A(B^0 \to K^+\pi^-) &= -\lambda_u^{(s)}(T + P_{uc}) - \lambda_t^{(s)}(P_{ct} + P^{EW}) \\ &+ \frac{3}{4}\lambda_t^{(s)}\left[\frac{c_9 - c_{10}}{c_1 - c_2}(-T + C + A - E) - \frac{c_9 + c_{10}}{c_1 + c_2}(-T - C + A + E)\right] \\ &= -|\lambda_u^{(s)}|(T + P_{uc})\left(e^{i\gamma} - \delta'_{EW}\right) - \lambda_t^{(s)}(P_{ct} + P^{EW}) \ , \end{aligned} \tag{47}$$

where P^{EW} is defined as in (40), and δ'_{EW} (containing the EWP contribution) is defined by

$$\begin{aligned} \delta'_{EW} &= -\frac{3}{4}\left|\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}\right|\left[\frac{c_9 - c_{10}}{c_1 - c_2}\cdot\frac{-T + C + A - E}{T + P_{uc}} - \frac{c_9 + c_{10}}{c_1 + c_2}\cdot\frac{-T - C + A + E}{T + P_{uc}}\right] \\ &\simeq -\frac{3}{2}\left|\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}\right|\kappa\frac{C - E}{T + P_{uc}} \ . \end{aligned} \tag{48}$$

Here we made use of the approximate equality (28).

The ratio R (38) can then be written as

$$R = \frac{|\epsilon' e^{i\phi'_T}(e^{i\gamma} - \delta'_{EW}) - e^{i\phi_P}|^2 + |\epsilon' e^{i\phi'_T}(e^{-i\gamma} - \delta'_{EW}) - e^{i\phi_P}|^2}{|\epsilon_A e^{i\phi_A} e^{i\gamma} - e^{i\phi_P}|^2 + |\epsilon_A e^{i\phi_A} e^{-i\gamma} - e^{i\phi_P}|^2} \ , \tag{49}$$

where

$$\epsilon' e^{i\phi'_T} = \frac{|\lambda_u^{(s)}|(T + P_{uc})}{|\lambda_t^{(s)}||P_{ct} + P^{EW}|} \ . \tag{50}$$

Expanding again in powers of ϵ' and keeping only the linear terms, we obtain

$$R = 1 - 2\epsilon' \cos(\Delta\phi' + \delta\phi)|\cos\gamma - \delta'_{EW}| + \mathcal{O}(\epsilon'^2) + \mathcal{O}(\epsilon_A) \ , \tag{51}$$

where $\Delta\phi' = \phi'_T - \phi_P$, $\delta\phi = \mathrm{Arg}(\cos\gamma - \delta'_{EW})$.

Let us compare the structure of the two ratios R (51) and R_*^{-1} (45) to first order in the small parameter $\epsilon' \approx \epsilon$. (These two parameters are equal up to corrections of order $|C/T| \simeq 0.2$ and $|P_{uc}/T|$). First, we note that R depends on final state rescattering (ϵ_A) whereas R_*^{-1} is unaffected by such effects. This feature was already noted in [11]. The dependence of these ratios on EWP contributions is encoded in the parameters δ'_{EW} and δ_{EW}. Whereas δ_{EW} (41) is real and is given in terms of known Wilson coefficients and CKM factors, δ'_{EW} (48) is in general complex and contains also the ratio $(C-E)/(T+P_{uc})$ depending on tree-level hadronic matrix elements. One usually assumes that this ratio is smaller than one, given roughly by the color-suppression factor measured in $B \to \bar{D}\pi$ [24]. Thus

$$|\delta'_{EW}/\delta_{EW}| \simeq |C/T| \simeq 0.2 \ . \tag{52}$$

Namely, EWP effects in R are smaller than in R_*^{-1} by a factor of about 5, in accord with [9,18]. A much smaller value than (52) was obtained in a model-dependent calculation [17].

