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Aspects of Quenched Chiral Perturbation Theory for Matter Fields

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Abstract

Three topics about the application of quenched chiral perturbation theory to matter fields are studied. It is proved that the hairpin axial current couplings in quenched chiral perturbation theories do not contribute to the quenched chiral singularities for one chiral loop renormalization of matter field properties. The modification of mass corrections in the chiral limit due to nonzero mass splittings are studied, and selection rules for hadron decays in quenched QCD are obtained.

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Most lattice QCD simulations are performed under the quenched approximation, where the fermionic determinant is set to unity. While in the real world the u and d quarks have masses much less than $\Lambda_{\rm QCD}$, lattice QCD are often simulated with $m_q \sim \Lambda_{\rm QCD}$, and physical results are extracted at the chiral limit $m_q \rightarrow 0$. As a result, it is important to understand the chiral limit of quenched QCD. Over the past few years, quenched chiral perturbation theory (QχPT) has been formulated as the low energy effective theory of quenched QCD [1–3]. In QχPT, the cancelation of internal quark loops is enforced by the introduction of ghost quarks (quarks with wrong – bosonic – statistics) [4]. Since the contributions of ghost loops to path integrals are equal in magnitude but opposite in sign to those of quark loops, there will be a complete cancelation of loop contribution if there are as many quarks as ghosts, with the same masses. (One may also consider partially quenched chiral perturbation theories [5], in which the number of quarks and the number of ghosts are different.) Just as in ordinary chiral perturbation theory (χPT) the low energy dynamics is dominated by the Goldstone bosons (pions for two flavor QCD) which are quark-antiquark bound states, in QχPT the low energy dynamics is dominated by the Goldstone "bosons" which may be quark-antiquark, ghost-antighost, quark-antighost or ghost-antiquark bound states.

The main difference between χPT and QχPT lies in the the role played by the η' meson. In ordinary QCD, η' acquires a heavy mass due to the necklace diagrams and hence are integrated out in the standard formulation of χPT. Since the necklace diagrams involve quark loops, however, they do not contribute in the quenched theory. As a result, the η' have the same mass as the other Goldstone bosons (massless in the chiral limit) and cannot be integrated out. It does differ from the other Goldstone bosons in having an extra term in its propagator due the the hairpin diagram.

$$\frac{1}{p^2-m^2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}+\frac{-nA_0p^2+nM_0^2}{(p^2-m^2)^2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}, \tag{1}$$

where the first row/column represents the η' meson and the second row/column represents the $\tilde{\eta}'$, the ghost-antighost analog of η', and n is both the number of quark flavors and the number of ghost flavors. The first term is the usual propagator where η' and $\tilde{\eta}'$ propagate independently with a single pole at $p^2 = m^2$, which denotes both $m_{\eta'}$ and m_π as π and η' are degenerate in quenched QCD. The second term carries a double pole and represents a possible mixing between η' and $\tilde{\eta}'$, characterized by the parameters A_0 and M_0^2. [1]

Besides the Goldstone boson fields, one can also study the dynamics of matter fields (vector and tensor mesons [6,7], baryons [8–10], heavy mesons [11–13], heavy baryons [14], *etc.*) under the framework of QχPT. In this paper, we will discuss three theoretical issue around the application of QχPT on matter fields. The first section will prove the (ir)relevance of hairpin axial couplings (which will be defined below) to quenched chiral corrections of matter field quantities. Then the effects of mass splittings in QχPT will be discussed, followed by the last section which study the possibility of understanding hadron decay in QχPT.

I. HAIRPIN AXIAL COUPLINGS AND QUENCHED CHIRAL CORRECTIONS

Since the matter fields are significantly heavier than Λ_{QCD}, it is customary to introduce the matter fields as heavy particles, *i.e.*, static fields with definite velocities. Under the heavy particle formalism, the propagators of the matter fields are always $(v \cdot p)^{-1}$, and in the leading order of a derivative expansion the matter fields couples to the Goldstone bosons through the axial currents.

$$\mathcal{L}_{\mathrm{standard}} = ig \,\mathrm{Str}\, [X^\mu_{(H^\dagger_{(f)}, H_{(i)})} A_\mu], \tag{2}$$

where $A_\mu = \frac{i}{2}(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi)$ is the Goldstone boson axial current, "Str" denotes the supertrace over all the quark and ghost flavors, and X^μ is the axial current operator between the initial state H_i and the final state H_f. The form of X^μ depends on the matter fields in question. For example, the axial current of the vector meson fields V^λ takes the following form [6,7]:

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[1] Here we are considering quenched QCD with two light flavors and we have set $m_u = m_d$. With three light flavors and $m_s \neq m_q$, the form of the double pole term will be more complicated. However, our analysis is independent of the particular form of the double pole term.

$$X^\mu_{(V^{\lambda}_{(f)}{}^\dagger, V^\sigma_{(i)})} = \epsilon^{\mu\nu\lambda\sigma} v_\nu \{V^\dagger_{\lambda(f)}, V_{\sigma(i)}\}, \tag{3}$$

where g is an undetermined coupling constant, while the exact forms of the baryon axial current couplings can be found in Ref. [8].

