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Injector for a Laser Linear Collider[1]

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Abstract. The injector for 2×1 *km* long laser driven linac considered. Injector is a race track with long straight sections. These sections squeezed together for a compact size (a Kayak-Paddle like cooler). In straight section the short period wigglers and RF cavities installed in series one by one for keeping the energy along the straight section practically constant. This injector is able to provide the invariant emittances of the order $5\cdot 10^{-8} cm\cdot rad$ and $2\cdot 10^{-9} cm\cdot rad$ for horizontal and vertical directions respectively. Bunch population required below 10^7 reduces the IBS effects.

1. INTRODUCTION

Injector –is a mostly important among components of a laser based accelerator. Tiny dimensions of accelerating structure require extremely small emittances. The pass holes must have dimensions as a fraction of accelerating wavelength. For example, if this wavelength is $\lambda_{ac} \cong 10\mu m$, then the passing holes dimensions need to be made as $\delta \cong 0.2\cdot \lambda_{ac}$. Taking into account that this must have a safety margins as big as ten sigma of the beam, one can estimate $\sigma \cong \sqrt{(\gamma\varepsilon)\cdot\beta/\gamma} \cong 0.1\cdot 0.2\cdot\lambda_{ac}$. For focusing elements displaced with period, say $3cm$, the envelope function could be β~$12cm$, so emittance required must be $\gamma\varepsilon \cong 4\cdot 10^{-4}\lambda_{ac}^2\cdot\gamma/\beta \cong 6\cdot 10^{-8} cm\cdot rad$, $\gamma \cong 2\cdot 10^3 (1GeV)$. With RF focusing, effective focusing parameter could reach $k \cong 2\cdot 10^5 \cdot Sin\varphi [m^{-2}]$ for the phase deflection φ. If this phase deflection applied along some part of the structure, the envelope function about β~$3cm$ could be reached. Together with increased energy up to ~$2GeV$ all this yields $\gamma\varepsilon \cong 5\cdot 10^{-7} cm\cdot rad$. Emittance of $\gamma\varepsilon \cong 10^{-6} cm\cdot rad$ is probably maximal allowed emittance value.

Supposed that the beams with extreme emittances could be prepared in a damping ring [1]. Incoherent synchrotron radiation carries out momenta parallel to instant direction of motion. RF, as usual, restores the longitudinal component of momenta, what yields an effective cooling. Equilibrium emittance depends on details of damping ring optics. In particular so called dispersion invariant[3] is important parameter in minimization of emittance. Mostly efforts were spend to find a structure, having dispersion invariant as small as possible.

As the repetition rate is an extensive parameter for the luminosity gain, one needs to prepare the bunches with such emittances as fast as possible. This in its turn requires high rate of energy losses in a cooler, as the cooling is going by synchrotron radiation. We will see, that equilibrium invariant emittances achievable *do not depend on the beam energy of the cooler*. So the working energy increase for the cooler helps much for

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[3] See formula (6) below.

reduction of Intra-Beam Scattering (IBS) process in a bunch. For such a tiny emittances even lowering the number of the particles in the bunch down to $N \cong 10^6$ does not help much against IBS. This could be overpassed dynamically, however.
So the process of preparing the bunch with extreme emittance, in brief, is the following. In a cooler the bunch of maximal phase density prepared. As the cooling mostly is going in straight sections filled with wigglers, the dispersion invariant is the smallest possible one. Length of the bunch is around 1- 10 *mm*. Then the bunching process arranged by the applying the inverse Free Electron Laser process in the ring, having the wiggler in it's structure. The initial bunch is subdivided into micro-bunches with the spacing of the laser wavelength. After that the bunch passed through special system where it is scrapped and all external (in phase space) particles are eliminated. After final acceleration this train of particles are directed into accelerating structure. Along the accelerating length in main linac some diaphragms required obviously.
The injector of the type described below is used in a Laser Linear Collider injection complex [2,3].

