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BPS Mass, Dirichlet Boundary Condition, and the Isotropic Coordinate System

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Abstract

We consider test strings and test branes ending on Dp-branes ($p \leq 6$) and NS5-branes in the background, for a heuristic understanding of the dynamics. Whenever some supersymmetry is preserved, a simple BPS bound appears, but the central charge in question is measured by certain isotropic coordinate system, rather than by the actual spacetime geometry. This way, the ground state energy is independent of the gravitational radii of the solitonic background. Furthermore, a perturbation around the supersymmetric ground states reveals that the appropriate Dirichlet boundary condition is *dynamically* induced. We close with comments.

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1 Motivation

In recent years, it became quite clear that fundamental strings are simply one of many different kind of branes. In particular, branes carrying Ramond-Ramond antisymmetric tensor charges have emerged as equally important building blocks of string theories. These objects are called D-brane, a named derived from the fact that they impose a Dirichlet boundary condition for open fundamental strings [1]. Such phenomena of Dirichlet boundary condition are not restricted to the case of fundamental string ending on D-branes. Rather, D-brane themselves can end on other kind of D-branes [2], and also D-branes may end on NS5-branes [3].

Physics behind open brane configurations, such as charge conservation, has been understood to some degree, and exploited at great length for studying supersymmetric Yang-Mills field theories. However, there are more details that need be further clarified. For instance, consider the well-known D-brane dynamics, which can be seen as induced by open strings stretched between them. In the low energy limit, the primary effect is induced by open strings in the ground states, and the dynamics is known to be described by Super-Yang-Mills theories in Coulomb phase [4]. One could understand this result from quantizing open strings as if the ambient geometry is completely trivial except a Dirichlet boundary condition imposed strings so that it always ends on the said D-branes [5]. Nevertheless, this prescription reproduces, via a loop expansion, some long-range curvature effects one D-brane causes on another [6][7].

A possible answer to this apparent quandary is that perhaps we did quantize the open string in the true curved background of D-branes but did not realize it because we were dealing with supersymmetric configurations only. Could we have started with the curved background induced by D-branes, and reached at the same low energy D-brane dynamics? But this begs for more questions. Supersymmetry tells us that the ground state energy of the stretched open string is determined by the length of the string. But which length? Given the curved background, do we mean the proper length? More generally what do we by “positions” of D-branes?

Our aim is to present a simple classical picture of what happens when a brane ends on another brane in a supersymmetric fashion. When the open string or D-branes end on branes of sufficiently higher dimensions, we may treat the former as test objects in the solitonic background of the latter and study its world-volume dynamics. A crucial fact that goes a long way clarifying some issues, is the existence of the so-called isotropic coordinate system. It is precisely with respect to this coordinate system that the notion of distances and positions must be defined, it turns out. We will also find that, although the ground state of stretched branes are insensitive to the curvature effect, the oscillator modes are not. As a byproduct, we recover the classical dynamics itself *impose* the expected Dirichlet boundary condition naturally on the stretched test object. We close with some afterthoughts.

2 Open String and Open Branes

All the branes in string theory, including the fundamental string itself, appear as solutions to the low energy supergravity. The BPS ground state solution can be found by requiring some fraction of SUSY to be preserved. There are spinors associated with the unbroken SUSY whose asymptotic values satisfy simple algebraic conditions [3][5].

$$\begin{array}{llcllcl} \mathrm{F1} & \Gamma^0\Gamma^1\eta_L & = & \eta_L, & \Gamma^0\Gamma^1\eta_R & = & -\eta_R \\ \mathrm{NS5} & \Gamma^0\cdots\Gamma^5\eta_L & = & \eta_L, & \Gamma^0\cdots\Gamma^5\eta_R & = & \pm\eta_R \\ \mathrm{D}p & \Gamma^0\cdots\Gamma^p\eta_L & = & \eta_R. & & & \end{array}$$

Table 1. Conditions on asymptotic value of spinors for unbroken SUSY

where the 10-dimensional spinors satisfy the usual chirality condition: $\Gamma^{11}\eta_L = \eta_L, \Gamma^{11}\eta_R = -\eta_R$ for IIA and $\Gamma^{11}\eta_{L,R} = \eta_{L,R}$ for IIB. The sign for NS5 branes is such that the 6-dimensional world-volume supersymmetry of NS5 branes is chiral $(2,0)$ for IIA and nonchiral $(1,1)$ for IIB. When two branes intersect, or when a brane ends on another, we expect the conditions for both branes to be satisfied simultaneously. We chose $(\Gamma^0)^2 = -1$.