The chiral lagrangian of QχPT is complicated by the existence of hairpin axial current couplings, which couplings η' and $\tilde{\eta}'$ to the flavor singlet axial current of matter fields.

$$\begin{aligned}\mathcal{L}_{\text{hairpin}} &= ih\,\text{Str}\,[X^\mu_{(H^\dagger_{(f)}, H_{(i)})}]\text{Str}[A_\mu] \\ &= in^{1/2}(h/f)\,\text{Str}\,[X^\mu_{(H^\dagger_{(f)}, H_{(i)})}]\partial_\mu(\eta' - \tilde{\eta}').\end{aligned} \tag{4}$$

After writing down the interaction lagrangians, one can study the chiral limit of different physical quantities by calculating the one chiral loop renormalizations. For example, the mass renormalization of the matter fields can be evaluated through the following integral,

$$\begin{aligned}\Delta m = \int \frac{d^4p}{(2\pi)^4} & \left[\frac{(\text{coup. const.})^2}{f^2} \sum_f X^\nu_{(H^\dagger_{(i)}, H_{(f)})} X^\mu_{(H^\dagger_{(f)}, H_{(i)})} \right] \\ & \frac{p_\nu p_\mu}{v \cdot p} \text{(propagator of the Goldstone boson)}.\end{aligned} \tag{5}$$

where the sum is over all possible intermediate states. Note that the expression inside the square brackets are independent of p and hence can be taken out of the integral. So ΔM just depends on the form of the propagators of the Goldstone bosons. For all the flavor non-singlet states the propagators are $(p^2 - m^2)^{-1}$, which is also the form of the "single pole" propagator of η' and $\tilde{\eta}'$. Putting these propagators in the integral with contribute to ΔM with the generic form

$$\Delta M \sim \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{v \cdot p} \frac{1}{p^2 - m^2} \sim m^3. \tag{6}$$

This $m^3 \sim m_q^{3/2}$ term has a singular dependence of m_q which is due to low-energy chiral dynamics and cannot be canceled by counterterms. Since such a term also appear in normal χPT, they are called standard chiral singularities. In contrast, if the contribution to the loop integral due to the "double pole" propagator is of the form:

$$\Delta M \sim \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{v \cdot p} \frac{-A_0 p^2 + M_0^2}{(p^2 - m^2)^2} \sim M_0^2 m. \tag{7}$$

This term $M_0^2 m \sim m_q^{-1/2}$ is more singular in the chiral limit than the m^3 term above and is a quenched artifact, which is usually called quenched chiral singularities. Since these quenched chiral singularities dominate over the standard ones in the chiral limit, understanding them may be helpful in the determination of the best extrapolation of lattice QCD results to the zero mass limit.

Naively, both the standard and quenched chiral singularities may depend on the normal axial coupling g and the hairpin axial coupling h. It turns out that the quenched chiral singularities are independent of the value of h. The reasoning is very simple: the quenched chiral singularities come from η' and $\tilde{\eta}'$ loops, and these Goldstone bosons can couple to the matter field through either the standard or hairpin axial current couplings. The matter field, made entirely of quarks and/or antiquarks, cannot couple to $\tilde{\eta}'$ through the standard axial coupling, hence

$$g_{\text{standard}\,\eta'} = n^{1/2} g, \qquad g_{\text{standard}\,\tilde{\eta}'} = 0. \tag{8a}$$

On the other hand, η' and $\tilde{\eta}'$ couple at the same strength through the hairpin coupling, but with opposite signs.

$$g_{\text{hairpin}\,\eta'} = n^{1/2} h, \qquad g_{\text{hairpin}\,\tilde{\eta}'} = -n^{1/2} h. \tag{8b}$$

Hence the total couplings are

$$g_{\text{total}\,\eta'} = n^{1/2}(g+h), \qquad g_{\text{total}\,\tilde{\eta}'} = -n^{1/2} h. \tag{8c}$$

The one loop integral is proportional to

$$\begin{aligned} n\big(g_{\text{total}\,\eta'} g_{\text{total}\,\eta'} + & g_{\text{total}\,\eta'} g_{\text{total}\,\tilde{\eta}'} + g_{\text{total}\,\tilde{\eta}'} g_{\text{total}\,\eta'} + g_{\text{total}\,\tilde{\eta}'} g_{\text{total}\,\tilde{\eta}'}\big) \\ &= n((g+h)^2 + 2(g+h)(-h) + (-h)^2) = g^2 n. \end{aligned} \tag{9}$$

All h dependences have canceled, as claimed.

Actually, the result can be obtained in a more elegant way. Let's define $\eta'_\pm = \eta' \pm \tilde{\eta}'$, where η'_+ and η'_- are orthogonal linear combinations. Notice that the hairpin axial couplings always couple to $\mathrm{Str}(A_\mu) = \partial_\mu(\eta' - \tilde{\eta}') = \partial\eta'_-$. On the other hand, since the "double pole" propagator mixes η' and $\tilde{\eta}'$, one can diagonalize it to find out its eigenstates. They turn out to be exactly $\eta'_\pm$.

$$
\begin{array}{c|cc} & \eta' & \tilde{\eta}' \\ \hline \eta' & 1 & 1 \\ \tilde{\eta}' & 1 & 1 \end{array}
\longrightarrow
\begin{array}{c|cc} & \eta'_+ & \eta'_- \\ \hline \eta'_+ & 2 & 0 \\ \eta'_- & 0 & 0 \end{array}
\tag{10}
$$

Since η'_- has eigenvalue zero, it does not propagate under the "double pole" propagator at all! In other words, quenched chiral singularities are solely due to η'_+ loops, while the hairpin axial couplings couple only to $\partial\eta'_-$. As a result, the hairpin axial current couplings cannot contribute to the quenched chiral singularities.