The straight sections squeezed together for a compact size, so the back and forward trajectories are congruent. The longer the straight section is–the smaller influence of bends could be made. The bends itself could be made to give a small input into cooling dynamics. Here the particle needs to re-radiate its full energy a few times. The rate of energy losses is given by

$$d\gamma / ds \cong -\frac{2}{3} r_0 \frac{K^2}{\lambda\!\!\!^{-2}} \gamma^2 \,, \tag{2}$$

where $K = eH_\perp \lambda\!\!\!^{-} / mc^2$, $H_\perp$–is a magnetic field value in the wiggler, $2\pi\lambda\!\!\!^{-}$–is the wiggler period. We would like to mention here, that $d\gamma / ds$ *is not a function of the wiggler period*. The γ– factor on right side of (2) needs to be considered as a constant. Substituting here for estimation $\gamma \cong 2 \cdot 10^3 (\mathit{1\ GeV})$, $K \approx 5$ ($H_\perp \cong 0.5\,T$, $2\pi\lambda\!\!\!^{-} \cong 10\,cm$), one can obtain $\frac{d\gamma}{ds} \cong -\frac{2}{3} 2.8 \cdot 10^{-13} \cdot 10 \cdot 4 \cdot 10^6 \cong 7.5 \cdot 10^{-6}$ [*1/cm*] or 0.00075 [*1/m*]. For so called characteristic damping length one can obtain

$$l_s = -\frac{\gamma}{d\gamma / ds} = \frac{3}{2} \frac{\lambda\!\!\!^{-2}}{r_0 K^2 \gamma} \,. \tag{3}$$

One can see, that it has *linear* dependence on energy. For parameters under discussion $l_s = -\frac{\gamma}{d\gamma / ds} \cong \frac{2 \cdot 10^3}{7.5 \cdot 10^{-4}} \cong 2.6 \cdot 10^6 m$. This means that after passing such a length, the energy re–radiated by the particle will be the same as it's initial one. The cooling time associated with this length is

$$\tau_{cool} \cong l_s / c = -\frac{\gamma}{c d\gamma / ds} = \frac{3}{2} \frac{\lambda\!\!\!^{-2}}{c r_0 K^2 \gamma} \tag{4}$$

One can see from here, that the cooling (damping) time *does not depend on the wiggler period*. From (4) one can obtain the estimation $\tau_{cool} \cong 2.6 \cdot 10^6 / 3 \cdot 10^8 \cong 8.6 \cdot 10^{-3} s$, or 8.6 *ms*. This cooling time obviously does not depend on the length of the straight section, as the influence of the bends was neglected –shorter the length, faster the revolution.

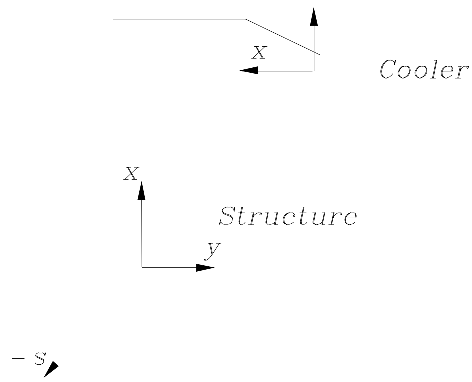
Emittance dynamics defined by equations (averaging over the period)

$$\frac{d\varepsilon_x}{ds} = \left\langle \left(H_x + \frac{\beta_x}{\gamma^2} \right) \frac{d(\Delta E / E)^2_{tot}}{ds} \right\rangle - 2\alpha_x \varepsilon_x \,, \tag{5}$$

with similar equation for vertical motion, where

$$H_{x,y} = \frac{1}{\beta_{x,y}} \left(\eta^2_{x,y} + (\beta_{x,y} \eta'_{x,y} - \frac{1}{2} \beta'_{x,y} \eta_{x,y})^2 \right), \tag{6}$$

$\eta_{x,y}$ –are dispersion functions. Derivatives are taken over longitudinal direction, which is *s*, see Fig. 2 below. In Fig. 2 there is represented the relationship between coordinate systems used in [2] and in this publication. One can suggest another orientation of the wiggler field polarization and accelerating structure.


x
Cooler
x
Structure
y
– s

$$\frac{d\varepsilon_x}{ds}=\left\langle\left(1+K_x^2 Cos^2(s/\lambda\!\!\!^-)\right)\frac{\beta_x}{\gamma^2}\frac{d(\Delta E/E)_{tot}^2}{ds}\right\rangle-2\alpha_x\varepsilon_x \qquad (9)$$

