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Improving Measurements of the CKM Phase γ Using Charm Factory Results

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Abstract

Several authors have proposed methods to measure the phase γ of the Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle using decays of the type $B \to DK$. We show how to remove uncertainties from these measurements and increase their sensitivity by measuring CP-conserving phases at a charm factory.

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CP-violation experiments can be used to test the standard model and probe new physics. Decays of the type $B \to DK$ provide several possibilities for carrying out such tests. Gronau, London and Wyler (GLW) [1] have shown that the CKM [2] phase $\gamma \approx \arg(V_{ub}^*)$ can be measured in the interference between the decays $B^+ \to \overline{D}^0 K^+$ and $B^+ \to D^0 K^+$, whose Feynman diagrams are shown in Figure 1. The phase difference between the two amplitudes is $\gamma + \delta_B$, where δ_B is a CP-conserving, final state interaction (FSI) phase. Interference occurs when the D is observed as one of the CP-eigenstates

$$D^0_\pm \equiv \frac{1}{\sqrt{2}} \left(D^0 \pm \overline{D}^0 \right), \tag{1}$$

which are identified by their decay products. Disregarding CP-violation in D decays and $D^0 - \overline{D}^0$ mixing, Equation (1) implies the triangle relations of Figure 2, from which γ is extracted. As with several other methods, γ is obtained up to a four-fold discrete ambiguity. This method can be applied to any decay mode of the type $B^+ \to D^0_\pm K^+ n\pi$, and in principle does not require δ_B to be different from zero.

Several variations of the GLW method have been developed. Dunietz [3] has suggested applying the method to $B^0 \to D^0_\pm K^{*0}$, where the subsequent decay $K^{*0} \to K^+\pi^-$ tags the flavor of the B. In this mode both amplitudes are color suppressed, and thus have similar magnitudes. This results in greater interference than one expects in $B^+ \to D^0_\pm K^+$, where the the $\overline{b} \to \overline{c}$ amplitude has a color allowed contribution, while the $\overline{b} \to \overline{u}$ amplitude is color suppressed. Atwood, Eilam, Gronau and Soni(AEGS) [4] have considered the decays $B^+ \to D^0_\pm [K^+]$, where $[K^+]$ is a state such as $K\pi$ or $K^*\pi$, with invariant mass around 1400 MeV, where several strange resonances can interfere. The different Breit-Wigner widths of the resonances lead to large and calculable CP-conserving phases, enabling the extraction of γ from decay rate asymmetries, in addition to using the GLW triangle method.

Atwood, Dunietz and Soni(ADS) [5] have pointed out a serious difficulty with the GLW method, originating from the fact that in order to measure the branching fraction

$\mathcal{B}(B^+ \to D^0 K^+)$, the D^0 must be identified in a hadronic[1] final state, $K^-\pi^+ n\pi$. To simplify the discussion, we explicitly refer to $K^-\pi^+$. ADS noted that the decay chain $B^+ \to D^0 K^+$, $D^0 \to K^-\pi^+$ results in the same final state as $B^+ \to \overline{D}^0 K^+$, $\overline{D}^0 \to K^-\pi^+$, where the $\overline{D}^0$ undergoes doubly Cabibbo suppressed decay. Using only measured quantities and factorization, they estimated the ratio between the interfering decay chains:

$$\begin{aligned}
&\left|\frac{A(B^+ \to \overline{D}^0 K^+)\ A(\overline{D}^0 \to K^-\pi^+)}{A(B^+ \to D^0 K^+)\ A(D^0 \to K^-\pi^+)}\right| \\
&\approx \left|\frac{V_{cb}^*}{V_{ub}^*}\ \frac{V_{us}}{V_{cs}}\ \frac{a_1}{a_2}\right| \sqrt{\frac{\mathcal{B}(\overline{D}^0 \to K^-\pi^+)}{\mathcal{B}(D^0 \to K^-\pi^+)}} \\
&\approx \frac{1}{0.08}\ 0.22\ \frac{1}{0.26}\ \sqrt{0.0077} \approx 0.9.
\end{aligned} \tag{2}$$

Evidently, sizable interference makes it practically impossible to measure $\mathcal{B}(B^+ \to D^0 K^+)$, and the GLW method fails. For $B^0 \to D^0_\pm K^{*0}$, Equation (2) yields the ratio 0.25. The problem is thus less severe in this mode, but it nevertheless introduces a significant systematic uncertainty to the measurement of γ.