The simplest supersymmetric configuration is that of a fundamental string (F1) ending on a D-brane. From this, we can reconstruct other configurations by duality chasing. **Table 2.** summarizes the result when the branes involved are at most of 6+1 dimensions. In this note, we will not consider the other cases, where the behavior of the background solutions are qualitatively different.

				F1	on	D6				
D1	on	NS5	← U →	F1		D5				
D2		NS5		F1		D4				
D3		NS5		F1		D3	← U →	D1	on	D3
D4		NS5		F1		D2		D2		D4
D5		NS5		F1		D1		D3		D5
(D6		NS5)		(F1		D0)		D4		D6

Table 2. Supersymmetric configurations of brane ending on brane. $SL(2,Z)$ U-duality

of IIB theory is used to jump across the columns, while T-duality relates items within each of three columns.

Finally, the cases of D6 ending on NS5 or F1 ending on D0, which can be found by a naive T-duality, cannot be realized unless there is a cosmological constant in the background as well. But since we will treat open D6 and open F1 strictly as test objects, one may consider the following consideration applicable for those as well.

3 Branes as Supergravity Background

There are two distinct classes of backgrounds where branes can end, namely those of D-branes and NS5-branes. Both are realized as extremal limits of black p-brane solutions cataloged by Horowitz and Strominger[8]. Let X^i $(i = p+1, \ldots, 10)$ be spatial coordinates orthogonal to the brane and Y^n $(n = 1, \ldots, p)$ be those along the brane. T is the time coordinate.

The Dp-brane solutions are characterized by a single harmonic function H_p with isolated sources at $\vec{X} = \vec{X}_a$'s. For $p \leq 6$, we have

$$H_p = 1 + \sum_a \frac{k_p}{|\vec{X} - \vec{X}_a|^{7-p}}, \tag{1}$$

where the k_p's are appropriately quantized and of dimension of length to the power of $7-p$. In the string frame, the Dp-branes has the following universal form,

$$G = H_p^{-1/2}(-dT^2 + d\vec{Y}^2) + H_p^{1/2} d\vec{X}^2. \tag{2}$$

The same harmonic function H_p dictates the dilaton behavior;

$$e^{\phi} = g_s H_p^{(3-p)/4}, \tag{3}$$

with the asymptotic string coupling constant g_s, and similarly determines the $(p+1)$-form field or its dual in the Ramond-Ramond sector. See [9] for a review.

One interesting aspect of these supersymmetric solitons is that the solution can be written in an isotropic manner as above. Note that the D-brane geometry is asymptotically flat provided $p \leq 6$. For small enough gravitational size or for large enough distances from solitonic core, the isotropic coordinates $(T, \vec{Y}, \vec{X})$ play the role of asymptotically flat coordinate with respect to which far away closed strings can be quantized. Later, we will see that the same X^i and Y^n coordinates appear flat as seen by the lowest lying open string ending on the D-brane soliton, as well, despite their proximity.

NS5-brane solutions are even simpler and can be again written with a single harmonic function $\tilde{H}$,

$$\tilde{H} = 1 + \sum_a \frac{\tilde{k}}{|\vec{X} - \vec{X}_a|^2}, \tag{4}$$

where

$$G = -dT^2 + d\vec{Y}^2 + \tilde{H} d\vec{X}^2, \tag{5}$$

and

$$e^\phi = g_s \tilde{H}^{1/2}. \tag{6}$$

NS5-brane is magnetically charged with respect to the NS-NS antisymmetric tensor B, so that only nonvanishing components are the B_{ij}'s.