Neglecting rescattering effects in $B^+ \to K^0\pi^+$ [21,22], (51) implies the bound

$$|\cos\gamma - \delta'_{EW}| \geq \frac{|1-R|}{2\epsilon'} \ , \tag{53}$$

quite similar to (46). δ'_{EW} has a very small magnitude, $|\delta'_{EW}| \simeq 0.2\delta_{EW} = 0.13$, where we used $\delta_{EW} = 0.63$ [11]. Therefore, in spite of the uncertainty in the phase of δ'_{EW}, this constraint on γ can potentially become useful provided that a value for R is measured which is *different* from 1 (not necessarily smaller than 1 as required by [17]). For a given value of $|\delta'_{EW}|$, the allowed region for $\cos\gamma$ is given by the constraint

$$|\cos\gamma| > \frac{|1-R|}{2\epsilon'} - |\delta'_{EW}| \ , \tag{54}$$

provided that

$$1 + |\delta'_{EW}| \geq \frac{|1-R|}{2\epsilon'} \geq |\delta'_{EW}| \ . \tag{55}$$

Eqs. (54) and (55) exclude a region around $\cos\gamma = 0$. For $\epsilon' \simeq 0.24$, $|\delta'_{EW}| = 0.13$, this requires $0.06 \leq |1-R| \leq 0.54$. The presently measured value of R, $R = 1.0 \pm 0.4$ [23], largely overlaps with this region. We note that fixing the strong phase of δ'_{EW} by theoretical arguments can further sharpen these bounds.

It is possible to improve this constraint on γ by combining the data on R with a measurement of the pseudo-asymmetry A_0 [18]

$$A_0 = \frac{B(B^0 \to K^+\pi^-) - B(\bar{B}^0 \to K^-\pi^+)}{B(B^+ \to K^0\pi^+) + B(B^- \to \bar{K}^0\pi^-)} \ . \tag{56}$$

One finds, to first order in ϵ',

$$A_0 = 2\epsilon' \sin\gamma \sin\Delta\phi' + \mathcal{O}(\epsilon'^2) + \mathcal{O}(\epsilon_A) \ . \tag{57}$$

For $\delta'_{EW} = 0$ and for given ϵ', R (51) and A_0 (57) determine γ up to a fourfold ambiguity [18]. In reality, since $|\delta'_{EW}| \simeq 0.13$ is very small, the solutions for γ are given by narrow bands corresponding to the uncertainty in the strong phase of δ'_{EW}.

VI. CONCLUSIONS

Electroweak penguin amplitudes play an important role in various attempts to determine CKM phases from rate measurements. In $\Delta S = 1$ B decays their contribution is comparable to that arising from the current-current terms in the weak Hamiltonian. It is therefore important to have an accurate theoretical control over their effect. Based on flavor SU(3) and dominance of Q_9 and Q_{10} EWP operators, we presented a general method for relating the EWP contributions to tree-level amplitudes in B decays to a pair of charmless mesons. This reduces in a model-independent way the number of hadronic amplitudes parametrizing B decays. SU(3) breaking effects on these relations were studied in some cases in a model-dependent way and were found to be small [11].

We applied these relations to three cases, a determination of α from $B \to \pi\pi$ and two ways of constraining γ from $B \to K\pi$ decays. In the first case (where only isospin was used) and when studying the ratio R_*^{-1} in $B^\pm$ decays [11] (where SU(3) flavor was employed), constraints were obtained which were free of hadronic uncertainties. On the other hand, a study of γ through the ratio R in $B^{0,\pm} \to K\pi^\pm$ depends on the knowledge of the ratio of certain tree-level amplitudes. Neglecting rescattering effects, we used the smallness of this ratio to argue that useful constraints on γ can be obtained from R provided that R is different from one.

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APPENDIX A: FOUR-QUARK OPERATORS WITH WELL-DEFINED SU(3) TRANSFORMATION PROPERTIES

We give in this Appendix the four-quark operators appearing in the weak Hamiltonian for b decays. They are defined as (in notation $\bar{q}_1\bar{q}_3 q_2 \simeq (\bar{b}q_1)(\bar{q}_2 q_3)$)

- $\Delta S = +1$ operators

$$\overline{\mathbf{15}}_{I=1} = -\frac{1}{2}(\bar{u}\bar{s}u + \bar{s}\bar{u}u) + \frac{1}{2}(\bar{d}\bar{s}d + \bar{s}\bar{d}d) \tag{A1}$$

$$\overline{\mathbf{15}}_{I=0} = -\frac{1}{2\sqrt{2}}(\bar{u}\bar{s}u + \bar{s}\bar{u}u) - \frac{1}{2\sqrt{2}}(\bar{d}\bar{s}d + \bar{s}\bar{d}d) + \frac{1}{\sqrt{2}}\bar{s}\bar{s}s \tag{A2}$$

$$\mathbf{6}_{I=1} = -\frac{1}{2}(\bar{u}\bar{s}u - \bar{s}\bar{u}u) + \frac{1}{2}(\bar{d}\bar{s}d - \bar{s}\bar{d}d) \tag{A3}$$