After proving that the hairpin axial current couplings do not contribute to quenched chiral singularities for mass renormalizations, it is time to study the possible generalizations and/or limitation of our result. First of all, can our analysis be extended to more than two flavors, with possibly different masses for different flavors? In the presence of different quark masses, it is more convenient to use instead of the singlet basis $(\eta', \tilde{\eta}')$, the flavor basis $(u\bar{u}, \tilde{u}\bar{\tilde{u}}, d\bar{d}, \tilde{d}\bar{\tilde{d}}, s\bar{s}, \tilde{s}\bar{\tilde{s}}, \ldots)$, where $\tilde{q}$ is the ghost degenerate with quark q. The hairpin axial current couplings go through $\partial_\mu\eta_- = \partial_\mu \sum_{k=1}^n (q\bar{q} - \tilde{q}\bar{\tilde{q}})$, in the leading order of flavor symmetry breaking. On the other hand, the "double pole" propagator now takes the form $(-A_0p^2 + M_0^2)\mathcal{M}$, where $\mathcal{M}$ is a $2n \times 2n$ matrix.

$$
\mathcal{M} = \begin{array}{c|cccccc}
 & u\bar{u} & \tilde{u}\bar{\tilde{u}} & d\bar{d} & \tilde{d}\bar{\tilde{d}} & s\bar{s} & \tilde{s}\bar{\tilde{s}} \\ \hline
u\bar{u} & P_uP_u & P_uP_u & P_uP_d & P_uP_d & P_uP_s & P_uP_s \\
\tilde{u}\bar{\tilde{u}} & P_uP_u & P_uP_u & P_uP_d & P_uP_d & P_uP_s & P_uP_s \\
d\bar{d} & P_dP_u & P_dP_u & P_dP_d & P_dP_d & P_dP_s & P_dP_s \\
\tilde{d}\bar{\tilde{d}} & P_dP_u & P_dP_u & P_dP_d & P_dP_d & P_dP_s & P_dP_s \\
s\bar{s} & P_sP_u & P_sP_u & P_sP_d & P_sP_d & P_sP_s & P_sP_s \\
\tilde{s}\bar{\tilde{s}} & P_sP_u & P_sP_u & P_sP_d & P_sP_d & P_sP_s & P_sP_s
\end{array}
$$

$$
= \begin{array}{c} \\ u\bar{u} \\ d\bar{d} \\ s\bar{s} \end{array}
\begin{array}{c} \begin{array}{ccc} u\bar{u} & d\bar{d} & s\bar{s} \end{array} \\
\begin{pmatrix} P_u P_u & P_u P_d & P_u P_s \\ P_d P_u & P_d P_d & P_d P_s \\ P_s P_u & P_s P_d & P_s P_s \end{pmatrix} \end{array}
\otimes
\begin{array}{c} \\ q\bar{q} \\ \tilde{q}\bar{\tilde{q}} \end{array}
\begin{array}{c} \begin{array}{cc} q\bar{q} & \tilde{q}\bar{\tilde{q}} \end{array} \\
\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{array}, \tag{11}
$$

where $P_q = (p^2 - m^2_{q\bar{q}})^{-1}$. It is now easy to check that $\mathcal{M}$ annihilate η'_-, especially if one notice that by the same representation η'_- can be expressed as

$$
\eta'_- = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \tag{12}
$$

and the last two dimensional column submatrix is annihilated by the 2×2 square submatrix in $\mathcal{M}$. Hence our result holds even when the quarks are not degenerate. In fact it only depends on the one-to-one degeneracy between the quarks and the ghosts. It is also clear in this representation that our result *cannot* be generalized to partially quenched theories, *i.e.*, theories in which the number of quarks and ghosts are different. In fact it is straightforward to show that, in partially quenched theories with n quarks and k ghosts, the quenched chiral singularities [2] have the generic form $(ng + (n-k)h)^2$, where g and h are the standard and hairpin axial current couplings respectively.

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[2]In partially quenched chiral perturbation theories, the η' and $\tilde{\eta}'$ mesons do not have double pole in their propagators. Instead they have two different single poles, one at the original position $p^2 = m^2$, while the other is shifted to $p^2 = m^2 + \delta m^2$. Please refer to Ref. [5] for details. Here we refer to the renormalization due to the shifted pole propagator as quenched chiral singularities, but in fact they are no more singular than the standard corrections unless one goes to the fully quenched limit.