ADS proposed to use this interference to obtain γ from the decay rate asymmetries in $B^+ \to [f_i]K^+$, where f_i, $i = 1, 2$, are two different hadronic states, at least one of which is of the type $K^-\pi^+ n\pi$, which can arise from the Cabibbo allowed decay of a D^0 or the doubly Cabibbo suppressed decay of a $\overline{D}^0$. Measuring the four branching fractions

$$\mathcal{B}(B^+ \to [f_i]K^+), \qquad \mathcal{B}(B^- \to [\overline{f}_i]K^-), \tag{3}$$

one calculates the four unknowns which cannot be measured in other B decays, namely $\mathcal{B}(B^+ \to D^0 K^+)$, γ (up to a discrete ambiguity), and the CP-conserving phase differences of the two final states. For a given final state $[f]$, the CP-conserving phase is

$$\delta = \delta_B + \delta_D, \tag{4}$$

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[1]Full reconstruction is impossible in semileptonic decays, resulting in unacceptably high background levels.

where

$$\delta_D = \arg\left[A(D^0 \to [f])A(\overline{D}^0 \to [f])^*\right] \tag{5}$$

can be quite large when $[f]$ is a flavor eigenstate [6], resulting in CP-asymmetries of order 100%.

ADS noted that since δ_B is identical for all D decay modes, an independent, charm factory measurement of δ_D would add a constraint to Equation (4), increasing the sensitivity of the method. The need to use two decay modes still causes significant loss of statistical power, however, since the only modes for which the product of the efficiency and branching fraction is sizable are $K^-\pi^+$, $K^-\pi^+\pi^0$ and $K^-\pi^+\pi^-\pi^+$. The benefits of the δ_D measurements are much greater, however, if one makes the additional hypothesis

$$\delta_B \approx 0. \tag{6}$$

It has recently been shown [7] that rescattering phases should not be strongly suppressed in B decays, contrary to previous expectations [8]. Tight experimental limits on FSI phases in $B \to D\pi$, $D^*\pi$, $D\rho$ and $D^*\rho$ [9], however, provide good indication that the phases could be small in two body $B \to D^{(*)}$ + light hadron decays. Tests of this hypothesis specifically in the case of $B \to DK$ are discussed below.

Given Equation (6), an independent measurement of the phase δ_D would enable the extraction of γ and $\mathcal{B}(B^+ \to D^0K^+)$ from the branching fractions of Equation (3) using a single flavor eigenstate D decay mode, provided $\sin\delta_D$ is not too close to 0. This would free other modes to be used for increasing statistics and resolving discrete ambiguities. In addition, we note that knowledge of δ solves the problem which Equation (2) implies for the GLW method. The method requires measuring the lengths of all sides of the triangle ABC of Figure 3. As discussed above, the side BC cannot be measured directly, since $\mathcal{B}(B^+ \to [K^-\pi^+]K^+)$ actually measure BD, which is the interference of BC with CD, the $\overline{b} \to \overline{c}$ transition amplitude followed by a doubly Cabibbo suppressed D decay. However, given δ,CD and BD, the length BC is determined. The triangle ABC is now fully constructed, and γ is obtained using the GLW method.

We proceed to study the measurement of δ_D at a charm factory, operating at the $\psi(3770)$ resonance. Equation (5) is graphically represented in Figure 4, demonstrating how to obtain δ_D from the the Cabibbo allowed D decay amplitude, A_{CA}, the doubly Cabibbo suppressed amplitude, A_{DCS}, and their interference, $A_\pm \equiv A_{CA} \pm A_{DCS}$. While A_{CA} and A_{DCS} have been measured at CLEO [10] for the $K^-\pi^+$ mode by using D^{*+} decays to tag the D^0 flavor, measuring $A_\pm$ requires producing $D^0\overline{D}^0$ pairs in a known coherent state. It is therefore best to perform all three measurements at the charm factory, canceling many systematic errors in the construction of the triangles of Figure 4. To measure the amplitude A_+ (A_-), one of the $\psi(3770)$ daughters is tagged as a D^0_- (D^0_+) by observing it decay into a CP-odd (CP-even) state, such as $K_S\,\pi^0$ (K^+K^-). The other daughter is then D^0_+ (D^0_-), and the fraction of the time that it is seen decaying into $K^-\pi^+$ gives the interference amplitude $\frac{1}{2}|A_\pm|^2$. We immediately find

$$\cos\delta_D = \pm\frac{|A_\pm|^2 - |A_{CA}|^2 - |A_{DCS}|^2}{2|A_{CA}||A_{DCS}|}. \tag{7}$$