$$\overline{\mathbf{3}}^{(a)}_{I=0} = -\frac{1}{2}(\bar{u}\bar{s}u - \bar{s}\bar{u}u) - \frac{1}{2}(\bar{d}\bar{s}d - \bar{s}\bar{d}d) \tag{A4}$$

$$\overline{\mathbf{3}}^{(s)}_{I=0} = \frac{1}{2\sqrt{2}}(\bar{u}\bar{s}u + \bar{s}\bar{u}u) + \frac{1}{2\sqrt{2}}(\bar{d}\bar{s}d + \bar{s}\bar{d}d) + \frac{1}{\sqrt{2}}\bar{s}\bar{s}s\,. \tag{A5}$$

- $\Delta S = 0$ operators

$$\overline{\mathbf{15}}_{I=\frac{3}{2}} = -\frac{1}{\sqrt{3}}(\bar{u}\bar{d}u + \bar{d}\bar{u}u) + \frac{1}{\sqrt{3}}\bar{d}\bar{d}d \tag{A6}$$

$$\overline{\mathbf{15}}_{I=\frac{1}{2}} = -\frac{1}{2\sqrt{6}}(\bar{u}\bar{d}u + \bar{d}\bar{u}u) + \frac{1}{2}\sqrt{\frac{3}{2}}(\bar{s}\bar{d}s + \bar{d}\bar{s}s) - \frac{1}{\sqrt{6}}\bar{d}\bar{d}d \tag{A7}$$

$$\mathbf{6}_{I=\frac{1}{2}} = \frac{1}{2}(\bar{d}\bar{s}s - \bar{s}\bar{d}s) + \frac{1}{2}(\bar{u}\bar{d}u - \bar{d}\bar{u}u) \tag{A8}$$

$$\overline{\mathbf{3}}^{(a)}_{I=\frac{1}{2}} = -\frac{1}{2}(\bar{u}\bar{d}u - \bar{d}\bar{u}u) + \frac{1}{2}(\bar{d}\bar{s}s - \bar{s}\bar{d}s) \tag{A9}$$

$$\overline{\mathbf{3}}^{(s)}_{I=\frac{1}{2}} = \frac{1}{2\sqrt{2}}(\bar{u}\bar{d}u + \bar{d}\bar{u}u) + \frac{1}{2\sqrt{2}}(\bar{d}\bar{s}s + \bar{s}\bar{d}s) + \frac{1}{\sqrt{2}}\bar{d}\bar{d}d\,. \tag{A10}$$

We also list the relative normalization between SU(3) reduced matrix elements used in this paper and in [3]:

$$a_1 = -\frac{1}{\sqrt{3}}\{1\}, \quad a_2 = -2\sqrt{\frac{2}{3}}\{8_1\}, \quad a_3 = -\frac{2}{\sqrt{3}}\{8_2\}, \quad a_4 = \frac{4}{\sqrt{5}}\{8_3\}, \quad a_5 = 3\sqrt{\frac{6}{5}}\{27\}\ . \tag{A11}$$

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FIGURES

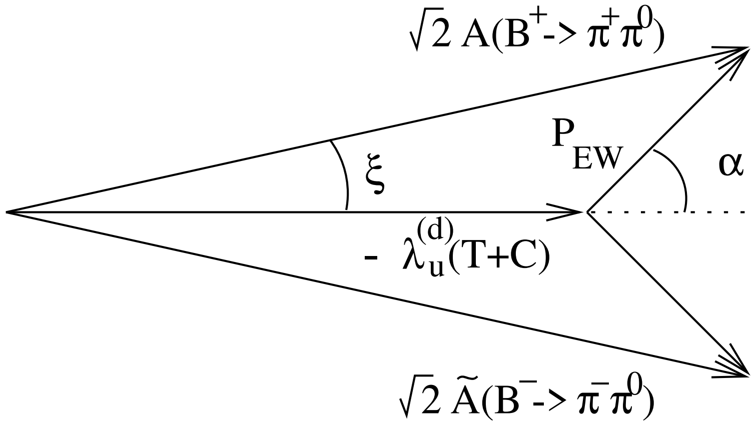

FIG. 1. EW penguin effects in the decay amplitude $A(B^+ \to \pi^0\pi^+)$ and its charge conjugate $\tilde{A}(B^- \to \pi^0\pi^-) \equiv e^{2i\gamma}A(B^- \to \pi^0\pi^-)$.