So far we have been studying exclusively mass renormalization. Does our result hold for renormalization of other physical quantities? Before answering this question, we must ascertain what we mean by standard and/or quenched chiral singularities under such circumstances. Suppose we are interested in studying the chiral correction to a certain operator $\mathcal{O}$ which acts on matter fields. The "single pole" propagator will lead to operator renormalization with the following integral form:

$$Z_{\mathcal{O}} = \int \frac{d^4p}{(2\pi)^4} \left[\frac{(\text{coup. const.})^2}{f^2} \sum_f X^{\nu}_{(H^{\dagger}_{(i)}, H_{(f)})} X^{\mu}_{(H^{\dagger}_{(f)}, H_{(i)})} \right] \frac{p_\nu}{v \cdot p} \mathcal{O} \frac{p_\mu}{v \cdot p} \frac{1}{p^2 - m^2}. \tag{13}$$

We will refer whatever chiral correction induced by this term as standard chiral corrections. On the other hand, the "double pole" will induce corrections of another form:

$$Z_{\mathcal{O}} = \int \frac{d^4p}{(2\pi)^4} \left[\frac{(\text{coup. const.})^2}{f^2} \sum_f X^{\nu}_{(H^{\dagger}_{(i)}, H_{(f)})} X^{\mu}_{(H^{\dagger}_{(f)}, H_{(i)})} \right] \frac{p_\nu}{v \cdot p} \mathcal{O} \frac{p_\mu}{v \cdot p} \frac{-nA_0 + nM_0^2}{(p^2 - m^2)^2}, \tag{14}$$

which we will refer as quenched chiral corrections. Generically, if the standard chiral correction takes the form of $m^\alpha \sim m_q^{\alpha/2}$ in the chiral limit, the quenched chiral correction is more singular with the form $M_0^2 m^{\alpha-2} \sim m_q^{\alpha/2-1}$. It is straightforward to generalize our result to the following statement: *Quenched one-chiral loop corrections for matter field properties are independent of the values of hairpin couplings.* This is our main result, and there is one important exception to this rule, which will be discussed later in this paper.

The author does *not* claim to be the first to note this pattern of cancelation of hairpin axial coupling in quenched chiral divergences. Our result is "immediate" (to quote Steve Sharpe) from the quark flow diagrams in Ref. [13]. When one surveys the existing literature it is probable that the authors of previous articles on QχPT on matter field are aware of such a pattern (although, as far as the author is aware of, this is the first systematic discussion of this result). For example, Labrenz and Sharpe has calculated the quenched chiral singularities (Eq. (97–99) in Ref. [8]) for baryons in three flavor QχPT and indeed they are independent of the the hairpin axial couplings. So are the quenched chiral corrections to the octet baryon axial charges as calculated by Kim and Kim [9]. Our rule also holds for heavy mesons, where Booth [11] as well as Sharpe and Zhang [12,13] has calculated the renormalizations of heavy meson decay constants, masses, bag constants, and Isgur–Wise form factors. In all these cases the hairpin axial couplings do not contribute to the quenched chiral corrections. The same can be said for Chiladze's work on heavy baryons [14]. For easy comparison with the literature we list the standard and hairpin axial coupling constants in Ref. [6–14] in the following table:

Matter Fields	Reference	Standard	Hairpin
vector mesons	Booth, Chiladze, Falk [6]	g_2	g_4
tensor mesons	Chow, Rey [7]	$\tilde{g}_2$	$\tilde{g}_4$
baryons ($N_c = 3$)	Labrenz, Sharpe [8]	D,F,$\mathcal{H}$,$\mathcal{C}$	γ,γ'
baryons ($N_c = 3$)	Kim, Kim [9]	D,F,H,C	γ,γ'
baryons ($N_c \to \infty$)	Chow, Rey [10]	g	not considered
heavy mesons	Booth [11]	g	γ
heavy mesons	Sharpe and Zhang [12,13]	g	g'
heavy baryons	Chiladze [14]	g_2,g_3	g_1

More interesting are the cases where this result actually fails. This is the case for Booth, Chiladze and Falk's study of QχPT for vector mesons [6], which is subsequently extended to the tensor mesons case by Chow and Rey [7]. For these cases the hairpin axial current couplings, which is denoted as g_4 in Ref. [6], does contribute to the quenched chiral singularities. The reason for this breakdown of our result is clear. In our analysis above we have assumed that the only source of quenched chiral singularities is the "double pole" term in the η' and $\tilde{\eta}'$ propagators, while for the case of vector and tensor mesons there are additional sources of quenched singularities, namely the possible hairpin diagrams for these matter fields themselves, which may lead to "double pole" propagators for these matter fields. However, if one explicitly setting the hairpin diagrams on these matter fields to be zero (*i.e.*, if one set the parameters $\mu_0 = 0$ and $A_N = 0$ in Ref. [6]), then our results are applicable. In other words, the hairpin axial current coupling g_4 cannot contribute to quenched chiral singularities which involve the η' and $\tilde{\eta}'$ "double pole" propagators (I_2 and

I_4 in Ref. [6]) but in general will contribute to the standard chiral singularities (I_1) as well as the quenched chiral singularities which do not involve the η' "double pole" propagator (I_3). Also note that there are more hairpin couplings for QχPT for vector and tensor mesons, which are called g_1 and g_3 in Ref. [6]. The definition of g_3 does involve $\mathrm{Str}[A_\mu] \sim \partial_\mu \eta'_-$ and hence our result is applicable to this g_3 coupling. Indeed it does not contribute to any of the quenched chiral singularities. On the other hand, the definition of g_1 does *not* involve $\mathrm{Str}[A_\mu]$ and hence is not constrained by our result.