Due to the low statistics tagging scheme of the $|A_\pm|$ measurement and the fact that $|A_{DCS}| \ll |A_{CA}|$, the error in $\cos\delta_D$ is dominated by the $|A_\pm|$ measurement error. Hence

$$\begin{aligned}\Delta\cos\delta_D &\approx \frac{\Delta|A_\pm|}{|A_{DCS}|} \approx \frac{\Delta|A_\pm|}{|A_\pm|}\,\left|\frac{A_{CA}}{A_{DCS}}\right| \\ &= \frac{1}{2\sqrt{N_{A_\pm}}}\,\left|\frac{A_{CA}}{A_{DCS}}\right|,\end{aligned} \tag{8}$$

where $N_{A_\pm}$ is the number of events observed in the $A_\pm$ channels, and we made use of $|A_{CA}| \approx |A_\pm|$. Since the event is fully reconstructed, background is expected to be small, and its contribution to $\Delta\cos\delta_D$ is neglected in this discussion. $N_{A_\pm}$ is given by

$$\begin{aligned}N_{A_\pm} &= N_{D\overline{D}}\,\mathcal{B}(D^0 \to K^-\pi^+)\,\epsilon(K^-\pi^+) \\ &\quad\times \sum_i \mathcal{B}(D^0 \to t_i)\,\epsilon(t_i),\end{aligned} \tag{9}$$

where $N_{D\overline{D}}$ is the number of $\psi(3770) \to D^0\overline{D}^0$ events, t_i are the CP-eigenstates used for tagging, and $\epsilon(X)$ is the reconstruction efficiency of the state X. From the list of $D^0 \to$CP-eigenstate branching fractions and efficiencies in Table I we obtain $\sum_i \mathcal{B}(D^0 \to t_i)\,\epsilon(t_i) \approx$

0.02. Taking $\mathcal{B}(D^0 \to K^-\pi^+) \approx 0.04$, $\epsilon(K^-\pi^+) \approx 0.8$ and $|A_{CA}/A_{DCS}| = 1/\sqrt{0.0077}$, Equation (8) becomes

$$\Delta \cos\delta_D \approx \frac{230}{\sqrt{N_{D\overline{D}}}}. \tag{10}$$

It is expected that in one year the charm factory will collect 10 fb^{-1}, or $N_{D\overline{D}} = 2.9\times 10^7$ [11], resulting in $\Delta\cos\delta_D \approx 0.04$. $\cos\delta_D$ can thus be measured with more than sufficient precision, even in the presence of background and with relatively modest luminosity. We note that the same measurement technique can be used with multi-body D^0 decays, in which $\cos\delta_D$ varies over the available phase space. While the relative statistical error in every small region of phase space will be large, its effect on the measurement of γ in $B \to DK$ will be proportionally small. The total error in γ due to $\Delta\cos\delta_D$ will be as small as in the two body $K^-\pi^+$ mode, up to differences in D^0 branching fractions, reconstruction efficiencies and backgrounds.

Several tests of the hypothesis (6) can be conducted. First, the decay rate asymmetry in $B^+ \to D^0_\pm K^+$ is proportional to $\sin\delta_B \sin\gamma$, hence its measurement can be used to put a limit on $\sin\delta_B$. The best limit should come from $B^0 \to D^0_\pm K^{*0}$, where factorization predicts the ratio between the interfering amplitudes to be ~ 2.8, compared to ~ 11 in $B^+ \to D^0_\pm K^+$. In addition, since both $B^0 \to D^0 K^{*0}$ and $B^0 \to \overline{D}^0 K^{*0}$ proceed through color suppressed amplitudes, one expects the relation

$$\frac{\mathcal{B}(B^0 \to D^0 K^{*0})}{\mathcal{B}(B^0 \to \overline{D}^0 K^{*0})} \approx \left|\frac{V^*_{ub}}{V^*_{cb}}\,\frac{V_{cs}}{V_{us}}\right|^2 \tag{11}$$

to hold well. Agreement between this expectation and the length BC, as obtained from the triangle BCD of Figure 3 (replacing $B^+ \to B^0$, $K^+ \to K^{*0}$ and applying Equation (6)), would support our theoretical picture of such decays, including the hypothesis (6). Disagreement, on the other hand, would be strong evidence against this hypothesis.

AEGS have also shown how to test Equation (6) in $B^+ \to D^0_\pm[K^+]$ decays using the sum of the decay widths $\Sigma = d^2[\Gamma(B^+ \to D^0_\pm[K^+]) + \Gamma(B^- \to D^0_\pm[K^-])]/dsdz$, where z is the $[K^+]$ decay angle and s is the difference between the invariant mass of the $[K^+]$ state and a weighted average mass of the interfering resonances. If $\delta_B \approx 0$, then Σ is an even function of s. Otherwise, an s-odd term is introduced.

Finally, assuming Equation (6) , the values of δ_D measured at the charm factory can be applied to the measurement of γ in all the $B \to DK$ modes and methods discussed above. Inconsistencies among the different γ measurements would thus invalidate Equation (6).