4 BPS Mass and Dirichlet Boundary Condition

4.1 Fundamental String on Dp-branes

We are interested in understanding classical dynamics of test objects, so we may safely ignore the fermions on the world-sheet. One possible form of the classical action is that of Nambu-Goto. Denoting the induced metric on the world-sheet by $h_{\mu\nu}$, the action is

$$S = m_s^2 \int d\sigma^2 \sqrt{-\mathrm{Det}\, h}, \tag{7}$$

up to couplings to the dilaton ϕ and to the antisymmetric tensor B. B is absent in the D-brane background, while the dilaton coupling occurs at higher order in $\alpha' = 1/2\pi m_s^2$ and will be subsequently ignored. Calling the spacetime coordinates Z^I collectively, the induced metric is

$$h_{\mu\nu} = \partial_\mu Z^I \partial_\nu Z^J G_{IJ}. \tag{8}$$

For the sake of simplicity, we will choose a static gauge where the world-volume time $\sigma_0 = \tau$ is identified with T. Also we choose $\sigma_1 = \sigma$ to run from 0 to 1.

Consider a pair of parallel Dp-branes located at $\vec{X} = 0$ and $\vec{X} = \vec{L}$.

$$H_p = 1 + \frac{k_p}{|\vec{X}|^{7-p}} + \frac{k_p}{|\vec{X} - \vec{L}|^{7-p}}, \tag{9}$$

Let an open string segment be stretched between such a pair. The induced metric consist of three pieces

$$h = -H_p^{-1/2} d\tau^2 + H_p^{-1/2} \partial_\mu Y^n \partial_\nu Y^n d\sigma^\nu d\sigma^\nu + H_p^{1/2} \partial_\mu X^i \partial_\nu X^i d\sigma^\mu d\sigma^\nu \tag{10}$$

Taking the determinant, we find

$$-\mathrm{Det}\, h = (\partial_\sigma X^i)^2 + H_p^{-1}(\partial_\sigma Y^n)^2 - \mathrm{Det}\left(H_p^{-1/2}\partial_\mu Y^n \partial_\nu Y^n + H_p^{1/2}\partial_\mu X^i \partial_\nu X^i\right) \tag{11}$$

Note that the third term contains two factors of time derivatives $\partial_\tau X$, $\partial_\tau Y$. This implies that there exists the following static solution

$$\vec{X} = \sigma \vec{L}, \quad \partial_\mu \vec{Y} = 0, \tag{12}$$

which corresponds to a straight BPS string segment that is located at some constant $\vec{Y}$. The action per unit time for a static configuration is the energy, so we find the ground state energy to be

$$m_s^2 L, \tag{13}$$

with $L = |\vec{L}|$. We find the BPS mass of the stretched open string is insensitive to the gravitational size of the background. But there is a subtlety here in that the distance that enters is not the proper distance but rather a coordinate distance in a well-defined isotropic coordinate system.

When one considers the two D-branes themselves as dynamical objects, their non-relativistic motion is dictated by a Yang-Mills theory [4]. "Positions" are realized as the eigenvalues of the adjoint $U(2)$ Yang-Mills field, while other heavy fields, of mass proportional to inter-brane "distances," reproduces the effect of the open BPS strings stretched among them. A comparison with the above clearly shows that "positions" and "distances" in the Yang-Mills description are based on their counterparts in the isotropic coordinate system. The lowest lying open string appears quite insensitive to the actual spacetime geometry induced by D-branes.

(One may have expected to find BPS mass proportional to the proper length of the string as measured by the string metric, which would have given a different answer. In fact, the proper length of the string is divergent due to a rather singular behavior of the p-brane extremal horizon.)

In case of D0-branes, this identification between isotropic coordinates and eigenvalues of Yang-Mills scalar matrices is implicit in the test of M(atrix) theory [10] against 11-dimensional supergravity, where, in effect, loop amplitudes induced by these open BPS string modes reproduce the leading long-range interaction between the two D-branes [6][10][11].