In summary, we have proved that the quenched chiral corrections to matter field properties are independent of the values of the hairpin axial current couplings. Besides serving as a useful check of the correctness of QχPT calculation results, this observation may be useful in the eventual determination of parameters in the QχPT lagrangians. All the coupling constants appearing in the QχPT lagrangians are nonperturbative quantities which cannot be calculated from first principles from quenched QCD and has to be extracted from lattice results (just as the paramaters in standard χPT lagrangians have to be extracted from experimental data). While one can extract the "double pole" parameters from lattice results on Goldstone boson systems, it may be difficult to disentangle the effects of the standard axial couplings from their hairpin counterparts. Our result suggests the procedure that one should extract the values of the standard axial couplings from the quenched chiral corrections, and with those pieces of information one can proceed to extract the values of the hairpin axial couplings from the standard chiral divergences.

II. MASS SPLITTING AND CHIRAL MASS CORRECTIONS

Chiral perturbation theory, both the standard and quenched versions, provides a systematic framework to describe low energy interactions between hadrons. Before one constructs a chiral lagrangian, however, one must decide which fields to include. There is no clear cut criteria on which fields should be included, but the conventional wisdom suggested that one should *at least* include all fields which are related by light flavor SU(n) (n = 2 or 3, depending on the scenario), as well as all the fields which are degenerate in the large N_c limit. If the hadron in question contains a heavy quark, then we should also include all the fields related by heavy quark spin symmetry as well. For example, in Ref. [6] both the vector meson octet and the singlet appeared in the chiral lagrangian, as the octet and the singlet are degenerate in the large N_c limit. In fact they can be conveniently combined into a U(3) nonet. Similarly in Ref. [8] the baryon chiral lagrangian contains both the octet and the decuplet, which are again degenerate in the large N_c limit. It is well known that, in standard χPT, the failure to include the baryon decuplet will lead to huge chiral corrections to baryon octet, while the chiral loops in a χPT with both the octet and the decuplet is much smaller [15]. Physically these degenerate states are strongly coupled to each other, and the failure to include them will lead to an incomplete representation of the interaction, causing a badly-behaved loop expansion.

In both the real world and its quenched approximation on the lattice, however, all these symmetries (chiral, heavy quark, and large N_c) are broken and the degeneracies are only approximate. In (Q)χPT, one often (though not always) treats these mass splittings as small perturbations and include them leading to the leading order. While such approximations are usually justified, they are not entirely satisfactory as it is not clear if the higher order effects will change the chiral behavior drastically. The question can be put more impendingly in the following way. Let's take the chiral corrections to hadron masses as an example. Since the one loop diagram has two axial current couplings, each giving a factor of $1/f$, one can obtain from dimensional analysis that

$$\Delta M \sim m^3/f^2, \tag{15}$$

simply on the basis that the Goldstone boson mass m is *the only other mass scale in the theory*. In other words, the chiral behavior of the mass correction cannot be changed unless some other mass scale is introduced in the theory. In QχPT, the new mass scale is M_0^2, and the quenched chiral singularities are proportional to $M_0^2 m$, which dominate in the chiral limit. Now the mass splitting Δ between states is another new mass scale and can potentially change the chiral behavior. The question is, will it?

Fortunately, it is possible to study the Feynman integral exactly. This has been done in χPT in Ref. [15] for baryons (though the treatment is general and can be applied to other systems as well), and the generalization to QχPT is trivial.

Let's review the analysis in Ref. [15], which study the integral [3]

$$I = \frac{i}{f^2}\int \frac{d^4p}{(2\pi)^4}\frac{p^2}{v\cdot p-\Delta}\frac{1}{p^2-m^2}, \tag{16}$$

where we will assume $\Delta > 0$ so that the intermediate state is heavier than the external state and hence the integral does not have any cuts corresponding to real particle decays (which will be the subject matter of the following section). Then the integral can be evaluated exactly. It is instructive to separate the result into two parts:

$$I = I^{(\mathrm{a})} + I^{(\mathrm{na})} \tag{17}$$

The analytic contribution

$$I^{(\mathrm{a})} = \frac{1}{12\pi f^2}(\frac{5}{3\pi}\Delta^3 + \frac{2}{\pi}\Delta m^2) \tag{18}$$

can be canceled by counterterms, while the nonanalytic contribution cannot be modified by counterterms and is genuine signature of low energy chiral dynamics.

$$I^{(\mathrm{na})} = -\frac{1}{12\pi f^2}\begin{cases} \frac{2}{\pi}(m^2-\Delta^2)^{3/2}\sec^{-1}\frac{m}{\Delta} + \Delta(\Delta^2-\frac{3}{2}m^2)\frac{1}{\pi}\ln\frac{m^2}{\mu^2}, & \Delta \le m; \\ \frac{-1}{\pi}(\Delta^2-m^2)^{3/2}\ln\frac{\Delta-\sqrt{\Delta^2-m^2}}{\Delta+\sqrt{\Delta^2-m^2}} + \Delta(\Delta^2-\frac{3}{2}m^2)\frac{1}{\pi}\ln\frac{m^2}{\mu^2}, & \Delta \ge m, \end{cases} \tag{19}$$

where μ is the subtraction point. This nonanalytic contribution is plotted in Fig. 1, with μ set to be equal to Δ and 2Δ. It is clear that, while $I^{(\mathrm{na})}$ does behave like m^3 at large m for both values of μ,

$$I^{(\mathrm{na})} = -\frac{1}{12\pi f^2}\left[m^3 - \frac{3}{2\pi}m^2\Delta\ln\frac{m^2}{\mu^2} + \dots\right], \qquad m \gg \Delta, \tag{20}$$

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[3] There is a factor of i difference between our definition of I and that in Ref. [15].