For vertical emittance $K_x = 0$ (and the wiggler field has no horizontal polarization). Equilibrium emittances defined by condition $d\varepsilon_{x,y}/ds = 0$. For quantum excitation the source (7a) does not depend on emittance. Combining (2), (7), (9) for quantum excitation *only* we obtain

$$(\gamma\varepsilon_x)\cong(1/2)\cdot\lambda\!\!\!^-_C\bar{\beta}_x(1+K_x^2/2)\gamma/\rho_x\cong(1/2)\cdot\lambda\!\!\!^-_C\bar{\beta}_x(1+K_x^2/2)K_x/\lambda\!\!\!^- \qquad (10)$$

$$(\gamma\varepsilon_y)\cong(1/2)\cdot\lambda\!\!\!^-_C\bar{\beta}_y\gamma/\rho_x\cong(1/2)\cdot\lambda\!\!\!^-_C\bar{\beta}_y K_x/\lambda\!\!\!^-, \qquad (11)$$

where $\bar{\beta}_{x,y}$ – are averaged envelope functions in the wiggler. The last formulas together with (3), $\tau_{cool}\cong(3/2)\cdot(\lambda\!\!\!^{-2}/cK^2\gamma)$, define the cooling dynamics under SR. One can see that equilibrium invariant emittances *do not depend on energy*. In addition, quantum equilibrium *vertical emittance* and the cooling time do not depend on the wiggler period. According (10), (11), the ratio $(\gamma\varepsilon_x)/(\gamma\varepsilon_y)\cong(1+K_x^2/2)\bar{\beta}_x/\bar{\beta}_y$.

Substitute for estimation $\bar{\beta}_{x,y}\approx 1m$, $\lambda\!\!\!^-\cong 5cm$, $K\cong 5$, one can obtain for *quantum emittances* the following estimations

$$(\gamma\varepsilon_x)\cong 2.5\cdot 10^{-8}\ cm\cdot rad,$$

$$(\gamma\varepsilon_y)\cong 2\cdot 10^{-9}\ cm\cdot rad.$$

Let us compare the emittances from (10), (11) with the fundamental one (1). Combining (10), (11), one can obtain

$$(\gamma\varepsilon_x)(\gamma\varepsilon_y)(\gamma l_b(\Delta p/p_0)\cong(1/4)\cdot(\lambda\!\!\!^-_C)^2\cdot(1+K_x^2/2)K_x^2(\bar{\beta}_x\bar{\beta}_y/\lambda\!\!\!^{-2})(\gamma l_b(\Delta p/p_0). \qquad (12)$$

To reach a quantum limit, one need to have

$$(1+K_x^2/2)K_x^2(\bar{\beta}_x\bar{\beta}_y/\lambda\!\!\!^{-2})(\gamma l_b(\Delta p/p_0))\approx\lambda\!\!\!^-_C N\cdot(2\pi)^3. \qquad (13)$$

Substitute here in extreme case $(\bar{\beta}_x\bar{\beta}_y/\lambda\!\!\!^{-2})\approx 1$, $(1/2)(1+K_x^2/2)K_x^2\approx(2\pi)^3$ one need to have

$$\gamma l_b(\Delta p/p_0)\approx\lambda\!\!\!^-_C N. \qquad (14)$$

As $\delta'=\gamma l_b/N$ –is an average distance between particles in the rest frame, (14) has a clear physical meaning like individual phase space cell must be of the order of Compton length , $\delta'\cdot(\Delta p/p_0)\approx\lambda\!\!\!^-_C$. Substitute for estimation, $l_b\cong 1cm$, $\gamma\cong 2\cdot 10$

Even in absence of artificial coupling, using (15), (16), we are coming to conclusion, that there is a coupling between horizontal and vertical degrees of freedom like[7]

$$\left(1+K_x^2/2\right)\frac{\varepsilon_y}{\beta_y}=\frac{\varepsilon_x}{\beta_x}. \tag{17}$$

This ratio of emittances is the same as for quantum excitation[8], but here we have *a real* coupling. Substitute (17) into (15), (16) we obtain

$$\varepsilon_x^{5/2}\cong\frac{3}{2}\cdot\frac{\left(1+K_x^2/2\right)^{3/2}}{K^2}\frac{N}{\gamma^6}\frac{\beta_x^{3/2}\cdot r_0^2\cdot\lambda\!\!\!^-{}^2\ln_C}{l_b\beta_y^2}\ . \tag{18}$$

One can see, that there is a strong dependence on the beam energy $\varepsilon_x\sim\gamma^{-12/5}$.