Consider small fluctuations of the open string[2] around this supersymmetric ground state. Let $\vec{X} = \sigma\vec{L} + \vec{f}(\tau,\sigma)$ with $\vec{f}$ orthogonal to $\vec{L}$, and $\vec{Y} = \vec{g}(\tau,\sigma)$. To the first nonvanishing order, the determinant can be expanded as follows,

$$\mathrm{Det}\, h \;=\; -L^2 \quad + \quad L^2\,(\partial_\tau g^n)^2 - H_p^{-1}(\partial_\sigma g^n)^2 \tag{14}$$

[2] A similar consideration can be found in [12].

$$+\quad H_pL^2\,(\partial_\tau f^i)^2-(\partial_\sigma f^i)^2 \tag{15}$$

$$+\quad \cdots. \tag{16}$$

The ellipsis represents terms at least quartic in the small fluctuations, and H_p here is actually $H_p(\vec{X}=\sigma\vec{L})$. The Lagrangian is obtained by taking square root and expanding in small f and g.

Note that, for fluctuations orthogonal to the background Dp-branes, the combination H_pL is the effective (inertial) mass density. A finite energy motion must have a square integrable $H_p\,(\partial_\tau f^i)^2$, and in particular for any eigenmode of the Hamiltonian, $H_p\vec{f}^2$ must be square-integrable. With the divergence of $H_p\sim(\Delta\sigma)^{p-7}$ near either end of the string, this immediately implies that $\vec{f}=\vec{X}-\vec{L}$ must obey the Dirichlet boundary condition wherever it ends on the Dp-brane. Note that, in contrast, no such condition is imposed on the other fluctuations $\vec{Y}=\vec{g}$, which are parallel to the background Dp-brane.

The f^i perturbation represents fluctuation away from the background Dp-brane. Thus, the string cannot break away from the Dp-brane, in that the end points cannot move away from $\vec{X}=0$ or $\vec{L}$ without costing an infinite amount of energy. One may be troubled by the fact that f^i isn't by itself the proper distance. However, if the test string is to break away completely from D-brane, its boundary should be able to escape the gravitational radius of the latter. And this happens only if one can have finite $f^i=\delta X^i$ while approaching the boundary, which is impossible.

Perhaps more to the point, the identification of L as the string "length" above has shown that, in translating to the standard prescription for D-brane conformal field theory, the X^i's themselves should correspond to the world-sheet fields that must be quantized with Dirichlet boundary condition. In this sense, we have derived the Dirichlet boundary condition from classical dynamics.

4.2 D$(p-2)$-branes on Dp-branes

Other classes of allowed configurations include D$(p-2)$-brane ending on background Dp-branes. From the viewpoint of Dp-brane world-volume, such an endpoint behaves as magnetic monopoles and this situation has been studied in various contexts.

The world-volume action of D$(p-2)$-brane is the celebrated Dirac-Born-Infeld action [13], but again as with fundamental string we will ignore the part irrelevant to the classical motion,

$$S_{p-2}=T_{p-2}\int d\sigma^{p-1}g_s\,e^{-\phi}\sqrt{-\mathrm{Det}\,(h+b+m_s^{-2}\mathcal{F})}, \tag{17}$$

where h is again the induced metric and b is the pull-back of the antisymmetric tensor B to the world-volume. B does not appear in the supergravity solution of Dp-branes, so it is safe to ignore b as well. $\mathcal{F}$ is the world-volume $U(1)$ gauge field strength. The Dp-brane tension is $T_p = 2\pi\,(m_s/\sqrt{2\pi})^{p+1}/g_s$.

Again we choose a static gauge where $\sigma_0 = \tau$ is identified with T, and will consider $\vec{X}$ and $\vec{Y}$ as dynamical. The background remains identical to the one we considered previously, so we have a pair of Dp-branes located at $\vec{X} = 0$ and $\vec{L}$. The induced metric of a stretched D$(p-2)$-brane is similarly given as in Eq.(10),

$$h = -H_p^{-1/2} d\tau^2 + H_p^{-1/2} \partial_\mu Y^n \partial_\nu Y^n d\sigma^\nu d\sigma^\nu + H_p^{1/2} \partial_\mu X^i \partial_\nu X^i d\sigma^\mu d\sigma^\nu. \tag{18}$$