there are nontrivial corrections in the chiral limit due to a non-vanishing Δ for $\mu = \Delta$, while the chiral behavior is much better-behaved with $\mu = 2\Delta$. It should be understood that changes in μ generate extra analytic terms (m^2 and m^0 terms in this case), which should be absorbed into $I^{(\mathrm{a})}$. To see it more explicitly one can perform a Taylor expansion around $m = 0$ [15],

$$I^{(\mathrm{na})} = -\frac{1}{12\pi f^2}\frac{1}{\pi}\Big[(-\Delta^3 + \frac{3}{2}\Delta m^2)\ln\frac{\mu^2}{4\Delta^2} - \frac{3}{8}\frac{m^4}{\Delta}\ln\frac{m^2}{4\Delta^2} + \mathcal{O}(\frac{1}{\Delta^2})\Big] \tag{21}$$

The $\ln \mu^2/4\Delta^2$ term is actually analytic in m^2 and can be absorbed into the analytic part. It vanishes, however, at $\mu = 2\Delta$, explaining why the $\mu = 2\Delta$ curve approaches the chiral limit so nicely. The leading nonanalytic contribution is $m^4 \ln m^2 \sim m_q^2 \ln m_q$, quite different from the m^3 behavior when $\Delta = 0$. In fact the $1/\Delta$ suppression factor is reminding us that this is a higher dimensional operator produced on integrating out the heavier intermediate state.

The corresponding integral in QχPT is

$$I_q = \frac{i}{f^2}\int\frac{d^4p}{(2\pi)^4}\frac{p^2}{v\cdot p - \Delta}\frac{-A_0p^2 + M_0^2}{(p^2 - m^2)^2}, \tag{22}$$

where the subscript q stands for "quenched". As discussed above, the quenched chiral singularities comes from the M_0^2 term. So for simplicity we will set $A_0 = 0$ is the discussion below, and

$$I_q = \frac{iM_0^2}{f^2}\int\frac{d^4p}{(2\pi)^4}\frac{p^2}{v\cdot p - \Delta}\frac{1}{(p^2 - m^2)^2} = M_0^2\frac{dI}{dm^2} \tag{23}$$

We are only interested in the nonanalytic part $I_q^{(\mathrm{na})}$, which is

$$I_q^{(\mathrm{na})} = -\frac{M_0^2}{12\pi f^2}\begin{cases} -\frac{3\Delta}{2\pi}\ln\frac{m^2}{\mu^2} + \frac{3}{\pi}(m^2 - \Delta^2)^{1/2}\sec^{-1}\frac{m}{\Delta}, & \Delta \le m; \\ -\frac{3\Delta}{2\pi}\ln\frac{m^2}{\mu^2} + \frac{3}{2\pi}(\Delta^2 - m^2)^{1/2}\ln\frac{\Delta - \sqrt{\Delta^2 - m^2}}{\Delta + \sqrt{\Delta^2 - m^2}}, & \Delta \ge m, \end{cases} \tag{24}$$

We see the expected $I \sim m \sim m_q^{1/2}$ nonlinear behavior for large m.

$$I_q^{(\mathrm{na})} = -\frac{1}{12\pi f^2}\Big[\frac{3}{2}M_0^2 m - \frac{3}{2\pi}M_0^2\Delta\ln\frac{m^2}{\mu^2} + \dots\Big], \qquad m \gg \Delta. \tag{25}$$

And for small m,

$$I_q^{(\mathrm{na})} = -\frac{M_0^2}{12\pi f^2}\left[\frac{3\Delta}{2\pi}\ln\frac{\mu^2}{4\Delta^2} - \frac{3}{4\pi}\frac{m^2}{\Delta}\ln\frac{m^2}{4\Delta^2} + \mathcal{O}(\frac{1}{\Delta^2})\right], \tag{26}$$

so the leading nonanalytic term is $m^2 \ln m^2 \sim m_q \ln m_q$, again quite different from the $m \sim m_q^{1/2}$ behavior with vanishing Δ. Another interesting point is shown in Fig. 2, where $I_q^{(\mathrm{na})}$ is plotted with $\mu = \Delta$ and 2Δ. In this case a change in μ just shifts the curve vertically but do not change the shape at all, as dictated by the Taylor expansion above, where the coefficient in front of the $\ln \mu^2$ term is just a constant in m.

What does the analysis above imply for real lattice simulations? Note that the nonlinear behavior in Fig. 2 appears around $m/\Delta \sim 0.5$. So the implication of Fig. 2 is twofold. Firstly, it suggests that, for $m > \Delta/2$, the extrapolation of hadron masses to the chiral limit should be linear, a fact which is consistent with existing lattice results. On the other hand, it predicts the breakdown of such linear behavior at $m \leq \Delta/2$, a regime which is not yet well-explored. In real lattice simulation, the application of this analysis is complicated by the existence of more than one possible intermediate states, and hence more than one Δ. However, one can still make some qualitative predictions. For example, since the N–Δ splitting is around 300 MeV, one expects the extrapolation for the nucleon mass departs from a linear behavior at around 150 MeV (about the physical pion mass). On the other hand, the small ρ–ω splitting (17 MeV) means that probably a simple linear extrapolation is sufficient in the vector meson sector. Certainly more detailed study along this line, especially for physical quantities other than masses, would be very useful to extraction of information from lattice data.