For $N\cong 10^9$, $\gamma\cong 2\cdot 10^3$, $l_b\cong 1cm$, $\overline{\beta}_{x,y}\approx 1m$, $\lambda\!\!\!^-\cong 5cm$, $K\cong 5$, we obtain for *IBS emittances*

$$\gamma\varepsilon_x\cong 2\cdot 10^{-5}\ cm\cdot rad\ ,$$
$$\gamma\varepsilon_y\cong 1.5\cdot 10^{-6}\ cm\cdot rad\ .$$

So the only way to come to emittances as required, compared with ones defined by quantum process only– is increasing the energy of the beam and following *scrapping* the extra particles. For example raising energy up to, say 10 *GeV* will drop emittance about three orders of magnitude. For successful operation of Laser Linear Collider the only $N\approx 10^6$ particles are required [2].

In [8] there was considered the conditions required under which the system comes to equilibrium. General output is that the system must have *positive* longitudinal mass, $m_s=m\gamma^3/(1-\eta\gamma^2)$ i.e. *negative* momentum compaction factor η. KPC satisfies this requirement, having $\eta\cong-0.0001$.

3. OSC METHOD

From previous part we could see, that the only source of damping is the classical synchrotron radiation. Namely this radiation is responsible for damping time like (3). If some other source of damping is present in the system, then resulting damping time τ_{res} will be described

$$\frac{1}{\tau_{res}}\cong\frac{1}{\tau_{cool}}+\frac{1}{\tau_{osc}}\cong\frac{c}{l_s}+\frac{1}{\tau_{osc}}=\frac{2}{3}\frac{cr_0K^2\gamma}{\lambda\!\!\!^-{}^2}+\frac{1}{\tau_{osc}}, \tag{19}$$

where τ_{osc}– is effective damping time associated with this additional cooling process. This additional cooling process might be Optical Stochastic Cooling (OSC) method [7]. This is basically a stochastic cooling with the bandwidth extended to optical one. The cooling time in this method is associated with the number of the particles in the bandwidth as the following

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[7] In [10] the condition was fond for the coupling, arising from IBS in an ordinary damping ring as $(\gamma\varepsilon_y)/(\gamma\varepsilon_y)\cong<\sqrt{\beta_y}>/(\gamma^2<H_x/\sqrt{\beta_y}>\approx R/(\gamma l)$, where R -is an average radius of the ring, l- is the length of period. One can see that these expressions are the same in the wiggler.

[8] As the functions in the angular brackets in formula (5) are, formally, the same.

$$\tau_{cool} \cong N_u \cdot T\,, \tag{20}$$

where N_u – is the number of the particles in the bandwidth, T–is period of revolution.

$$N_u \cong \frac{f}{\Delta f} N \frac{\lambda_u}{l_b} \cong \frac{f}{\Delta f} N \frac{2\pi\hat{\lambda}}{l_b\,\gamma^2}\,, \tag{21}$$

where $2\pi\hat{\lambda}$ – is the wiggler period, λ_u – is the central wavelength of radiation amplified. Amplification coefficient κ of optical amplifier can be expressed as following [7]

$$\kappa \cong \frac{(\gamma\varepsilon_s)}{r_0 N}\frac{\Delta f}{f}\,, \tag{22}$$

where $(\gamma\varepsilon_s)$ – is an invariant longitudinal emittance, see (1), $\Delta f / f$ – is a relative bandwidth, N – is the number of the particles. One can see the only phase density in longitudinal direction $\rho_\varepsilon = (\gamma\varepsilon_s)/N$ is important for the level of amplification.

The decrease of emittance is equal to

$$\frac{\varepsilon_f}{\varepsilon_0} \cong \frac{1}{\alpha N_u}\,. \tag{23}$$

For $N \cong 10^{10}$, $\lambda_u \cong 10^{-4}\ cm = 1\,\mu m$, $l_b \cong 1 cm$, $(\Delta f\ /\ f) \cong 20\%$ one can expect $N_u \cong 5\cdot 10^{10}\cdot 10^{-4} \cong 5\cdot 10^{6}$ and $\varepsilon_f / \varepsilon_0 \cong 2.7\cdot 10^{-5}$. This is of course the maximal possible value of cooling.