It is not difficult to see that there exists a static solution with $\mathcal{F} \equiv 0$, and

$$\vec{X} = \sigma_1 \vec{L}, \tag{19}$$

while remaining noncompact $p-3$ directions of the D$(p-2)$-branes are stretched parallel to the Dp-brane. (That is, along Y^n coordinates.) Evaluating the action density on such a configuration, we find

$$T_{p-2}\, g_s\, e^{-\phi} \sqrt{-\mathrm{Det}\, h}\, d\sigma^{p-1} \to T_{p-2}\, d\tau \wedge L d\sigma_1 \wedge V_{p-3}, \tag{20}$$

where the volume form V_{p-3} is induced from the flat metric $d\vec{Y}^2$. The action density contains an extra factor $H_p^{(p-3)/4}$ from the dilaton, when compared to the analog in the string case, but this is exactly canceled by contribution from extra noncompact $\sigma_2, \ldots, \sigma_{p-2}$ directions. Thus, we again find that the BPS mass density is given by the naive formula, except that the central charges are measured in terms of the isotropic coordinate distance rather than in terms of the proper distance.

Fluctuations around this configuration can be treated similarly as in the case of test string. For definiteness, let $X^i = \sigma_1 L^i + f^i(\sigma_\mu)$, and $Y^n = \epsilon^n \sigma_{n+1} + g^n(\sigma_\mu)$ with $\epsilon^n = 1$ for $n \leq p-3$ and $\epsilon^n = 0$ for the rest. With $L^i f^i = \epsilon^n g^n = 0$, we find

$$\begin{aligned} g_s^2\, e^{-2\phi}\, \mathrm{Det}(h + m_s^{-2}\mathcal{F}) \;&=\; -L^2 && (21)\\ &+\; L^2 \left((\partial_\tau \vec{g})^2 - (\partial_{\sigma_2} \vec{g})^2 - \cdots - (\partial_{\sigma_{p-2}} \vec{g})^2 \right) - H_p^{-1} (\partial_{\sigma_1} \vec{g})^2 && (22)\\ &+\; H_p L^2 \left((\partial_\tau \vec{f})^2 - (\partial_{\sigma_2} \vec{f})^2 - \cdots - (\partial_{\sigma_{p-2}} \vec{f})^2 \right) - (\partial_{\sigma_1} \vec{f})^2 && (23)\\ &+\; \cdots, && (24) \end{aligned}$$

where we suppressed terms of higher order in $\vec{f}$ and $\vec{g}$ as well as those with factors of $\mathcal{F}$. The latter starts at $\sim \mathcal{F}^2$. Again, a finite energy eigenmode of $\vec{f}$ must be square-integrable against the weight $H_p(\vec{X} = \sigma_1 \vec{L})$ which is severely divergent at either end of the open D$(p-2)$ brane: The boundaries of the test D$(p-2)$-brane are stuck on the Dp-branes, so that the Dirichlet boundary condition is again dynamically imposed.

4.3 Dp-branes on NS5-branes

Now we consider the final case on our list, where Dp-branes with $p \leq 5$ end on NS5-branes. Let us put a pair of NS5-branes in the background, as given by the harmonic function,

$$\tilde{H} = 1 + \frac{\tilde{k}}{|\vec{X}|^2} + \frac{\tilde{k}}{|\vec{X} - \vec{L}\,|^2}. \tag{25}$$

The induced metric of the stretched Dp-brane is then

$$h = -d\tau^2 + \partial_\mu Y^n \partial_\nu Y^n d\sigma^\nu d\sigma^\nu + \tilde{H} \partial_\mu X^i \partial_\nu X^i d\sigma^\mu d\sigma^\nu, \tag{26}$$

while the pull-back of B is given by

$$b = B_{ij}\, \partial_\mu X^i \partial_\nu X^j \; d\sigma^\mu d\sigma^\nu, \tag{27}$$

where we used the fact that the B_{ij}'s are only nonvanishing components of the NS-NS antisymmetric tensor.