III. HADRON DECAY IN QUENCHED QCD

In this previous section, we have studied two point functions with the intermediate state X_i *heavier* than the external state X_e, *i.e.*, $\Delta = X_i - X_e > 0$. In this section, we will consider the opposite, where $\Delta < 0$ and the intermediate state is *lighter* than the external state. In particular, we are interested in the case which $\Delta < -m$, and at large Euclidean time the two point function will be dominated the $X_i\pi$ state, resulting in a cut in the two point function. However, often these decays involve creating a quark–antiquark pair from the vacuum. The question is, can hadron decay be studied on lattice under the quenched approximation?

In this section, we are going to derive "selection rules" for the hadron decays under quenched approximation. For example, an obvious selection rule is *the daughter hadrons cannot contain any valence quark which does not appear in the mother hadron as valence quark*, as these new valence flavors have to from an internal quark loop. We will see below that quenching does impose strong constraints on the possible decay mechanisms. We will focus at system with the standard and hairpin axial current couplings (denoted by g and h respectively), but no other kind of couplings (*i.e.*, g_1 and g_3 in Ref. [6] are out). Then there are two classes of diagrams representing hadron decays:

(1) Diagrams which all the vertices are standard couplings g, and no hairpin propagators. The corresponding integrals will be proportional to g^2. Since these decays have clear analogs in standard (unquenched) QCD, we will call them "standard decays".

(2) Decays with one hairpin axial vertex (integral $\sim gh$), and/or diagrams with the "double pole" propagator on the Goldstone boson line (integral $\sim g^2M_0^2$). These decays may either have analogs in QCD or be merely quenched artifacts. We will call these "hairpin decays". Note that the diagrams with hairpin axial coupling at both vertices vanish identically as the Goldstone boson line will form an internal quark loop.

We will first consider standard decays for meson. It is trivial to see that OZI suppressed decays, like $\phi \to \rho\pi$, cannot take place under the quenched approximation. However, not all OZI allowed decays can survive quenching. Decays through Fig. 3a, which contains an internal quark loop, are not allowed in quenched QCD, while decays through Fig. 3b are preserved. Note, however, that the quark and the antiquark from an external meson line in Fig. 3b annihilate, demanding that the external meson must be flavor neutral. However, isospin symmetry, which is a good symmetry of QχPT, can rotate flavor neutral states into flavor states *unless the flavor neutral state has isospin zero.* Hence, we have the selection rule that for *the only possible standard meson decays are OZI allowed decays of $I = 0$ mesons.* The amplitude of such decays in quenched QCD is identical to its counterpart in standard QCD. An example of such decay is $\omega \to \rho\pi$ in the vector meson sector, and its tensor meson analog $f_2 \to a_2\pi$; unfortunately these processes do not happen in the real world where the total mass of the daughter particles is larger than the mother particle. It turns out that light mesons, which form U(3) nonets, usually mix ideally so that the $I = 0$ mass eigenstates are the $s\bar{s}$ state and the $u\bar{u} - d\bar{d}$ states. The $s\bar{s}$ states, like the ϕ vector meson and the f_2' tensor meson, can either decay through OZI suppressed modes like $\phi \to \rho\pi$, which is clearly disallowed in quenched QCD, or modes like $\phi \to 2K$, which is described by Fig. 3a and hence also do not survive quenching. And while the $u\bar{u} - d\bar{d}$ state, like the ω vector meson and the f_2 tensor meson in principle can decay through Fig. 3b, they are so light that they cannot decay into the $I = 1$ state in the same nonet (ρ and a_2, respectively). So the only possibility for standard decays in the meson sector are decays of an $I = 0$ state from a highly excited nonet to two lighter $I = 1$ states in other nonets. These decays cannot be described by QχPT as they involve transitions between different nonets, and it is probably difficult to really observe these decays on the lattice as these highly excited states may be very dirty. In passing we note that heavy mesons do not have standard decays in quenched QCD.

In the baryon sector we are interested in studying decays like $\Delta \to \mathrm{N}\pi$ and $\Sigma_c^{(*)} \to \Lambda_c\pi$. The former decay has been studied in detail in Ref. [8], from which we will borrow heavily. The main complication in studying these baryon decays is that the initial and final baryons have different isospin wave functions to satisfy the spin-statistics theorem. For concreteness, Ref. [8] has chosen to study the decay of Δ^{++}, which in standard QCD the only possible decay mode is:

$$\Delta^{++} = uuu \to u\bar{d} + duu = \pi^+ + p^+ \tag{27}$$

Note that a $d\bar{d}$ pair is created. So *naively* Δ^{++} decay is prohibited by quenching. This argument, however, was shown to be wrong in Ref. [8].