If this cooling system is forced to work with electron/positron gas close to generation, one can see from (1) that amplification (22) becomes

3.2. Optical amplifier

Formulas (19), (20), (21) give an idea about amplification required and the power contained in the laser flash. Two examples considered below use these formulas.

Example 1.

For $N \cong 10^{10}$, $l_B \cong 15cm$, $M = 5$, $(\Delta E / E) \cong 10^{-3}$, $\gamma \cong 10^3$ (500 *MeV*), $\lambda \cong 1\mu\ m$, optical amplifier must be able to have amplification about $\kappa \approx 300$, peak power about 5 *kW*, average power about 25 *W* with repetition rate *f* of the order of 10 *MHz*. Number of the particles in the bandwidth $N_S \cong 3 \cdot 10^5$ defines the number of the turns and the damping time $\tau_C \cong N_S / f \cong 30ms$. Emittance reduction $\varepsilon_f / \varepsilon_0 \cong 1/\alpha N_S \cong 10^{-3}$.

Example 2.

For $N \cong 10^8$, $l_B \cong 5cm$, $M = 5$, $(\Delta E / E) \cong 10^{-4}$, $\gamma \cong 10^3$ (500 *MeV*), $\lambda \cong 1\mu\ m$, optical amplifier must be able to have amplification about $\kappa \approx 100$, peak power about 225 *W*, average power about 0.075 *W* with repetition rate of 2 *MHz*[9]. Number of the particles in the bandwidth $N_S \cong 2 \cdot 10^3$ defines the number of the turns and the damping time $\tau_C \cong N_S / f \cong 1ms$. Emittance reduction $\varepsilon_f / \varepsilon_0 \cong 1/\alpha N_S \cong 7 \cdot 10^{-2}$.

The parameters described above look as realistic from the energetics. Optical amplifier needs to be done with lowest phase distortion and minimal time delay.

3.3. Installation in a cooler

An example of installation of OSC system for electron/positron cooler is represented in the Fig. 3 below.

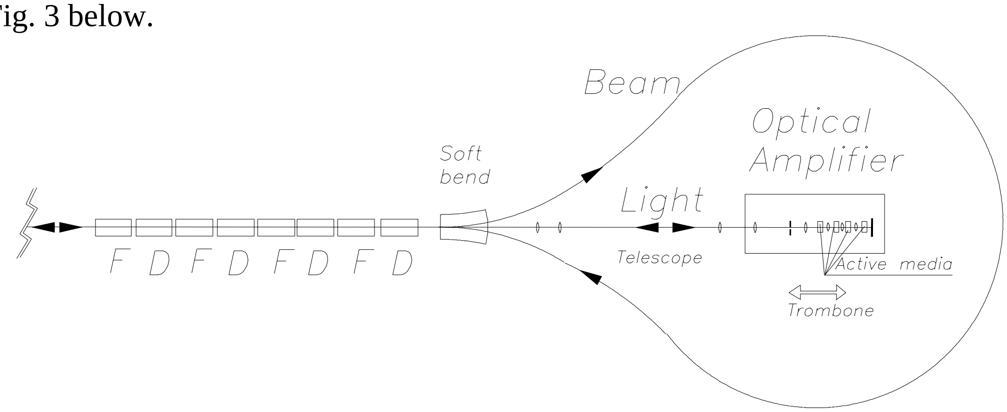

FIGURE 3. Example of installation of OSC system in a cooler.

For the parameters we used above, the period of the lenses must be $2L \cong 2\gamma^2\lambda / (1 + K^2)$ what is about 2 *m*. More hard radiation from the dipole wigglers installed in straight section has *different* wavelength and does not interfere with optical amplifier operating at lower wavelength around $1\,\mu m$. Optical amplifier installed at stabilized platform. For the fine phase adjustments one can use the dual prism system. Initial phase adjustments made by movement the table (trombone). Optical telescopes at both ways used for proper conjunction the radiation from the wiggler and amplifier.

[9] Low frequency of revolution is result of the presence of long straight sections in a Kayak-Paddle type cooler.