Despite the presence of the antisymmetric tensor field, the same kind of ground state solution is possible. Write $X^i = \sigma_1 L^i + f^i(\sigma_\mu)$, and $Y^n = \epsilon^n \sigma_{n+1} + g^n(\sigma_\mu)$ with $\epsilon^n = 1$ for $n \leq p-1$ and $\epsilon^n = 0$, and we can easily expand the determinant with the constraint $L^i f^i = \epsilon^n g^n = 0$,

$$\begin{aligned}
g_s^2\, e^{-2\phi}\, \mathrm{Det}(h+b) \;\; &= \;\; -L^2 && (28)\\
&+ \;\; L^2 \left((\partial_\tau \vec{g})^2 - (\partial_{\sigma_2} \vec{g})^2 - \cdots - (\partial_{\sigma_p} \vec{g})^2 \right) - \tilde{H}^{-1} (\partial_{\sigma_1} \vec{g})^2 && (29)\\
&+ \;\; \tilde{H} L^2 \left((\partial_\tau \vec{f})^2 - (\partial_{\sigma_2} \vec{f})^2 - \cdots - (\partial_{\sigma_p} \vec{f})^2 \right) - (\partial_{\sigma_1} \vec{f})^2 && (30)\\
&+ \;\; L^2 \tilde{H}^{-1} \left((B_{ij} L^i \partial_\tau f^j)^2 - (B_{ij} L^i \partial_{\sigma_2} f^j)^2 - \cdots - (B_{ij} L^i \partial_{\sigma_p} f^j)^2 \right) && (31)\\
&+ \;\; \cdots. && (32)
\end{aligned}$$

We again recover the naive BPS mass formula for straight open Dp-branes from the leading term.

In contrast to the previous examples there are new terms from B_{ij}. While this appears to modify the dynamics, for B_{ij} has exactly the same divergence as $\tilde{H}$, it is easy to see that the inner product with $\vec{L}$ reduces the divergence of the antisymmetric tensor field and that we may neglect its presence near the boundaries. The effective inertial mass density for the f^i modes diverges with $\tilde{H}$ near the boundaries, as before, and the test Dp-brane are stuck on the NS5-branes.

5 Comments

Adopting an elementary picture of test strings and test branes ending on background supergravity solitons we illustrated how the two crucial properties of open strings and open branes arise. Background independent central charge emerged from the isotropic coordinate system. Most notably, the defining property of Dirichlet boundary condition is found to be dynamically induced, thanks to a divergent effective *inertial* mass density of the test object. The boundaries are too heavy to move along certain directions, in effect.

There could be many problems if we want to elevate the present considerations to a more rigorous level. For instance, the solution to a low energy effective theory of type II string theories may not be trustworthy near the core due to a potentially divergent curvature. While we will not address such questions fully, there are many cases when the potential problems can be removed, say, by going to M-theory.

Consider a fundamental string ending on a pair of D6-branes. In M-theory, this corresponds to a membrane wrapping around a nontrivial 2-cycle in a hyper-Kähler space with two Taub-NUT centers. A Taub-NUT center, corresponding to a D6-brane, has only a coordinate singularity. The 11-dimensional curvature tensor is in fact bounded above by $\sim 1/R^2$, where R is the radius of the compact 11th direction. We may proceed similarly as above, starting with the Nambu-Goto action for membrane. The area computation can be found in Ref. [14], whose result obviously agrees with ours. In this picture, the Dirichlet boundary condition follows from the topology, for one cannot move the string (or the spherical membrane) away from the Taub-NUT centers without changing the topology of the membrane.

Another set of examples would be D4-brane ending on NS5-branes, or F1 ending on D2. In these cases, the Dirichlet boundary condition is naturally imposed since D4 and F1 grow out of the latter, which are M5 and M2 branes respectively, by developing a compact direction [15]. Also, Ref. [16] depicts how F1 grows out of a Dp-brane as a solution to the Born-Infeld action, where again the Dirichlet boundary condition would follow automatically.

The point is that the picture we try to present is simply one of many possible alternatives. It just happens to be the simplest one that also can be applied uniformly across the board. Our purpose is not in a rigorous proof, but rather in giving the phenomena of branes ending on branes a more familiar look.

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