Standard decays for baryons can go through either Fig. 4a, which has an internal quark loop and hence does not survive quenching, and Fig. 4b, which does. We will denote the amplitude of Δ^{++} decaying through Fig. 4a as Γ_q, where q is the quark on the internal loop, while Γ_0 is the amplitude of its decay through Fig. 4b. In the unquenched world, the total decay amplitude is

$$\Gamma^{\text{unquenched}} = \Gamma_0 + \sum_q \Gamma_q = \Gamma_0 + \Gamma_u + \Gamma_d. \tag{28}$$

On the other hand, fermionic statistics of the quark forbids the existence of an intermediate uuu octet. Hence we have

$$\Gamma_0 = -\Gamma_u, \tag{29}$$

and hence

$$\Gamma^{\text{unquenched}} = \Gamma_d = \Gamma_q. \tag{30}$$

The Δ^{++} decay amplitude after quenching is

$$\Gamma^{\text{quenched}} = \Gamma_0 + \sum_{q,\tilde{q}} \Gamma_q = \Gamma_0 + \Gamma_u + \Gamma_{\tilde{u}} + \Gamma_d + \Gamma_{\tilde{d}}. \tag{31}$$

The cancelation of quark loops and ghost loops demands that

$$\Gamma_q + \Gamma_{\tilde{q}} = 0, \tag{32}$$

which gives

$$\Gamma^{\text{quenched}} = \Gamma_0 = -\Gamma_q = -\Gamma^{\text{unquenched}}. \tag{33}$$

So the decay amplitude changes sign under quenching. A more illuminating way to present the result is to note that

$$\Gamma^{\text{quenched}} = (\Gamma_0 + \Gamma_u) + \Gamma_{\tilde{u}} + (\Gamma_d + \Gamma_{\tilde{d}}) = \Gamma_{\tilde{u}} = \Gamma_{\tilde{q}}, \tag{34}$$

as the sum in each pair of parentheses vanishes. So in quenched QCD Δ^{++} decays to $(u\bar{\tilde{u}})(\tilde{u}uu)$,which clearly has the opposite sign as the QCD decay.

How does this lesson generalize to other baryon decays? We will focus our attention to the orbitally unexcited baryons, which are the states with most physical interest. In other words, we will consider the octet-decuplet light baryon system, and the antitriplet-sextet heavy baryon system. Now we make the phenomenological observation that, while baryon decays through pion emission are possible in both of these systems (*e.g.*, $\Delta \to \mathrm{N}\pi$ and $\Sigma_c^{(*)} \to \Lambda_c\pi$), in neither of these system can baryon decay through K or η decay as physically these mesons are too massive. We will make the assumption that this pattern continue to hold in quenched QCD. Then Fig. 4b demands that, for decay through pion emission to take place, the initial state must has at least two light valence quarks. So the selection rule is: *standard decays for baryons are possible if the decaying baryon has at least two light valence quarks*, and the list includes $\Delta \to$N and $\Sigma_{(c)}^{(*)} \to \Lambda_{(c)}$. And for all of these decays, the quenched amplitude has the same magnitude but opposite sign as the unquenched counterpart. On the other hand, orbitally unexcited $\Xi_{(Q)}$'s and $\Omega_{(Q)}$'s do not have standard decays under the quenched approximation. Our rule does not apply to orbitally excited baryons, which may have enough phase space to decay through K and η emission, but again studying these states on the lattice may be quite difficult.

After studying the standard decays, let's now move on to the hairpin decays. The important observation here is, hairpin decays always have the form $X_e \to X_i\eta'$, as only η' couples through hairpin coupling, or propagator through the "double pole" propagator. Since η' is a singlet under flavor SU(2) and SU(3), however, it means that *hairpin decays can take place only between multiplets with the same representation under the flavor group.*

In the view of this result, the most natural places to look for these hairpin decays are transitions between a heavy quark doublet, like $D* \to D\eta'$ and $\Sigma_c^* \to \Sigma_c\eta'$. Since the two states in a heavy quark doublet is related by just flipping the heavy quark spin, they always have identical flavor representation. As discussed above, there are no standard decay for $D*$, and this hairpin decay is the only possible mode under the quenched approximation. The Σ_c^* baryon can decay to $\Lambda_c\pi$, which would probably dominate over this hairpin mode, as the latter in suppressed in the large N_c limit [10]. In the real world, however, Σ_c^* is only

80 MeV above Σ_c, so probably this hairpin decay cannot take place after all. The strange quark is not massive enough to be considered a heavy quark, and while the "strange quark doublet" Σ^* and Σ have the same representation under flavor SU(2) (both have $I = 1$), their flavor SU(3) representations are different (Σ is in the octet, Σ^* is in the decuplet). Hence the hairpin decay $\Sigma^* \rightarrow \Sigma\eta'$ is flavor SU(3) suppressed. Lastly, note that hairpin decays from the tensor meson nonet to the vector meson nonet are allowed under this selection rule, and in principle one can study these decays on the lattice. These decays, however, are not described within the framework of (Q)χPT, where the tensor meson number and vector meson number are separately conserved.

In conclusion, we have chosen that quenching provides severe "selection rules" on hadron decays. Studying decay on lattice are important as it provide one of the most direct way to determine the values of the coupling constants in the QχPT lagrangian (just as experimental decay rate can lead to determinations of coupling constants in the standard chiral lagrangian). One can obtained $\mathcal{C} \sim g_{\Delta \mathrm{N}\pi}$ [8] from $\Delta \rightarrow \mathrm{N}\pi$, and $g_3 \sim g_{\Sigma_Q^{(*)}\Lambda_Q\pi}$ [14] from $\Sigma_c^{(*)} \rightarrow \Lambda_c\pi$. Regretfully, hadron decays are not observable in most of the lattice QCD simulations in the present generation as the light quark masses are still relatively large, and hence $m > m_\Delta - m_\mathrm{N}$. But it is probable that, with the improvement in algorithms and increase in available computational power, probing the small m region would be a feasible endeavor in the not-too-distant future.

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