TABLE 1. Parameters of amplifiers

Amplifier	**Dye**	**Ti:Al_2O_3**
Wavelength	$\lambda \approx 340 \div 540\ nm$	
Life time	$\tau_L \cong 5\ ns$	$\tau_L \cong 3.5\,\mu s$
Absorption cross section	$\sigma_{01} \cong 2\cdot 10^{-16} \div 4\cdot 10^{-16}\ \mathrm{cm}^2$	$\sigma_{01}(490nm) \cong 10^{-19}\ cm^2$
Emission cross section	$\sigma_{10} \cong 2\cdot 10^{-16} \div 4\cdot 10^{-16}$	$\sigma_{10}(790nm) \cong 3\cdot 10^{-19}\ cm^2$
Density	$n_0 \approx 10^{17}\ cm^{-3}$	$n_0 \approx 10^{20}\ cm^{-3}$
Absorption length, $l_{ab} \cong 1/n_0\sigma_{01}$	$0.05 \div 0.1\ cm$	$0.1\ cm$
Pumping area, $S_{pump} \cong \lambda \cdot l_{ab}$	$5\cdot 10^{-6} cm^2$	$7\cdot 10^{-6} cm^2$
Saturation power density, $P_{sat} \cong \hbar\omega\ n_0 l_{ab} / \tau_L$	$\leq 100\ kW / cm^2$	$\leq 1\ MW / cm^2$
Pumping power, $I = P_{sat} \cdot S_{pump}$ /stage	0.5 *W/stage*	7

The other one with gradient wigglers. As the trajectory is basically a straight line radiation instability does not activated up to significant gradients. Both structures give about the same values of emittances in agreement with calculated in section 2. For bending arcs also few optics tested. The best result gives FD structure. Radiation instability compensated in straight sections. Investigation of dynamic aperture and chromaticity compensation is in progress.

5. OTHER APPLICATIONS OF THE COOLER

The ring of the type described has an interesting application–as a gamma quanta's and positronium generator. For this purposes electron and positron beam made to *move in the same direction*. In the bending arcs the electrons and positrons are going face to face bypassing each other in the vertical bump, arranged with electrostatic separators. Interaction of electrons and positrons is going in a straight section. For proper longitudinal spacing (congruence) of e^+ and e^- bunches, the RF cavities must be moved to the bending arcs.

Cross-section have a dependence like $\sigma_{\uparrow\downarrow} \propto \pi r_0^2 c / v_{rel}$ for para positronium (arrows mark e^+, e^- spin orientations), having zero momentum, where v_{rel} – is a relative velocity of particles and formally could reach a big value [11]. For ortho-positronium, which decays in three gammas, the cross section is about $\alpha = e^2 / \hbar c$ times smaller. The luminosity for a single bunch in a cooler could be estimated like

$$Lum \approx \frac{N^2 Q_x c}{L \cdot 4\pi l_b \sqrt{(\gamma \varepsilon_y)\beta_y / \gamma}}\,, \tag{26}$$

where L– is a length of straight section, Q_x –is a number of transverse oscillations per section. This is not a function of the length L, as $Q_x / L = Const$ [10]. With emittances of $(\gamma \varepsilon_y) \cong 2 \cdot 10^{-8}\ cm \cdot rad$, number of the particles $N \cong 10^{10}$, $\beta_y \cong 100 cm$, $\gamma \cong 2 \cdot 10^3$, the luminosity could reach $Lum \approx 10^{31} cm^{-2} s^{-1}$. So the number of para-positroniums could reach $\approx 10^6 s^{-1}$. For ortho-positronium the yield will be 1% of this. As the luminosity depends on emittance as a square root function on emittance, but as a square on the bunch population, one can increase the number of the particles and/or the number of the bunches scarifying the emittance. 100 bunches with 10 times increased population will give increase in the yield as much as 10^4 times, what brings it to $\approx 10^{10} s^{-1}$ (para).

This cooler gives two rays of gammas and relativistic positroniums directed along a straight section[11]. Hypothetically this could be used for nuclear waste neutralization in closed volumes, for example[12], or for positronium chemistry. As the positronium with it's zero total charge does not interact with the matter and atmosphere, it easily penetrates the walls. The life time of positroniums are

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[10] In assumption that the length of the closing loop, Fig.3, is less than the length of the straight section.

[11] With introduction of some time dependent kick at the entrance of straight section, one can arrange, however, a bypass the beams in one direction, so the only one ray remains.

[12] From distances up to 100 *km*.

$$\tau_{\uparrow\downarrow} \cong \frac{2}{{}^{2}\alpha^{5}} \cong 123 \cdot 10^{-}$$