To summarize, we have shown that the problem with the Gronau-Wyler method is solved by conducting relatively low-luminosity phase measurements at a charm factory. This will enable the exploitation of more B decay modes for the measurement of γ than otherwise possible, and enhance the sensitivity of the Atwood-Dunietz-Soni method. We have also indicated how to test the hypothesis, which our analysis depends on, that the FSI phase is negligible in $B \to DK$.

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t	$\mathcal{B}(D^0 \to t)$	$\epsilon(t)$	$\mathcal{B} \times \epsilon$
$K_S\ \pi^0$	0.0106	0.3	0.003
$K_S\ \eta(\to \gamma\gamma)$	0.007	0.1	0.0007
$K_S\ \rho^0$	0.012	0.4	0.005
$K_S\ \omega(\to \pi^+\pi^-\pi^0)$	0.021	0.2	0.004
$K_S\ \eta'(\to \pi^+\pi^-\eta)$	0.017	0.1	0.0017
$K_S\ \eta'(\to \rho^0\gamma)$	0.017	0.1	0.0017
K^+K^-	0.004	0.8	0.0032
$\pi^+\pi^-$	0.0015	0.8	0.0012
total			0.021

TABLE I. Branching fractions [12] of D^0 decays to CP-eigenstates, reconstruction efficiencies, and their products. Efficiencies include sub-mode branching fractions, such as $K_S \to \pi^+\pi^-$, and are constructed assuming 90% track and photon efficiency and 50% π^0 efficiency.

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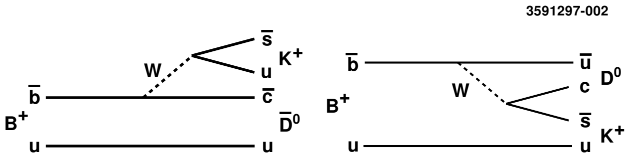

FIG. 1. Dominant Feynman diagrams of $B^+ \to \overline{D}^0 K^+$ and $B^+ \to D^0 K^+$.

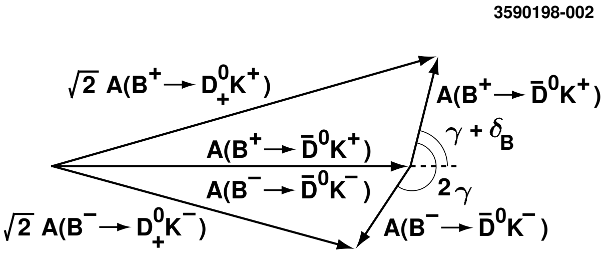

FIG. 2. The triangle relation $D^0_+ \equiv \frac{1}{\sqrt{2}}\left(D^0 + \overline{D}^0\right)$ applied to the B^+ and B^- decay amplitudes. Similar triangles exist for $D^0_- \equiv \frac{1}{\sqrt{2}}\left(D^0 - \overline{D}^0\right)$.

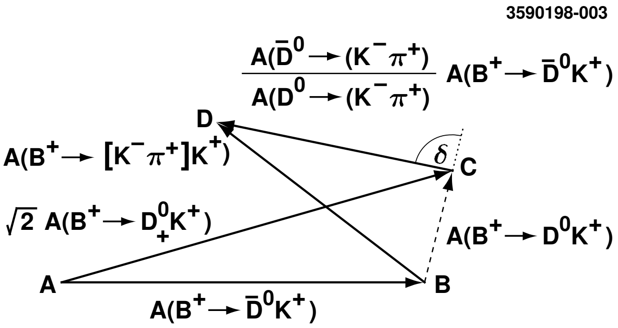

FIG. 3. Solution of the problem with the GLW method: Given δ, BD and CD, the length BC is determined, enabling the construction of the triangle ABC.

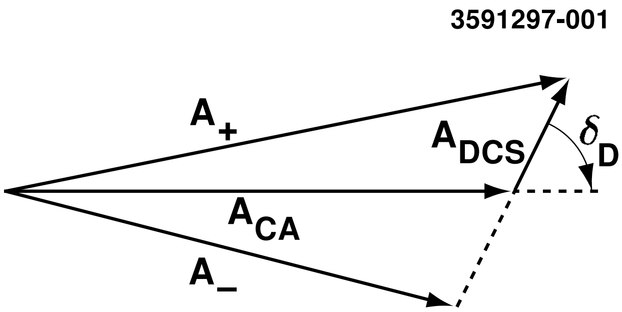

FIG. 4. Obtaining the phase δ_D of Equation (5) from the Cabibbo allowed D^0 decay amplitude, A_{CA}, the doubly Cabibbo suppressed amplitude, A_{DCS}, and their interference, $A_\pm \equiv A_{CA} \pm A_{DCS}$.